Name:

ALGORITHMICS UNIT 3 & 4

Trial Exam 1: 2022

Reading Time: 15 minutes
Writing time: 120 minutes (2 hours)

QUESTION AND ANSWER BOOK

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	8	8	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers and one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape

Materials supplied

- Question and answer book of ?? pages
- Answer sheet for multiple-choice questions

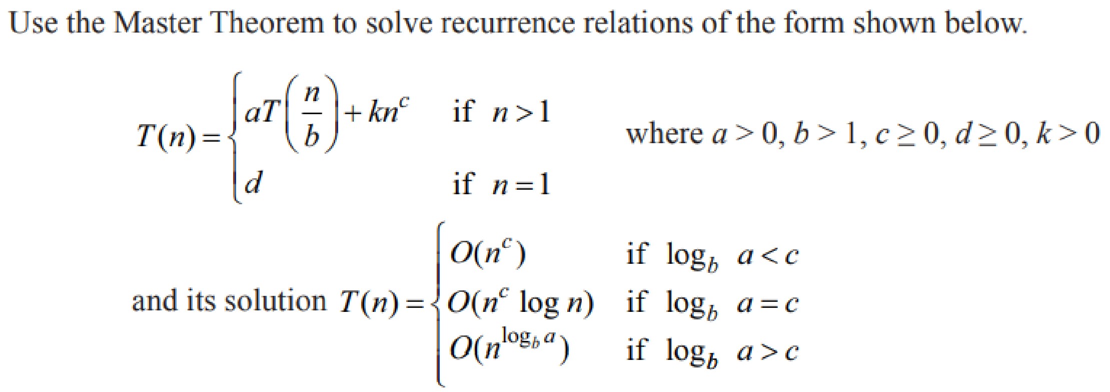
Instructions

- Write your student number in the space provided above on this page.
- Check that your name and student number as printed on your answer sheet for multiple-choice questions are correct, and sign you name in the space provided to verify this.

- All written responses must be in English, point form is preferred.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

The VCAA Exam will include the Master Theorem in this form.



Use the Master Theorem to solve recurrence relations of the form shown below.

$$T(n)=\begin{cases} aT\left(\dfrac{n}{b}\right)+kn^{c} & \text{if } n>1 \\ d & \text{if } n=1 \end{cases} \qquad \text{where } a>0,\ b>1,\ c\geq 0,\ d\geq 0,\ k>0$$

$$\text{and its solution } T(n)=\begin{cases} O(n^{c}) & \text{if } \log_b a<c \\ O(n^{c}\log n) & \text{if } \log_b a=c \\ O(n^{\log_b a}) & \text{if } \log_b a>c \end{cases}$$

The VCAA form of Master Theorem is equivalent to the form of Master Theorem taught in our class by consideration of log laws. $\log_b a = c \iff a = b^c \iff \dfrac{a}{b^c} = 1$ $\log_b a < c \iff a < b^c \iff \dfrac{a}{b^c} < 1$ $\log_b a > c \iff a > b^c \iff \dfrac{a}{b^c} > 1$	$T(n) = aT\left(\dfrac{n}{b}\right) + f(n^k)$ • $\dfrac{a}{b^k} < 1$ then $O(n^k)$ • $\dfrac{a}{b^k} = 1$ then $O(n^k \log_b n)$ • $\dfrac{a}{b^k} > 1$ then $O(n^{\log_b a})$

SECTION A – Multiple Choice – circle one option only

Question 1

What value is returned by the following pseudocode?

```
Function f1(input  x)
      If (x<0) then
            Report -1*x
      Else
            Report x
      End if
End function
```

A. |x|

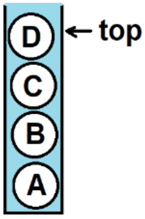
B. –x

C. X

D. x<0

Question 2

A stack X contains the following items:



What outputs are expected after the following commands are executed in sequence.

