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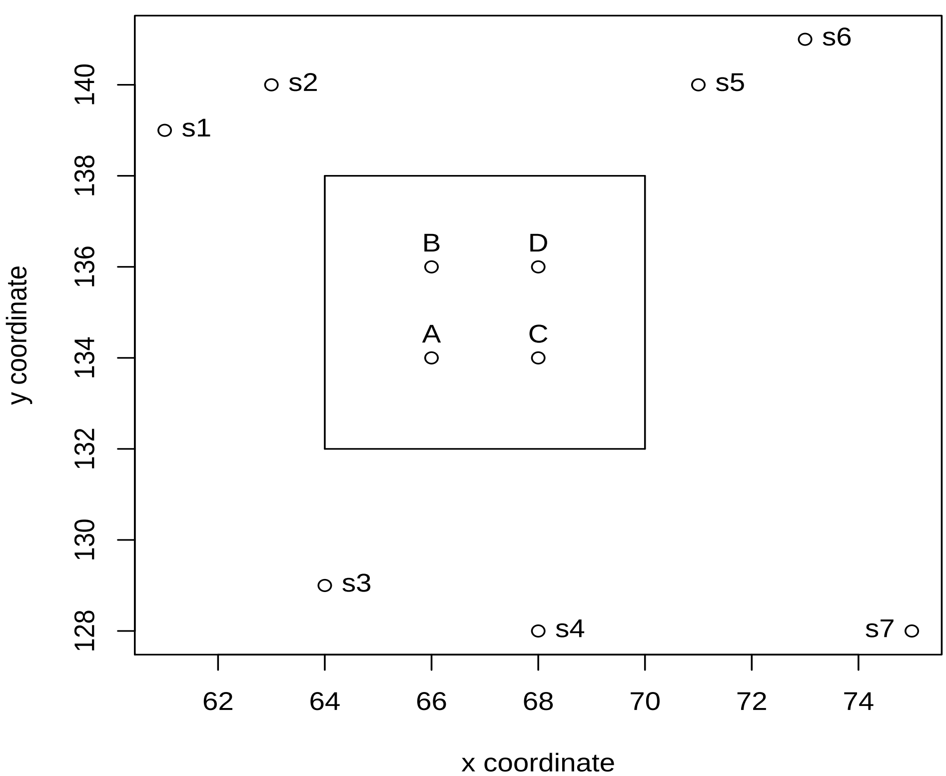
Statistics C173/C273 **Instructor: Nicolas Christou**

Block kriging exampe

We will use the 7-point data from earlier lectures. Here they are:

s_i	x	y	$z(s_i)$
s_1	61	139	477
s_2	63	140	696
s_3	64	129	227
s_4	68	128	646
s_5	71	140	606
s_6	73	141	791
s_7	75	128	783

Here is the $x-y$ plot:



For these data, let's assume that we use the exponential semivariogram model with parameters $c_0 = 0, c_1 = 10, \alpha = 3.33$, $\gamma(h) = 10(1 - e^{-\frac{h}{3.33}})$. The corresponding covariance function is:

$$C(h) = \begin{cases} 10 & h = 0 \\ 10e^{-\frac{h}{3.33}} & h > 0 \end{cases}$$

Suppose we want to estimate the average of the block defined by the coordinates shown on the figure of page 1: $(64, 132), (64, 138), (70, 132), (70, 138)$ (see figure on page 1).

One way to do this is to "krige" many points inside the block and at the end average these estimates. Let's use only 4 points inside the block defined by the following coordinates:
$A(66, 134), B(66, 136), C(68, 134), D(68, 136)$.

