MTH112M Quiz

Graded

Student

SHAURYA JOHARI

Total Points

13 / 20 pts

Question 1

Question 1

5 / 5 pts

+ 1 pt The centroid lies on the vertical line $x = 2$ or assuming $(2, \bar{y})$ is the centroid.

+ 2 pts Applying Pappus's theorem, $\frac{4}{3}\pi = 2\pi\bar{y}(Area(D)) = 2\pi\bar{y}(\pi/2)$

+ 1 pt Computing $\bar{y} = \frac{4}{3\pi}$ or centroid is $(2, \frac{4}{3\pi})$

+ 2 pts Finding correctly the centroid of the semi-circle instead of semi-disc.

+ 1 pt Stating that the solid of revolution of D is a sphere of radius 1 with volume $\frac{4}{3}\pi$.

+ 1 pt Stating the x-coordinate of centroid $C_x = \frac{1}{Area(D)} \int_1^3 x\sqrt{1-(x-2)^2}dx$

+ 1 pt Stating the y-coordinate of centroid $C_y = \frac{1}{Area(D)} \int_0^1 y(2\sqrt{1-y^2})dy$

+ 1 pt Computing area of D

+ 1 pt Finding $C_x = 2$

+ 1 pt Finding $C_y = \frac{4}{3\pi}$

+ 0 pts Completely Incorrect Answer or No Answer

✔ **+ 5 pts** Completely correct answer

1

Question 2

Question 2

0 / 5 pts

+ 2 pts For the observation that the total surface area of the solid is = the curved outer surface area + inner cylindrical surface area+ area of the outer washer like surface

+ 1 pt Correct computation of the curved outer surface area $= \int_{\frac{3}{4}}^{2} \left(2\pi\sqrt{x} \times \sqrt{1+\frac{1}{4x}}\right) dx = \frac{19\pi}{6}$

+ 1 pt Correct computation of inner cylindrical surface area $= 2\pi\frac{\sqrt{3}}{2} \times \frac{5}{4}$

+ 1 pt Correct computation of the outer washer like surface area $= \pi\left(2 - \frac{3}{4}\right)$.

✔ **+ 0 pts** Completely Incorrect Answer or No Answer

Question 3

Question 3

5 / 5 pts

+ 1 pt For the observation that the required line is obtained by finding a point lies on both planes and a direction perpendicular to both of the given planes.

+ 1 pt Correct computation of a point lies on both planes.

+ 1 pt the normal direction to the first plane is given by the vector $n_1 = (2, 0, -3)$ and the normal direction to the second plane is given by the vector $n_2 = (1, 6, 0)$.

+ 1 pt Correct computation of the direction perpendicular to both of the given planes. The vector $n_1 \times n_2$ is parallel to the direction of the required line of intersection. Here, $n_1 \times n_2 = 18i - 3j + 12k$

+ 1 pt Finding the correct equation of straight line

+ 0 pts Completely Incorrect Answer or No Answer

✔ **+ 5 pts** Completely Correct Answer

Question 4

Question 4

3 / 5 pts

✔ **+ 1 pt** Correct computation of $R'(t) = \left(\frac{4t}{(1+t^2)^2}, \frac{2(1-t^2)}{(1+t^2)^2}, 0\right)$

✔ **+ 1 pt** The arc length representation of the given curve starting from $t = 0$ is $R(s) = (-cos\ s,\ sin\ s,\ 0)$.

+ 2 pts ****The arc length parameter s is given by $s = \int_0^t \frac{2}{1+u^2}\ du = 2\ tan^{-1}\ t.$

✔ **+ 1 pt** Correct computation of $||R'(t)|| = \frac{2}{1+t^2}$.

+ 0 pts Completely Incorrect Answer or No Answer

+ 0 pts Not attempted/ completely incorrect answer.

