South China University of Technology

The Experiment Report of Machine Learnin

SCHOOL: SCHOOL OF SOFTWARE ENGINEERING

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《逻辑回归、线性分类与随机梯度下降》

Abstract—
该报告是关于逻辑回归、线性分类与随机梯度下降实验。采取了 **NAG**，**RMSProp**，**AdaDelta** 和 **Adam** 方法优化 **SGD**。实验使用的是 **LIBSVM Data** 的中的 **a9a** 数据，包含 **32561 / 16281(testing)**个样本，每个样本有 **123/123 (testing)**个属性。

I. INTRODUCTION

实验目的**:**

- 对比理解梯度下降和随机梯度下降的区别与联系。
- 对比理解逻辑回归和线性分类的区别与联系。
- 进一步理解 **SVM** 的原理并在较大数据上实践。

II. METHODS AND THEORY

1）逻辑回归和随机梯度下降：

- 读取实验训练集和验证集。
- 逻辑回归模型参数初始化,可以考虑全零初始化，随机初始化或者正态分布初始化。
- 选择 **Loss** 函数及对其求导,过程详见课件 **ppt**。
- 求得部分样本对 **Loss** 函数的梯度 **G**。
- 使用不同的优化方法更新模型参数（**NAG**，**RMSProp**，**AdaDelta** 和 **Adam**）。
- 选择合适的阈值，将验证集中计算结果大于阈值的标记为正类，反之为负类。在验证集上测试并得到不同优化方法的 **Loss** 函数值 **LNAG**，**LRMSProp**，**LAdaDelta** 和 **LAdam**。
- 重复步骤 **4-6** 若干次，画出 **LNAG**，**LRMSProp**, **LAdaDelta** 和 **LAdam** 随迭代次数的变化图。

2）线性分类和梯度下降：

- 读取实验训练集和验证集。
- 支持向量机模型参数初始化，可以考虑全零初始化，随机初始化或者正态分布初始化。
- 选择 **Loss** 函数及对其求导,过程详见课件 **ppt**。
- 求得部分样本对 **Loss** 函数的梯度。
- 使用不同的优化方法更新模型参数（**NAG**，**RMSProp**，**AdaDelta** 和 **Adam**）。
- 选择合适的阈值，将验证集中计算结果大于阈值的标记为正类，反之为负类。在验证集上测试并得到不同优化方法的 **Loss** 函数值 **LNAG**，**LRMSProp**，**LAdaDelta** 和 **LAdam**。
- 重复步骤 **4-6** 若干次，画出 **LNAG**，**LRMSProp**, **LAdaDelta** 和 **LAdam** 随迭代次数的变化图。

loss 函数及其导数**:**

- 逻辑回归和随机梯度下降：

Log-likehood loss function:

$$J(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n}\log(1+e^{-y_i\cdot\mathbf{w}^\top\mathbf{x}_i}) + \frac{\lambda}{2}\|\mathbf{w}\|_2^2$$

导数计算：

$$\mathbf{w}' \to \mathbf{w} - \eta\frac{\partial J(\mathbf{w})}{\partial\mathbf{w}} = (1-\eta\lambda)\mathbf{w} + \eta\frac{1}{n}\sum_{i=1}^{n}\frac{y_i\mathbf{x}_i}{1+e^{y_i\cdot\mathbf{w}^\top\mathbf{x}_i}}$$

- **线性分类和随机梯度下降：**

Hinge loss:

$$Hinge\ loss = \xi_i = \max(0, 1-y_i(\mathbf{w}^\top\mathbf{x}_i+b))$$

导数计算：

$$\frac{\partial f(\mathbf{w},b)}{\mathbf{w}} = \begin{cases}\mathbf{w}^\top - C\mathbf{y}^\top\mathbf{X} & 1-y_i(\mathbf{w}^\top\mathbf{x}_i+b) >= 0\\ \mathbf{w}^\top & 1-y_i(\mathbf{w}^\top\mathbf{x}_i+b) < 0\end{cases}$$

$$\frac{\partial f(\mathbf{w},b)}{b} = \begin{cases}-C\sum_{i=1}^{N}y_i & 1-y_i(\mathbf{w}^\top\mathbf{x}_i+b) >= 0\\ 0 & 1-y_i(\mathbf{w}^\top\mathbf{x}_i+b) < 0\end{cases}$$

