JOHN MONASH SCIENCE SCHOOL

VICTORIAN CERTIFICATE OF EDUCATION

2021

ALGORITHMICS (HESS)

Practice Exam

2021

SOLUTIONS

SECTION A – Multiple-choice Questions

Question	Comments
1	**Correct Answer :** B
2	**Correct Answer :** C
3	**Correct Answer :** B
4	**Correct Answer :** A
5	**Correct Answer :** A
6	**Correct Answer :** D
7	**Correct Answer :** A
8	**Correct Answer :** C
9	**Correct Answer :** D
10	**Correct Answer :** C
11	**Correct Answer :** B
12	**Correct Answer :** A
13	**Correct Answer :** B
14	**Correct Answer :** A
15	**Correct Answer :** C
16	**Correct Answer :** B
17	**Correct Answer :** B
18	**Correct Answer :** D
19	**Correct Answer :** D
20	**Correct Answer :** A

END OF QUESTION AND ANSWER BOOK

SECTION B
Question 1 (4 marks)

Matilda is running a commission service for paintings. Customers may request a painting for $12 and are offered an additional $10 'express painting' option (which guarantees quicker completion of the painting).

a. Name an appropriate ADT that Matilda could use and how it would model the painting requests that she receives. 2 marks

Example answer: An appropriate ADT to use is a Priority Queue, as customers are 'served' in order of when they placed their order, but are also able to be assigned a priority and 'served' earlier.

1 mark for identifying an appropriate ADT

1 mark for an appropriate justification.

Matilda tires of the painting world and decides to enter the world of fast food. She ponders how she could model customers waiting to be served.

b. What is an appropriate ADT that Matilda could use to model the order in which customers should be served at a fast food restaurant? 2 marks

Example answer: An appropriate ADT to use would be a queue as customers will generally be served in a first come first served manner however elements of a priority queue can potentially be used as for some smaller orders the completion time is much shorter than larger orders so it may be put at a higher priority do be done sooner.

1 mark for identifying an appropriate ADT

1 mark for an appropriate justification.

Question 2 (4 marks)

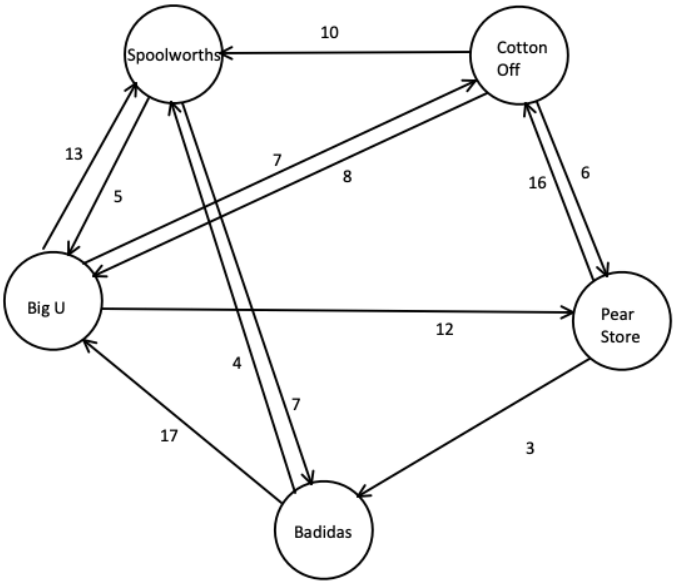
Paige is trying to travel between as many stores in a shopping centre as possible, in the shortest time possible.

She collects the following information regarding the travel time between stores:

To / From	Spoolworths	Cotton Off	Big U	Pear Store	Badidas
Spoolworths	-	-	5	-	7
Cotton Off	10	-	8	6	-
Big U	13	7	-	12	-
Pear Store	-	16	-	-	3
Badidas	4	-	17	-	-

Create a graph to represent the provided information about the shopping centre.

Example answer: the graph should contain the nodes representing the stores and the edges representing the paths between the stores, with the weights representing the distances. The edges should be directed to show the starting and ending points.



