Distance Education Centre Victoria
Flexible learning for all

2015

NAME:

DECV Student ID:

ALGORITHMICS UNIT 3 & 4

Trial Exam 1: 2015 DECV

Reading Time: 15 minutes
Writing time: 120 minutes (2 hours)

QUESTION AND ANSWER BOOK

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	8	8	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers and one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape

Materials supplied

- Question and answer book of ?? pages
- Answer sheet for multiple-choice questions

Instructions

- Write your student number in the space provided above on this page.
- Check that your name and student number as printed on your answer sheet for multiple-choice questions are correct, and sign you name in the space provided to verify this.

- All written responses must be in English, point form is preferred.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

SECTION A – Multiple Choice – select one option only

Question 1

Time complexity of an algorithm is defined as:

A. the relationship between the *size of the input* and the *run time* for the algorithm
B. exact count of operations T(n) as a function of input size n
C. The amount of time an algorithm takes to run
D. The average case run time of an algorithm

Question 2

Factors that affect time complexity analysis are:

A. The programming language chosen to implement the algorithm
B. The quality of the compiler
C. The speed of the computer on which the algorithm is to be executed
D. None of the above

Question 3

The time complexity for the following nested loop fragment is:

```
x := 0
for j = 1 to n/2 do
      for k = 1 to n*n do
            x := x + j + k
      end do
end do
```

A. $O(n^2)$
B. $O\left(\frac{n}{2}\right)$
C. $O\left(\frac{n^2}{2}\right)$
D. $O(n^3)$

Question 4

The time complexity for the following nested loop fragment is:

```
x := 0
for j = 1 to n do
      for k = 1 to k < 3*j do
            x = x + j
      end do
end do
```

A. $O(n^2)$
B. $O\left(\frac{n}{2}\right)$
C. $O\left(\frac{n^2}{2}\right)$
D. $O(n^3)$

Question 5

Which of the following is **not** true for terms representing time complexity?

A. n dominates logn
B. n^2dominates n*log (n)
C. n^m dominates n^k when k > m
D. a^n dominates n^m for any a > 1 and m >= 0

Question 6

Consider the pop operation for a Stack data structure, the time complexity of this operation is:

A. $O(n)$
B. $O(1)$
C. $O(logn)$
D. $O(n^2)$

Question 7

If the number of operations does not depend on specific items, it depends only on the number of items, then the algorithm is said to be deterministic therefore:

A. all possible instances of the problem ("best case", "worst case", "average case") give the same number of operations
B. The worst case has the highest time complexity
C. The average case is the most difficult time complexity to calculate
D. The best case time complexity is said to be linear.

Question 8

Let L be an empty list, the state of L after executing these operations

$$L \leftarrow \text{append}(L, 1)$$
$$L \leftarrow \text{append}(L, 5)$$
$$L \leftarrow \text{prepend}(L, 8)$$
$$L \leftarrow \text{append}(L, L)$$

is:

A. 1,5,8,L
B. 8,L,1,5
C. 8,1,5,L
D. 8,5,1,L

Question 9

Consider a restaurant kitchen. When the kitchen receives orders for food they are processed in the order that they are received. Occasionally an order will need to be rushed and done ahead of the other orders. An abstract data type that could be used to model the food orders being processed by the kitchen is:

A. A stack
B. A list
C. A queue
D. A priority queue

Question 10

The formal definition of the connected graphs is:

A. has every pair of vertices joined by one edge.
B. All vertices have a degree of 1 or higher
C. There are (V-1) edges for V vertices
D. All vertices have a degree of 2 or higher

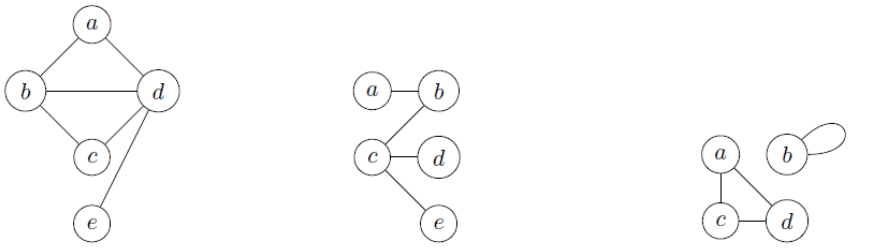
Question 11

The formal definition of a tree is:

A. A graph that contains at least one cycle
B. A graph of $|V|$ vertices and $|V-1|$ edges
C. A connected graph with no cycles
D. A graph of forests

Question 12

Which of the following graphs are cyclic graphs?