Pop(X)
IsEmpty(X)
Size(X)
Push(X,H)

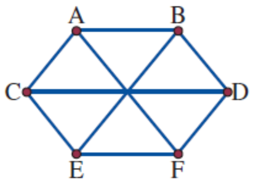
A. [A|B|C], False, 3, [A|B|C|H]

B. [A|B|C|D], False, 4, [A|B|C|D]

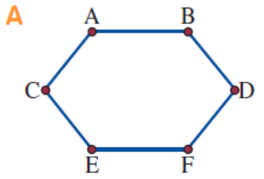
C. [A|B|C], False, 3, [A|B|C|D]

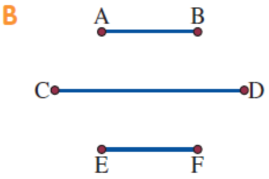
D. [A|B|C|D], False, 4, [A|B|C|D|H]

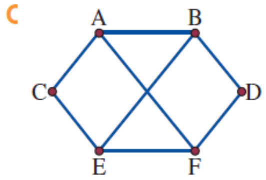
Question 3

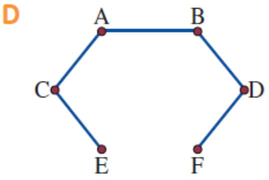


When converted to a spanning tree, the graph shown above will resemble which of the following?



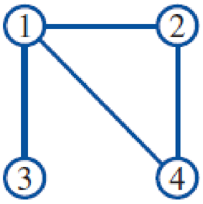






Question 4

Which formal definitions correctly define the graph G=(V,E) shown:



A. V={1,2,3,4}, E={1-2,1-4,2-3,2-4}, connected, cyclic

B. V={1,2,3,4}, E={1-2,1-3,1-4,2-4}, unconnected, cyclic

C. V={1,2,3,4}, E={1-2,1-3,1-4,2-4}, connected, cyclic

D. V={1,2,3,4}, E={1-2,1-3,1-4,2-4}, connected, acyclic

Question 5

The diameter or width of a graph G is:

A. the length of the shortest path between any two nodes

B. the largest distance between any pair of vertices. If G is disconnected, then its diameter is infinite.

C. the number of edges in a circuit, or cycle in the graph

D. a tour of a graph G which contains every edge of G.

Question 6

Problems that cannot be solved by any algorithm are called?

A. Tractable problems

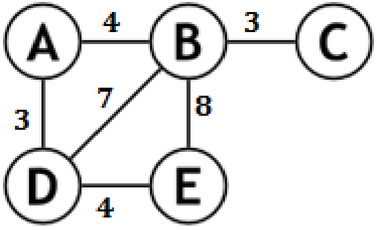
B. Intractable problems

C. Undecidable problems

D. Decidable problems

Question 7

When Prim's is executed on the following graph, starting at any vertex:



The number of edges in and the weight of the MST respectively is:

A. 4,14

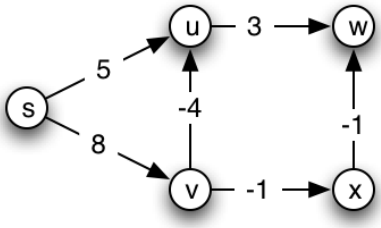
B. 4,17

C. 5,14

D. 5,17

Question 8

Running the Bellman-Ford Algorithm on the following digraph:



Gives the following distances on the vertices after the algorithm terminates:

A. S=0,U=4,V=8,W=6,X=7

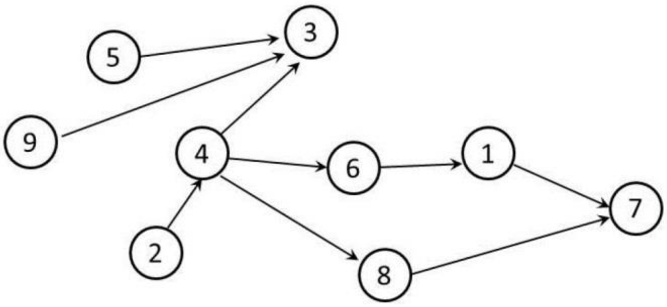
B. S=0,U=1,V=8,W=4,X=7

C. S=0,U=4,V=8,W=8,X=7

D. S=0,U=5,V=8,W=8,X=7

Question 9

Which of the following sequences are topological sorts for the directed acyclic graph below?



