Sampling and Monitoring Networks

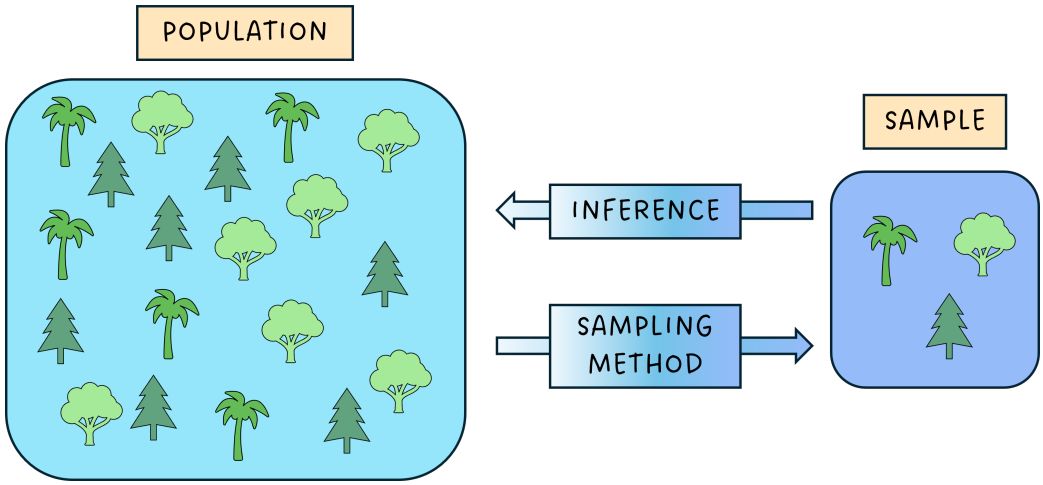
1 Overview

This session will focus on how environmental and ecological data are obtained, specifically through sampling. Sampling is a fundamental concept in data analysis, especially in environmental data.

We use samples in environmental data in situations where it is not possible to measure the entire population. In environmental settings, this could be because:

- The population is too large.
- Some or all of the population is difficult, expensive, or even impossible to reach.
- The samples may be *destructive*, i.e., taking the sample causes permanent damage to the object being measured.

We want to use the information we obtain on the **sample** in order to make *inference* on the **population**.



2 Designing an ecological/environmental study

When we design an environmental or an ecological study we should focud on these steps:

1. Define the study objectives.
2. Summarize the environmental context.
3. Identify the target population.
4. Select an appropriate sampling design.
5. Implement and summarize.

Step 1 — Define the study objectives

We need to define clear and simple objectives for our study. What is the key question or hypothesis?These will be driven by the properties of our data that we would like to measure:

- Characteristics of a variable, e.g. mean, median, variance.
- Temporal or spatial trends of a variable.
- Frequency of events, e.g. number of pollution events, species abundance or occurrence.

i Example: water quality

What is the spatial or temporal variability of water quality across a River Surveillance Network (RSN)?

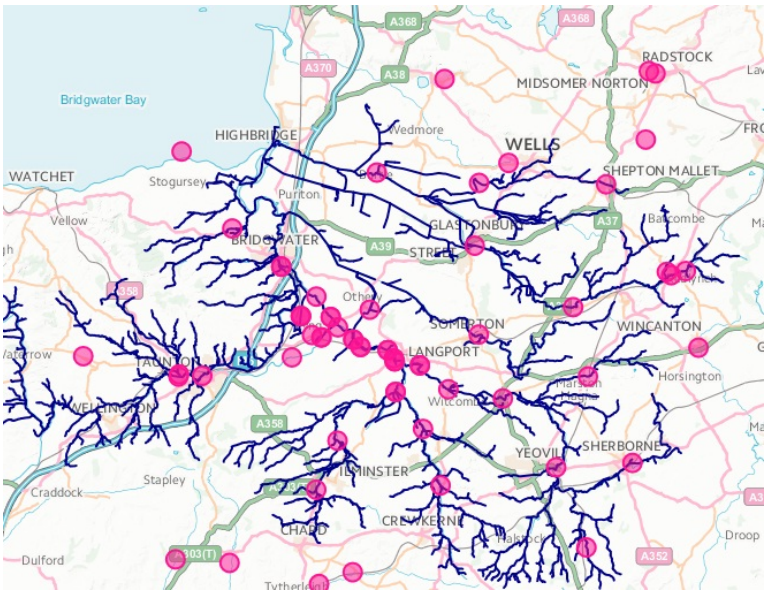


Figure 1: OS Open Rivers network represented in blue lines and the pink dots representing the Environment Agency monitoring stations.

Step 2 — Consider the context

We have to think about the context of the question we are asking. This means understanding the nature of our data, which is essential to ensuring we have a **representative** sample.

For example, if we're measuring a river, we need to know about the depth, width, and current. If we are sampling in a forest we need to know about vegetation and wildlife.

Step 3 — Identify the target population

The population is the set of all possible objects that could be sampled.

- All the fish in a lake.
- All oak trees over 5m tall in a particular part of a forest.
- Every river within a particular water network.

Sometimes the population is actually what we are trying to measure, e.g. “How many red squirrels live in the Cairngorms National Park?”

> Example: Water quality
>
> **Target population**: RSN 1:250k with over 1.4 million reaches (a discrete segment of a river with relatively uniform characteristics) .
> **Characterise environmental conditions** of the target population such as Water Quality Indicators, i.e., we need to define our **response variable**:
>
> - Macroinvertebrates composition obtained from the RICT Model 44 network (1:50k scale and trimmed to match the RSN network). E.g., **WHPT-ASPT** (*Walley Hawkes Paisley Trigg Average Score Per Taxon*) is a **biological metric** used to evaluate the ecological health of rivers based on the presence and sensitivity of macroinvertebrate (e.g., insects, worms, snails) communities.
> - Orthophosphate $\left[PO_4\right]^{3-}$ concentrations (mg/L)

Step 4 — Select a sampling design

There are a number of sampling designs which are commonly used for environmental data:

- Simple random sampling.
- Stratified random sampling.
- Systematic sampling.
- Spatial sampling.

We will discuss some of these in more detail during the course.

Step 5 — Implement and summarise

- **Data collection** - what information is being collected and how? E.g., biological elements, river habitat surveys, physico-chemical elements, toxic chemicals, invasive non-native species, physical properties, etc. What technical equipment or techniques are used to measure these elements?
- **Implementation** - deploying the network and measuring the quantities of interest. Be aware of practical challenges that might impact the subsequent analysis such as: accessibility contains to the selected sampling site, landowner permission, safety issues, unfavorable environmental conditions, etc.

