Math 2D

Chapter 10.1

Derivatives

- $(fg)' = f'g + fg'$
- $(\frac{f}{g})' = \frac{f'g - fg'}{g^2}$
- $\tan x' = \sec^2 x$
- $\sec x' = \sec x \tan x$
- $\csc x' = -\csc x \cos x$

Trigonometry

- $\sin^2 x + \cos^2 x = 1$
- $1 + \cot^2 x = \csc^2 x$
- $\tan^2 x + 1 = \sec^2 x$
- $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - \sin^2 x$

Concavity Test

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$$

Chapter 10.2

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\frac{\mathrm{d}}{\mathrm{d}t}(\frac{\mathrm{d}y}{\mathrm{d}x})}{\frac{\mathrm{d}x}{\mathrm{d}t}}$$

Cycloid

$$\begin{cases} x_p = r(\theta - \sin\theta) \\ y_p = r(1 - \cos\theta) \end{cases}$$

Arc length

$$\int_a^b \sqrt{(\frac{\mathrm{d}x}{\mathrm{d}t})^2 + (\frac{\mathrm{d}y}{\mathrm{d}t})^2} \mathrm{d}t$$

Chapter 10.3

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$\begin{cases} x^2 + y^2 = r^2 \\ \tan\theta = \frac{y}{x} \end{cases}$$

Symetric rules

- Unchanges when θ replaced by $-\theta$:
  - symetric about x axis
- Unchanged when r replaced by -r or θ replaced by $\theta + \pi$:
  - symetric about the pole(origin)
- Unchanged when θ replaced by $(\pi - \theta)$:
  - symetric about y axis

Tangents

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}r}{\mathrm{d}\theta}\sin\theta + r\cos\theta}{\frac{\mathrm{d}r}{\mathrm{d}\theta}\cos\theta - r\sin\theta}$$

Chapter 12.1-12.4

Right Hand Rule

Distance

$$P_1(x_1, y_1, z_1), P_2(x_2, y_2, z_2)$$

$$P_1P_2 = l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Scalar

$$\vec{v} = < a, b, c > |\vec{v}| = \sqrt{a^2 + b^2 + c^2}$$

Unit Vector ($|\vec{u}| = 1$)

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$$

Dot product

$\vec{a} = < a_1, a_2, a_3 >, \vec{b} = < b_1, b_2, b_3 >$

1. $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\vec{a}||\vec{b}|\cos\theta$
2. $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

Orthogonal=> perpendicular=> $\theta = \frac{\pi}{2}$

Projection

Scalar projection of $\vec{b}$ onto $\vec{a}$

$$Comp_{\vec{a}}^{\vec{b}} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

Vector projection of $\vec{b}$ onto $\vec{a}$

$$Proj_{\vec{a}}^{\vec{b}} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \cdot \vec{a}$$

Cross Product

$\vec{a} = < a_1, a_2, a_3 >, \vec{b} = < b_1, b_2, b_3 >$

$\vec{a} \cdot \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} + j \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} =< a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 >$$ The vector $\vec{a} \times \vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$ $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

Chapter 12.5

Line:

$\vec{r} = \vec{r}_0 + t \cdot \vec{v} < x, y, z > = < x_0, y_0, z_0 > + t \cdot < a, b, c > \begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases} \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

Skew Lines:

Neither intersect nor parallel

Plane

$\vec{n}$=>Normal Vector A plane through $P(x_0, y_0, z_0)$ with $\vec{n} =< a, b, c >$ is

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Find Angles between two plance:

Find the angle between normal vectors

Find plane pass through P_1 P_2 and P_3

$\vec{n} = \overrightarrow{P_1 P_2} \times \overrightarrow{P_2 P_3}$ and with P_1

Distance from $P(x_1, y_1, z_1)$ to plane $ax + by + cz = 0$

$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

Find the distance between two lines:

1. Find normal vector
2. Establish a plane
3. Use formula