A. 9, 2, 5, 4, 3, 8, 1, 6, 7

B. 9, 2, 5, 4, 8, 5, 1, 7, 3

C. 9, 2, 5, 3, 4, 6, 8, 1, 7

D. 2, 9, 5, 4, 6, 3, 8, 1, 7

Question 10

Dijkstra’s algorithm calculates:

A. Topological sort.

B. A spanning tree.

C. Connected component.

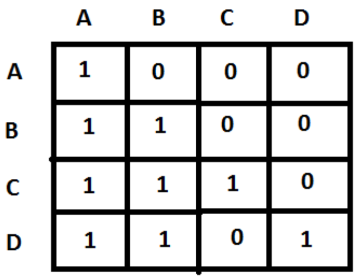
D. Single source shortest path.

Question 11

The Transitive Closure matrix below matches which graph?

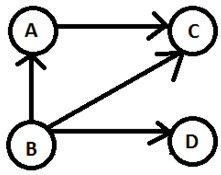
To nodes

from nodes

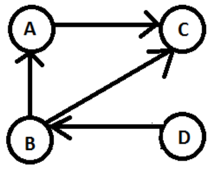


	A	B	C	D
A	1	0	0	0
B	1	1	0	0
C	1	1	1	0
D	1	1	0	1

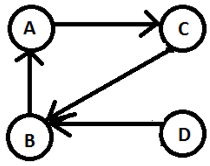
A



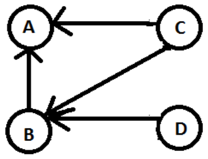
B



C



D



Question 12

The Floyd-Warshall Transitive Closure Algorithm performs the following main action:

A. Transition Matrix T(i,j) is updated if a path exists from i-j OR a (path exists from i-k AND path exists from k-j)

B. Transition Matrix T(i,j) is updated if shortest path is found via intermediate node k

C. Transition Matrix T(i,j) is updated if a path exists from i-j AND a (path exists from i-k OR path exists from k-j)

D. Transition Matrix T(i,j) is updated if a path exists from i-j OR a (path exists from i-k OR path exists from k-j)

Question 13

Which one of the following algorithms **does not** use a Greedy strategy to move toward the solution?

A. Best First Search

B. Dijkstra’s Shortest Path

C. Prim’s Minimum Spanning Tree

D. Floyd Warshall’s Transitive Closure

Question 14

In a recursive function or algorithm the **base case** is a condition that:

A. Stops the function from calling itself.

B. Initiates the recursive calls to the function

C. Causes the recursion to skip a value.

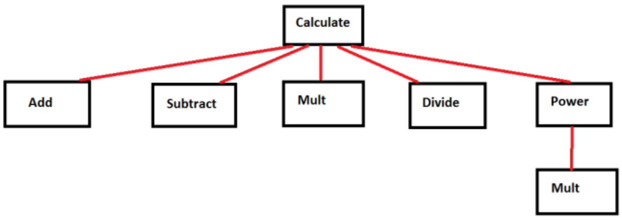
D. Initiates a conversion to an iterative algorithm.

Question 15

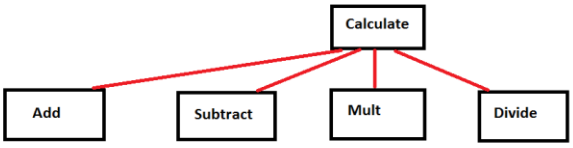
In the following Pseudocode, the call graph/tree showing the hierarchy of the calls that the algorithm Calculate makes is best represented by:

```
Function Power(arg1, arg2)
        for i=1 to arg2 do
            result:=Mult(arg1,arg1)
        end do
        report result
End function


Algorithm Calculate(arg1, arg2, operation, result)
        if operation is addition then
              result:=Add(arg1,arg2)
        else if operation is subtraction then
              result:=Subtract(arg1,arg2)
        else if operation is multiplication then
              result:=Mult(arg1,arg2)
        else if operation is division then
              result:=Divide(arg1,arg2)
        else if operation is power then
              result:=Power(arg1,arg2)
        else
              report “Error unknown operation”
        end if
End Algorithm
```

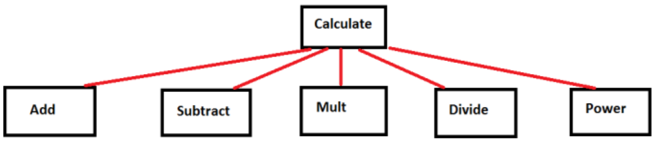
A



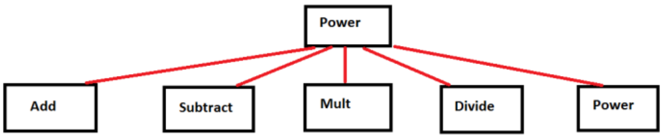
B



C



D



Question 16

The running time of an algorithm is represented by the following recurrence relation:

$$T(n) = \begin{cases} n & n \leq 3 \\ T\left(\frac{n}{3}\right) + cn & otherwise \end{cases}$$

Which one of the following represents the time complexity of the algorithm?