III. EXPERIMENT

◆ **实验代码**

1）逻辑回归和随机梯度下降：

```python
# !python3
# -*- coding:utf-8 -*-
from numpy import *
import math
import pandas as pd
import sklearn.datasets
import matplotlib.pyplot as plt
from numpy import random
# %matplotlib inline

NUM_ITERATIONS = 100


def get_data(filename):
    data = sklearn.datasets.load_svmlight_file(filename)
    return data[0], data[1]


def compute_loss(X, y, theta):
    loss = 0
    for dx, dy in zip(X, y):
        loss += log1p(exp(-(dy*dot(dx, theta))))
    n = shape(X)[0]
    loss = loss / n + (linalg.norm(theta) ** 2) / 2
    return mean(loss)
```

```python
def compute_gradient(X, y, theta, rand):
    grad = zeros(shape=(1, len(theta)))
    for i in rand:
        grad += (y[i]*X[i]/(1+exp(y[i]*dot(X[i],theta))))
    grad /= len(rand)
    grad = -grad + theta.T
    return grad.T


def SGD(X_train, y_train,X_test, y_test, ini_theta, num_iter):
    learning_rate = 0.1
    theta = ini_theta
    Ltest = []
    random.seed(0)
    for i in range(num_iter):
        rand = random.randint(shape(X_train)[0], size=10)
        theta = theta - learning_rate*compute_gradient(X_train,y_train,theta,rand)
        Ltest.append(compute_loss(X_test,y_test,theta))
        print('SGD: Loss of the ', i+1, ' iteration for test:', Ltest[i])
    return Ltest
```

```python
def NAG(X_train, y_train,X_test, y_test, ini_theta, num_iter):
    theta = ini_theta
    learning_rate = 0.01
    momentum = zeros(shape=(shape(theta)[0], 1))
    miu = 0.9

    Ltest = []
    for i in xrange(num_iter):
        rand = random.randint(shape(X_train)[0], size=10)
        grad = compute_gradient(X_train, y_train, theta-miu*momentum, rand)
        momentum = momentum*miu + learning_rate*grad
        theta = theta-momentum
        Ltest.append(compute_loss(X_test, y_test, theta))
        print('NAG:Loss of the ', i+1, ' iteration for test:', Ltest[i])
    return Ltest


def RMSprop(X_train, y_train, X_test, y_test, ini_theta, num_iter):
    theta = ini_theta
    learning_rate = 0.1
    g = zeros(shape=(len(theta), 1))
    eps = 1e-5
    dr = 0.9  # decay_rate
    Ltest = []

    for i in xrange(num_iter):
        rand = random.randint(shape(X_train)[0], size=10)
        grad = compute_gradient(X_train, y_train, theta, rand)
        g = dr*g +(1-dr)*grad*grad
        theta = theta - learning_rate*grad/sqrt(g+eps)
        Ltest.append(compute_loss(X_test, y_test, theta))
        print('RMSprop: Loss of the ', i+1, ' iteration for test:', Ltest[i])
    return Ltest
```

```python
def Adam(X_train, y_train,X_test, y_test, ini_theta, num_iter):
    theta = ini_theta
    m = zeros(shape=(len(theta), 1))
    g = zeros(shape=(len(theta), 1))
    learning_rate = 0.1
    eps = 1e-5
    beta1 = 0.9
    beta2 = 0.999
    Ltest = []

    for i in range(1,num_iter+1):
