Complex Fourier series coefficients

Why calculate c_0 and c_n separately?

In the lecture notes, we have that the formula

$$c_n = \frac{1}{2L} \int_{-L}^{L} f(x) \exp\left(-\frac{in\pi x}{L}\right) \mathrm{d}x$$

applies for all n, including $n = 0$, so why do we calculate c_0 separately? Well, it turns out when we try to do the generic integral for c_n, we get some really complicated stuff that is (in the best cases) resolved with integration by parts, due to the presence of the exponential function. The result of the integration is valid for any value of n that makes the exponential term appear (otherwise we wouldn't even need to do integration by parts, etc).

So, we see that exactly when $n = 0$, the exponential term equals 1 and drops out of the integrand. That means for the specific instance $n = 0$, and only for that instance, we are left with

$$c_0 = \frac{1}{2L} \int_{-L}^{L} f(x) \, \mathrm{d}x = a_0$$

So while the initial integral is still *true* for the $n = 0$ case, this integral takes an entirely different form, so we should calculate it separately. Then we put all the solutions for different n together to get

$$f(x) = c_0 + \sum_{n=-\infty;\ n \neq 0}^{\infty} c_n \exp\left(\frac{in\pi x}{L}\right)$$