A. O(n)

B. O(nlogn)

C. O(n^2)

D. O(n^2logn)

Question 17

What does the following recursive function do?

```
function fun3(input: integer n)
   if (n == 0 or n == 1)then
         return n
   else if (the remainder of n/3 != 0)then
         return 0
   else
         return fun3(n/3)
   end if
end function
```

A. It returns 1 when n is a multiple of 3, otherwise returns 0

B. It returns 1 when n is a power of 3, otherwise returns 0

C. It returns 0 when n is a multiple of 3, otherwise returns 1

D. It returns 0 when n is a power of 3, otherwise returns 1

Question 18

What is the value of the variable “Answer” for the following code when called with ***OK(3, Answer)*** if Answer is initialised with a value of zero?

```
Function OK(integer n, integer result)
   if (n == 0) then
     return 0
   else
     return OK(n - 1, n + result)
  end if
end function
```

A. 0
B. 6
C. 5
D. 3

Question 19

If L(X), L(Y), L(Z) is the count of outgoing links from page X, Y,Z respectively and Pr(X), Pr(Y), Pr(Z) is the probability of landing on page X, Y, Z respectively. Then theoretically the published Google Page rank for X in a system of 3 pages X, Y, Z can be determined from which of the following equations:

A $Pr(X) = \frac{0.15}{3} + 0.85(\frac{Pr(Y)}{L(Y)} + \frac{Pr(Z)}{L(Z)})$

B $Pr(X) = \frac{Pr(X)}{L(X)} + \frac{Pr(Y)}{L(Y)} + \frac{Pr(Z)}{L(Z)}$

C $Pr(X) = \frac{0.15}{3} + 0.85(\frac{Pr(X)}{L(X)} + \frac{Pr(Y)}{L(Y)} + \frac{Pr(Z)}{L(Z)})$

D $Pr(X) = \frac{0.15}{2} + 0.85(\frac{Pr(Y)}{L(Y)} + \frac{Pr(Z)}{L(Z)})$

Question 20

What is the recurrence relation for the time complexity as a function of the input size “n” for the recursive Function XYZ shown below?

```
Function XYZ (Input NumberList)
// Input NumberList a list of numbers
// the k-th element of NumberList is NumberList[k]
// a subset of the list between item j and item k is NumberList[j..k]
      Len:=length(NumberList)   // length returns the number of items in list
      If (Len > 0) then
           Sum:=NumberList[Len] + return XYZ(NumberList[1..(Len-1)]
      End if
      Return Sum
End Function
```

A. $\boldsymbol{T(n) = T(n-1) + c}$

B. $\boldsymbol{T(n) = 2T(n) + c}$

C. $\boldsymbol{T(n) = 2(T(n/2)) + c}$

D. $\boldsymbol{T(n) = T(n-1) + n}$

SECTION B – Extended Response Questions Answer all questions in the space provided.

Question 1 (12 marks)

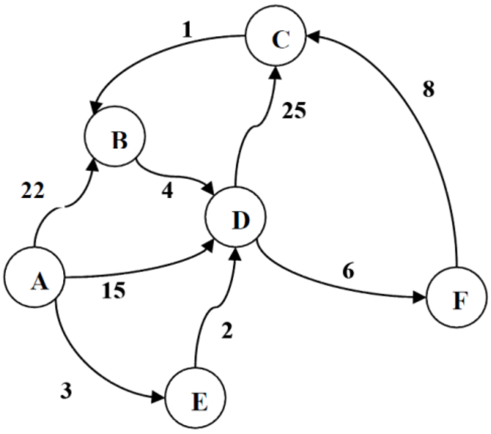
a) Concisely state the main purpose of Breadth First Search (BFS) and Depth First Search (DFS) algorithms and briefly compare their strategies. (2 marks)

