Name:

ALGORITHMICS UNIT 3 & 4

Trial Exam 1: 2021

Reading Time: 15 minutes
Writing time: 120 minutes (2 hours)

QUESTION AND ANSWER BOOK

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	7	7	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers and one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape

Materials supplied

- Question and answer book of 24 pages
- Answer sheet for multiple-choice questions

Instructions

- Write your student number in the space provided above on this page.
- Check that your name and student number as printed on your answer sheet for multiple-choice questions are correct, and sign you name in the space provided to verify this.

- All written responses must be in English, point form is preferred.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

IMPORTANT NOTE:
The VCAA Exam will include the Master Theorem in this form.

Use the Master Theorem to solve recurrence relations of the form shown below.

$$T(n) = \begin{cases} aT\left(\frac{n}{b}\right) + kn^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases} \qquad \text{where } a > 0, b > 1, c \geq 0, d \geq 0, k > 0$$

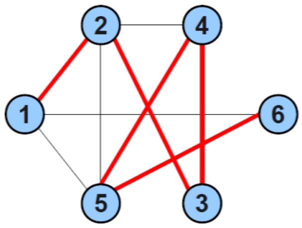
$$\text{and its solution } T(n) = \begin{cases} O(n^c) & \text{if } \log_b a < c \\ O(n^c \log n) & \text{if } \log_b a = c \\ O(n^{\log_b a}) & \text{if } \log_b a > c \end{cases}$$

The VCAA form of Master Theorem is equivalent to the form of Master Theorem taught in our class by consideration of log laws. $\log_b a = c \Leftrightarrow a = b^c \Leftrightarrow \frac{a}{b^c} = 1$ $\log_b a < c \Leftrightarrow a < b^c \Leftrightarrow \frac{a}{b^c} < 1$ $\log_b a > c \Leftrightarrow a > b^c \Leftrightarrow \frac{a}{b^c} > 1$	$T(n) = aT\left(\frac{n}{b}\right) + f(n^k)$ • $\frac{a}{b^k} < 1$ then $O(n^k)$ • $\frac{a}{b^k} = 1$ then $O(n^k \log_b n)$ • $\frac{a}{b^k} > 1$ then $O(n^{\log_b a})$

SECTION A – Multiple Choice – select one option only

Question 1

Consider the following problem:
For a graph G of nodes and edges, is there a simple path that goes through every node exactly once?



Which description best fits this problem.

A. Feasible, P-class

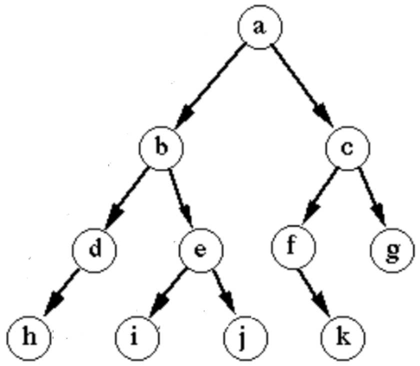
B. Uncomputable, NP-Complete

C. Undecidable, NP-Hard

D. Intractable, NP

Question 2

If the following graph is traversed using Depth First Search starting at node a, and using alphabetic order where multiple options exist.



The order of nodes processed will be:

A. `a b d e h i j c f k g`

B. `a b c d e f g h i j k`

C. `a b e i j d h c g f k`

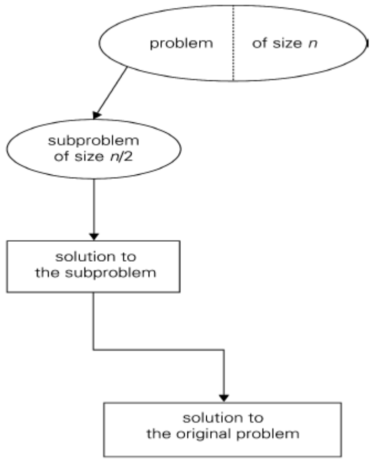
D. `a b d h e i j c f k g`

Question 3

A recursive algorithm has the property of one recursive call which decreases the size of the problem by 0.5 until the base case is reached, where a constant amount of work is done.

Outside of the recursion a linear amount of work is done relative the input size.