Often statisticians will not actually carry out the sampling, but will instead rely on field experts in many cases. Once we receive the data, it’s important to assess the data for censoring, outliers, missingness. We can then fit an appropriate statistical model. Finally, we should report our results in clear language, including uncertainty where appropriate.

In this session we will look at different sampling schemes commonly used in environmental and ecological monitoring. A sampling strategy integrates both sample selection methods from a target population and estimation techniques to infer population attributes from sample measurements.

Such attributes can be viewed as a quantitative combination of population values such as the mean Orthophosphate $[PO_4]^{3-}$ concentration or the total number of macroinvertebrates in a river. Likewise, an *estimator* is a mathematical expression or a function of the sample that provide us with a *estimate* of the population parameter. For instance, let θ denote the population parameter of interest (e.g., the population mean), then $\hat{\theta}$ represents its estimator (see [additional notes]{about.qmd} on estimator properties). Typically, the value of θ is unknown and it is unfeasible to measure all N elements of the population. Thus, we select and measure $n < N$ units to estimate θ. The question now is how do we select such units?

3 Sampling Methods

3.1 Simple Random Sampling

As the name suggests, this is the simplest form of sampling. Every object in our population has an **equal probability** of being included in the sample. This requires us to have a complete list of the population members, or a sampling frame covering the entire region. We then generate a set of n random digits which identify the individuals or objects to be included in a study.

For a sample of size n, denoted $y_1, \dots, y_n$, we can compute the sample mean as

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}.$$

We can then compute the estimated population variance as

$$s^2 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1}.$$

As well as estimating the population mean and variance, we also have to think about the uncertainty surrounding these estimates. This is what a confidence interval is typically representing.

Our sample of size n is just one of many possible samples of size n which we could have obtained. We must take this into account when considering the uncertainty associated with our sample mean. This is known as **sampling variability**. We can compute this as:

$$Var(\bar{y}) = \frac{s^2}{n}\left(1 - \frac{n}{N}\right).$$

Here, $(1 - \frac{n}{N})$ is what is known as a **finite population correction** (FPC), which accounts for the proportion of the data that remains unknown.

Task 1

To monitor patterns of weekly water consumption, a town conducted a SRS of $n = 100$ homes. The town records showed that there were $N = 5392$ residential dwellings with water meter in town. Water consumption was recorded in 100-gallon units. The sample average consumption was $\bar{y} = 12.5$ 100-gallons units per week and the sample variance was $s^2 = 1352$ (100-gallon units)^2. The local authority is interested in estimating the town's total weekly residential water usage. An unbiased estimator of a population total under SRS was given by Horvitz-Thompson (1963) as:

$$\hat{\tau} = N\bar{y} \tag{1}$$

Using HT estimator, calculate the town's total weekly residential water consumption. Then. derive the variance of this estimator and compute the standard deviation (square-root of the variance) for the town's totals.

See Solution

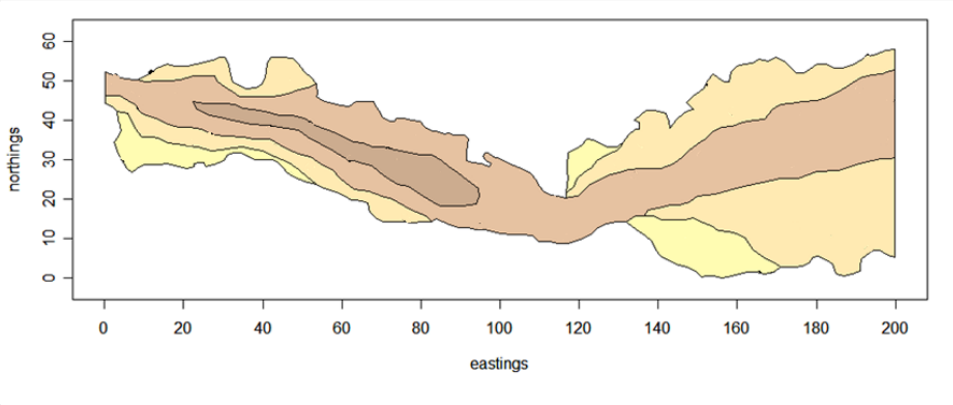
Using Equation 1, the estimated weekly residential water use town totals is $5392 \times 12.5 = 67400$ 100-gallon units. Then, the variance and standard deviation of $\hat{\tau}$ are :

$$\begin{aligned} Var(\hat{\tau}) &= Var(N\bar{y}) \\ &= N^2 Var(\bar{y}) \\ &= N^2 \times \frac{s^2}{n}\left(1 - \frac{n}{N}\right) \\ &= 5392^2 \times \frac{1352}{100}\left(1 - \frac{100}{5392}\right) \\ &\Rightarrow \\ \sqrt{Var(\tau)} &= 5392 \times \sqrt{\frac{1352}{100}\left(1 - \frac{100}{5392}\right)} \\ &= 19,641 \text{ 100-gallon units.} \end{aligned}$$

3.2 Stratified Sampling

Example: Cobalt-60 in sediment

Cobalt-60 is a synthetic radioactive isotope of cobalt produced in nuclear reactors. We may be interested in estimating how much of this is in the sediment of a river estuary. This map is colour coded by different sediment types. How might we make use of this information when sampling?



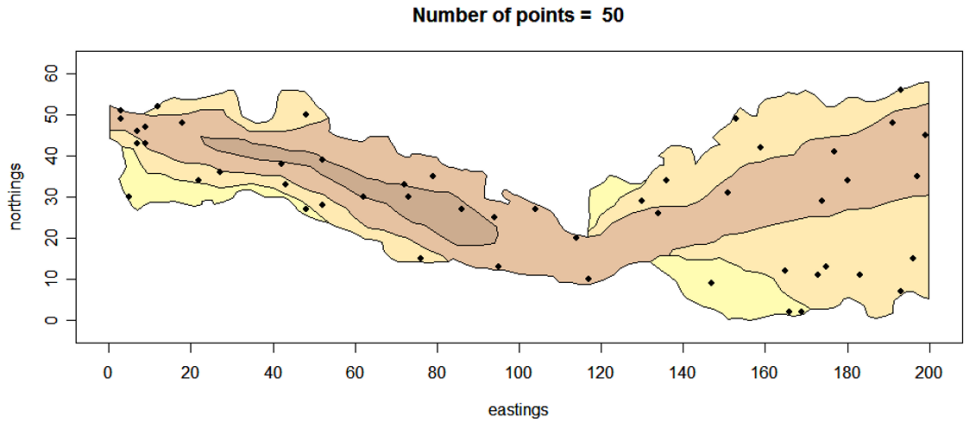
Stratified sampling involves dividing the population into two or more groups (or strata) which have something in common. Divide the dataset of size N into L non-overlapping strata such that within-strata variability is less than between-strata variability. We then ensure that each of our strata are represented in our sample, and take this into account in our final estimates.