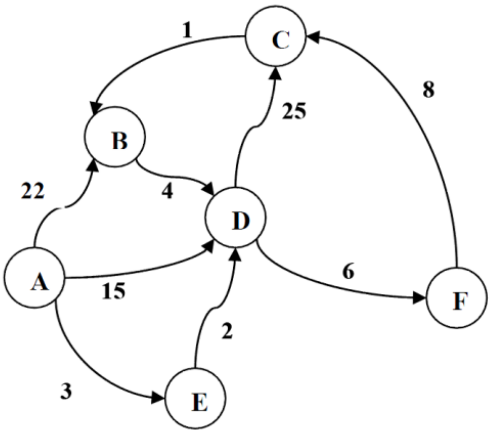
b) Which elementary abstract data structure is used in BFS? Describe how is it used to work toward the solution. (2 marks)

c) Throughout the entire execution of a Breadth First Search, how many actions as a function of the input size to the algorithm is spent adding and removing data to the abstract data structure from 1b)? Justify your response. (2 marks)

Question 1 (continued)

Use the following directed graph (printed twice) to answer the questions below:





d) Apply Breadth First Search with node **A** as the start node. List the nodes in order of visitation using ***alphabetic order where a choice exists*** until all nodes visited.

(2 marks)

e) Apply Depth First Search with node **A** as the start node. List the nodes in order of visitation using ***alphabetic order where a choice exists*** until all nodes visited.

(2 marks)

f) Explain how the Best First Search algorithm works compared to Breadth First Search and Depth First search and compare the main aims of each type of traversal. (2 marks)

Question 2 (12 marks)

Fancy Farming Fencing suppliers need to find best way to cut wire fencing of length n in order to maximise the price earned.

- Given: wire fencing of width w and length n, Assume that each length wire fencing has a **price** p
- Find best set of cuts to get maximum price
  - Each cut is integer length,
  - Can use any number of cuts, from 0 to n−1
  - No cost for a cut, fencing only sold by length

For example the table below shows the price that can be obtained for wire fencing of length 8 units.

length i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
price p	1	5	8	9	10	17	17	20

This recursive algorithm CutF below will compute the answer.

```
Function CutF(p, n)
// Input p is an indexed array holding the price of cuts
// Input n is the length of the Wire fencing
        if n == 0 then
            return 0
        end if
        q := -1000000
        for i in 1 .. n loop
        // function maximum returns the maximum value of two inputs
            q = maximum(q, p[i] + return CutF (p, n-i))
        end loop
        return q
end Function
```

a) Show the call tree that will result when the above Function **CutF** is called with a price table p and n=4. (2 marks)

b) What is the recurrence relation describing the recursive call as a function of the input size n? (1 mark)

Question 2 (continued)

c) Hence determine the worst case time complexity of the above Function CutF. (2 marks)

d) Describe how the time complexity of this problem be improved using Dynamic Programming. (2 marks)

e) Complete the following Function that uses Dynamic Programming to find the solution to the problem for the general case. (3 marks)

```
Function DPCutF(p, n)
// Input p is an indexed array holding the price of cuts
// Input n is the length of the Wire fencing
// Output r is an array showing the maximum revenue cut position for length i
// function maximum (input1, input2) returns the maximum value of the two inputs
  r[0] := 0
  for j in 1 .. n loop
       q := -1000000










  end loop
  return r[n]

end Function
```

f) What is the worst case time complexity of DPCutF? Justify your answer. (2 marks)

Question 3 (8 marks)

a) Describe the main components of a Turing machine and how they function. (2 marks)

Here's a Turing machine defined with a transition table and a transition diagram.

state 0: (**0**, **0**, right, 1)

state 1: (**1**, **1**, right, 1)
(**0**, **0**, left, 2)

state 2: (**1**, **0**, left, 3)
(**0**, **0**, right, 4)

state 3: (**1**, **0**, left, 2)

state 4: (**0**, **1**, left, 6)

state 5: [no instructions]