        rand = random.randint(shape(X_train)[0], size=10)
        grad = compute_gradient(X_train, y_train, theta, rand)
        m = beta1*m+(1-beta1)*grad
        g = beta2*g+(1-beta2)*grad*grad
        alpha = learning_rate*sqrt(1-beta2**i)/(1-beta1**i)
        theta = theta - alpha*m/sqrt(g+eps)
        Ltest.append(compute_loss(X_test, y_test, theta))
        print('Adam: Loss of the ', i, ' iteration for test:', Ltest[i - 1])
    return Ltest
```

```python
def AdaDelta(X_train, y_train,X_test, y_test, ini_theta, num_iter):
    theta = ini_theta
    g = zeros(shape=(len(theta), 1))
    s = zeros(shape=(len(theta), 1))
    eps = 1e-5
    r = 0.95
    # learning_rate = 50

    Ltest = []
    for i in xrange(num_iter):
        rand = random.randint(shape(X_train)[0], size=10)
        grad = compute_gradient(X_train, y_train, theta, rand)
        g = r*g+(1-r)*grad*grad
        diff_theta = -sqrt(s+eps)*grad/sqrt(g+eps)
        theta=theta+diff_theta
        s = r*s+(1-r)*diff_theta*diff_theta
        Ltest.append(compute_loss(X_test, y_test, theta))
        print('AdaDelta: Loss of the ', i+1, ' iteration for test:', Ltest[i]
    return Ltest


def main():
    # Logistic regression
    X_train, y_train = get_data('./data/a9a')
    X_train = X_train.todense()
    temp = ones(shape=[shape(X_train)[0], 1], dtype=float32)
    X_train = concatenate([X_train, temp], axis=1)
    y_train = y_train.reshape(len(y_train), 1)
```

```python
    X_test, y_test = get_data('./data/a9at')
    X_test = X_test.todense()
    X_test = append(zeros(shape=(shape(X_test)[0], 1)), X_test, 1)
    temp = ones(shape=[shape(X_test)[0], 1], dtype=float32)
    X_test = concatenate([X_test, temp], axis=1)
    y_test = y_test.reshape(len(y_test), 1)
    for i in range(shape(y_test)[0]):
        if y_test[i,0] == -1:
            y_test[i,0] = 0

    # initialize parameters
    num_iter = NUM_ITERATIONS
    initial_theta = random.rand(shape(X_test)[1])
    initial_theta = initial_theta.reshape(len(initial_theta),1)

    LSGD = SGD(X_train,y_train,X_test,y_test,initial_theta,num_iter)
    LNAG = NAG(X_train, y_train, X_test, y_test, initial_theta, num_iter)
    LRMSprop = RMSprop(X_train, y_train,X_test, y_test, initial_theta, num_iter)
    LAdam = Adam(X_train, y_train, X_test, y_test, initial_theta, num_iter)
    LAdaDelta = AdaDelta(X_train, y_train, X_test, y_test, initial_theta, num_iter)

    # visualization
    num_iter=xrange(num_iter)
    plt.plot(num_iter, LSGD, label='SGD')
    plt.plot(num_iter, LNAG, label='NAG')
    plt.plot(num_iter,LRMSprop, label='RMSprop')
    plt.plot(num_iter, LAdam,label='Adam')
    plt.plot(num_iter, LAdaDelta,label='AdaDelta')
    plt.title('Loss for test')
    plt.xlabel('iteration')
    plt.ylabel('loss')
    plt.legend()
    plt.show()
```

```python
if __name__=='__main__':
    main()
```