We ensure that each of these strata are represented proportionally within our sample

(known as **proportional allocation**). Samples are still taken randomly within each stratum.

Example: Cobalt-60 in sediment (continued)

The map below shows black dots at the sampling locations that were chosen using stratified sampling. There are 50 sampling locations in total, and we can see that these are found proportionally across the strata. E.g., there are very few sampling locations within the light yellow region, reflecting that region's smaller area. Stratified sampling has ensured that we do, however, have at least some sampling locations within each strata.



Let $N_1, \ldots, N_L$ be the populations of our L strata, and $n_1, \ldots, n_L$ be the number of samples taken from each. It is straightforward to obtain sample means $y_1, \ldots, y_L$ and sample variances $s_1^2, \ldots, s_L^2$ for each stratum.

Then we compute the overall sample mean as

$$\bar{y} = \frac{\sum_{l=1}^{L} (N_l y_l)}{N}.$$

We can also compute the variance of the sample mean as

$$Var(\bar{y}) = \sum_{l=1}^{L} \left[\left(\frac{N_l}{N} \right)^2 \frac{s_l^2}{n_l} \left(1 - \frac{n_l}{N_l} \right) \right].$$

3.3 Systematic Sampling

Systematic sampling is a sampling method which makes use of a natural ordering that exists in data. We wish to take a sample of size n from a population of size N, which means every $k = \frac{N}{n}$ objects are sampled. For systematic sampling, we select our first unit at random, then select every kth unit in a systematic way.

For example, if we have $N = 50$ and $n = 5$, then $k = 10$. If our first unit is 2, our sample becomes units 2, 12, 22, 32, 42.

Advantages

- Convenient and quick.
- Well spaced across the study.

- Sort of random — every object has an equal chance of selection.

Disadvantages

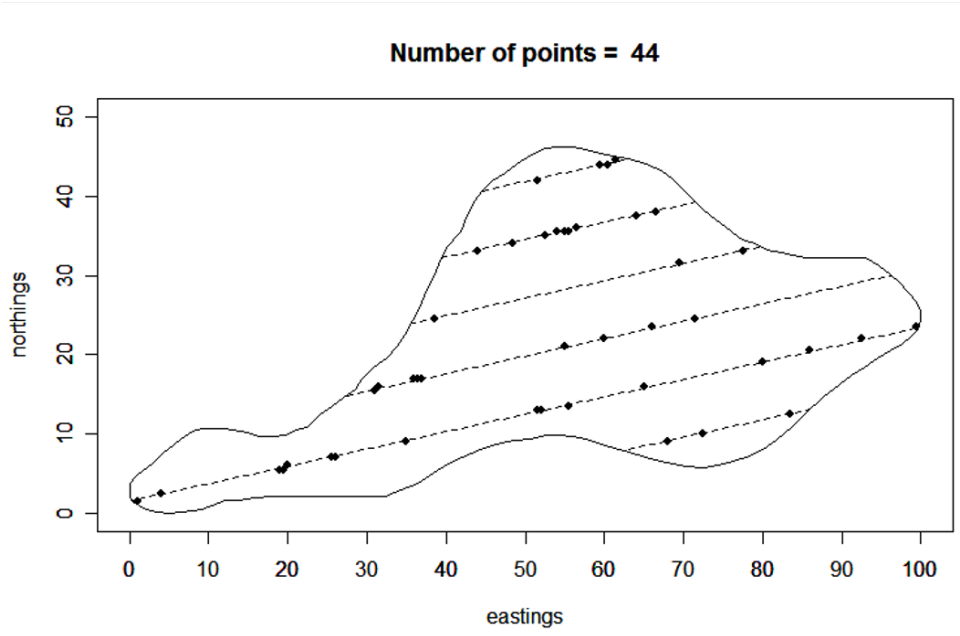
- May not be representative.
- Systematic patterns in the data can be overlooked.
- Extremely deterministic — estimation of variance particularly difficult.

3.4 Spatial Sampling

Spatial sampling is required when our data have an attribute that is spatially continuous. For example, if we are measuring water quality in a lake, we may have a three-dimensional coordinate system of *length*, *width* and *depth*. In some cases, it is possible to measure at any one of these locations, and simple random sampling or stratified sampling can be used. There are many examples where it is not possible or convenient to do so, in which case some form of systematic sampling may be used.

Spatial sampling often uses a systematic sampling scheme based on **transects**. A transect is a straight line along which samples are taken. The starting point, geographical orientation and number of samples are chosen as part of the sampling scheme. Samples will then be either taken at random points along the length of the line (*continuous sampling*) or systematically placed points (*systematic sampling*).

Suppose we need to take samples of water quality on a lake. Our sampling scheme may use multiple transects simultaneously.

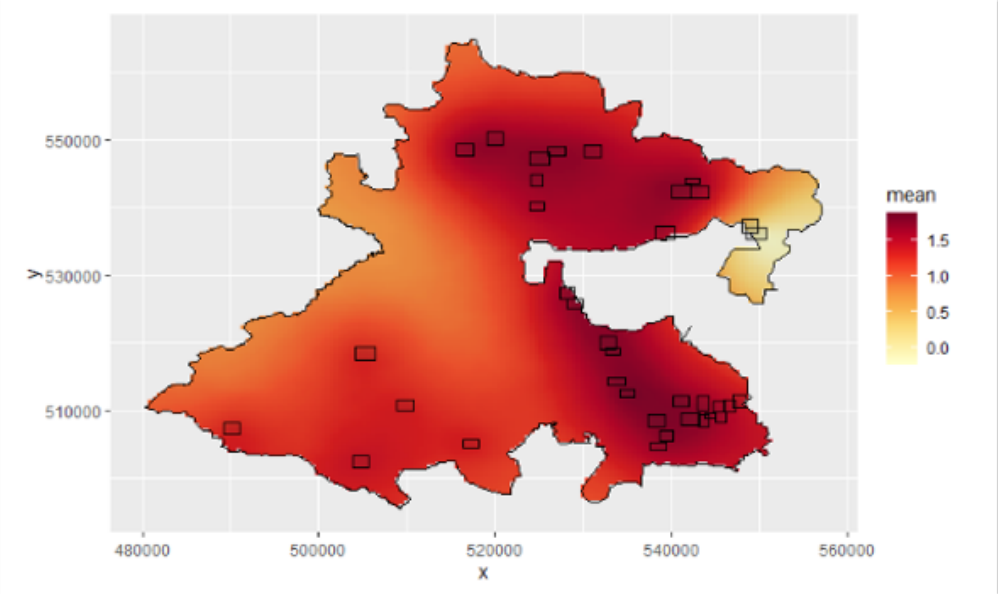


3.4.1 Distance sampling

In ecology, *distance sampling,* is a widely used spatial sampling method for estimating animal abundance or population density by measuring the perpendicular distances from a transect line or point to detected individuals. The methods assumes that the probability of observing animals decreases with increasing distance from the observer according to a specific detection function. The distance data are modelled using this detection function to account for imperfect detection and estimate the proportion of missed individuals in the study area.