Predicting point A:

$$\boldsymbol{W} = \begin{pmatrix} 10 & 5.103 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 & 1 \\ 5.103 & 10 & 0.362 & 0.202 & 0.905 & 0.489 & 0.061 & 1 \\ 0.435 & 0.362 & 10 & 2.902 & 0.199 & 0.111 & 0.362 & 1 \\ 0.199 & 0.202 & 2.902 & 10 & 0.244 & 0.152 & 1.222 & 1 \\ 0.489 & 0.905 & 0.199 & 0.244 & 10 & 5.103 & 0.224 & 1 \\ 0.259 & 0.489 & 0.111 & 0.152 & 5.103 & 10 & 0.193 & 1 \\ 0.048 & 0.061 & 0.362 & 1.222 & 0.224 & 0.193 & 10 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1.196 \\ 1.334 \\ 1.985 \\ 1.497 \\ 0.958 \\ 0.512 \\ 0.388 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.130 \\ 0.146 \\ 0.240 \\ 0.164 \\ 0.136 \\ 0.072 \\ 0.112 \\ -1.077 \end{pmatrix}.$$

Therefore,
$\hat{z}(s_A) = 0.130(477) + 0.146(696) + 0.240(227) + 0.164(646) + 0.136(606) + 0.072(791) + 0.112(783) \Rightarrow$
$\hat{z}(s_A) = 551.036$.

Predicting point B:

$$\boldsymbol{W} = \begin{pmatrix} 10 & 5.103 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 & 1 \\ 5.103 & 10 & 0.362 & 0.202 & 0.905 & 0.489 & 0.061 & 1 \\ 0.435 & 0.362 & 10 & 2.902 & 0.199 & 0.111 & 0.362 & 1 \\ 0.199 & 0.202 & 2.902 & 10 & 0.244 & 0.152 & 1.222 & 1 \\ 0.489 & 0.905 & 0.199 & 0.244 & 10 & 5.103 & 0.224 & 1 \\ 0.259 & 0.489 & 0.111 & 0.152 & 5.103 & 10 & 0.193 & 1 \\ 0.048 & 0.061 & 0.362 & 1.222 & 0.224 & 0.193 & 10 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1.736 \\ 2.228 \\ 1.123 \\ 0.841 \\ 1.462 \\ 0.755 \\ 0.269 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.143 \\ 0.227 \\ 0.161 \\ 0.117 \\ 0.181 \\ 0.068 \\ 0.104 \\ -1.050 \end{pmatrix}.$$

Therefore,
$\hat{z}(s_B) = 0.143(477) + 0.227(696) + 0.161(227) + 0.117(646) + 0.181(606) + 0.068(791) + 0.104(783) \Rightarrow$
$\hat{z}(s_B) = 582.733$.

Predicting point C:

$$\boldsymbol{W}=\begin{pmatrix} 10 & 5.103 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 & 1\\ 5.103 & 10 & 0.362 & 0.202 & 0.905 & 0.489 & 0.061 & 1\\ 0.435 & 0.362 & 10 & 2.902 & 0.199 & 0.111 & 0.362 & 1\\ 0.199 & 0.202 & 2.902 & 10 & 0.244 & 0.152 & 1.222 & 1\\ 0.489 & 0.905 & 0.199 & 0.244 & 10 & 5.103 & 0.224 & 1\\ 0.259 & 0.489 & 0.111 & 0.152 & 5.103 & 10 & 0.193 & 1\\ 0.048 & 0.061 & 0.362 & 1.222 & 0.224 & 0.193 & 10 & 1\\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1}\begin{pmatrix} 0.755\\ 0.958\\ 1.462\\ 1.650\\ 1.334\\ 0.755\\ 0.627\\ 1 \end{pmatrix}=\begin{pmatrix} 0.100\\ 0.125\\ 0.183\\ 0.197\\ 0.176\\ 0.081\\ 0.137\\ -1.120 \end{pmatrix}.$$

Therefore,
$\hat{z}(s_C)=0.100(477)+0.125(696)+0.183(227)+0.197(646)+0.176(606)+0.081(791)+0.137(783)\Rightarrow$
$\hat{z}(s_C)=582.293.$