The **recurrence relation** representing the number of actions with respect to the input size of n would be:



A. T(n)=T(n/2)+n, T(1)=O(1)

B. T(n)=T(n/2) + O(1), T(1)=O(1)

C. T(n)=O(nlogn)

D. T(n)=T(n-1)+n, T(1)=O(1)

Question 4

If all edges have the same weight in an undirected graph, which algorithm will find the shortest path between two nodes more efficiently?

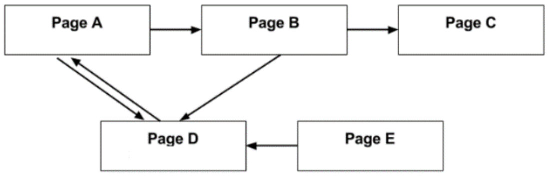
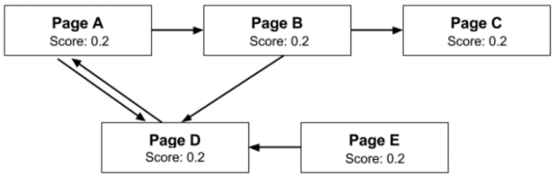
A. Dijkstra

B. Bellman-Ford

C. Depth-First Search

D. Breadth-First Search

Question 5

The Page rank algorithm is run on the graph shown below	After initialisation the Page rank scores for each page are shown below
	

After iteration 1 the Page rank score for Page D is:**.**

A. 0.40400

B. 0.06400

C. 0.14900

D. 0.23400

Question 6

Consider the following problem:

Given a graph G and an integer k, is there a spanning tree whose edges sum to less than k?

What is the classification of the problem above?

A. Decision problem

B. Minimum Spanning Tree problem

C. NP problem

D. Hard problem

Question 7

If x, x^2 and x^3 lie on a number line in the order shown below, which of the following could be the value of x?

A. -2

B. $-\frac{1}{2}$

C. $\frac{3}{4}$

D. 1

Question 8

Which of the following best describe simulated annealing used as a heuristic?

A. Returns an approximation to the optimal solution using a cooling schedule of randomisation.

B. Returns an approximation to the optimal solution where there is no randomisation.

C. It will not return an optimal solution when there is a cooling schedule of randomisation.

D. Uses purely greedy methods for an approximation to the optimal solution.

Question 9

The missing terms in order for the Priority Queue signature shown below are:

$$
\begin{array}{ll}
\mathit{name} & \mathit{priorityQueue}; \\
\mathit{ops} & \mathit{newPriorityQueue} : \rightarrow \mathit{priorityQueue}; \\
 & \mathit{enqueue} : \mathit{priorityQueue} \times \mathit{element} \times \boxed{\qquad\qquad} \rightarrow \mathit{priorityQueue}; \\
 & \mathit{minElement} : \mathit{priorityQueue} \rightarrow \mathit{element}; \\
 & \mathit{dequeueMin} : \mathit{priorityQueue} \rightarrow \boxed{\qquad\qquad\qquad}; \\
 & \mathit{isEmpty} : \mathit{priorityQueue} \rightarrow \boxed{\qquad\qquad\qquad};
\end{array}
$$

A. item, item, Boolean

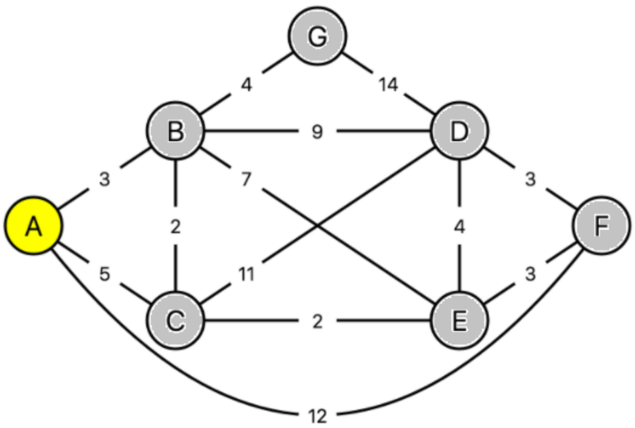
B. rank, item, priorityQueue

C. rank, priorityQueue, Boolean

D. rank, element, element

Question 10

Using the graph below,



if we apply Dijkstra's algorithm to find the shortest distance between node A and all the others, in what order do the nodes get included into the visited set (i.e their distances have been finalized)?

A. B C F G E D

B. B C G E F D

C. C B E F G D

D. C B E G F D

Question 11

Which of the following is NOT an example of an NP problem?