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Question 4. Consider the curve $R(t) = \left(\frac{t^2-1}{t^2+1}, \frac{2t}{t^2+1}, 0\right)$, where $t \geq 0$. Reparametrize the curve R in terms of arc length. (5 Marks)

Answer 4:

Let $x = \frac{t^2-1}{t^2+1}$ & $y = \frac{2t}{t^2+1}$

Observe that $x^2+y^2 = \frac{t^4+2t^2+1}{t^4+2t^2+1} = 1 \Rightarrow$

$R(t)$ is a circle centered at origin $(0,0,0)$ of radius 1

Also, $R(0) \equiv (-1,0,0)$ $R(1) \equiv (0,1,0)$

$\lim_{t\to\infty} R(t) \equiv (1,0,0)$. Let $t = \tan\theta$, $\theta \in (0, \frac{\pi}{2})$

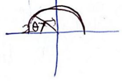
$R(t) = (-\cos 2\theta, \sin 2\theta, 0)$

$\because \frac{\tan^2\theta - 1}{\sec^2\theta} = -\cos 2\theta$ & $\frac{2\tan\theta}{\sec^2\theta} = \sin 2\theta$

$\Rightarrow$ Parameterized curve (in θ): $(-\cos 2\theta, \sin 2\theta, 0)$

$\theta \in [0, \frac{\pi}{2})$

$k\frac{\pi}{4} = \frac{\pi}{2}$



And distance travelled along arc $= 2\theta$

$(k\frac{\pi}{4} = \frac{\pi}{2}$ & $k\frac{\pi}{2} = \pi \Rightarrow k = 2)$

$\Rightarrow$ Reparameterized Curve:

~~$(-\cos(2\tan^{-1}t), \sin(2\tan^{-1}t), 0)$~~

$(-\cos t, \sin t, 0)$

t: Distance along arc Travelled

MTH112M QUIZ

Date : 17/10/2023 | Time : 7:20 - 8:50 pm | Total Marks : 20

Name : SHAURYA JOHARI	Roll : 230959

Please enter your NAME and ROLL NUMBER in the space provided above. Booklets without name and roll number will not be graded. There are four compulsory questions in this booklet. Write the answer in the space provided after each question. For rough work, separate sheets will be provided to you. Write your name and roll number on the rough sheets.

Question 1. Consider the semi-disc $D = \{(x,y) \in \mathbb{R}^2 : (x-2)^2 + y^2 \leq 1, y \geq 0\}$ in xy plane. Find the centroid of D. (5 Marks)

Answer 1:

Consider the given semi disc D & the X axis (Line $y=0$)

Let G be the centroid of D.

$\because$ D is symmetric about $x=2$, if G is k distance away from X-axis, then G is at $(2, k)$

Revolving D about X-axis gives us a sphere of radius $R=1$

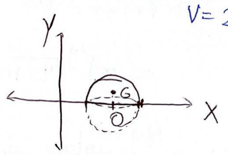
So $V = \frac{4}{3}\pi R^3 = \frac{4\pi}{3} = V$

Also, area of semi disc $= \pi R^2/2 = \pi/2 = A$

Applying Papus thm. $\because$ k is dist btw centroid & line of revolution (NOT CROSSING D), we have

$V = 2\pi A k \Rightarrow \frac{4\pi}{3} = \pi^2 k$

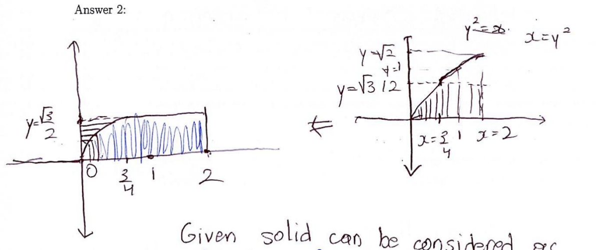
OR $k = \frac{4}{3\pi}$



$\therefore$ Centroid is located at $(2, \frac{4}{3\pi})$

Question 2. Let R be the region in the first quadrant of xy plane enclosed by the curves $y = \sqrt{x}$, $y = \frac{\sqrt{3}}{2}$ and $x = 2$. Consider the solid obtained by revolving the region R about the x-axis. Evaluate total surface area of the solid. (5 Marks)