3.4.2 Quadrats

In some cases, we will instead be interested in trying to understand the frequency of a certain species across space. A **quadrat** is a tool used in ecology and other settings for this purpose. A series of squares (quadrats) of a fixed size are placed in the habitat of interest, and the species within the quadrats are counted. The number of quadrats, and their positions and orientations are chosen as part of the sampling scheme.

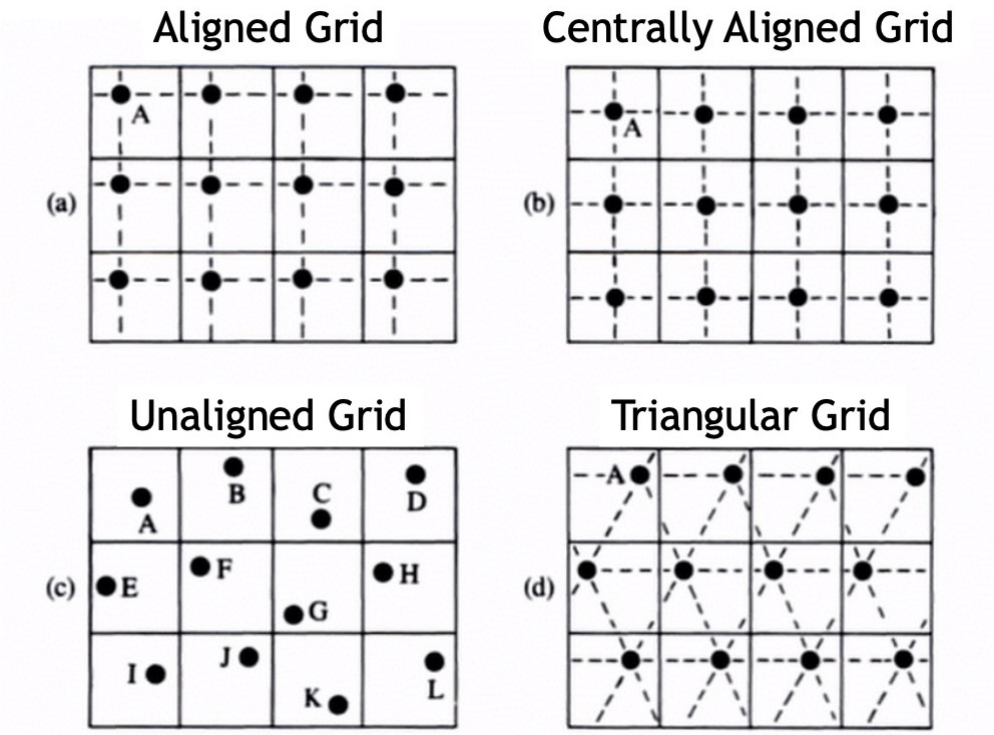


3.4.3 Grid Sampling

It may often be useful to use a regular grid to make sampling convenient and efficient. The grid is overlaid on the spatial region, and a fixed number of samples (usually one) is taken from each grids square. We choose the size of the grid such that the number of squares relates to the number of samples we require. For example, for a region of size 5km $\times$ 5km, choosing 1km $\times$ 1km grid squares would give us 25 squares in total.

There are a few different types of grid sampling.

- **Aligned Grid** — we take a sample from the *same* (randomly selected) coordinates within each square.
- **Centrally Aligned Grid** — we take a sample from the *central* coordinates of each square.
- **Unaligned Grid** — each grid square has a sample taken from *different* randomly selected coordinates.
- **Triangular Grid** — this is a modified version of the aligned grid where the points are fixed based on a triangular arrangement.



The aligned and centrally aligned grids are convenient but may miss systematic patterns in the data. The unaligned grid avoids this, and combines the advantages of simple random sampling and stratified sampling. However, it can be inefficient for collection. The triangular grid can perform well in specific cases where the spatial correlation structures varies with direction.

Exercise 1: Heights of trees

Aim: Estimate the average height of trees which are uniformly distributed within a 10 km^2 forest.

- What is the population here?

Solution
The **population** is all trees in the forest.

- What are the sampling units?

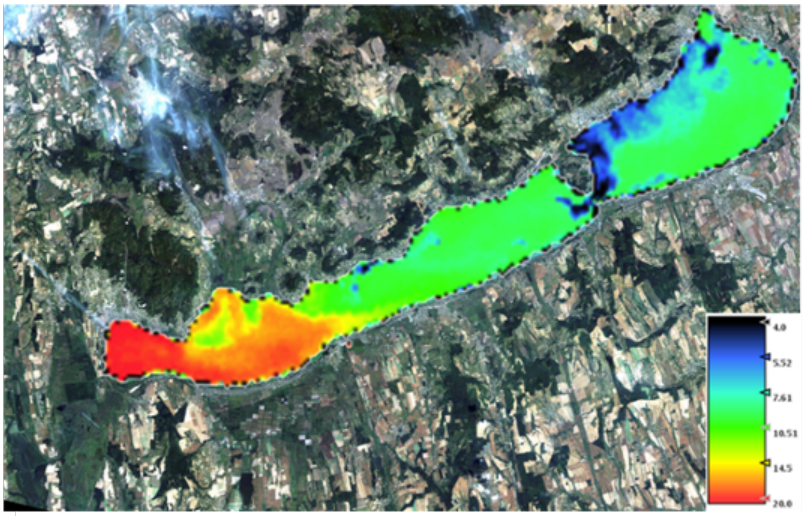
Solution
Our **sampling units** are individual trees.

- What type of sampling scheme would be appropriate here?

Solution
Simple random sampling would likely be appropriate here, since it allows us to cover the large area of the forest at low "cost" (time and financial). The trees are uniformly distributed across the forest, so no need for strata. You could argue for systematic sampling using a grid or quadrats, but this may make analysis more complex.

