Model Structures and Fitting Criteria for System Identification with Neural Networks

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Motivations

Recurrent Neural Networks are commonly used to model dynamical systems. However, they seldom exploit available a priori knowledge.

In this work:

- We present tailor-made model structures for system identification with Neural Networks
- We develop efficient algorithms to fit these model structures to data.

Motivations

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In this work:

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Settings

The data-generating system S_o is assumed to have the discrete-time state-space representation:

$$\begin{aligned} x_{k+1} &= f(x_k, u_k) \\ y_k^{\mathrm{o}} &= g(x_k) \\ y_k &= y_k^{\mathrm{o}} + e_k \end{aligned}$$

Training dataset $\mathcal{D}$ consisting of N input samples $U = \{u_0,\ u_1, \ldots,\ u_{N-1}\}$ and output samples $Y = \{y_0,\ y_1, \ldots,\ y_{N-1}\}$ available.

Objective: estimate a dynamical model of S_o.

Neural Dynamical Models

A very generic neural model structure:

$$
\begin{aligned}
x_{k+1} &= \mathcal{N}_f(x_k, u_k;\ \theta) \\
y_k &= \mathcal{N}_g(x_k; \theta)
\end{aligned}
$$

where $\mathcal{N}_f$, $\mathcal{N}_g$ are feed-forward neural networks. Can be specialized:

- Linear approximation available $\Rightarrow$

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\begin{aligned}
x_{k+1} &= Ax_k + Bu_k + \mathcal{N}_f(x_k, u_k; \theta) \\
y_k &= Cx_k + \mathcal{N}_g(x_k, u_k; \theta)
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- State fully observed $\Rightarrow$

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$$x_{k+1} = \mathcal{N}_f(x_k, u_k;\ \theta)$$
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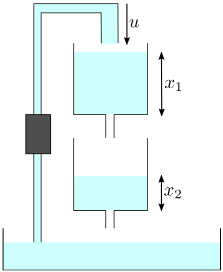
Neural Dynamical Models

Physics-inspired model structures

Two-tank system, input=flow u in upper tank, output=lower tank level x_2.

- The system has two states: x_1 and x_2
- The state x_1 does not depend on x_2
- The state x_2 does not depend directly on u
- The state x_2 is observed



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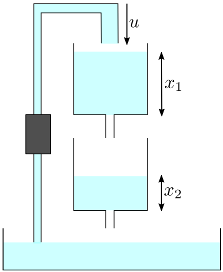

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Training Neural Dynamical Models

In principle, simulation error minimization is a valid strategy:

$$\theta^o = \arg\min_{\theta, \hat{x}_0} \sum_{k=0}^{N-1} \left\| \hat{y}_k(\theta, \hat{x}_0) - y_k \right\|^2$$

where

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for $k = 0, 1, \ldots, N-1$.

However, it is not convenient from a computational perspective:

- Simulation is not parallelizable. Several neural network evaluations have to be performed sequentially.
- Back-propagation cost increases with the sequence length

In this work, we minimize instead the simulation error over batches of q subsequences, each one of length $m \ll N$.

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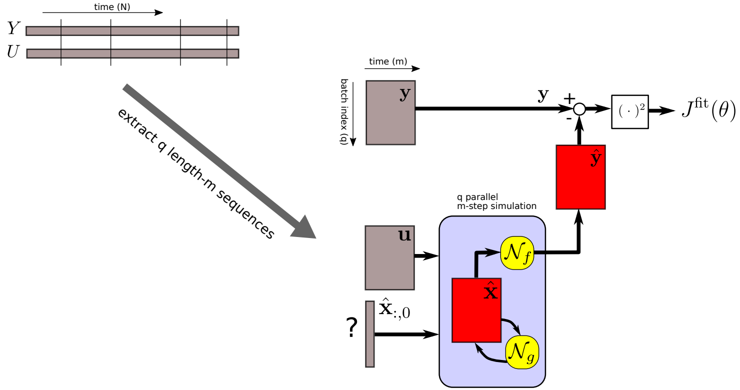
Training Neural Dynamical Models

for each iteration of gradient-based optimization:



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We do not know it, but we need it to initialize all simulations.