Answer 2:



Given solid can be considered as combination of 2 solids:

1- $y=\sqrt{x}$ curve revolved around X-axis $(0 \leq x \leq 3/4)$

2- $y=\sqrt{3}/2$ curve revolved around X-axis $(3/4 < x \leq 2)$

Net surface area of 2^{nd} solid: $\pi(y)^2 + 2\pi y(2-\frac{3}{4}) = \frac{3}{4}\pi + \frac{5}{4}\sqrt{3}\pi = A_2$

Net surface area of 1^{st} Solid: ~~Vol generated~~ =

$y=\sqrt{x}$ $\quad y' = \frac{1}{2\sqrt{x}}$ $\quad \cancel{\int_0^{3/4} \pi(\sqrt{x})^2 dx = \frac{9\pi}{32}}$ $\quad \int_0^{3/4} 2\pi y\sqrt{1+(y')^2}\,dx$

$= \pi\int_0^{3/4}\sqrt{4x+1}\,dx = \frac{\pi}{4}\times\frac{2}{3}(4^{3/2} - 1^{3/2})$

$= 7\pi/6 = A_1$

Net Surface area $= A_1 + A_2 = \frac{23\pi}{12} + \frac{5\sqrt{3}\pi}{4}$

Question 3. Find the parametric equation of the line of intersection of two planes $2x - 3z = 4$ and $x + 6y = 5$ in $\mathbb{R}^3$. (5 Marks)

Answer 3:

Given planes: $\bar{r}\cdot\bar{a}_1 = b_1$ & $\bar{r}\cdot\bar{a}_2 = b_2$

Where $\bar{a}_1 = 2\hat{i} - 3\hat{k}$ & $\bar{a}_2 = \hat{i} + 6\hat{j}$

$b_1 = 4$ & $b_2 = 5$

$\because$ $\bar{a}_1 \times \bar{a}_2 \neq 0$, The planes will intersect along a common line with direction vector $\vec{d}$.

$\because$ All ~~poin~~ lines in planes $\bar{r}\cdot\bar{a}_1 = b_1$ & $\bar{r}\cdot\bar{a}_2 = b_2$ are perpendicular ~~aton~~ to $\bar{a}_1$ & $\bar{a}_2$ respectively,

$\bar{d}\cdot\bar{a}_1 = \bar{d}\cdot\bar{a}_2 = 0 \Rightarrow \exists\, k \in \mathbb{R}$ s.t. $\bar{d} = k(\bar{a}_1 \times \bar{a}_2)$

$(2\hat{i} - 3\hat{k}) \times (\hat{i} + 6\hat{j}) = 18\hat{i} - 3\hat{j} + 12\hat{k}$

$\Rightarrow$ Given line: $\bar{r} + \gamma\bar{d}$ where $\gamma \in \mathbb{R}$.

And $\bar{r}\cdot\bar{a}_1 = b_1$ & $\bar{r}\cdot\bar{a}_2 = b_2$.

Consider $\bar{r} = -\hat{i} + \hat{j} - 2\hat{k}$. $\bar{r}\cdot\bar{a}_1 = -2+6 = 4 = b_1$.

$\bar{r}\cdot\bar{a}_2 = -1+6 = 5 = b_2 \Rightarrow -\hat{i} + \hat{j} - 2\hat{k}$ lies on line.

$\therefore$ Parametric eq^n of line:

$\hat{i}(-1+6t) + \hat{j}(1-t) + \hat{k}(-2+4t)$ Where $t \in \mathbb{R}$

$$\Rightarrow \left[\begin{array}{l} x = -1+6t \\ y = 1-t \\ z = -2+4t \end{array}\right] t \in \mathbb{R} \text{ is the line}$$