Name:

ALGORITHMICS UNIT 3 & 4

Trial Exam 2: 2019

Reading Time: 15 minutes
Writing time: 120 minutes (2 hours)

QUESTION AND ANSWER BOOK

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	9	9	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers and one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape

Materials supplied

- Question and answer book of 21 pages
- Answer sheet for multiple-choice questions

Instructions

- Write your student number in the space provided above on this page.
- Check that your name and student number as printed on your answer sheet for multiple-choice questions are correct, and sign you name in the space provided to verify this.

- All written responses must be in English, point form is preferred.

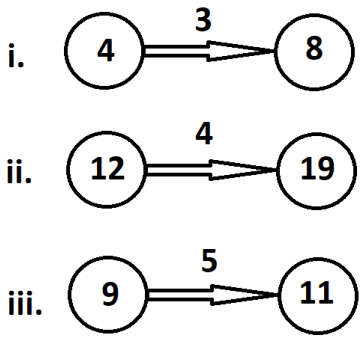
Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the test room.

The Master Theorem as provided on VCAA exam papers is:	$T(n) = \begin{cases} aT\left(\frac{n}{b}\right) + kn^c, & \text{if } n > 1 \\ d, & \text{if } n = 1 \end{cases}$ where $a > 0, b > 0, c \geq 0, d \geq 0, k > 0$ and its solution $T(n) = \begin{cases} O(n^c), & \text{if } \log_b a < c \\ O(n^c \log n), & \text{if } \log_b a = c \\ O(n^{\log_b a}), & \text{if } \log_b a > c \end{cases}$
Georgia's note: *Recall the log laws, for an alternate representation that shows the growth of the recursion in the layers. Also easier to work out!*	$\begin{cases} \log_b a < c \Rightarrow \frac{a}{b^c} < 1 \\ \log_b a = c \Rightarrow \frac{a}{b^c} = 1 \\ \log_b a > c \Rightarrow \frac{a}{b^c} > 1 \end{cases}$

SECTION A – Multiple Choice – select one option only

Question 1

What is the result of relaxing each of the following weighted edges shown at right, where the node labels show the distances calculated so far?



A. 8 becomes 7, 19 becomes 16, 11 becomes 14

B. 8 becomes 8, 19 becomes 16, 11 becomes 14

C. 8 becomes 8, 19 becomes 19, 11 becomes 11

D. 8 becomes 7, 19 becomes 16, 11 becomes 11

Question 2

Will loop infinitely	Pseudocode
(i)	01 x = 0 02 while True 03 print x 04 endwhile
(ii)	01 x = 0 02 while x < 10 03 print x 04 endwhile
(iii)	01 x = 0 02 while x < 10 03 print x 04 x = x + 1 05 endwhile
(iv)	01 y = 5 02 for x = 1 to y 03 print x 04 next

Which of the numbered pseudocode fragments will loop without terminating in reasonable time?

A. (i),(iii)

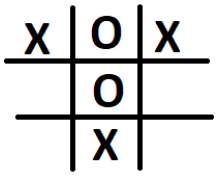
B. (ii),(iv)

C. (i),(ii)

D. (i),(iv)

Question 3

Consider the directed graph where nodes are reachable tic-tac-toe board positions and edges represent valid moves. What are the in-degree and the out-degree respectively of the following game node when it is O's turn to play?



A. In-degree is 3, out-degree is 4

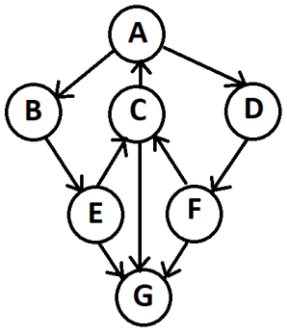
B. In-degree is 4, out-degree is 4

C. In-degree is 6, out-degree is 4

D. In-degree is 9, out-degree is 9

Question 4

If you perform a depth first search on the directed graph shown above, starting at node A and whenever faced with a decision of which node to pick from a set of nodes, choose the one that is earliest in the alphabet, the sequence of node traversal will be:



A. A B E G C D F

B. A B E C G D F

C. A B D E F C G

D. A B D C F E G

Consider the following pseudocode fragment to answer Question 5 and Question 6

```
01 for k = 1 to 10 do
02    for p = 1 to k do
03        print (k + p)
04    end do
05 end do
06 m = 7
07 print m * m
```

Question 5

The first three numbers that will be printed by this pseudocode fragment are:

A. 1,2,49

B. 2,6,8

C. 1,2,3

D. 2,3,4

Question 6

Line 03 is executed

A. 35 times

B. 55 times

C. 10 times

D. 100 times

Question 7

With regard to Artificial Intelligence, which of the following statements is FALSE?

