Euclidean

Auxilary Theorem.

$a = b.q + r \qquad \underbrace{gcd(a, b)}_{d} \stackrel{?}{=} gcd(b, r)$

$d \stackrel{?}{=} d'$

d still divides $b.(-q)$

(i) $d|a \wedge d|b \Rightarrow d|a \wedge d|b.(-q)$

$\Rightarrow d|(a - b.q) \Rightarrow d|r$

when we sum them d still divides the sum of them.

(ii) $d'|b \wedge d'|r \Rightarrow d'|b.q \wedge d'|r$

$\Rightarrow d'|(bq + r) \Rightarrow d'|a$

common divisor

(*) $\overbrace{gcd(a,b)}^{d} = \underbrace{cd(b, r)} \leq \overbrace{gcd(b, r)}^{d'} \Rightarrow d = d \leq d'$

(**) $\overbrace{gcd(b, r)}^{d'} = \overbrace{cd(a, b)}^{d'} \leq \overbrace{gcd(a, b)}^{d} \Rightarrow d' = d' \leq d$

Therefore, $\boxed{d = d'}$