A. 3 colouring of a given graph

B. Travelling salesman problem

C. Knapsack

D. Halting Problem

Question 12

What is Searle's “Chinese room” thought experiment supposed to show?

A. That computers aren't yet able to simulate the human ability to understand

B. That understanding involves more than the ability to formally reproduce appropriate outputs

C. That it is only possible for systems to demonstrate understanding

D. That no machine can demonstrate genuine understanding

Question 13

Assuming P ≠ NP, which of the following is true?

A. NP-Complete = NP

B. NP-Complete ∩ P = ψ {empty set}

C. NP-Hard = NP

D. P = NP-Complete

Question 14

The following recurrence relation $T(n) = 2^n\, T(n/2) + n^n$ gives information about how much work is done by a particular recursive algorithm.

Using the Master Theorem it can be represented by the following function of the input size n.

A. O(nlogn)

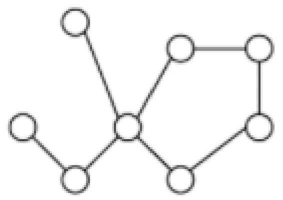
B. $O(2^n)$

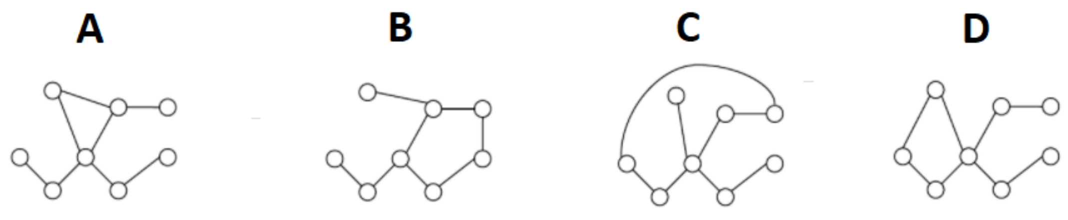
C. O(n!)

D. Master Theorem does not apply as the number of recursive calls made is not a constant value

Question 15

Which graph below is isomorphic to this graph:





Question 16

What does the following mystery function defined in pseudocode below do?

```
Function mystery(a, b)
// Input a an integer
// Input b an integer

    if (b == 0) then
       return 0
    else
       return (a + mystery(a, b-1))
    end if
end function
```

A. Returns $a + b$

B. Returns $a + ab$

C. Returns ab

D. Returns a^b

Question 17

Consider a situation where you have to write a function to calculate a number raised to a power such as x^n where x can be any number and n is a positive integer. What can be the best possible time complexity of your power function using an advanced design pattern?

A. O(n)

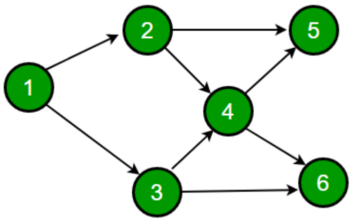
B. O(logn)

C. O(log(logn))

D. O(nlogn)

Question 18

Consider the Directed Graph shown



Which of the following is NOT a topological ordering?

A. 1 2 3 4 5 6

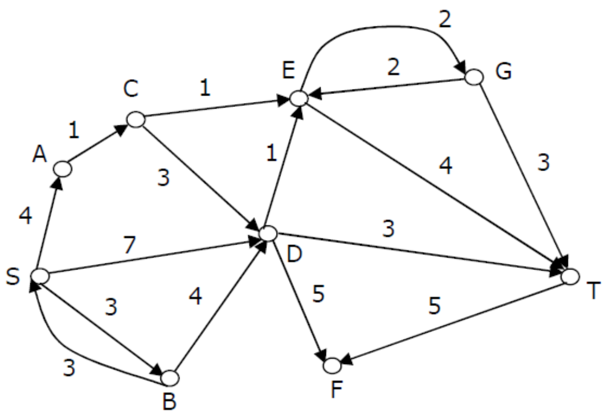
B. 1 3 2 4 5 6

C. 3 2 4 1 6 5

D. 1 3 2 4 6 5

Question 19

Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra?s shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex v is updated only when a strictly shorter path to v is discovered.



A. SACET

B. SDT

C. SBDT

D. SACDT

Question 20

Consider the following three functions:

$$f1(n) = 10^n$$

$$f2(n) = n^{logn}$$

$$f3(n) = n^{\sqrt{n}}$$

Which one of the following options arranges the functions in the increasing order of asymptotic growth rate?