A. Only (i)
B. Only (i) and (ii)
C. Only (i) and (iii)
D. Only (ii) and (iii)

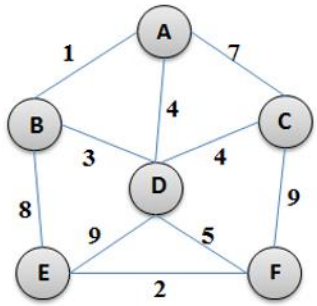
Question 13

A path that passes through every vertex of a graph exactly once without returning to starting vertex is an:

A. Euler Path
B. Shortest Path
C. Minimum Cost Path
D. Hamiltonian Path

Question 14

Consider the following weighted Graph:

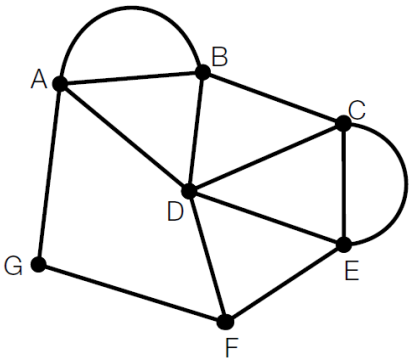


The minimum spanning tree has the cost:

A. 18
B. 15
C. 14
D. 17

Question 15

A Hamiltonian Path for the graph shown that begins at *F* is:



A. G-A-B-D-C-E-F

B. F-E-C-D-B-A-G

C. F-E-C-E-D-F-G-A-B-A-D-B-C-D

D. F-G-A-B-D-E-F

Question 16

Which data structure is used in breadth first search of a graph to hold the nodes?

A. stack

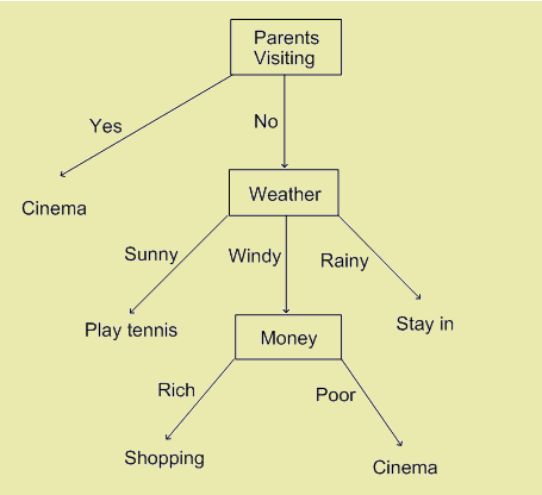
B. queue

C. dictionary

D. array

Question 17

The decision tree below represents a set of activities that are done depending on whether parents are visiting :



Which action below <u>cannot</u> occur from this tree?

A. If the parents are visiting, go to the cinema.

B. If the parents are not visiting and it is sunny, then play tennis.

C. If the parents are not visiting and it is windy and you're rich, then go shopping.

D. If the parents are not visiting and it is windy and you're poor, then stay in.

Question 18

The signature for a Dictionary Abstract Data type looks like:

$$
\begin{array}{l}
name\ Dictionary; \\
import\ key, value; \\
ops \quad newDictionary \ : \rightarrow dictonary; \\
\qquad insertDictionary : key \times value \times dictionary \rightarrow dictionary; \\
\qquad removeDictionary \ : key \times dictionary \rightarrow dictionary; \\
\qquad lookupDictionary \ : key \times dictionary \rightarrow value;
\end{array}
$$

Removing an item from this Dictionary has the inputs and outputs of:

A. INPUT: key, value OUTPUT: dictionary

B. INPUT: dictionary OUTPUT: dictionary

C. INPUT: key, dictionary OUTPUT: value

D. INPUT: key, dictionary OUTPUT: dictionary

Question 19

The definition of Transitive closure in graph theory is:

A. A directed path between two nodes.

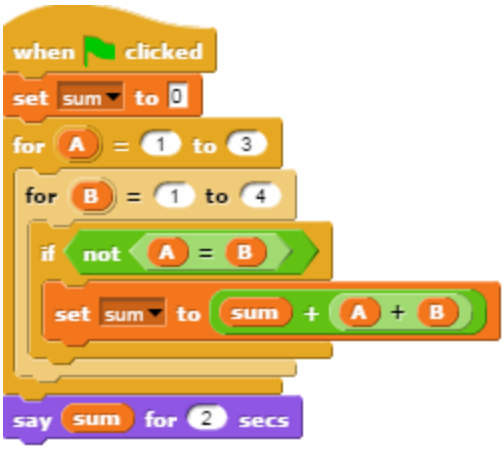
B. A True or False relation that informs if a path exists between two nodes.

