Faculty of Mechanical and Aerospace Engineering
Institut Teknologi Bandung

WF2202 Partial Differential Equation and Numerical Method

Take Home Exam

Given out: WED 4 JUN 2025, Due Date WED 18 JUN 2025 (17.00 WIB)

Please write this etical statement on the top of your answer sheet and sign:

"I hereby declare that all answers in the exam are from my independent work. I did not commit or facilitate any improper conduct during exam. If i am proven to be in violation, I am ready to accept the consequences in accordance with the applicable regulations"

sign

(Your Name)

The objective of the learning on the numerical method is to educate the students to be able to solve many mathematical problems computationally such as system of linear equations, partial differential equations, finite difference, and integral. In this take home exam, the students are expected to provide **theoretical basis, flow chart, and programming source code** for the following problems.

1. Create a flowchart and programming code to solve the system simultaneous linear algebraic equations, which is defined as the following:

$$
\begin{array}{l}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1j}x_j + \ldots + a_{1(n-1)}x_{(n-1)} + a_{1n}x_n = b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2j}x_j + \ldots + a_{2(n-1)}x_{(n-1)} + a_{2n}x_n = b_2 \\
\quad\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ij}x_j + \ldots + a_{i(n-1)}x_{(n-1)} + a_{in}x_n = b_i \\
\quad\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
a_{(n-1)1}x_1 + a_{(n-1)2}x_2 + \ldots + a_{(n-1)j}x_j + \ldots + a_{(n-1)(n-1)}x_{(n-1)} + a_{(n-1)n}x_n = b_{(n-1)} \\
a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nj}x_j + \ldots + a_{n(n-1)}x_{(n-1)} + a_{nn}x_n = b_n
\end{array}
\tag{E.1.1}
$$

where the a's are constant coefficients and the b's are constants, and x are unknown variables. Create a matlab programming code that can be used to solve the system simultaneous linear algebraic equations. Please include in your programming solution the following:

a) Create the solve the system simultaneous linear algebraic equations that will have inputs from the keyboards. Inputs are :

   a.1. the number of equations/number of unknown variables =n

   a.2. the a's constant coefficients and

   a.3. the b's constants.

b) There are two choice for the method:

   Choice A = Using Gauss Elimination

   Choice B = Using Gauss Seidel

c) Print the system simultaneous linear algebraic equations, and validate the code for the solution given below

   Number of equations = 4

   10 X1 - 2 X2 - X3 - X4 = 3

   -2X1 + 10 X2 - X3 - X4 = 15

   - X1 - X2 + 10 X3 - 2 X4 = 27

   - X1 - X2 - 2 X3 + 10X4 = -9

   The solutions are printed in the format as the following

   The answers after 7 iterations are the following (4 digits of accuracy):

   X1= 0.9999

   X2= 1.9999

   X3= 2.9999

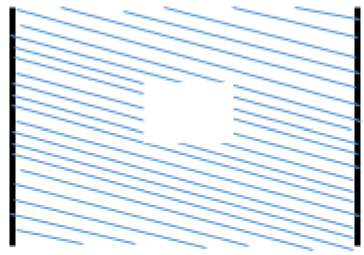
   X4= -0.0002

   (for cases of n=7)

d) Use the validated code to solve the following system of equations using both Gauss Elimination and Gauss-Seidel Methods.

$$\text{Consider } \mathbf{Ax} = \mathbf{b} \text{ where } \mathbf{A} = \begin{bmatrix} 6 & 1 & 1 & 1 & 1 \\ 1 & 7 & 1 & 1 & 1 \\ 1 & 1 & 8 & 1 & 1 \\ 1 & 1 & 1 & 9 & 1 \\ 1 & 1 & 1 & 1 & 10 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -10 \\ -6 \\ 0 \\ 8 \\ 18 \end{bmatrix}.$$

2. A wall 1 ft. thick and infinite in other directions, as shown in figure below, has an initial uniform temperature (T_i) of 100 °F. The surface temperature (Ts) at the two sides are suddenly increased and maintained at 300 °F . The wall is composed of nickel steel (40% Ni) with diffusivity of $\alpha = 0.1$ ft^2/hr.