A. $f3, f2, f1$

B. $f2, f1, f3$

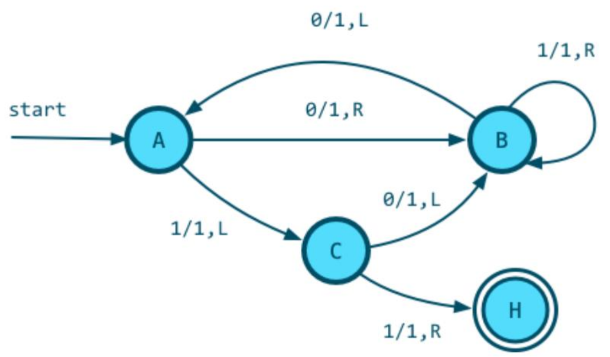
C. $f1, f2, f3$

D. $f2, f3, f1$

SECTION B – Extended Response Questions Answer all questions in the space provided.

Question 1 (14 marks)

The following Turing Machine (TM) has been as a directed graph, with states represented by nodes and transitions labelled on the edges.



Edge transition description:	TM Symbols: {1,0}
in/out,R *in/out*,L	*in* is the input symbol *out* is the output symbol R moves one symbol to the right L move one symbol to the left

a) For the above Turing Machine describe the steps taken to the output when it is given the following input tape, with the current state and position on the input tape shown. (3 marks)

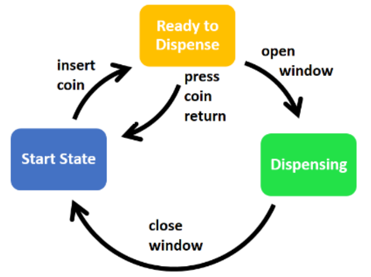
	A ↓					
	0	1	1	0	0	1

b) Convert the Turing Machine shown above into a table of instructions. (3 marks)

Current State	Scanned Symbol	Print Action	Move Action	Next State

Finite State Automata (FSA) are very simple machines, an example is a simple vending machine that will dispense a candy bar when a dollar coin has been inserted.

- There are four actions possible: insert a coin, press `coin return', open the window to take the candy bar, and close the window again.
- There are three states: the start state, ready to dispense state, dispensing state.
- The FSA is shown in the diagram at the right.



c) With consideration of the definition of a FSA provided above, outline the similarities and the differences between a FSA and a TM. (3 marks)

d) Formally what are the definition of problems in terms of solvability and verifiability in **class P** and in **class NP** with respect to Turing machines? (3 marks)

Consider this lock that requires a combination to be opened.

e) Can this lock be opened by a randomly chosen student? Using Computer Science conventions describe what kind of problem this is and discuss which complexity class this problem belongs to with justifications of your selection. (2 marks)

Question 2 (15 marks)

Consider the following problem: Given a wine wholesaler has a wine barrel of capacity n litres and an array of prices that includes prices of wine volumes that are sold to restaurants of size smaller than n. Determine the maximum value obtainable by dividing up the wine and selling the smaller volumes.

Example 1: If the wine barrel capacity is 8 litres and the values of different volumes that can be sold are given as the follows, then the maximum obtainable value is 22 (by two wine volumes of 2 and 6)

Volume (litres)	1	2	3	4	5	6	7	8
Price	1	5	8	9	10	17	17	20

Example 2: If the prices are as follows, then the maximum obtainable value is 24 (by eight wine volume sales of 1 litre)

Volume (litres)	1	2	3	4	5	6	7	8
Price	3	5	8	9	10	17	17	20

a) Explain how many ways are there to get different volumes for sale from a wine barrel of capacity n litres? (2 marks)

b) Briefly outline how a naïve Brute Force solution could to solve this problem. (2 marks)

c) If the wine barrel contains a volume of 10 litres and the volumes for sale are=[1,2,3,4] and the price=[1,5,8,9] show all the possible decisions using the naïve Brute Force method as a decision tree for 2 levels of decisions. (2 marks)

d) What features of this problem can be identified to solve it more efficiently? Explain these features and which design pattern can be used to improve time complexity. (2 marks)