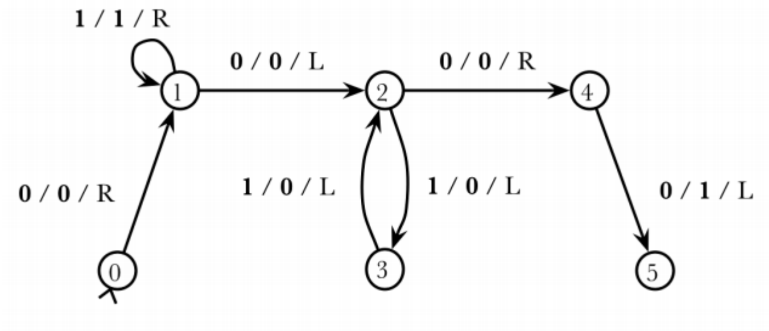


Figure 1

b) Given the initial tape shown below starting at the left most cell in the state of "0" show the final tape that will be returned by the Turing machine described above. (2 marks)

initial tape:

0	1	1	1	1	1	1	0	0	0

Question 3 (continued)

Finite state machines and Turing machines are models of computation, or hypothetical examples of machines, used as examples to state how much a certain machine/coding language can or needs to do.

A finite state machine (FSM) is a machine that has something called a state, and can receive inputs. The inputs change the state. For example: A turnstile can be modeled as a FSM.

It has two states: either it is locked or unlocked.
If it's locked and it gets a coin, it goes into the unlocked state.
If it's unlocked and it gets a coin, nothing happens, so it stays unlocked.
If it's locked and it gets pushed, nothing happens, so it stays locked.
If it's unlocked and it gets pushed, it goes into the locked state.

c) Show the information for the Turnstile FSM as a connected graph. (2 marks)

A few more cases may occur that need to be considered for the turnstile to operate correctly and need to be included in the operation.

- The coin inserted must have a certain value for the turnstile to open.
- In the case of a fault there needs to be a manual override to open the turnstile.

d) Create a new transition graph depicting the FSM to handle the existing and the extra cases. (2 marks)

Question 4 (11 marks)

a) What was Hilbert’s program and what prompted Hilbert to initiate a program to make Mathematics a system that was complete, consistent and decidable? (2 marks)

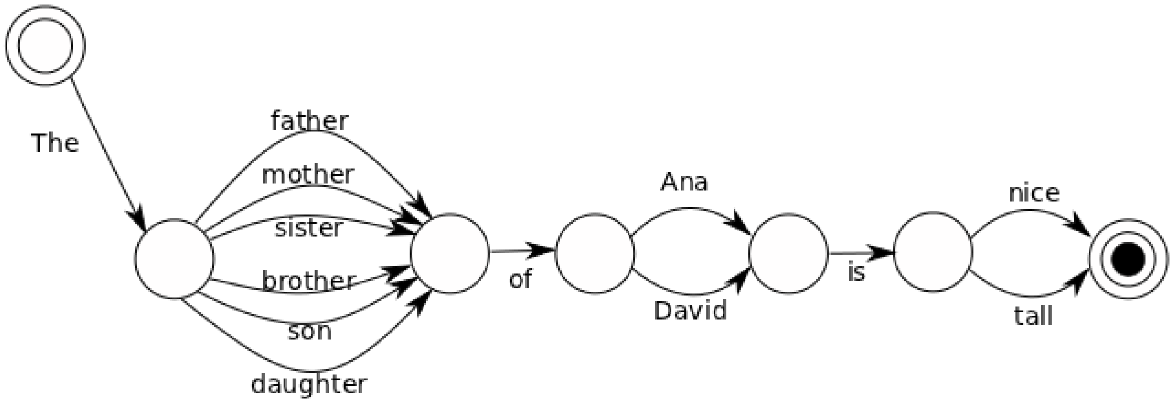
b) What are the implications of Godel’s incompleteness theorem, how does it impact Hilbert’s goal of a complete, consistent and decidable Mathematical language and system? (3 marks)

c) What are the implications of Turing’s Halting problem for the creation of algorithms to solve problems? How does it impact Hilbert’s goal of a complete, consistent and decidable Mathematical language and system? (3 marks)

d) Explain how Hilbert’s program contributed to the foundations of Computer Science and to the definition of algorithms and what is computable and decidable? (3 marks)

Question 5 (10 marks)

a) Using the following directed graph of sequential nodes defining symbols to be used in order for correct sentences, complete the Algorithm **CheckSentence** that will check any sentence made up of a list of words for correctness using a directed graph with a nominated starting and end point. (6 marks)