C. A directed acyclic graph.

D. A Brute Force algorithm performed on a directed graph.

Question 20

What is the final "sum" that will be calculated for the following Edgy code?



A. 54

B. 52

C. 34

D. 42

SECTION B – Extended Response Questions Answer all questions in the space provided.

In questions where more than one mark is available, appropriate working **must** be shown.

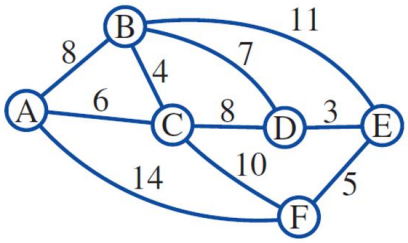
Question 1 (8 marks)

Consider the following algorithm for finding the minimum spanning tree of a Graph

```
Algorithm Prims(Input Graph G, Output Tree T)
Input: G=(V,E) a weighted graph

Initialize MST:  T=(Vnew = { V0},  Enew = {} )
//Start the MST by selecting any vertex in the Graph
Repeat until Vnew = V:
      Choose minimal weighted edge {u, v} where u is in Vnew and v is not
      Add v to Vnew and {u, v} to Enew
Output: T=(Vnew , Enew )
End Algorithm
```

Consider the following weighted graph:



a. Execute the algorithm showing the set T and E_{new} at each iteration. (2 marks)

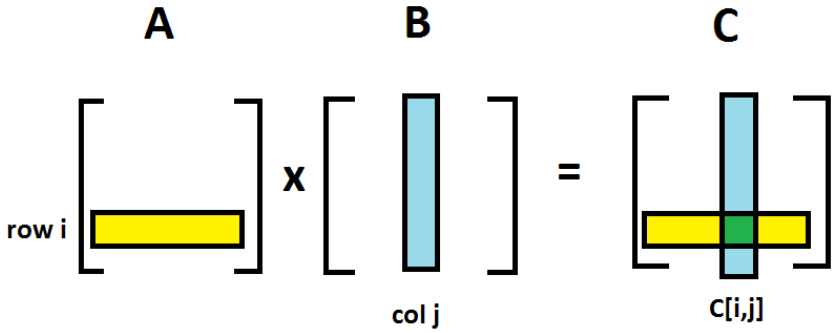
b. Are there any non trivial loop invariants that you can identify? List and describe. (2 marks)

Question 1 (continued)

c. Show the correctness of the algorithm in finding the Minimum Spanning Tree. (2 marks)

d. What is the algorithm design pattern used by Prim's Algorithm? Describe the general properties of this design pattern. (2 marks)

Question 2 (5 marks)
Given two n x n matrices A and B, find the time complexity of the algorithm below for computing their product C=AB.




```
Algorithm MatrixMult (Input matrices: A (nxn), B (nxn),Output matrix: C(nxn))
// multiply two square matrices and gives result C
// the conventional notation for describing
// each element of matrix is described by its row,column
// For example A[1,3] is the element in row 1, column 3
for i=1 to n do
      for j=1 to n do
            for k=1 to n do
                  
            end do
      end do
end do
return C
end Algorithm
```

a. Complete the required actions using conventional notation for this algorithm to work correctly. (1 marks)

b. What is the time complexity of the algorithm MatrixMult? Show all your reasoning for your answer. (2 marks)

Question 2 (continued)

c. Compare the time complexity for the best, average and worst cases for this algorithm. (2 marks)

Question 3 (7 marks)

A Transport plane has to deliver the most valuable set of items to a remote location without exceeding the plane's capacity.