Predicting point D:

$$\boldsymbol{W}=\begin{pmatrix} 10 & 5.103 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 & 1\\ 5.103 & 10 & 0.362 & 0.202 & 0.905 & 0.489 & 0.061 & 1\\ 0.435 & 0.362 & 10 & 2.902 & 0.199 & 0.111 & 0.362 & 1\\ 0.199 & 0.202 & 2.902 & 10 & 0.244 & 0.152 & 1.222 & 1\\ 0.489 & 0.905 & 0.199 & 0.244 & 10 & 5.103 & 0.224 & 1\\ 0.259 & 0.489 & 0.111 & 0.152 & 5.103 & 10 & 0.193 & 1\\ 0.048 & 0.061 & 0.362 & 1.222 & 0.224 & 0.193 & 10 & 1\\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1}\begin{pmatrix} 1.016\\ 1.462\\ 0.888\\ 0.905\\ 2.228\\ 1.196\\ 0.411\\ 1 \end{pmatrix}=\begin{pmatrix} 0.101\\ 0.168\\ 0.139\\ 0.132\\ 0.263\\ 0.078\\ 0.120\\ -1.090 \end{pmatrix}.$$

Therefore,
$\hat{z}(s_D)=0.101(477)+0.168(696)+0.139(227)+0.132(646)+0.263(606)+0.078(791)+0.120(783)\Rightarrow$
$\hat{z}(s_D)=596.347.$

Note: All the covariances above are shown on the next page.

We can now average these 4 estimates to get an estimate of the mean of the block:

$$\hat{Z}_{BLOCK}=\frac{551.036+582.733+582.293+596.347}{4}=578.102.$$

Comment: As we observe we need to run kriging 4 times. And if there are many points within the block we will have to run kriging many times. This may be computationally expensive. The other way to do this is to use block kriging. With block kriging you need only to run kriging once. Here are the details: The covariance matrix $\boldsymbol{\Sigma}$ which constructed using the observed data points will be the same as shown above. What changes, are the entries of the vector $\boldsymbol{c}$. Each entry of this vector is the average of the covariances between each observed data point with every point in the block.

Here is the distance matrix. This is an 11×11 matrix (4 points to be estimated plus the 7 observed points).

Distance =

	A	B	C	D	s_1	s_2	s_3	s_4	s_5	s_6	s_7
A	0.000	2.000	2.000	2.828	7.071	6.708	5.385	6.325	7.810	9.899	10.817
B	2.000	0.000	2.828	2.000	5.831	5.000	7.280	8.246	6.403	8.602	12.042
C	2.000	2.828	0.000	2.000	8.602	7.810	6.403	6.000	6.708	8.602	9.220
D	2.828	2.000	2.000	0.000	7.616	6.403	8.062	8.000	5.000	7.071	10.630
s_1	7.071	5.831	8.602	7.616	0.000	2.236	10.440	13.038	710.050	12.166	17.804
s_2	6.708	5.000	7.810	6.403	2.236	0.000	11.045	13.000	8.000	10.050	16.971
s_3	5.385	7.280	6.403	8.062	10.440	11.045	0.000	4.123	13.038	15.000	11.045
s_4	6.325	8.246	6.000	8.000	13.038	13.000	4.123	0.000	12.369	13.928	7.000
s_5	7.810	6.403	6.708	5.000	10.050	8.000	13.038	12.369	0.000	2.236	12.649
s_6	9.899	8.602	8.602	7.071	12.166	10.050	15.000	13.928	2.236	0.000	653.547
s_7	10.817	12.042	9.220	10.630	17.804	16.971	11.045	7.000	12.649	13.153	0.000

And here are the covariances between all the points:

Covariance =

	A	B	C	D	s_1	s_2	s_3	s_4	s_5	s_6	s_7
A	10.000	5.485	5.485	4.277	1.196	1.334	1.985	1.497	0.958	0.512	0.388
B	5.485	10.000	4.277	5.485	1.736	2.228	1.123	0.841	1.462	0.755	0.269
C	5.485	4.277	10.000	5.485	0.755	0.958	1.462	1.650	1.334	0.755	0.627
D	4.277	5.485	5.485	10.000	1.016	1.462	0.888	0.905	2.228	1.196	0.411
s_1	1.196	1.736	0.755	1.016	10.000	5.109	0.435	0.199	0.489	0.259	0.048
s_2	1.334	2.228	0.958	1.462	5.109	10.000	0.363	0.202	0.905	0.489	0.061
s_3	1.985	1.123	1.462	0.888	0.435	0.363	10.000	2.899	0.199	0.111	0.363
s_4	1.497	0.841	1.650	0.905	0.199	0.202	2.899	10.000	0.244	0.153	1.222
s_5	0.958	1.462	1.334	2.228	0.489	0.905	0.199	0.244	10.000	5.109	0.224
s_6	0.512	0.755	0.755	1.196	0.259	0.489	0.111	0.153	5.109	10.000	0.000
s_7	0.388	0.269	0.627	0.411	0.048	0.061	0.363	1.222	0.224	0.193	10.000

The block kriging system:

$$\begin{pmatrix} c(s_1,s_1) & c(s_1,s_2) & c(s_1,s_3) & \cdots & c(s_1,s_n) & 1 \\ c(s_2,s_1) & c(s_2,s_2) & c(s_2,s_3) & \cdots & c(s_2,s_n) & 1 \\ \cdots & \cdots & \ddots & \cdots & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \cdots & 1 \\ c(s_n,s_1) & c(s_n,s_2) & c(s_n,s_3) & \cdots & c(s_n,s_n) & 1 \\ 1 & 1 & \cdots & \cdots & 1 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ \vdots \\ w_n \\ -\lambda \end{pmatrix} = \begin{pmatrix} c(s_1,A) \\ c(s_2,A) \\ \vdots \\ \vdots \\ c(s_n,A) \\ 1 \end{pmatrix}$$

$$\boldsymbol{W} = \begin{pmatrix} 10 & 5.103 & 0.435 & 0.199 & 0.489 & 0.259 & 0.048 & 1 \\ 5.103 & 10 & 0.362 & 0.202 & 0.905 & 0.489 & 0.061 & 1 \\ 0.435 & 0.362 & 10 & 2.902 & 0.199 & 0.111 & 0.362 & 1 \\ 0.199 & 0.202 & 2.902 & 10 & 0.244 & 0.152 & 1.222 & 1 \\ 0.489 & 0.905 & 0.199 & 0.244 & 10 & 5.103 & 0.224 & 1 \\ 0.259 & 0.489 & 0.111 & 0.152 & 5.103 & 10 & 0.193 & 1 \\ 0.048 & 0.061 & 0.362 & 1.222 & 0.224 & 0.193 & 10 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1.176 \\ 1.495 \\ 1.365 \\ 1.223 \\ 1.495 \\ 0.805 \\ 0.424 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.118 \\ 0.166 \\ 0.181 \\ 0.152 \\ 0.189 \\ 0.075 \\ 0.118 \\ -1.084 \end{pmatrix}.$$

Therefore,
$\hat{z}_{BLOCK} = 0.118(477)+0.166(696)+0.181(227)+0.152(646)+0.189(606)+0.075(791)+0.118(783) \Rightarrow$
$\hat{z}_{BLOCK} = 578.102$.

Note: Each element of the vector $\boldsymbol{c}$ is the average of the covariances between each observed point with each point in the block. For example the first entry 1.176 was computed as follows (from the covariance matrix on page 4):

$$\frac{1.196 + 1.736 + 0.755 + 1.016}{4} = 1.176$$

We can also observe that the weights of the block kriging system are the averages of the weights of the four ordinary kriging systems from pages 2-3.