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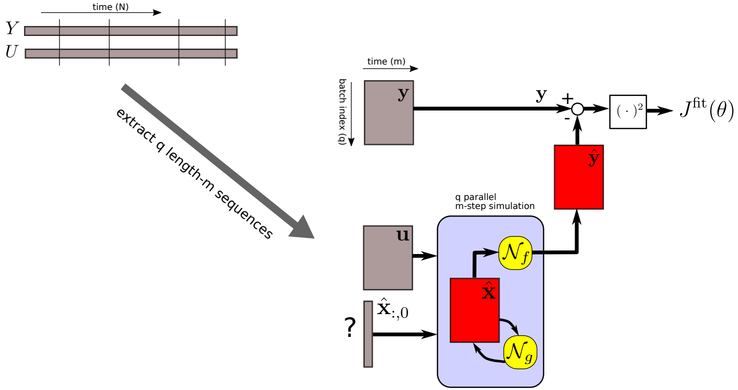

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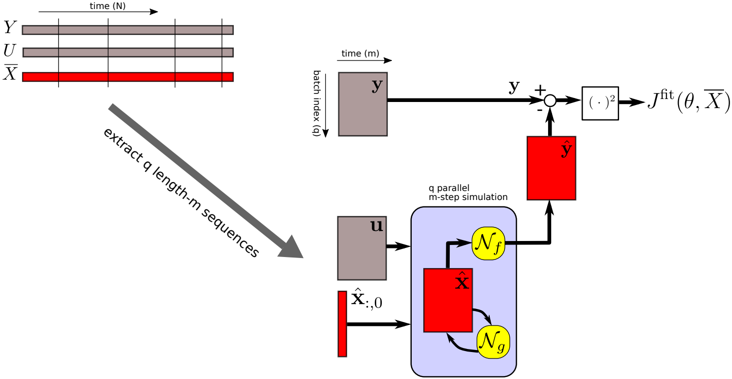
Training Neural Dynamical Models

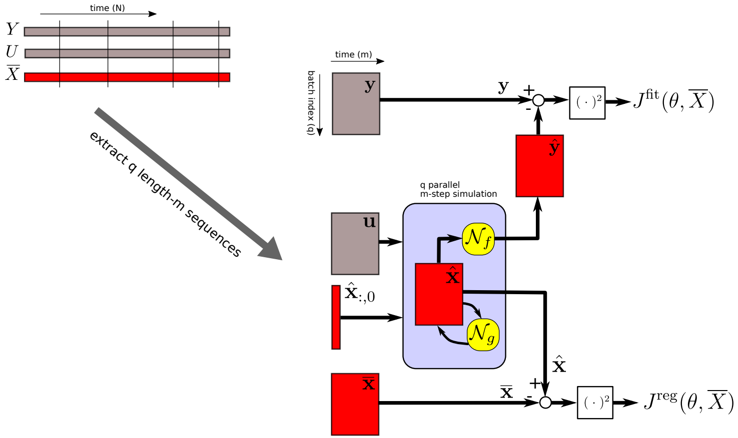
We consider the unknown state sequence $\overline{X}$ as an optimization variable.
We sample from $\overline{X}$ to obtain the initial state for simulation in each batch.



J^{fit} is now a function of both θ and $\overline{X}$. We optimize w.r.t. both!

Training Neural Dynamical Models

The hidden state sequence $\overline{X}$ should also satisfy the identified dynamics!
We enforce this by adding a regularization term in the cost function.



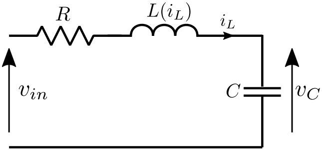
We minimize a weighted sum of J^{fit} and J^{reg} w.r.t. both θ and $\overline{X}$.

Simulation example

RLC circuit

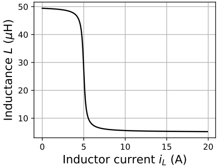
We consider a nonliner RLC circuit:

$$\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{-1}{L(i_L)} & \frac{-R}{L(i_L)} \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L(i_L)} \end{bmatrix} v_{in}$$



with nonlinear inductance $L(i_L)$

$$L(i_L) = L_0 \left[\left(\frac{0.9}{\pi} \arctan\left(-5(|i_L| - 5) + 0.5 \right) \right) + 0.1 \right]$$