A. In order to manipulate symbols, a computer needs to understand what they mean.

B. Solving a problem involves using actions to transition from a start state to a goal state.

C. Problems are solved through the manipulation of symbols.

D. Artificial Intelligence programs reduce problems to symbols.

Question 8

John Searle created a thought experiment where he sat in a room and was passed messages written in Chinese. He would look up the symbols in a book and write down the corresponding symbols as a reply. To an outside observer it would appear that he understood Chinese. Searle used this thought experiment to illustrate that computers were not intelligent. What was his **main** point?

A. Humans can understand Chinese symbols but computers can't.

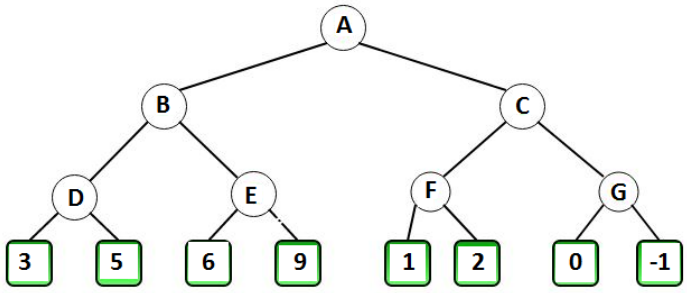
B. Manipulating symbols requires intelligence.

C. The replies were used to determine if Searle was a human or a computer.

D. Computers just manipulate symbols without understanding them.

Question 9

A game tree for the maximising player is shown at point A in the game for a turn taking game. According to the minimax algorithm, which is the most favourable path that the maximising player should choose against an equally capable opponent?



A. A-B-E

B. A-C-F

C. A-B-D

D. A-C-G

Question 10

Consider the following pseudocode snippet

```
y := 0 ;
while y < n do (
     y := y + 1 ;
)
```

The best and worst time complexity respectively is:

A. O(1), O(n)

B. Ω(1), O(n)

C. Ω(n), O(n)

D. Ω(n), O(1)

Question 11

Consider the following pseudocode snippet

```
y := 1 ;
while y < n do (
     x := n ;
     while x > 0 do (
         x := x - y ;
     );
     y := y + 1 ;
)
```

The number of actions performed in the worst case as a function of n, is most closely represented by:

A. $\frac{n(n+1)}{2}$

B. $\frac{n(n+1)(2n-1)}{6}$

C. $\frac{n}{1!}+\frac{n}{2!}+\frac{n}{3!}+\frac{n}{4!}+\frac{n}{5!}+.....+\frac{n}{n!}$

D. $n+\frac{n}{2}+\frac{n}{3}+\frac{n}{4}+\frac{n}{5}+.....+1$

Question 12

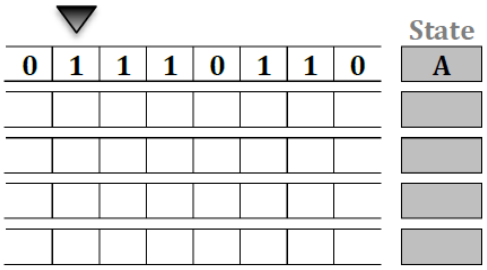
A Turing machine with 6 states is defined as follows:

State A				State B			
Read	Write	Move	State	Read	Write	Move	State
0	0	R	A	0	0	R	C
1	0	R	B	1	1	R	B

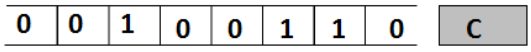
State C				State D			
Read	Write	Move	State	Read	Write	Move	State
0	0	R	C	0	0	L	D
1	0	R	F	1	1	L	E

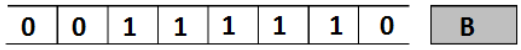
State E				State F			
Read	Write	Move	State	Read	Write	Move	State
0	0	R	A	0	0	Halt	
1	1	L	E	1	1	L	D

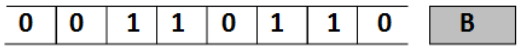
For the following tape in state A, at the position indicated by the arrow.