Question 2 (continued)

e) Complete the assignment to the **value array** for the more efficient solution for the wine sales problem using structured pseudocode below. (4 marks)

```
Algorithm wineSales(volumes, price, n)
    // volumes array. volumes for sale, volumes[1] is first element
    // price array, price[1] is the first element
    // n is the size of the volumes and price arrays

    // initialise the value array, where value[1] is the first element
    For i=1 to n do
      If (volumes[1] == 1) then
        // then all discrete volumes can be sold from 1 to n
        value[i] :=
      Else
        value[i] :=
      End if
    End do

    // Build the array of n elements value[]
    for i = 2 to n do
        for j = 1 to i do
            if (i – volumes[j]==0) then
                value[i] = maximum(value[i],                    )
            else if (i – volumes[j] > 0) then
               value[i] = maximum(value[i],                     )
            end if
        end do
    end do

end algorithm
```

f) State the time complexity and the space complexity of the solution from part e). Justify your responses. (2 marks)

g) With reference to the pseudocode in part e), in which ADT **precisely** is the solution for wine sales from a wine barrel of n litres found? (1 mark)

Question 3 (12 marks)

Genevieve and Bacchus consider the following number puzzle problem:

There is a three digit integer, let the number be abc. What is the largest three digit number with the property that the number is equal to the sum of its hundreds digit, the square of its ten digit and the cube of its units digit?	Written algebraically a solution for: $abc = a + b^2 + c^3$ where a, b, c are integers between 0 and 9

Genevieve does a little bit of algebra before trying to solve the problem with the following results.

$$\begin{aligned} 100a + 10b + c &= a + b^2 + c^3 \\ 99a + 10b - b^2 &= c(c^2 - 1) \\ 99a + b(10 - b) &= (c-1)c(c+1) \end{aligned}$$

And starts making a table to try out all the combinations

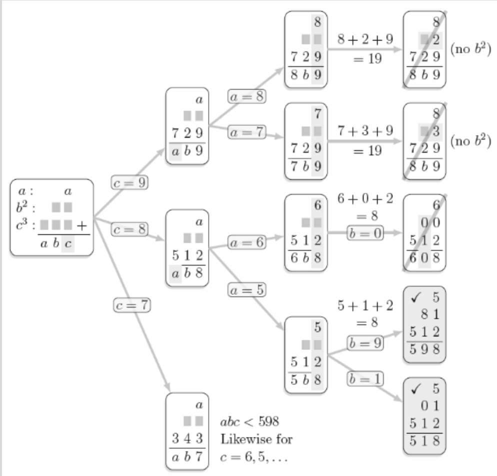
$99a$	$b(10-b)$	$(c-1)c(c+1)$
$99 \times 1 = 99$	$1 \times 9 = 9$	$1 \times 2 \times 3 = 6$
$99 \times 2 = 198$	$2 \times 8 = 16$	$2 \times 3 \times 4 = 24$
$99 \times 3 = 297$	$3 \times 7 = 21$	$3 \times 4 \times 5 = 60$
$99 \times 4 = 396$	$4 \times 6 = 24$	$4 \times 5 \times 6 = 120$
$99 \times 5 = 495$	$5 \times 5 = 25$	$5 \times 6 \times 7 = 210$
$99 \times 6 = 594$	$6 \times 4 = 24$	$6 \times 7 \times 8 = 336$
$99 \times 7 = 693$	$7 \times 3 = 21$	$7 \times 8 \times 9 = 504$
$99 \times 8 = 792$	$8 \times 2 = 16$	$8 \times 9 \times 10 = 720$
$99 \times 9 = 891$	$9 \times 1 = 9$	

a) Write a Generate-and-Test algorithm in structured commented pseudocode to solve this problem for Genevieve using the table algebra method above. (6 marks)

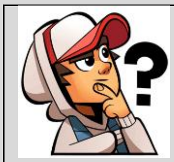
Question 3 (continued)

Bacchus has a different approach and tries to solve this problem by considering the possible values of b^2 and c^3.
Where the number $abc = a + b^2 + c^3$

Digit	0	1	2	3	4	5	6	7	8	9
Square	0	1	4	9	16	25	36	49	64	81
Cube	0	1	8	27	64	125	216	343	512	729



Bacchus looks for conditional logic in solving this problem



$a \neq 0, b \neq 0, c \neq 0$, since $abc \neq 0$
c can only be $\{8,9\}$
a can take values from the first digit of c^3 to $(c-1)$
The last digit of b^2 could be $\{1,4,5,6,9\}$