2）线性分类和梯度下降：

```python
# !python3
# -*- coding:utf-8 -*-
from numpy import *
import math
import pandas as pd
import sklearn.datasets
import sklearn.model_selection
import matplotlib.pyplot as plt
from numpy import random
# %matplotlib inline

NUM_ITERATIONS = 200


def get_data(filename):
    data = sklearn.datasets.load_svmlight_file(filename)
    return data[0], data[1]


def compute_loss(X, y, theta):
    loss = 0.0
    for i in range(shape(X)[0]):
        pred = dot(X[i], theta)
        loss += max(0, y[i,0]*pred)
    loss += 0.5 * linalg.norm(theta) ** 2
    return mean(loss)/shape(X)[0]

def compute_gradient(X, y, theta,i):
    grad = zeros(shape=(1, len(theta)))
    gamma = 1
    pred = dot(X[i],theta)
    grad = max(0,1-y[i,0]*pred)*(-y[i,0]*X[i])+gamma*theta.T
    return grad.T


def SGD(X_train, y_train,X_test, y_test, ini_theta, num_iter):
    learning_rate = 0.05
    theta = ini_theta
    Ltest = []
    for i in range(num_iter):
        j = random.randint(0,shape(X_train)[0])
        theta = theta - learning_rate*compute_gradient(X_train,y_train,theta,j)
        Ltest.append(compute_loss(X_test,y_test,theta))
        print('SGD: Loss of the ', i+1, ' iteration for test:', Ltest[i])
    return Ltest

def NAG(X_train, y_train,X_test, y_test, ini_theta, num_iter):
    theta = ini_theta
    learning_rate = 0.01
    momentum = zeros(shape=(shape(theta)[0], 1))
    miu = 0.9

    Ltest = []
    for i in xrange(num_iter):
        j = random.randint(0, shape(X_train)[0])
        grad = compute_gradient(X_train, y_train, theta-miu*momentum,j)
        momentum = momentum*miu + learning_rate*grad
        theta = theta-momentum
        Ltest.append(compute_loss(X_test, y_test, theta))
        print('NAG:Loss of the ', i+1, ' iteration for test:', Ltest[i])
    return Ltest


def RMSprop(X_train, y_train, X_test, y_test, ini_theta, num_iter):
    theta = ini_theta
    learning_rate = 0.01
    g = zeros(shape=(len(theta), 1))
    eps = 1e-7
    dr = 0.95  # decay_rate
    Ltest = []

    for i in xrange(num_iter):
        j = random.randint(0, shape(X_train)[0])
        grad = compute_gradient(X_train, y_train, theta,j)
        g = dr*g +(1-dr)*square(grad)
        theta = theta - learning_rate*grad/sqrt(g+eps)
        Ltest.append(compute_loss(X_test, y_test, theta))
        print('RMSprop: Loss of the ', i+1, ' iteration for test:', Ltest[i])
    return Ltest
```

```python
def Adam(X_train, y_train, X_test, y_test, ini_theta, num_iter):
    theta = ini_theta
    m = zeros(shape=(len(theta), 1))
    g = zeros(shape=(len(theta), 1))
    learning_rate = 0.01
    eps = 1e-8
    beta1 = 0.9
    beta2 = 0.999
    Ltest = []

    for i in xrange(num_iter):
        j = random.randint(0, shape(X_train)[0])
        grad = compute_gradient(X_train, y_train, theta,j)
        m = beta1*m+(1-beta1)*grad
        g = beta2*g+(1-beta2)*square(grad)
        arpha = learning_rate*sqrt(1-beta2)/(1-beta1)
        theta = theta - arpha*m/sqrt(g+eps)
        Ltest.append(compute_loss(X_test, y_test, theta))
        print('Adam: Loss of the ', i+1, ' iteration for test:', Ltest[i])
    return Ltest

def AdaDelta(X_train, y_train, X_test, y_test, ini_theta, num_iter):
    theta = ini_theta
    g = zeros(shape=(len(theta), 1))
    s = zeros(shape=(len(theta), 1))
    eps = 1e-8
    r = 0.95
    learning_rate = 50
    Ltest = []
    for i in xrange(num_iter):
        j = random.randint(0, shape(X_train)[0])
        grad = compute_gradient(X_train, y_train, theta,j)
        g = r*g+(1-r)*square(grad)
        diff_theta = -multiply(sqrt(s+eps),grad)/sqrt(g+eps)
        theta=theta+learning_rate*diff_theta
        s = r*s+(1-r)*square(diff_theta)
        Ltest.append(compute_loss(X_test, y_test, theta))
        print('AdaDelta: Loss of the ', i+1, ' iteration for test:', Ltest[i])
    return Ltest

def main():
    # Logistic regression
    X_train, y_train = get_data('./data/a9a')
    X_train = X_train.todense()
    temp = ones(shape=[shape(X_train)[0], 1], dtype=float32)
    X_train = concatenate([X_train, temp], axis=1)
    y_train = y_train.reshape(len(y_train), 1)
    for i in range(shape(y_train)[0]):
        if y_train[i,0] == -1:
            y_train[i,0] = 0

    X_test, y_test = get_data('./data/a9at')
    X_test = X_test.todense()
    X_test = append(zeros(shape=(shape(X_test)[0], 1)), X_test, 1)
    temp = ones(shape=[shape(X_test)[0], 1], dtype=float32)
    X_test = concatenate([X_test, temp], axis=1)
    y_test = y_test.reshape(len(y_test), 1)
    for i in range(shape(y_test)[0]):
        if y_test[i,0] == -1:
            y_test[i,0] = 0

    # initialize parameters
    num_iter = NUM_ITERATIONS
    initial_theta = random.rand(shape(X_test)[1])
    initial_theta = initial_theta.reshape(len(initial_theta),1)
    LSGD = SGD(X_train,y_train,X_test,y_test,initial_theta,num_iter)
    LNAG = NAG(X_train, y_train, X_test, y_test, initial_theta, num_iter)
    LRMSprop = RMSprop(X_train, y_train,X_test, y_test, initial_theta, num_iter)
    LAdam = Adam(X_train, y_train, X_test, y_test, initial_theta, num_iter)
    LAdaDelta = AdaDelta(X_train, y_train, X_test, y_test, initial_theta, num_iter)

    # visualization
    num_iter=xrange(num_iter)
    plt.plot(num_iter, LSGD, label='SGD')
    plt.plot(num_iter, LNAG, label='NAG')
    plt.plot(num_iter,LRMSprop, label='RMSprop')
    plt.plot(num_iter, LAdam,label='Adam')
    plt.plot(num_iter, LAdaDelta,label='AdaDelta')
    plt.title('Loss for test')
    plt.xlabel('iteration')
    plt.ylabel('loss')
    plt.legend()
    plt.show()


if __name__=='__main__':
    main()
```