The unsteady one-dimensional heat conduction equation in cartesian coordinates is written as follows :

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

a. Find analytical solution of the above PDE.
b. Derive the discrete equation of the PDE using Forward Time Central Space (FTCS) scheme.
c. Generate a numerical algorithm of the discretized equation.
d. Generate a numerical coding based the developed algorithm.
e. Compute temperature distribution within the wall as a function of time using the following sets of step sizes : $\Delta x = 0.05\ ft\ with\ various\ \ \Delta t$: $(i)\ \Delta t = 0.005\ hr$; (ii) $\Delta t = 0.01\ hr$ and $(iii)\ \Delta t = 0.05\ hr$
f. Make analysis of the results
g. Compare the numerical result with the analytical result.

3. Use Gauss-Seidel Method and the code that you have developed in question no. 1 to solve for the temperature of the steady state heated plate in figure P.3.1. Do the iteration with the error to ε_s = 1%. The temperatures on all four sides are stated by the numbers on the part of your NIM. For example, NIM 13623020 will give these boundary condition. **Plot your results of temperature distribution on the plate**.

NIM =

1	3	6	2	3	0	2	0

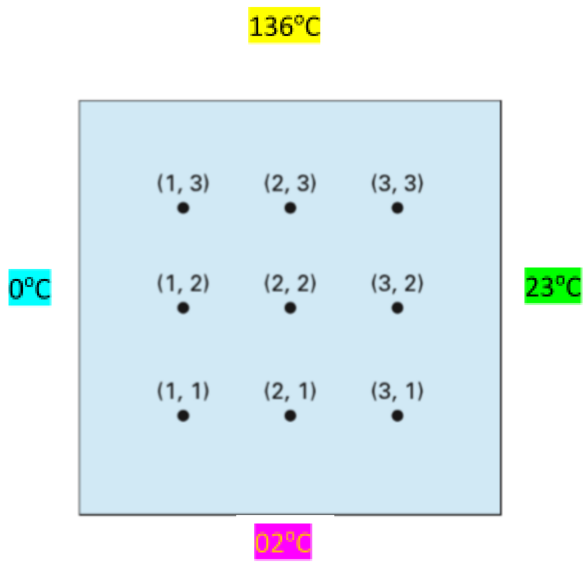


Figure P.3.1. Heated Plate

4. Deflection of aluminum beam that is clamped at one end and given the load as shown in figure P.4.1 can be calculated using the Bernoulli-Euler theory as follows:

$$\frac{M}{EI} = \frac{y''}{\left[1+(y')^2\right]^{\frac{3}{2}}} \tag{E.4.1}$$

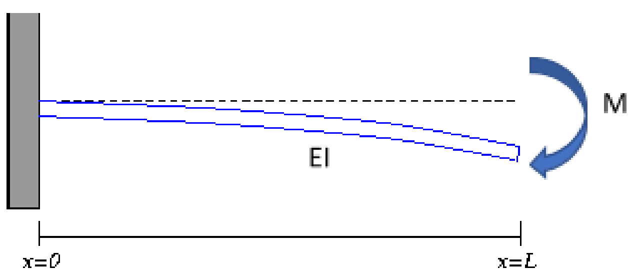


Figure P.4.1.

Equation (E.4.1) can be solved numerically by writing it into:

$$y'' = \frac{M}{EI}\left[1 + \left(y'\right)^2\right]^{\frac{3}{2}} \tag{E.4.2}$$

and then, equation (E.4.2) can be written into two equations:

$$y' = z \tag{E.4.3}$$

$$z' = \frac{M}{EI}[1 + z^2]^{\frac{3}{2}} \qquad \text{(E.4.4)}$$

with the boundary condition y (0) = z (0) = 0

Data of beams are : E= Five last numbers of your NIM kg/mm^2 , I=1500 mm^4 , L=200 mm, M= Four last numbers of your NIM mm kg. For Example: NIM 13621021, E = 21021 kg/mm^2 and M=1021 mm kg.

a) Write the formula of Runge-Kutta with c/p coefficient using Modified Euler for second order R-K method and sketch the graphic to get y_{m+1} from y_m and z_{m+1} from z_m

b) Using the boundary condition $y_0 = y(0) = 0$ dan $z_0 = z(0) = 0$, determine y_1 and z_1 if h=20 mm. (use the 2nd order R-K method).

c) Using the same boundary conditions in b), determine the y and z with the 4th order Runge-Kutta R-K Method

d) Plot your results in b) and c) in the same graph

Good Luck for the exam.