CME 100 Final topics[1]

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December 8, 2018

Note: This is a list of topics that *I personally think* you should know for the final. It's more or less everything you've learned this quarter, but the actual final might test more or less. Yes, the final is cumulative, just like your other life experiences. I tried to be comprehensive, yet succinct. In any case, the one thing you should think is, "Wow, I've learned so much this quarter!" :)

1 Vectors

- Vector representation in 2D and 3D
- Magnitude (norm, $\|\vec{u}\|$) and direction (unit vector, $\vec{u}/\|\vec{u}\|$) of a vector
- Addition, subtraction, and scalar multiplication of vectors
- Dot product (scalar), cross product (vector), and outer product (matrix) of vectors
- Projections, distance from point to {line, plane}, area of parallelogram
- Equations of lines and planes in space
- Parallel and perpendicular vectors, intersections of planes
- Vector-valued functions, trajectories ($\vec{r}$), velocity ($\vec{v}$), acceleration ($\vec{a}$)
- Arc length (s), unit tangent vector ($\vec{T}$), unit normal vector ($\vec{N}$), binormal vector ($\vec{B}$)
- Curvature (κ), tangential and normal acceleration (a_T and a_N), torsion (τ)

2 Linear Algebra

- Matrix dimensions, transpose (A^T), {identity (I), triangular (L and U), diagonal (D)} matrices
- Addition, subtraction, {scalar, matrix-vector, matrix-matrix} multiplication
- System of equations, Gaussian elimination, {no, unique, infinitely-many} solutions
- Rank, linear independence, singular
- Inverse matrix (A^{-1}), Gauss-Jordan elimination, determinants ($|A|$)

[1] As always, if you have any questions, or if you think of something I should add, let me know!

3 Multivariable Differentiation

- **Function** $\stackrel{\text{def}}{=}$ mapping from *domain* to *range*. Level sets, limits
- Partial derivatives (∂), chain rule, implicit differentiation, variables held constant
- Linearization, error of approximation
- Gradient (∇f), directional derivative, level surfaces
- Gradient descent, unconstrained optimization, {local, absolute} {max, min}, saddle points
- Method of least squares, constrained optimization, Lagrange multipliers (λ)

4 Multivariable Integration

- {Double, triple} integrals, area, volume, mass, center of mass ($\overline{x}$), moment of inertia (I_x)
- {Polar, cylindrical, spherical} coordinates, Jacobian, coordinate transformations
- Line integrals ($\int_C$), vector fields ($\vec{F}$), flux ($\int_C \vec{F} \cdot \hat{n}\, \mathrm{d}s$), circulation/work ($\int_C \vec{F} \cdot \vec{T}\, \mathrm{d}s$), conservative fields ($\vec{F} = \nabla f$)
- Divergence ($\nabla \cdot \vec{F}$), curl ($\nabla \times \vec{F}$), surface integrals ($\iint \mathrm{d}S$), {Green's, Stokes', Divergence} Theorem.

5 Miscellaneous

- Newton's second law: $\sum \vec{F} = m\vec{a}$.
- Simple mass-spring physics: $F = -kx \longleftrightarrow$ systems of equations
- Parabolic motion: $x = x_0 + v_0 t + \frac{1}{2}at^2$
- Torque: $\vec{\tau} = \vec{r} \times \vec{F}$
- Trig identities: I like this site.
- MATLAB: {`for`, `while`} loops, matrix ops, `.*` vs `*`, `if/else`, index vs. value (`n` vs. `t(n)`)

6 Flow charts

There's been some demand for flow charts to help organize the problem solving method. This is what I've attempted to make below, and you can use it however you see fit. If there's an error let me know. I'll also caution that there are multiple ways to solve a problem, so don't get pigeonholed into one method by the flow chart!

General approach to integration:

(1) Sketch the region of integration

(2) Write out the general formula for the integral

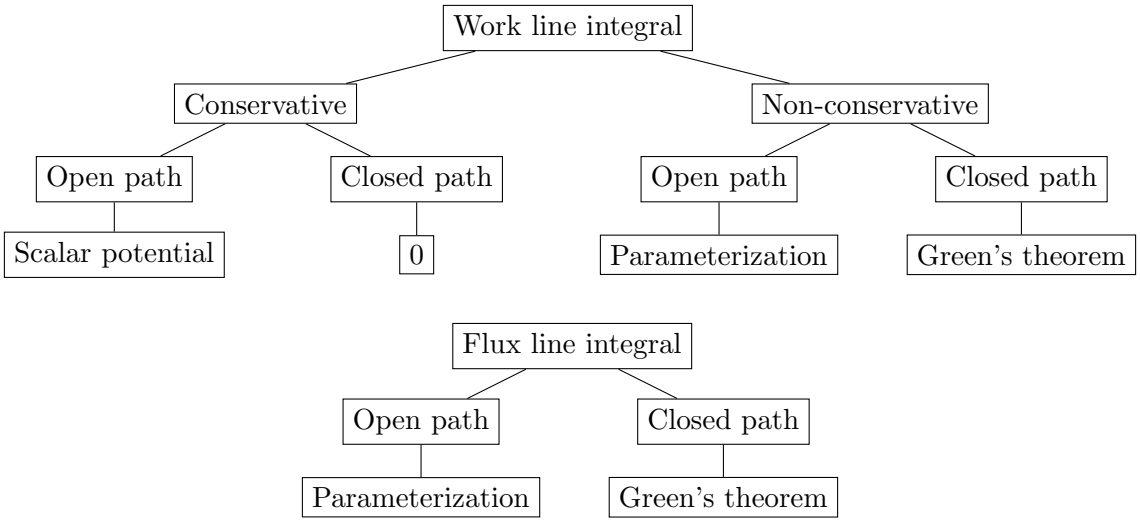
(3) Change coordinate system as needed (Cartesian to cylindrical, dS to dA, etc)

(4) Compute the value of every term of the integrand

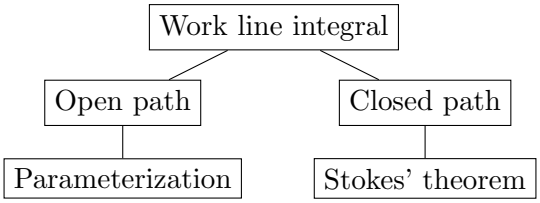
(5) Determine the limits of integration

(6) Do what you were born to do: integrate

6.1 2D Integrals



6.2 3D Integrals



The above is for non-conservative vector fields. If the vector field is conservative the 2D flow chart applies just fine.