- **模型参数的初始化方法：**
  - 逻辑回归和随机梯度下降：正态分布初始化
  - 线性分类和随机梯度下降：随机初始化

◆ 实验结果和曲线图:

● 逻辑回归和随机梯度下降:

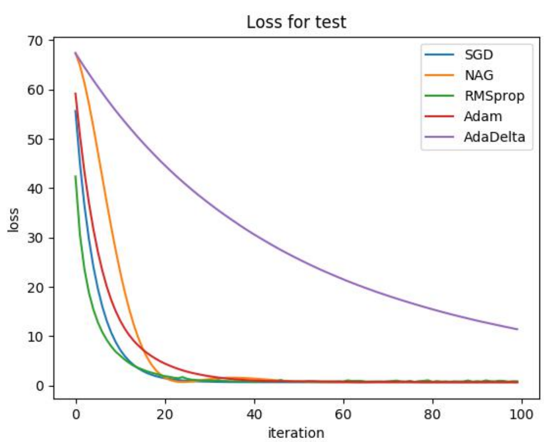
SGD:
learning_rate = 0.1

NAG:
learning_rate = 0.01
momentum = zeros(shape=(shape(theta)[0], 1))
miu = 0.9

RMSprop:
learning_rate = 0.1
g = zeros(shape=(len(theta), 1))
eps = 1e-5
dr = 0.9 # decay_rate

Adam:
m = zeros(shape=(len(theta), 1))
g = zeros(shape=(len(theta), 1))
learning_rate = 0.1
eps = 1e-5
beta1 = 0.9
beta2 = 0.999

AdaDelta:
theta = ini_theta
g = zeros(shape=(len(theta), 1))
s = zeros(shape=(len(theta), 1))
eps = 1e-5
r = 0.95