Exercise 2: Chlorophyll-a in Lake Balaton

Aim: Estimate the average level of chlorophyll-a on Lake Balaton, Hungary. Levels are heavily affected by differences in the levels of nutrients along the length of the lake (known as a 'trophic gradient').



MERIS Chlorophyll a – Lake Balaton, Hungary

- What is the population here?

Solution
The **population** is all possible water samples from the lake.

- What are the sampling units?

Solution
Our **sampling units** are individual water samples.

- What type of sampling scheme would be appropriate here?

Solution
Stratified random sampling seems appropriate here due to *heterogeneity* in the lake. We could design transects which cover most regions of the lake. However, may be difficulty accessing all areas by boat. Also, there are potential issues with the boat itself disrupting the levels.

4 Sample Size

A crucial part of sampling is identifying the appropriate sample size for our study. If the sample is *too small*, it will not be sufficiently representative of the population. If the sample is *too big*, it will be expensive and time consuming to collect, which may defeat the purpose of using sampling in the first place.

It is therefore important that we understand exactly what it is that we want from our sampling process. We can think about it in terms of two key aspects, **power** and **precision**.

- Precision — how accurately do I want (or need) to estimate the mean, median, variance etc?
- Power — how small a difference is it important to detect, and with what degree of certainty?

We can use the formula for the confidence interval to help compute sample sizes.

The general form of a 95% confidence interval for the population mean, μ, is

$$\bar{x} \pm t_{1-\alpha/2}\sqrt{Var(\bar{x})}$$

The width of the interval is determined by the estimated standard error, $\sqrt{Var(\bar{x})}$, and we know the formula for this contains n. Therefore, if we know how wide we need our interval to be (i.e. the precision), we can calculate the n required to do that.

Let our maximum required standard error be denoted as U. Then we know:

- $\sqrt{Var(\bar{x})} \leq U$
- $\frac{\sqrt{s^2}}{\sqrt{n}} \leq U$
- $\sqrt{n} \geq \frac{\sqrt{s^2}}{U}$

Here, s^2 is the **sample** variance. This calculation requires us to use s^2, the variance of our sample. But we don't have the sample yet — the whole point of this exercise is to determine the size of our sample. So what can we use instead? Typically we will use knowledge from prior studies where available, or will commission a small pilot study.

Exercise 3: PCB in salmon

Polychlorinated biphenyl (PCB) is a carcinogenic pollutant often found in fish. We wish to estimate the mean PCB level in the salmon in a fish farm, and require a precision level (estimated standard error) of ± 0.1mg/kg^2. We know from previous studies that the variation of PCB in salmon flesh is 3.19^2. How large a sample do we need?
Answer (as an integer): ____
Solution
We can solve our prior equation as

$$n \geq \left(\frac{s}{u}\right)^2 = \left(\frac{3.19}{0.1}\right)^2 = 1018.$$

We have estimated that we need a minimum sample size of 1018 to obtain the required precision.

In Exercise 3 above, we calculated the minimum sample size required to obtain a required precision. In some cases this may be impractical, in which case we may have to settle for a lower precision.

4.1 Sample Size in Stratified sampling

For stratified sampling, this process is much more complicated. As well as considering the sample size, we also have to think about how to allocate our samples across the strata. A common approach is to specify a **cost model**, where we take into account the different costs of sampling each stratum. The aim is to minimize the estimated standard error for a given total cost.

Let C be the overall cost, let c_0 be the fixed overhead costs of the survey and let c_l be the cost per sample in stratum l. Then our cost model is

$$C = c_0 + \sum_{l=1}^{L} c_l n_l$$

Typically C is fixed, and the goal is to select the values of n_l which allow us to obtain the best possible estimate.

Now let $\omega_l = N_l/N$ be the proportion of the overall population which is found within stratum l. Also let σ_l be the standard deviation for the population of stratum l.

We can then compute the optimum number of samples in each stratum as

$$n_l = n \frac{\omega_l \sigma_l / \sqrt{c_l}}{\sum_{k=1}^{L} \omega_k \sigma_k / \sqrt{c_k}}$$

If the costs are the same for all strata, the equation simplifies to what is known as the **Neyman allocation**

$$n_l = n \frac{\omega_l \sigma_l}{\sum_{k=1}^{L} \omega_k \sigma_k}$$

We often calculate n by a similar approach as that described for simple random sampling. In practice, the **population** standard deviation σ_l is often replaced by the **sample** standard deviation, s_l.

We now know how to determine the correct sample size to use to obtain estimates with the desired precision. This requires some advance knowledge, e.g. the tolerable level of error, the cost of the experiment, or the expected standard deviation. However, bear in mind that we have not considered the possibility of missing data in our sample, and losing some data will impact precision. We have also assumed all data points are independent, but this is not always true for time series or spatial data. (We will address this later in the course.)

5 Monitoring Networks

A **monitoring network** is a set of stations placed across a region of interest to gather information about one or more environmental variables. Standard sampling is adequate in many cases. However, the advantage of networks is that they can change over time. New sites can be added, different variables measured, and technology improved.

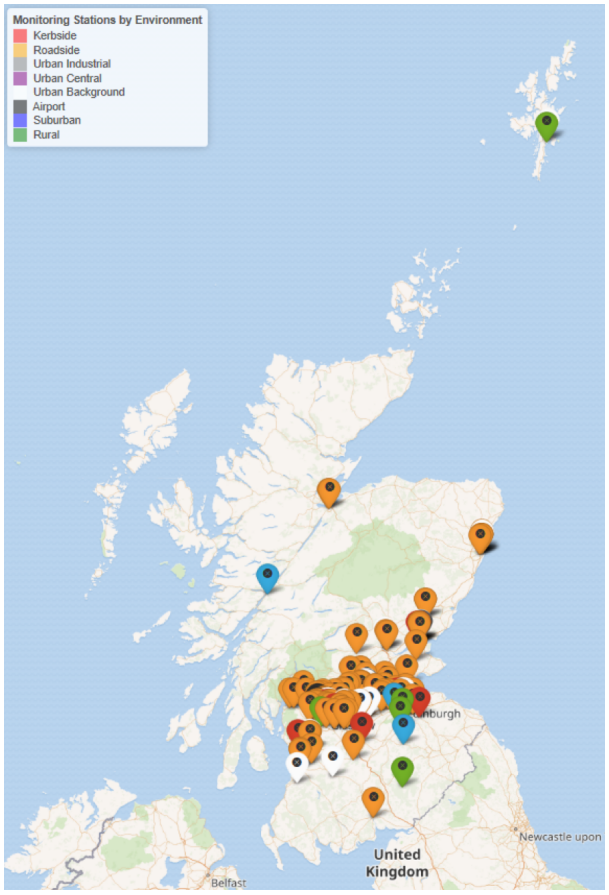


Figure 2: Scottish Air Quality monitoring sites

Example: UK Environmental Change Network (ECN)

The UK Environmental Change Network (ECN) is a long-term environmental monitoring and research programme. It started in 1992 and collects a wide range of data from a network of sites. The aim is to improve our understanding of how and why environments and ecosystems change. The sites are generally selected rather than randomly positioned.

