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2.16 Proof: $\forall (x_1, y_1+y_2)$, $(x_2, y_3+y_4) \in S$, $0 \le \theta \le 1$, ~~we have~~

$\cancel{\theta(x_1, y_1+y_2) + (1-\theta)}$

where $(x_1, y_1), (x_2, y_3) \in S_1$, $(x_1, y_2), (x_2, y_4) \in S_2$, we have

$$\theta(x_1, y_1+y_2) + (1-\theta)(x_2, y_3+y_4)$$
$$= (\theta x_1 + (1-\theta)x_2, [\theta y_1 + (1-\theta)y_3] + [\theta y_2 + (1-\theta)y_4])$$

$\because$ S_1, S_2 are convex sets.

$\therefore$
$$\theta(x_1, y_1) + (1-\theta)(x_2, y_3)$$
$$= (\theta x_1 + (1-\theta)x_2, \theta y_1 + (1-\theta)y_3) \in S_1$$
$$\theta(x_1, y_2) + (1-\theta)(x_2, y_4)$$
$$= (\theta x_1 + (1-\theta)x_2, \theta y_2 + (1-\theta)y_4) \in S_2$$

$\therefore$ $\theta(x_1, y_1+y_2) + (1-\theta)(x_2, y_3+y_4)$
$$= (\theta x_1 + (1-\theta)x_2, [\theta y_1 + (1-\theta)y_3] + [\theta y_2 + (1-\theta)y_4]) \in S$$

$\therefore$ S is convex

3.3 g is a concave function. Proof:

$\because$ f is convex on its domain (a,b), and increasing

$\therefore$ $\forall x, y \in (a,b)$, $0 \le \theta \le 1$, we have
$$f(\theta x + (1-\theta)y) \le \theta f(x) + (1-\theta)f(y)$$

~~$g(f(x)) = x$ for $a < x < b$~~

$\because$ $g(f(x)) = x$ for $a < x < b$

~~g is increasing~~

~~$g(f(\theta x + (1-\theta)y)) \le g$~~

$\therefore$ $f(\theta x + (1-\theta)y) \le f(g(\theta f(x) + (1-\theta)f(y)))$

~~$f(\theta x + (1-\theta)y \le$~~

$\because$ f is increasing

$\therefore$ $\theta x + (1-\theta)y \le g(\theta f(x) + (1-\theta)f(y))$

$\therefore$ $\theta g(f(x)) + (1-\theta)g(f(y)) \le g(\theta f(x) + (1-\theta)f(y))$

$\therefore$ g is a concave function