There are n items that can be selected

item	1	2	3	……..	i		n
weight	w1	w2	w3		wi		wn
value	v1	v2	v3		vi		vn

a. In how many ways can the n items be selected? Explain your reasoning. (2 marks)

b. What is the classification of this type of problem using conventions of Computer Science? (2 marks)

Question 3 (continued)

c. Describe the different classes of problems, and the criteria that determine those classes that are defined in Computer Science. (3 marks)

Question 4 (8 marks)

Consider a ternary search. This is an algorithm for searching for a key value K in a sorted array A[1..n]. If n=1, compare element with search key K, otherwise search recursively by comparing K with range of A[1..n/3] if larger compare K with A[n/3..2n/3] if larger still compare K with A[2n/3…n]. Search for K in the subset of A that has the appropriate range.

a. What design technique is this algorithm based on? Describe the principles of this design pattern. (2 marks)

b. Write out the algorithm in pseudocode. (3 marks)

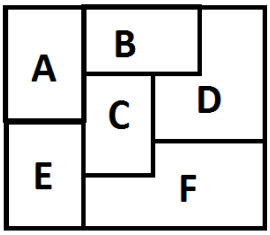
c. Set up a recurrence relation for the time complexity of the algorithm. (1 marks)

Question 4 (continued)

d. Find the worst case time complexity for this algorithm. How does this algorithm compare with the time complexity of Binary Search? (2 marks)

Question 5 (10 marks)

Consider the following map:



a. Represent the information shown in the diagram above as a connected graph with nodes and edges. (2 marks)

b. (i) Explain how we can use the graph-colouring problem to colour the map so that no two neighbouring regions are coloured the same. (2 marks)

(ii) What is the minimum number of colours required in this instance? (1 marks)

(iii) Write a greedy algorithm in pseudocode to colour the map so that no two neighbouring regions are the same colour. (3 marks)

(iv) What is the time complexity of your algorithm? (2 marks)

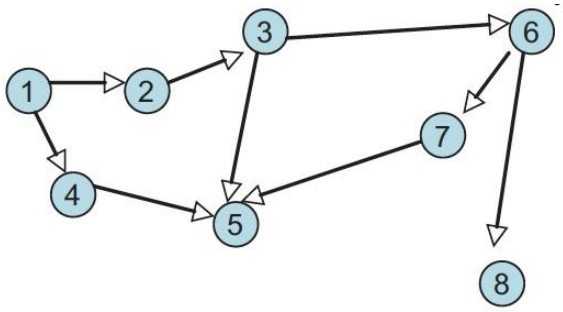
Question 6 – (14 Marks)

a. Explain how one can identify connected components of a graph by using:

i. Depth-first search (2 marks)

ii. Breadth-first search (2 marks)

b. Explain how one can check a graph's acyclicity by using Breadth-first search. (2 marks)



c. Consider the directed graph shown above:

(i) Show the order of nodes visited by Depth-First Search starting at node 1 and ending at node 8. (Always select in numeric order when given multiple options.) (2 marks)

(ii) Show the order of nodes visited by Breadth-First Search starting at node 1 and ending at node 8. (Always select in numeric order when given multiple options.) (2 marks)

Question 6 (-continued)

d. Write an algorithm in pseudocode that will perform a topological sort on a directed graph. (3 marks)

e. Demonstrate your algorithm on the directed graph shown above. (1 mark)

Question 7 – (16 Marks)

a. Given the following list of integers 66, 33, 40, 22, 55, 88, 60, 11. Show the stages of ordering produced by the Bubblesort algorithm to sort these integers. (2 marks)

b. Given the following list of integers 66, 33, 40, 22, 55, 88, 60, 11. Trace by hand the Quicksort algorithm that uses the **leftmost element** as the pivot to sort these integers. (2 marks)

c. Consider sorting the following of list n=8 items 10, 9, 8, 7, 6, 5, 4, 3, using Quicksort with the pivot at the **leftmost element.**

(i) How many actions would be needed to sort this list? (2 marks)

(ii) What strategy could be used to improve the performance of Quicksort for all cases of input? (2 marks)

Question 7 – (continued)

d. Show the splits and the merges done when the recursive Mergesort is used to sort the letters in the list {Q, U, E, S, T, I, O, N}. (2 marks)

e. Consider the following algorithm for Insertion Sort, that accepts as input an array A of values that need to be sorted. The array A has n elements and each element is referenced by the notation A[1]….A[n].

```
for i := 2 to length(A)
    j := i
    while j > 1 and A[j-1] > A[j]
        swap A[j] and A[j-1]
        j := j - 1
    end while
end for
```

(i) What is the run time complexity of this algorithm, assuming that the "swap" has a constant count of commands and has the order O(1)? Show all your reasoning. (2 marks)

(ii) Identify all loop invariants in this algorithm? (2 marks)

(iii) Give a justification for the correctness of this algorithm in sorting values. (2 marks)

Question 8 (12 marks)

Describe the following testing methodologies and their elements and how they are used to test algorithms.

a. Black Box (2 marks)

b. White Box (2 marks)

c. The following inputs determine the information required to book a flight.