● 线性分类和随机梯度下降:

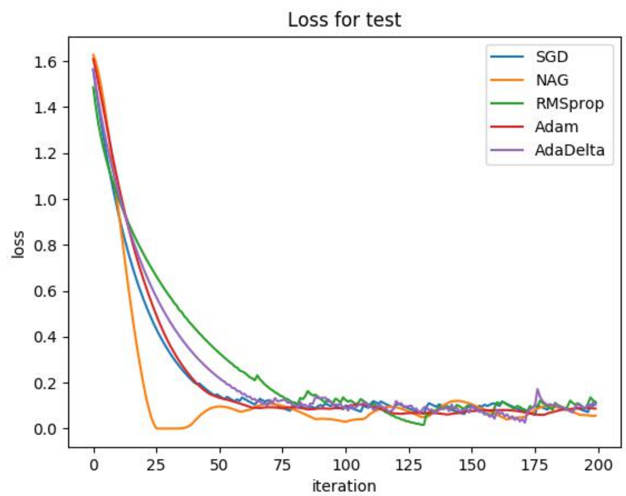
SGD:
learning_rate = 0.05

NAG:
learning_rate = 0.01
momentum = zeros(shape=(shape(theta)[0], 1))
miu = 0.9

RMSprop:
learning_rate = 0.01
g = zeros(shape=(len(theta), 1))
eps = 1e-7
dr = 0.95 # decay_rate

Adam:
m = zeros(shape=(len(theta), 1))
g = zeros(shape=(len(theta), 1))
learning_rate = 0.01
eps = 1e-8
beta1 = 0.9
beta2 = 0.999

AdaDelta:
theta = ini_theta
g = zeros(shape=(len(theta), 1))
s = zeros(shape=(len(theta), 1))
eps = 1e-8
r = 0.95
learning_rate = 50



IV. CONCLUSION

◆ 实验结果分析:

● 逻辑回归和随机梯度下降:

实验的结果从图像看，除 **AdaDelta** 方法是自适应调整学习率收敛速度比较慢外，其他优化方法都在一百次迭代内较快地完成了收敛。这个实验我采取了随机小批量梯度下降的方法为基础，可以看出相比任务二的震荡情况会好很多，收敛的效果更好些。

● 线性分类和随机梯度下降:

实验的结果从最终图像上看效果一般,**loss** 有轻微震荡的情况，六个随机梯度下降的方法在第七十五

次迭代时收敛。其中 **SGD**，**NAG**，**Adam** 的收敛速度相较较快。这个实验我采取的是随机梯度下降，相较任务一的实验，震荡情况会明显些。对于 **AdaDelta** 收敛速度慢的情况，我在更新 **theta** 时乘了个比较合适的 **learning_rate,** 实验结果的效果上看收敛速度基本能跟其他方法持平。

◆ 对比逻辑回归和线性分类的异同点：

● 相同点**:**

两种方法都是常见的分类算法**,**从目标函数来看**,**区别在于逻辑回归采用的是 **logistical loss**，**SVM** 采用的是 **hinge loss**。这两个损失函数的目的都是增加对分类影响较大的数据点的权重**,**减少与分类关系较小的数据点的权重。**SVM** 的处理方法是只考虑 **support vectors**，也就是和分类最相关的少数点**,** 去学习分类器。而逻辑回归通过非线性映射，大大减小了离分类平面较远的点的权重，相对提升了与分类最相关的数据点的权重。

● 不同点：

分类和回归的区别在于输出变量的类型。定量输出称为回归，或者说是连续变量预测；定性输出称为分类，或者说是离散变量预测。

◆ 实验总结：

首先这次实验体验了看论文实现代码的过程，对各算法的原理过程都深刻了解了许多，比起单单上课听讲要收益许多。整个实验过程，我对调参的过程也体验了很多，深刻感受到各参数的毫厘之差对最终的实验结果造成的影响确是巨大的。找到一组合适的参数是非常不容易的事情。