He starts by trying the largest possible values of c (c^3) first, then filling in possible values for a and finally trying logical values of b (b^2).
This trial-and-error search is presented here as a tree with the dead-ends paths crossed out.

b) Write a Backtracking algorithm in structured commented pseudocode to solve this problem using Bacchus' methods. (6 marks)
Note: Conditional logic can be indicated using comments and does not need to be given in detail

~~**Question 4 (10 marks)**~~

a) ~~Briefly explain what is DNA Computing. How does it differ from silicon computer technologies? (2 marks)~~

b) ~~What are the benefits of DNA Computing? Explain the implications for solving NP problems. (2 marks)~~

c) ~~In 1998 Adelman conducted an experiment to solve the Hamiltonian path problem using DNA computing. Outline the main steps of Adelman's algorithm that was executed with DNA strands. (3 marks)~~

~~To encode the solution to the Hamiltonian path problem Adelman worked with individual strands of DNA, the solutions formed quickly in parallel, thereafter several technical processes were needed to un-decipher the DNA solutions, anecdotally these processes took days.~~

d) ~~What would be the space complexity and time complexity required for using DNA computing successfully for solving NP problems? Hence, what are the implications for encoding information and understanding solutions? (3 marks)~~

Question 5 (5 marks)

A sequence of numbers is called a zig-zag sequence if the differences between successive numbers strictly alternate between positive and negative.

For example, 1,7,4,9,2,5 is a zig-zag sequence because the differences (6,-3,5,-7,3) are alternately positive and negative.	In contrast, 1,4,7,2,5 and 1,7,4,5,5 are not zig-zag sequences, the first because its first two differences are positive and the second because its last difference is zero.

Given a sequence of integers, sequence, write an algorithm in commented and structured pseudocode that returns the length of the longest subsequence of sequence in an array of length n that is a zig-zag sequence with O(n) time complexity.

Question 6 (12 marks)

Consider the Bellman-Ford algorithm which works to find shortest paths in a weighted graph G.

```
Algorithm Bellman-Ford(G,v)
// Input G a weighted graph
// Input node v the source node, all nodes have attribute of distance

For each node n in G do
    n.distance := ∞   // initialise all nodes to have infinite distance
End do
v.distance := 0  // initialise source node to zero

For i=1 to [__________] do
    For each (u,w) in E do
        If u.distance + length(u,w) < w.distance then
          [______________________________________________]
        End if
    End do
End do

For each (u,w) in E do
    If [________________________________________________] then
        Print Negative cycle detected – shortest path not found
    End if
End do
End Algorithm
```

a) Fill in the missing parts of the algorithm. What are the missing actions in the algorithm shown? (3 marks)

b) What is the time complexity of the Bellman-Ford algorithm in terms of |V| and |E|? (1 mark)

c) If there are no negative cycles, what is the largest possible diameter of a graph G? (1 mark)

d) Describe any modifications to the ADTs and the pseudocode that would be needed to keep track of the predecessor node for the shortest path in the algorithm above? (3 marks)

Question 6 (continued)

e) Assuming that a graph G={V,E} has no negative cycles, using induction give a proof of correctness for the Bellman-Ford algorithm that includes a justification for the main loop running |V|-1 times.

(4 marks)

Question 7 (12 marks)

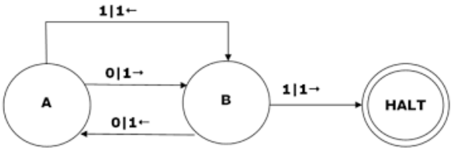
a) What is the definition of a computable function as defined in the context of mathematics and computer science? (2 mark)

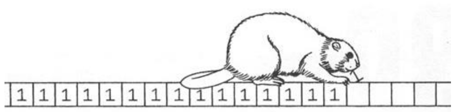
b) The Entscheidungsproblem (German for "decision problem") is a challenge posed by Hilbert's Program. What is the definition and main aim of the posing this problem? (2 marks)

c) Briefly describe the Halting problem. Is the Halting problem decidable? Explain. (2 marks)

d) Outline the contradiction using logic/pseudocode or a diagram that can be set up to show the decidability of the Halting problem. (3 marks)

Question 7 (continued)





~~e) Briefly describe the Busy Beaver problem. Is the Busy Beaver problem decidable? Explain.(3 marks)~~

END OF TRIAL EXAM