- Airline Type = {**S**cheduled, **L**ow Cost, **C**harter}
- Cabin= {**F**irst Class, **B**usiness, **E**conomy}
- Fare Type={**O**ne Way, **R**eturn}
- Fare Conditions = {**F**lexible, **R**estricted}

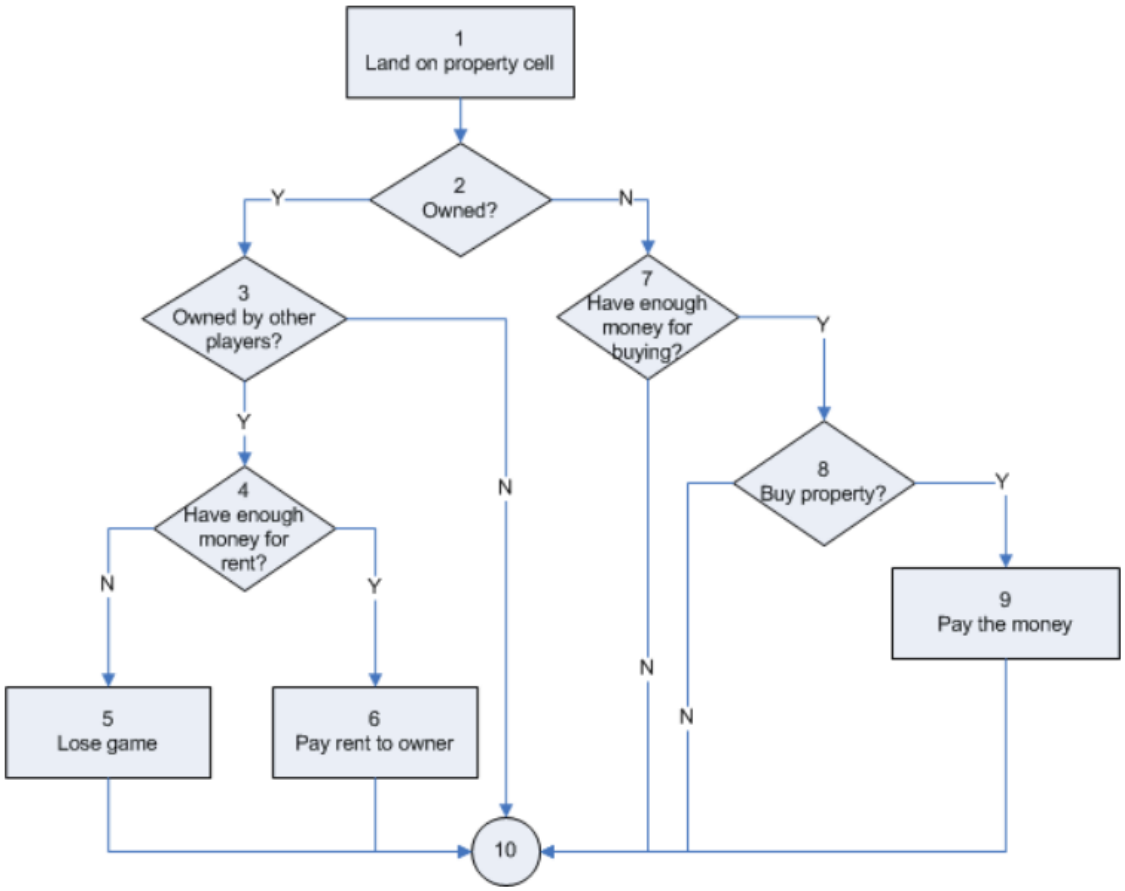
(i) What are the total possible combinations of booking a flight using these inputs? (1 mark)

(ii) Demonstrate how pairwise testing can be used for this example, showing how many tests will result using this method. (2 marks)

d. What is boundary and edge testing? Give an example of each type of this testing. (2 marks)

Question 8 (-continued)

e. Consider the following flowchart for the steps involved in purchasing a property.



Enumerate all the possible paths through this flowchart, using the numbers on the nodes. How many possible paths will need to be tested? (3 marks)

END OF TRIAL EXAM 1