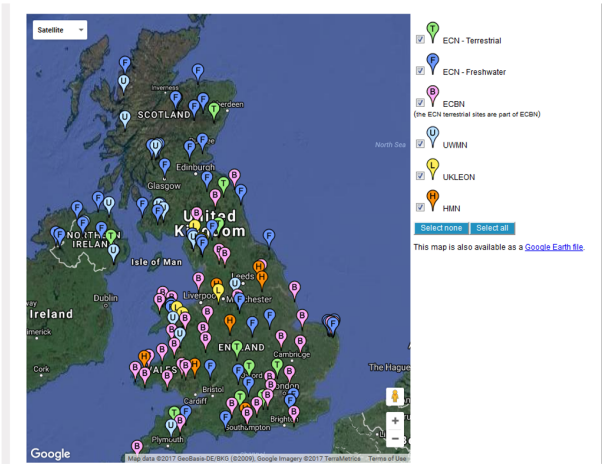


Figure 3: Map of UK environmental monitoring networks

The map above shows monitoring site locations for several long-term environmental monitoring networks in the UK, including the ECN and the Upland Waters Monitoring

Network (UWMN).

Example: UKCEH Countryside Survey

The UKCEH Countryside Survey is a 'census' of the natural resources of Great Britain's countryside. The first full survey was in 1978, and it was taken again at 6-10 year intervals until 2019. Since 2019, it has been funded as a 'rolling' survey, measuring locations on 5-yearly cycles. The goal is to map changes at various different scales, as well as to understand what is driving those changes.

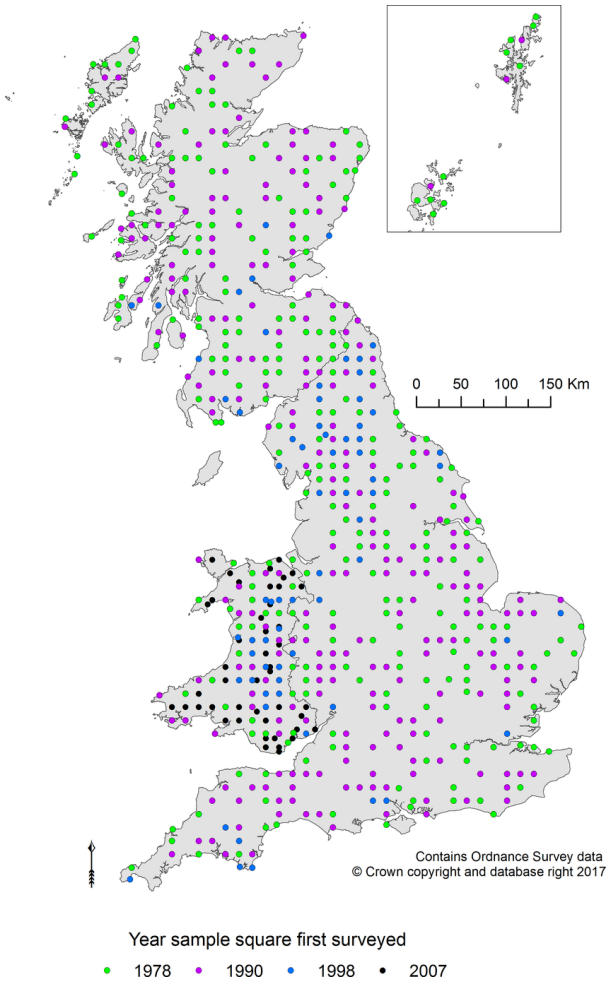


Figure 4: Map of UKCEH Countryside Survey 1km grid squares

There are several approaches to how sampling sites or resources are selected. Here, we will focus on one of popular method called Generalised Randomised Tessellation Stratified (GRTS)

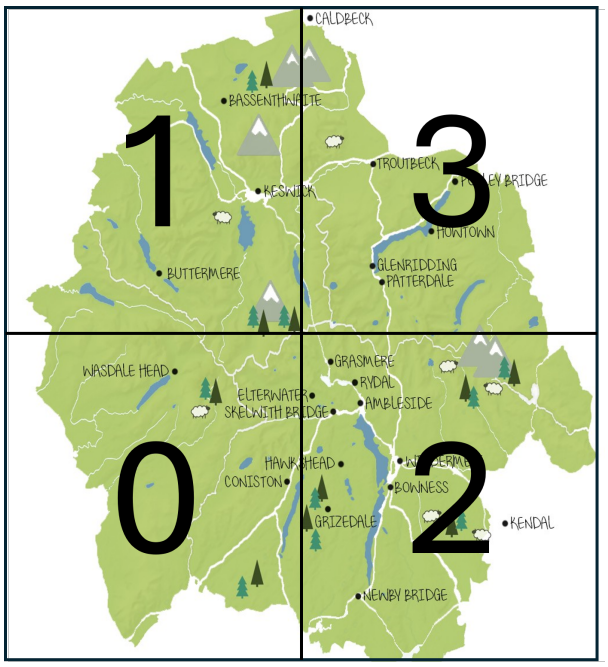
5.1 Generalised Randomised Tessellation Stratified

The Generalised Randomised Tessellation Stratified (GRTS) is a form of spatially balanced probability sampling scheme introduced by Stevens and Olsen (2004). It has been widely used for monitoring extensive environmental resources by USEPA and Marine Scotland. It builds on random, systematic and stratified sampling procedures while ensuring spatial balance (where every replicate of the sample exhibits a spatial density pattern that resembles the spatial density pattern of the resource of interest). The GRTS algorithm can be