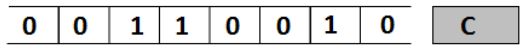


After 4 operations of the TM, the symbols on the tape and current state will be:

A. 

B. 

C. 

D. 

Question 16

The recurrence relation $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}, \text{where } T(1) = c$ representing the time complexity of an algorithm, where c is a constant, is equivalent to:

A. $T(n) = O\left(\sqrt{n}\right)$

B. $T(n) = O\left(n^3\right)$

C. $T(n) = O(n)$

D. $T(n) = O(1)$

Question 17

Using dynamic programming, the output to stage 9 of a coin change problem to find the minimum number of coins to be given as change where the coins denominations are coins={1^c,3^c,4^c} and the target change is 10 cents, is formed by:

A. Using stage n-1 of the calculations.

B. Using stage 8 of the calculations.

C. Using stage 8,6,5 of the calculations.

D. Using stage 9,8,7 of the calculations.

Question 18

Consider recursive function `geo` defined as follows (for positive integers input only):

```
Function geo(n)
// Input n is a natural number
If (n≤1)
    Return n
Else
    Return geo(n-1)+2n
Endif

End function
```

According to this function, if geo is called with `geo(3)` the value returned by the function will be:

A. 8

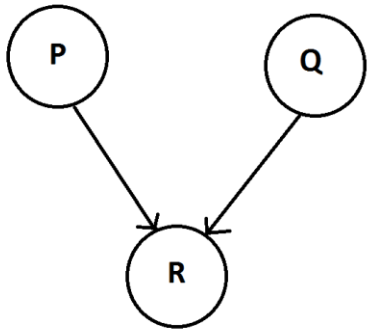
B. 9

C. 10

D. 11

Question 19

Suppose that P, Q, and R are different web pages in a system that are connected by directed edges as shown in the diagram.



Which possible Page Rank web page calculations matches the outcome **before** and **after** a directed link from P to Q is added?

A. Before: $\Pr(R_{i+1}) = \Pr(Q_i) + \Pr(P_i)$ After: $\Pr(R_{i+1}) = \Pr(Q_i) + \frac{1}{2}\Pr(P_i)$

B. Before: $\Pr(R_{i+1}) = \Pr(Q_i) + \Pr(P_i) + \frac{1}{3}\Pr(R_i)$ After: $\Pr(R_{i+1}) = \Pr(Q_i) + \frac{1}{2}\Pr(P_i) + \frac{1}{3}\Pr(R_i)$

C. Before: $\Pr(R_{i+1}) = \Pr(Q_i) + \frac{1}{2}\Pr(P_i) + \Pr(R_i)$ After: $\Pr(R_{i+1}) = \Pr(Q_i) + \frac{1}{3}\Pr(P_i)$

D. Before: $\Pr(R_{i+1}) = \Pr(Q_i) + \frac{1}{2}\Pr(P_i) + \frac{1}{3}\Pr(R_i)$ After: $\Pr(R_{i+1}) = \Pr(Q_i) + \Pr(P_i) + \frac{1}{2}\Pr(R_i)$

Question 20

'Push' and 'Pop' are appropriate operations for which data structure?

A. a list

B. a queue

C. a stack

D. an array

SECTION B – Extended Response Questions Answer all questions in the space provided.

Question 1 (7 marks)

Let F_1, F_2, ….F_n be the size of n computer files that need to be stored on a disk with capacity of D gigabytes.

We cannot store all these files in this case as $\sum_{i=1}^{n} F_i > D$.