Input: voltage v_{in}. Output: voltage v_C, current i_L. SNR=20

Neural model structure: fully observed state

$$x_{k+1} = \mathcal{N}_f(x_k, u_k; \theta)$$

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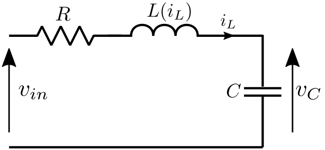

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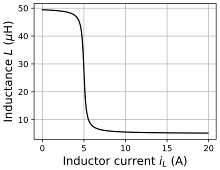

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Neural model structure: fully observed state

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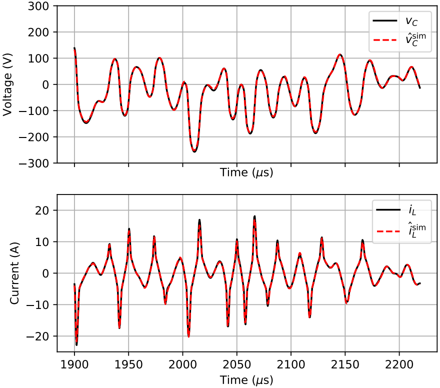
Numerical example

RLC circuit

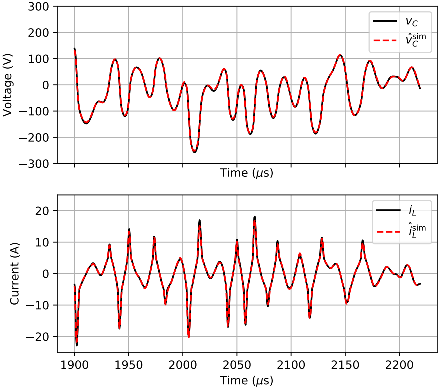
Results in simulation on the test dataset. Training with:

$q = 62$ sequences of length $m = 64$



train time: 320 s

simulation error minimization

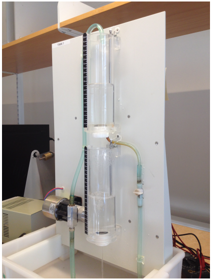


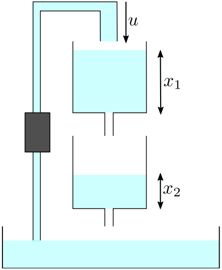
train time: 7000 s

Numerical example

Cascaded Tank System

Dataset with real measurements from `www.nonlinearbenchmark.org`





Neural model structure: physics-inspired

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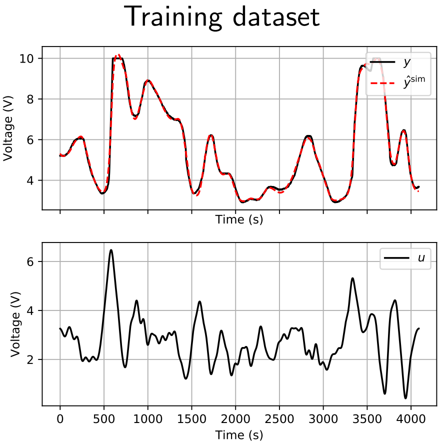
\dot{x}_2 &= \mathcal{N}_2(x_1, x_2, u) \\
y &= x_2
\end{aligned}
$$

The dependency of $\mathcal{N}_2$ on u models water overflow from upper tank.

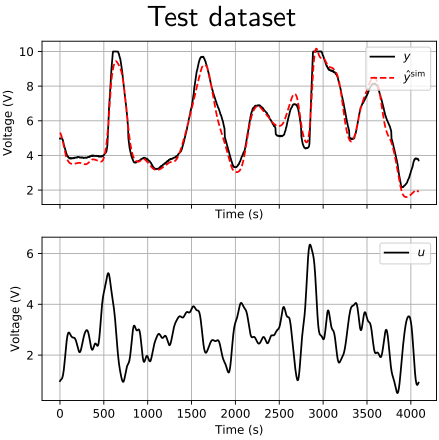
Numerical example

Cascaded Tank System

Training with $m = 128, q = 64$. Results on:



$R^2 = 0.99$, RMSE $= 0.08$ V



$R^2 = 0.97$, RMSE $= 0.33$ V

Conclusions

We have presented tailor-made neural structures for system identification embedding a priori knowledge.

We have shown how to parallelize the training using batches of short-size subsequences, and taking into account the effect of the initial condition.

Current/Future work

- Extension to the continuous-time setting
- Learning of Partial Differential Equations

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Thank you.
Questions?

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