# Math 2D

## Chapter 10.1

### Derivatives

- $(fg)' = f'g + fg'$
- $(\frac{f}{g})' = \frac{f'g - fg'}{g^2}$
- $\tan x' = \sec^2 x$
- $\sec x' = \sec x \tan x$
- $\csc x' = -\csc x \cos x$

### Trigonometry

- $\sin^2 x + \cos^2 x = 1$
- $1 + \cot^2 x = \csc^2 x$
- $\tan^2 x + 1 = \sec^2 x$
- $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - \sin^2 x$

### Concavity Test

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$$

## Chapter 10.2

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\frac{\mathrm{d}}{\mathrm{d}t}(\frac{\mathrm{d}y}{\mathrm{d}x})}{\frac{\mathrm{d}x}{\mathrm{d}t}}$$

### Cycloid

$$\begin{cases} x_p = r(\theta - \sin\theta) \\ y_p = r(1 - \cos\theta) \end{cases}$$

### Arc length

$$\int_a^b \sqrt{(\frac{\mathrm{d}x}{\mathrm{d}t})^2 + (\frac{\mathrm{d}y}{\mathrm{d}t})^2}\mathrm{d}t$$

## Chapter 10.3

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$\begin{cases} x^2+y^2=r^2 \\ \tan\theta=\frac{y}{x} \end{cases}$$

## Symetric rules

- Unchanges when $\theta$ replaced by $-\theta$:
  - symetric about x axis
- Unchanged when r replaced by -r or $\theta$ replaced by $\theta+\pi$:
  - symetric about the pole(origin)
- Unchanged when $\theta$ replaced by $(\pi-\theta)$:
  - symetric about y axis

## Tangents

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}r}{\mathrm{d}\theta}\sin\theta+r\cos\theta}{\frac{\mathrm{d}r}{\mathrm{d}\theta}\cos\theta-r\sin\theta}$$

# Chapter 12.1-12.4

## *Right Hand Rule*

## Distance

$$P_1(x_1,y_1,z_1),P_2(x_2,y_2,z_2)\ P_1P_2=l=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

## Scalar

$$\vec{v}=<a,b,c>|\vec{v}|=\sqrt{a^2+b^2+c^2}$$

## Unit Vector ($|\vec{u}|=1$)

$$\vec{u}=\frac{\vec{v}}{|\vec{v}|}$$

## Dot product

$\vec{a}=<a_1,a_2,a_3>,\vec{b}=<b_1,b_2,b_3>$

1. $\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3=|\vec{a}||\vec{b}|\cos\theta$
2. $\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

## Orthogonal=> perpendicular=> $\theta=\frac{\pi}{2}$

## Projection

### Scalar projection of $\vec{b}$ onto $\vec{a}$

$$Comp_{\vec{a}}^{\vec{b}}=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$$

### Vector projection of $\vec{b}$ onto $\vec{a}$

$Proj_{\vec{a}}^{\vec{b}} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \cdot \vec{a}$

### Cross Product

$\vec{a} =< a_1, a_2, a_3 >, \vec{b} =< b_1, b_2, b_3 >$

$\vec{a} \cdot \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} + j \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + k \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} =< a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 >$ The vector $\vec{a} \times \vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$ $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

## Chapter 12.5

### Line:

$\vec{r} = \vec{r}_0 + t \cdot \vec{v} < x, y, z >=< x_0, y_0, z_0 > +t \cdot < a, b, c > \begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases} \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

### Skew Lines:

Neither intersect nor parallel

### Plane

$\vec{n}$=>Normal Vector A plane through $P(x_0, y_0, z_0)$ with $\vec{n} =< a, b, c >$ is

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

### Find Angles between two planes:

Find the angle between normal vectors

### Find plane pass through $P_1$ $P_2$ and $P_3$

$\vec{n} = \overrightarrow{P_1 P_2} \times \overrightarrow{P_2 P_3}$ and with $P_1$

### Distance from $P(x_1, y_1, z_1)$ to plane $ax + by + cz = 0$

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

### Find the distance between two lines:

1. Find normal vector
2. Establish a plane
3. Use formula

## Chapter 13.3

### Arc Length in 3D

$$l = \int_a^b \sqrt{(\frac{\mathrm{d}x}{\mathrm{d}t})^2 + (\frac{\mathrm{d}y}{\mathrm{d}t})^2 + (\frac{\mathrm{d}z}{\mathrm{d}t})} \mathrm{d}t$$

if we assume $\overrightarrow{r(t)} =< f(t), g(t), h(t) >$=>$\overrightarrow{r'(t)} =< f'(t), g'(t), h'(t) >$

$$l = \int_a^b |\overrightarrow{r'(t)}| \mathrm{d}t$$

### Tangent Vector

$$\overrightarrow{T(t)} = \frac{\overrightarrow{r'(t)}}{|\overrightarrow{r'(t)}|} \; |\overrightarrow{T(t)}| = 1$$

### Normal Vector

$$\overrightarrow{N(t)} = \frac{\overrightarrow{T'(t)}}{|\overrightarrow{T'(t)}|}$$

### Binomal Vector

$$\overrightarrow{B(t)} = \overrightarrow{T(t)} \times \overrightarrow{N(t)}$$

### Normal Plane

Normal plane: The normal plane consists of all lines that are orthogonal to the tangent vector.

The normal vector of the normal plane is the tangent vector

$$\vec{n} = \vec{T}$$

### Osculating plane

The plane that comes the closest to containing the part of the curve near point P.

The normal vector of the osculating plane is the binomal vecotr

$$\vec{n} = \vec{B}$$

## Chapter 13.4

### Velocity

$$\overrightarrow{v(t)} = \overrightarrow{r'(t)}$$

### Speed

$$Speed = |\overrightarrow{v(t)}| = |\overrightarrow{r'(t)}|$$

## Chapter 14.1

### Level Curves / Contour Map

The level curves of a function $f$ of two variables are the curve with equations $f(x, y) = k$, where k is a constant (in the range of $f$).

### Functions of Three or more variables

## Chapter 14.2

## Limit of multi-var functions

## Prove the limit does not exist

Pick two different paths and show that the height or the value if the function is different if we transverse along these different paths.

## Tips

- Make sure the path you choose contains the point (a,b)
- It is highly recommended that one of the paths you choose is either by forcing x=0 and moving along y axis or forcing y=0 and moving along x axis.
- It is helpful to choose a path that makes the degree of the numerator and the denominator equal.

## Continuity

Sometimes we can use the concept of continuity for proving that the limit exist.

- If we have a region (two dimensional region in xy-plane) that the function is defined in that region, the function is continuous in that region. In other words, a function is continuous at any point inside its domain.
- Polynomials are continuous everywhere.
- Rational functions are continuous everywhere except where the denominator is zero.