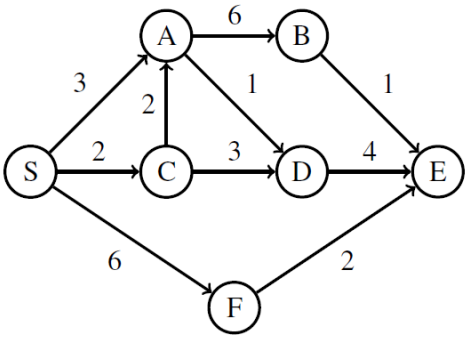
a. Does a greedy algorithm that selects files in order of non-decreasing F_i size maximise the possible **number of files** held on the disk? Justify your answer with a proof or counter example. (2 marks)

b. Does a greedy algorithm that selects files in order of non-decreasing file size F_i, use as much of the capacity of the disk as possible? Justify your answer with a proof or counter example. (2 marks)

c. With reference to Cobham's thesis and the Church Turing thesis, what is the time complexity and computability of an approach to finding the **optimal** solution for parts a) and b)? (3 marks)

Question 2 (6 marks)

Consider the following directed and weighted graph.



a. Show the distance information in each main step of the execution of Dijkstra's algorithm starting at node S, and state the order that the nodes are removed from the priority queue. (2 marks)

b. Show the resulting shortest path tree starting at node S with distances annotated on the nodes. (2 marks)

c. If the edge C-D is reversed on the original graph, and its weight multiplied by negative 1, explain the properties of the updated graph and discuss how or if Dijkstra's can be implemented. (2 marks)

Question 3 (11 marks)

Suppose that you are given 3 words in variables X, Y and Z, respectively, where length(X)=n, length(Y)=m and length(Z)=n+m. The word Z is said to be a shuffle of the letters in words X and Y formed by interleaving the letters from X and Y in such a way that the left-to-right ordering is maintained.

a. If X=chocolate and Y=chips show that Z=cchocohilaptes is a shuffle of X and Y, but chocochilatspe is not. (2 marks)

Here is an algorithm to work out if Z is a shuffle of X and Y.

```
Function IsShuffle (X, Y, Z)
// Inputs words X, Y, Z, treated as list of letters
// Outputs Boolean TRUE/FALSE if shuffle exists
If length(Z) is not equal to (length(X)+length(Y)) then
          Return FALSE
Else
     For i=1 to length(Z) do
        If ith element of Z is not in X or Y then
              Return FALSE
        End If
     End do
End if

If ((length(Z)=0) and length(X)=0 and length(Y)=0 then
         Return TRUE
Else If (1st element of Z equals 1st element of X) then
         Remove first element of X
         Remove first element of Z
         isShuffle(X,Y,Z)
Else if (1st element of Z equals 1st element of Y) then
         Remove first element of Y
         Remove first element of Z
         isShuffle(X,Y,Z)
Else
         Return FALSE
End if
```

To call the function `IsShuffle` we prepare by

Setting the global variables

```
Create list X
Set X=[c|h|o|c|o|l|a|t|e]
Create list Y
Set Y=[c|h|i|p|s]
Create list Z
Set Z=[c|c|h|o|c|o|h|i|l|a|p|t|e|s]
```

Then the algorithm is called

```
Print IsShuffle(X,Y,Z)
```

a. What are the main design patterns used in the algorithm? Justify your response using the modules defined and identify the lines of pseudocode for each design pattern described. (2 marks)

Question 3 (continued)

b. Prove by induction that IsShuffle will return the correct answer for any three words X, Y, Z where each variable is input as a list of letters.. (3 marks)

c. What is the time complexity of IsShuffle for the best and worst case? Justify your response. (2 marks)

d. Is it possible to improve the efficiency of IsShuffle. Describe how this could be done and outline the new time complexity for the improvement. (2 marks)

Question 4 (8 marks)

a. Write a function called mapSquares in structured pseudocode that takes a list of integers called keys as a parameter and creates and fills a dictionary mapping each of the elements in keys to their squares. For example, if 5 is an element of keys, the dictionary returned by the function would contain a mapping from the key 5 to the value 25. (3 marks)

b. Write an algorithm FindAnySymbol in structured pseudocode to find the first position in an array of symbols stored in variable **s** that holds any symbol from another array called **d**, or -1 if there is no such position. For example, if **s=[M|o|n|d|a|y]** and **d=[a|e|i|o|u]**, it will find the first vowel in **s** which is at position 2, if **s=[t|i|g|e|r]** and **d=[z|o|o]**, it will return -1 since none of the letters in **d** are in **s.**(5 marks)