1 mark for correctly using nodes

1 mark for correctly using edges

1 mark for correct use of weights

1 mark for correct use of directed edges

Question 3 (8 marks)

A teacher goes to collect the papers for the students latest assignment and is figuring out how to go through and mark them all

a. Name an appropriate ADT that they could use 1 mark

Example answer: Stack or Queue

1 mark for correctly identifying one of these ADTs

b. Some of her students need their results back early to apply for another course. Describe a new specification for the ADT named in part **a.** that will allow the teacher to accommodate this new requirement. As part of your answer, explain how the new operation could be implemented. 2 marks

Example answer: Students who need to be marked first need to be found and returned to the top of the stack.

Priorities: Stack x *element* x *Stack* x *Stack* → *Stack*

The way this specification would work is when called for a specific student, it will peek at the first element in the original stack and then add this to the new stack, let's call this stack 1, and pop it from the old one. It will repeatedly do this until finding the element with the same student name. It would then peek and add this to the other unused stack, call it stack 2. It will then peek and add each element from stack 1 back onto the original stack popping them from stack 1 after being added before then peeking and placing the element from stack 2 onto the original stack and popping stack 2. The student with priority should now be the top of the stack and next to be marked.

1 mark for correctly identifying a specification

1 mark for description of how the operation would be carried out.

Question 4 (3 marks)

Wei's hat store has run into a problem: All the hats are out of order. Wei would like to sort them based on their size.

Write pseudocode for an algorithm that will take as input a list of hats and their sizes and that will return the list of hat sorted from smallest to largest.

1 mark for an attempt to compare each element with another based on size

1 mark for an attempt to sort each element

1 mark for returning the correct output of a sorted list

Question 5 (3 marks)

Describe the method by which the Minimax algorithm attempts to find an optimal solution.

1 mark for correctly mentioning generation of all game states

1 mark for description of how the algorithm will assign scores based on end state

1 mark for correctly describing the backwards propagation of scores up the tree assuming both players are using minimax

Question 6 (3 marks)

Write pseudocode for an algorithm, that utilises dynamic programming, to find the nth Fibonacci number.

Example answer:

```
1:     Cache <- {}
2:     Procedure fib(n):
3:           // INPUT: n, a positive integer
4:           If n in Cache.keys:
5:                 Return Cache[n]
6:           Else If n <= 1:
7:                 Result <- 1
8:           Else:
9:                 Result <- fib(n-1) + fib(n-2)
10:          End If
11:          Cache[n] <- Result
12:          Return Result
13:    End Procedure
```

1 mark for an attempt to store results of previous calculations

1 mark for an attempt to use previous calculations

1 mark for returning the correct output

Question 7 (4 marks)

Prove the correctness of this Algorithm that returns the square of the input, *n*.

```
S ← 0
i ← 0
while i < n
   S ← S + n
   i ← i + 1
return S
```

1 mark for correct base case: s=0, i =0, k=0
1 mark for the induction hypothesis: For m=k, S=mn and i=m
1 mark for the induction step:
i=m+1
S=(m+1)n, i=m+1 as expected.
S=mn+n

1 mark for final statement :The loop stops when i=n and at this time S=nxn=n^2

Question 8 (5 marks)

Use induction to prove that the sum of the first *n* odd numbers is equal to n^2.

Example answer
1+3+5+...+(2n-1) = n^2
Base case: n=1
LHS = 1
RHS = 1
Assume n=k
1+3+5+...+(2k-1) = k^2
k = k+1
1+3+5+...+(2k-1)+(2(k+1)-1) = (k+1)^2
k^2+(2(k+1)-1) = (k+1)^2
LHS = k^2+2k+1
LHS = (k+1)^2 = RHS
Therefore the algorithm will always work regardless of which iteration it is up to and the sum of odd n number is equal to n^2

1 mark for correct base case
1 mark for the induction hypothesis
1 mark for an attempt at the induction step
1 mark for successfully carrying out the induction step
1 mark for final statement

Question 9 (4 marks)