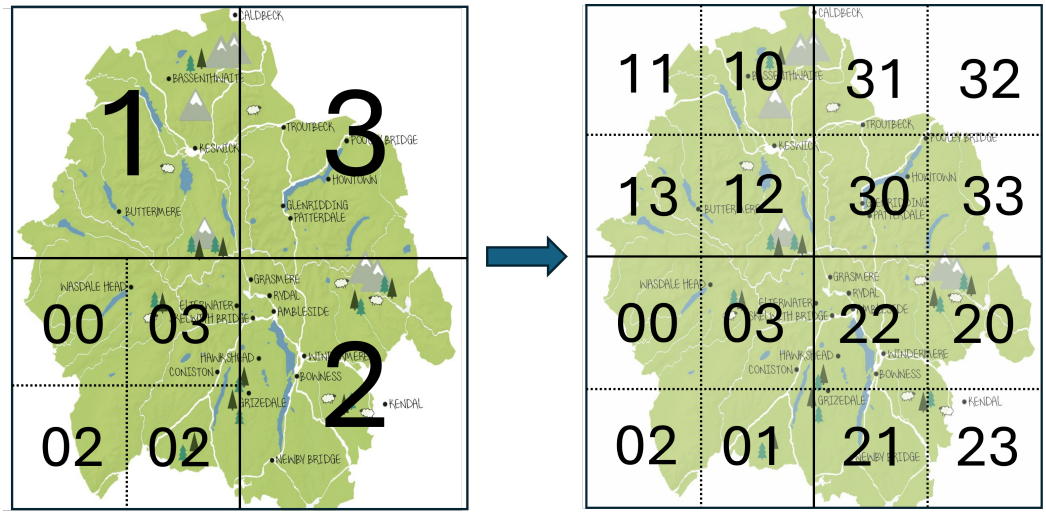
implemented as follows:

1. First one must determine the inclusion probability of each site/feature (e.g., trees, river breaches, lakes, etc.). For example, there are $N = 16$ main lakes in the lake district and a sample of $n = 4$ is desired. Assuming equal sampling probabilities the inclusion probabilities are $n/N = 4/16 = 0.25$ for each lake.

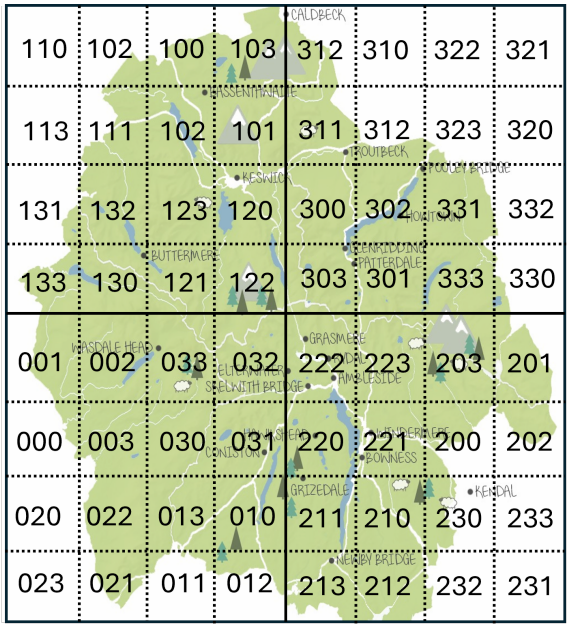
2. A square bounding box is superimposed onto the sampling frame and is divided into four equally sized square cells (level 1 cells). Each cell is then randomly labelled/numbered, e.g.,$\mathcal{A}_1 \equiv \{a_1 : a_1 - 0, 1, 2, 3\}$.



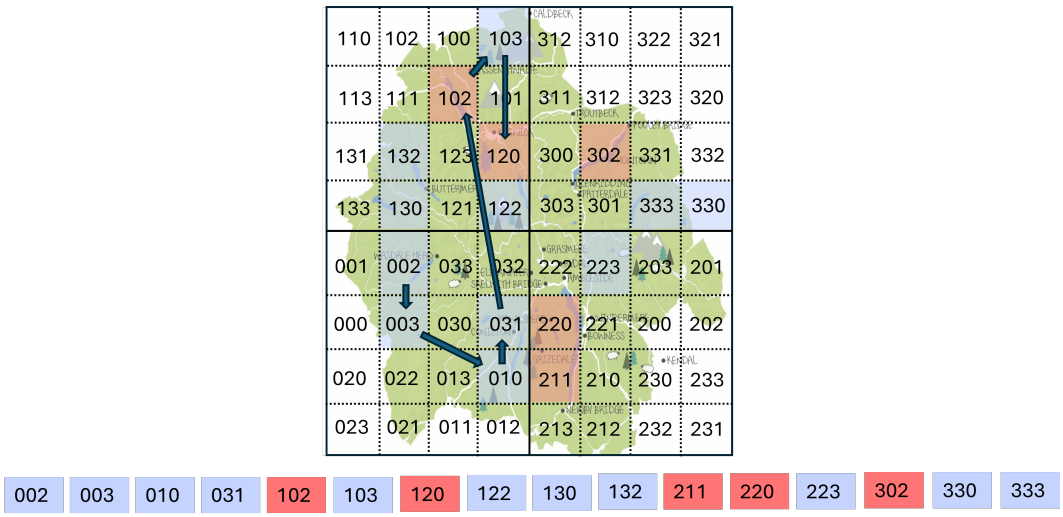
3. Each square is split into four more tessellations which are again randomly numbered while retaining the first-level label. The set of level-two cells is denoted by $\mathcal{A}_2 \equiv \{a_1 a_2 : a_1 = 0, 1, 2, 3;\ a_2 = 0, 1, 2, 3\}$.



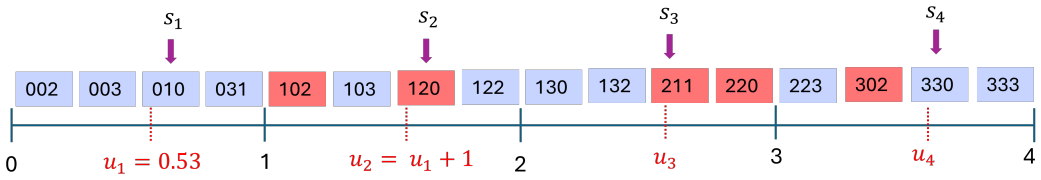
4. Continue this hierarchical randomisation to the desired spatial scale such that $\mathcal{A}_k \equiv \{a_1, \ldots, a_k : a_1 = 0, 1, 2, 3; \ldots; a_k = 0, 1, 2, 3\}$ until the sum of the inclusion probabilities of each element within a given square are less than one (i.e., (Number of samples needed from this cell) / (Number of sites in this cell) < 1).