Question 5 (10 marks)

a. Show the steps that the simplest Mergesort algorithm would take to put the following list of codes into ascending alphabetical order (from A to Z). P , B , F , J , T , A , Z , H (2 marks)

b. Describe the design pattern used by the Mergesort algorithm. Explain how it works and situations where it is appropriate to use this design pattern to make algorithms more efficient. (3 marks)

c. In general how is the time complexity worked out for this type of design pattern? Describe the mathematical constructions used? Demonstrate the technique for working out the time complexity of the simplest Mergesort implementation. (3 marks)

Question 6 (10 marks)
Happytown are holding an election with three candidates (A, B and C). The voting system is preferential and voters must mark each candidate in order of preference 1, 2, 3.

a. Using the graph abstract data type as a main building block, describe a model composed of abstract data types that can hold the information for the following:
   - keeps track of how many first, second, and third preferences are given to each candidate
   - keeps track for each first preference candidate where the second and third preferences have been allocated
   - keeps track for each second preference candidate where the third preference has been allocated

(3 marks)

b. Show the main operations required to construct and maintain the model you have described in part a. for holding the information for this problem. (3 marks)

Question 6 (continued)

The first 5 voting papers counted for this election are as follows:

Paper 1	Paper 2	Paper 3	Paper 4	Paper 5
1. A	1. B	1. B	1. B	1. C
2. B	2. C	2. A	2. A	2. A
3. C	3. A	3. C	3. C	3. B

c. Show how the model you have defined in parts a. and b. will hold this information, by using an annotated diagram or similar. (4 marks)

Question 7 (10 marks)

a. Briefly explain the following concepts and give an example problem to which it relates to:
   - uncomputability
   - undecidability (2 marks)

b. What is the Church-Turing Thesis? Provide an argument that testing whether a number is a prime number is a decidable or undecidable function. (3 marks)

In Mathematics the theory of addition of natural numbers is complete and decidable (Presburger 1929), as is the theory of the multiplication of positive integers (Skolem 1930).
These proofs of completeness and decidability for addition and multiplication of natural numbers resulted from Hilbert's program of formalisation of Mathematics.

c. Explain what Hilbert's program was trying to achieve? What was the motivation for the program? Explain the limitations and implications for Mathematics and Computer Science, that were subsequently realised or discovered. (3 marks)

Question 8 (8 marks)

a. . Describe the Travelling Salesman Problem (TSP) and explain, informally, why it is classed as a very hard, indeed, intractable, problem. (2 marks)

b. Explain what is meant by the term problem size. What is used as the measure of problem size for the TSP? (2 marks)

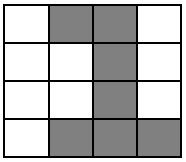
c. Explain a "brute force" method for obtaining a solution to the problem, then give and informally justify its worst case time performance measure in terms of the problem size (given in answer to part 8b above)? (2 marks)

d. A particular computer, can process 1000 tours a second. Give an estimate, with a justification for the time taken (clearly stating your units of time measure) to solve by your brute force method, a TSP with problem size 21. (You may assume that 20! is approximately 2.43×10^{18}.)? (2 marks)

Question 9 (10 marks)

Consider the case of recognising text and the use of Neural Networks.

A digital image of the number "1" represented by 16 shaded cells on a 4x4 matrix.



a. Describe in detail a method that can be used to input a digital image, like the one shown into the Neural Network based on the 4x4 matrix of 16 shaded cells. (1 mark)

b. Show a pictorial representation of part b) of your answer. (2 marks)

c. After the input step has completed, describe briefly the next 3 main stages of processing performed by the Neural Network. (3 marks)

Question 9 (continued)

	A neural network has been set up that recognises handwriting and detects mistakes in spelling and grammar, as well as making suggestions about how to restructure sentences to improve clarity and style. This is every English teacher's dream neural network.

After trialling this neural network for several months it appears so intelligent and so effective that Ms. Took, a very new English teacher starts using it to "read" and grade her students' essay submissions.

d. Using the **Robot** and **Brain Simulator** standard responses for and against the Chinese Room argument outline an argument for Ms. Took to either continue, or stop using this neural network to help her grade her students. (4 marks)

END OF TRIAL EXAM 2