Ben is trying to keep track of who interacted with each other in the GAT to model the spread of a rumour after 2 gossip spreaders went into the exam.

a. Describe a suitable ADT that could be used to model this task and how it could be used to store the information. 2 marks

No marking scheme provided

A primary recipient is someone who has been told the rumour by one of the gossip spreaders. A secondary recipient is someone who has been told the rumour by a primary recipient.

b. Rob suggests that to sort people into primary and secondary recipients, Dijkstra's algorithm would be best because it has a time complexity of O($n$log(n)). Discuss the suitability of Dijkstra's algorithm for this task. 2 marks

No marking scheme provided

Question 10 (2 marks)

Euler stated that there was no solution to the Konigsberg Bridge Problem. What conditions did Euler say must be satisfied for an Eulerian Trail to exist?

Example answer: There must be either 0 or 2 odd degree nodes only, any more than this, and it is not possible to traverse each edge exactly once. Only the start and end nodes can be of odd degree. In order to enter and exit every other node, each node must be of even degrees.

1 mark for 0 or 2 odd degree nodes
1 mark for the identification of start and end nodes being the only possible odd nodes.

Question 11 (6 marks)

Consider the following list L = {cookie, cake, brownie, penguin, cow, sheep, goat}.

a. Which element is referred to when we say L[3]? 1 mark

1 mark for "penguin"

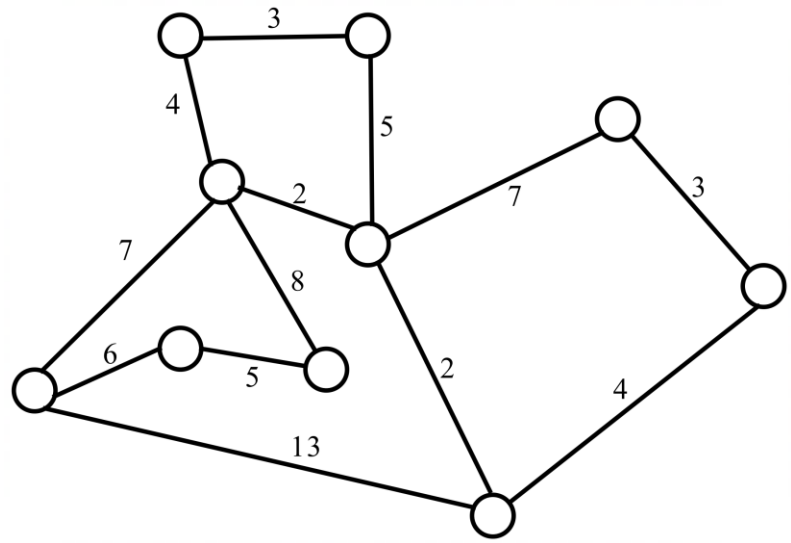
b. State the appearance of L after each of the following operations are carried out. 5 marks

1. L.append(pig)
2. L.prepend(cup)
3. L.get(L[1])
4. L.set(L[4], rabbit)
5. L.delete(L[0])

1 mark for L = {cookie, cake, brownie, penguin, cow, sheep, goat, pig}
1 mark for L = {cup, cookie, cake, brownie, penguin, cow, sheep, goat, pig}
1 mark for L = {cup, cookie, cake, brownie, penguin, cow, sheep, goat, pig}
1 mark for L = {cup, cookie, cake, brownie, rabbit, cow, sheep, goat, pig}
1 mark for L = {cookie, cake, brownie, rabbit, cow, sheep, goat, pig}

Question 12 (3 marks)

Aden lives in a cave system where he is tortured every day, suffering for eternity or until the end of this practice exam… whichever comes sooner.



Aden wishes to find the minimal spanning tree of his cave system.