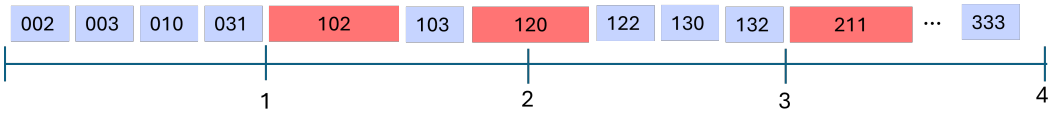
5. The elements (e.g., lakes) in $\mathcal{A}_k$ are placed in hierachichal order by sorting out $\mathcal{A}_k$ by the level 1 cells from smallest to largest, then by the level-two cells from smallest to largest, and so on. This will transform the level k grid cell to a **one-dimensional number** line. The length of each line-segment represents the inclusion probability for a given site (or lake in this example). Thus, the line's total length equals the sum of these inclusion probabilities (the sum should equal n since we do not allow the inclusion value within a cell to be greater than one). Here we have colored cells contianing larger lakes in red and small lakes in blue.



6. Then, we can use **systematic** sampling along the line to select the lakes to survey. E.g., you can randomly draw u_1 from an uniform [0,1] and place it on a line. Imagine we draw $u_1 = 0.53$, then location of u_1 on the line falls within some line segment that represents a site, which we denote s_1 (in the example this is the lake in cell 010). We include this sites as our first site in the sample and then continue with the next j sites by setting $u_j = u_{j-1} + 1$ for $j = 2, \dots, n$.



7. Suppose we would like larger lakes to be twice as likely to be selected as small lakes. Thus, instead of given all lakes the same unit length we can give large lakes twice the unit length of small lakes.



In addition to unequal inclusion probabilities we can also perform stratified sampling. Thus, instead of sampling from the entire sampling frame simultaneously, we divide a sampling frame into distinct sets of sites and select samples from each stratum independently of other strata -we apply the GRTS algorithm to obtain stratum-specific sample sizes. The R-package `spsurvey` implements GRTS algorithm to select spatially balanced samples via the `grts()` function.

5.2 Before-after-Impact approaches

The purpose of monitoring is to assess the changes in a particular variable over time. This can typically be carried out using standard statistical techniques, taking into account the structure of the data. Sometimes we are interested in whether a specific event has had an

impact on the variable, e.g., the effect of new regulations on the air pollution level. Typically this involves assessing the levels before and after the event.

It is generally very difficult to untangle the effects of a single event. Even if we identify a change in the mean or variance, how do we know that it is due to our event? Many environmental systems change naturally over time for any number of reasons. We do not have a statistical control, meaning that we can't turn back the clock and check what would have happened without the event.

However, this challenge is not unique to environmental data. We face it regularly in statistics. Often, we are only interested in the effect of one particular variable, but we have to account for other nuisance variables via regression or other techniques. We can also sometimes account for other unmeasured variability through random effects. The key is to acknowledge what you do and don't know, and to account properly for uncertainty.

Examples: Before-After Designs

1. **Before-After Single Site**

Often, we wish to assess the impact of an intervention on a site, but we only have data for that site.

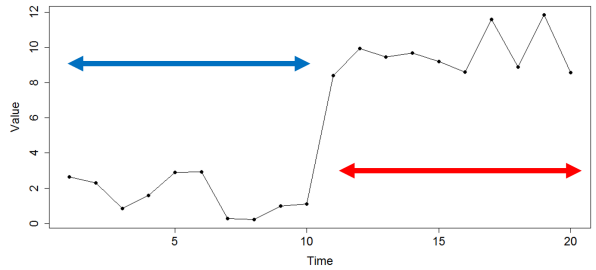


Figure 5: Line graph of value by time, illustrating the impact of an intervention. The blue arrows represent the time "before" the intervation and the red arrows represent the time "after" the intervantion.

We can fit a statistical model, such as:

$$X_{ik} = \mu + \alpha_i + \tau_{k(i)} + \varepsilon_{ik}$$

with μ representing the overall mean, α_i the effect of period (before/after), $\tau_{k(i)}$ the time within period, and ε_{ik} the errors.
This is not an ideal design, since we have no control to compare to.

2. **Before-After Multiple Sites**

We can sometimes improve upon the Before-After Single Site design, if we can collect data at multiple $j = 1, \dots, M$ sites that are likely to be impacted by the intervention. We can fit a statistical model, such as:

$$X_{ijk} = \mu + \alpha_i + \tau_{jk(i)} + \delta_j + \varepsilon_{ijk},$$

where μ is the overall mean; α_i is the effect of period (before/after); $\tau_{jk(i)}$ represents the time within the *i*th period; δ_j is the site j random effect and ε_{ijk} are the sampling errors.

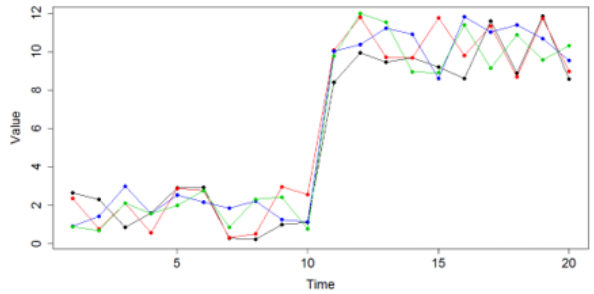


Figure 6: Line graph of value by time, illustrating the impact of an intervention. Each line connects the data points for a single site.

Treating the sites as *subsamples* allows the sites to be used to improve estimation of the effect's magnitude, compared to a single site design.

3. **Before-After Control-Impact (BACI)**

BACI is a common design, where one or more potentially impacted sites, and one or more sites that are thought not vulnerable to impact, are sampled before and after the time of the intervention.

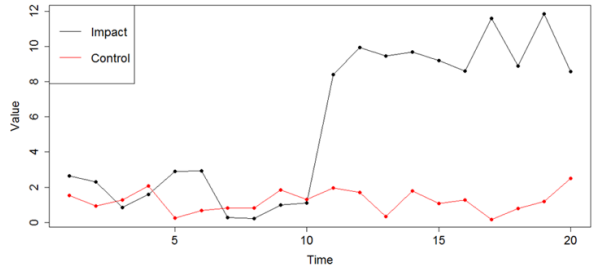


Figure 7: Line graph of value by time, illustrating the impact of an intervention on the "impact" site. The two lines connect the data points for two sites (one impact site and one control site).

We can fit a statistical model, such as:

$$X_{ij} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ij}$$

with μ representing the overall mean, α_i the effect of period (before/after), β_j the effect of location (control/impact), and ε_{ik} the errors.
Having the control sites allows us to better assess whether any impacts are *caused by* the intervention.

Stevens, Don L, and Anthony R Olsen. 2004. "Spatially Balanced Sampling of Natural Resources." *Journal of the American Statistical Association* 99 (465): 262–78. https://doi.org/10.1198/016214504000000250.