# Section 13.5, Problem 20

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Here's a more correct version of what I tried to explain in office hours. We want to find the force vector, and we know by Newton's second law that $\vec{F} = m\vec{a}$. So really we need the acceleration vector $\vec{a}$. Now, there are two approaches to this problem. One is to straight up find the vector by differentiating $\vec{r}(x)$ twice. Another way is to break it down into

$$\vec{a} = a_T\vec{T} + a_N\vec{N}$$

I showed the second way in OH and that's what I'll explain here. We have by the parametrization that

$$\begin{aligned}
\vec{r}(x) &= \begin{bmatrix} x & x^2 \end{bmatrix} \\
\vec{v}(x) &= \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \\
&= \begin{bmatrix} \frac{\mathrm{d}x}{\mathrm{d}t} & 2x\frac{\mathrm{d}x}{\mathrm{d}t} \end{bmatrix} \\
&= \begin{bmatrix} \dot{x} & 2x\dot{x} \end{bmatrix} \\
\|\vec{v}\| &= \sqrt{\dot{x}^2 + (2x\dot{x})^2} \\
&= \dot{x}\sqrt{1 + 4x^2} = 10
\end{aligned}$$

Note that we are differentiating with respect to $t$, and that is why we need to apply the chain rule when differentiating the $x$'s (implicitly assumed to be a function of $t$). This is what I **forgot** in OH but hopefully my steps above are clear as to how this is done. Doing it this way makes clear two things:

(1) The problem states that $\|\vec{v}\| = 10$, that the particle travels with constant speed, and now our expression for $\|\vec{v}\|$ should make it clear that it's totally possible for $\dot{x}\sqrt{1+4x^2}$ to be a constant. Previously I did not use chain rule and didn't have the $\dot{x}$.

(2) Now when we're find the unit tangent vector, we want to do

$$\vec{T} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\dot{x}\sqrt{1+4x^2}}\begin{bmatrix} \dot{x} & 2x\dot{x} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{1+4x^2}} & \frac{2x}{\sqrt{1+4x^2}} \end{bmatrix}$$

Now, we've shown that $\|\vec{v}\|$ is a constant, so you could divide by 10 instead of the full expression for $\|v\|$. But if you did that, $\dot{x}$ would still remain in the numerator, and then you'd have to solve for that, which is a pain, and... yeah, so normalizing by the full expression for $\|\vec{v}\|$ is the better way to go here (even though they're numerically equivalent). Note that the final expression for $\vec{T}$ is the same expression we derived in OH (the $\dot{x}$ cancels out woot woot). But this way is more correct.

Everything else I showed in OH still applies and directly follows after this step. With $\vec{T}$ you can get $\vec{N}$ with the handy rotation trick. This document just clears up some confusion with these steps specifically. Thanks for your patience. If you have more questions, feel free to reach out.