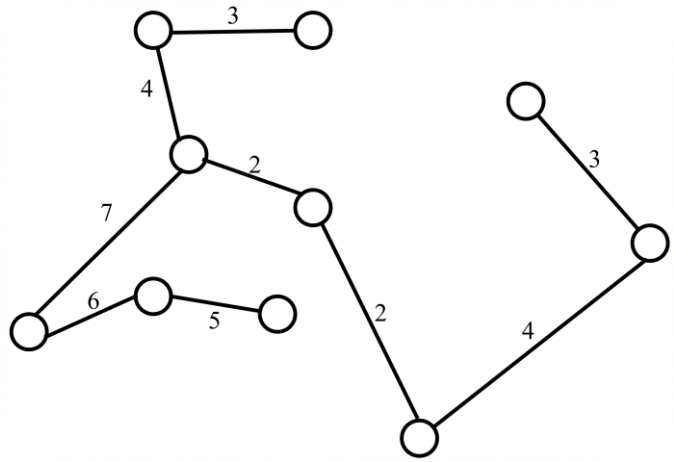
a. Describe what a minimal spanning tree is. 1 mark

Example answer: The shortest length tree that connects every node in the graph.

1 mark for correct description

b. Construct a minimal spanning tree for Aden's cave system. 2 marks

Example answer:



1 mark for attempt at a tree

1 mark for correct MST

Question 13 (4 marks)

Professor Bohni was writing a scientific paper on the uses of minimax in turn-based games. However, he has tripped and bumped his head after too many monkeys jumped on the bed so the doctor said, 'I'm sorry professor, you've messed up your head'. It's such a shame. The paper was nearly complete. If only **somebody** could help Professor Bohni finish his paper.

For the following games, state and explain with reasons why minimax would or would not be appropriate to use to help choose the best move to make.

a. Chess 1 mark

1 mark for inappropriate with valid reasons

b. Tic-Tac-Toe 1 mark

1 mark for appropriate with valid reasons

c. Connect-4 1 mark

1 mark for inappropriate with valid reasons

d. Rock-Paper-Scissors 1 mark

1 mark for appropriate with valid reasons

Question 14 (5 marks)

A student and close apprentice of Professor Bohni, Student Sharan, was too busy training for his baki physique and did not hear about Professor Bohni's blunder. After paying off so many freelancers to write his New York Times Best Selling Novel of 2043 'Algorithmics for dummies', he had gone broke. Sharan accidentally signed off his royalties to scam artist Julee-Ann and has no way to pay off the freelancers for a second novel. To make up for this, Sharan had the grand idea of hiring homeless people to run errands for him around the neighbourhood, checking everyone's house for Amazon Deliveries to snag. A new job, a new beginning. However, Homeless Amish is worried that his loan shark Kody will be mad if he does not make enough money before the deadline. He wishes to complete this trip as fast as possible.

Help Homeless Amish plan the fastest route before he becomes fish sticks for Loan Shark Kody.

a. Describe the NP-hard version of the travelling salesman problem. 2 marks

No marking scheme provided

b. Explain how Amish could make use of Randomised Heurtistics to assist him in finding the solution to the travelling salesman problem. As part of your answer discuss the optimality of the solution this approach would supply. 3 marks

No marking scheme provided

Question 15 (3 marks)
Outline a dynamic programming approach that Shiann could use to find pyramidal numbers to the nth sequence.

No marking scheme provided
Question 16 (5 marks)

Mr Bohni is in a rush to get to his favourite coffee shop before his lunch break runs out, but his GPS is malfunctioning hence has to reprogram it using graph theory.

a. Explain what his graph would represent, and what algorithm he could use to find the shortest cost to his favourite coffee shop, explain your choice 3 marks

Example answer: Using graph theory he would represent shops and other locations as nodes on the graph, and edges would represent the paths between these places with the weightings being the cost. He would use Dijkstra's algorithm to find the shortest path to his coffee shop; he only needs to find the distance between the starting point and the coffee shop.