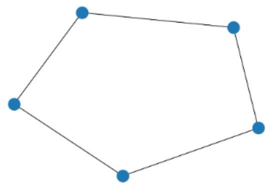
```
Algorithm CheckSentence(Words, Syntax)
// Input Words a list of words, input Syntax a directed graph showing permitted
sequence
// Function length returns the length of the input list
// Function diameter returns the diameter of the input graph
        Len=length(Words)
        Diam=diameter(Syntax)
        If (Len != Diam) then
                Report "Incorrect length sentence"
                Exit
        Else
        //  Assume each node of syntax is labelled in order 1,2,3….n
        //  Edges have permitted words as labels
                For i:=1 to Diam do




                End Loop
        End if
        Report "Sentence is ok"
End Algorithm
```

Question 5 (continued)

b) What other abstract datatypes could be used to support the words and order of use of “correct sentences”? Justify your choice and give an example of how it would be used to support the information in the connected graph above. (4 marks)

Question 6 (7 Marks)

A round graph is a graph with at least three nodes and in which a single cycle exists containing all vertices. An example of a round graph with five vertices is shown below.



Consider the algorithm RoundGraph shown below that takes one integer input $n \geq 3$.

```
1    Algorithm RoundGraph(n)
2    // Assumption n is at least 3
3        If (n=3) then
4            Return a complete graph with three nodes
5        Else
6            G = RoundGraph(n-1)  //G is a graph ADT
7            Add new node u to G
8            Randomly select edge p-q in G
9            Create edge p-u and u-q to G
10           Delete the edge p-q
11           Return G
12       End if
13   End Algorithm
```

a) Show the call tree/graph for RoundGraph(5). (1 mark)

b) What are the design pattern(s) used for the RoundGraph algorithm? Justify your response. (2 marks)

c) Demonstrate the correctness of the RoundGraph algorithm, the numbered lines of the algorithm and diagrams can be used to support the proof. (4 marks)

Question 7 (8 Marks)

The Restaurant Problem

You are the organiser for a Youth United Nations Conference, it is the first night and you've forgotten to book a restaurant. The rules for the first night dinner is that **no** two delegates from the same country can eat at the same restaurant. Due to the limited seating at some restaurants, this arrangement may not be possible and some delegates may not be able to eat at a restaurant. You would like to minimise the number of delegates that miss out so as not to cause an international incident.

Algorithm Input Variables to be used	Algorithm Output
• Integer **C** – the number of countries attending the conference • Input **CList** a list of countries attending eg. {PERU, NZ, EGYPT} • Input **DCList** is list of number of delegates attending from each corresponding country in **CList** • Integer **R** – the number of available restaurants • Input **SList** – giving the number of seats available at each restaurant	Your output must consist of a single value containing a single integer, the total number of delegates that cannot be seated at a restaurant.

Sample Input 1	Output 1	Explanation

C=2
DCList={3,3}
R=3
SList={4,3,4}
	0	The delegates are easy to seat in this case--simply place one member of each team in each restaurant. Since every restaurant has more than two seats available, everybody can be seated and nobody misses out.

Sample Input 2	Output 2	Explanation

C=3
DCList={4,3,3}
R=3
SList={5,2,3}
	2	Here the first country has more delegates than there are restaurants, so at least one of its delegates must miss out. The second restaurant only has space for delegates from two countries, and so a second delegate must miss out. It is possible however to arrange the seating so that only two delegates miss out overall, and so the final solution is two.

a) Using the data structures and inputs provided, create an algorithm in structured pseudocode that will give a solution for the Restaurant problem according to the constraints with the inputs specified in Sample Input 1, Sample Input 2 and in general input cases. (6 marks)

b) How can graph ADTs be used to model this problem? Explain the strategy. (2 marks)

Question 8 (12 marks)

a) Can Neural Networks after supervised training be categorized as “Weak” or “Strong” AI? Explain and justify your responses. (2 marks)

b) The philosopher Searle believes that Artificial Intelligence is not possible. Describe the thought experiment he devised to demonstrate that Artificial Intelligence is not possible. (2 marks)

c) Complete the table below describing “The Robot Reply” and “The Brain Simulator Reply” to Searle’s argument together with Searle’s response to the objections. (8 marks)

Robot Reply	**Searle’s response to Robot reply**
Brain Simulator Reply	**Searle’s response to Brain Simulator reply**

END OF TRIAL EXAM 1 (Good Luck!)