1 mark for explaining nodes are destinations and edges paths between them
1 mark for saying Dijkstra's
1 mark for giving a satisfactory answer to their choice

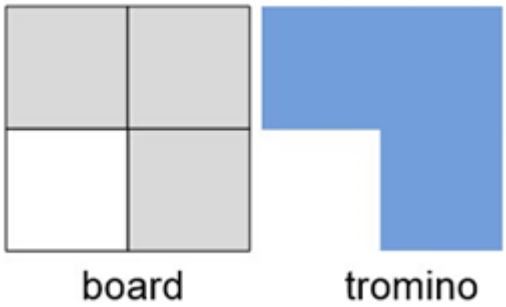
b. A one-way wormhole has appeared to the coffee shop, giving him extra "cost" to spare, how would this affect the graph, and would you still use Dijkstra's, if not what other alternatives would you choose. 2 marks

Example answer: The wormhole would be a directed edge with negative cost on the graph, Dijstra's might work but not guaranteed hence he should use Bellman ford.

1 mark for the wormhole creating a negative edge weight
1 mark for explaining that Dijkstra's may not work as a result and choice of either Flloyd-Warshall or Bellman Ford.

Question 17 (8 marks)

Dr Albrecht is attempting to tile his new bathroom with a unique tile shape.



He wishes to tile his 16 x 16 square bathroom but is unsure if this is possible given the tile's unusual shape.

Determine the viability of this tiling and explain how the following algorithms would reach a solution.

a. Backtracking 2 marks

No marking scheme provided

b. Divide and Conquer 2 marks

No marking scheme provided

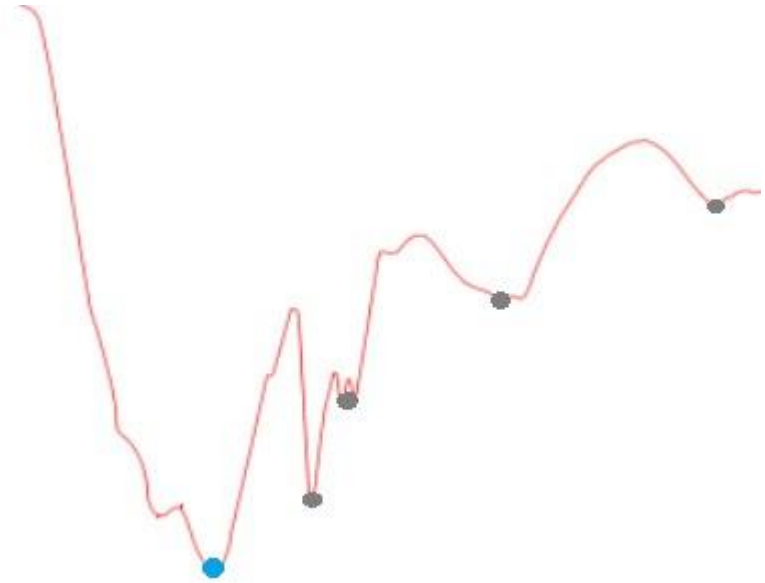
c. Decrease and Conquer 2 marks

No marking scheme provided

d. Brute Force 2 marks

No marking scheme provided

Question 17 (5 marks)



Professor Bohni finally awakens after knocking his head. But what's this? It seems he has become delusional and has sleep-walked into the middle of a mountainous desert. A large valley between sand dunes is in front of him. As he has been unconscious for many days, he is extremely dehydrated. He knows that somewhere in the valley, at the **lowest altitude**, lies an oasis with a river where Professor Bohni can quench his thirst. By modelling the valley using the above graph, using his advanced algorithmic skills, Mr Bohni knows he can calculate the exact location of the river using randomized heuristics.

a. Describe the basics of how randomised heuristics work and why it is useful. 2 marks

1 mark for a description of the general process of randomised heuristics including mention of its random starting point

1 mark for statement of how it can be used to solve problems that would normally be intractable

b. Explain how randomised heuristics could be applied to this specific problem. Ensure you include in your answer which algorithmic design component randomised heurists rely on that will result in the blue point being found instead of the grey points. 2 marks

1 mark for a description of how simulated annealing could be used in this problem

1 mark for description of how the temperature value is used to help find the global minimum and not a local minimum

c. Is randomised heuristics guaranteed to find the location marked with the blue dot? 1 mark

1 mark for stating that randomised heuristics may find the optimal solution but will be unable to verify this as being optimal