## Checkmating One, by Using Many: Combining Mixture of Experts with MCTS to Improve in Chess

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#### Abstract

This paper presents a new approach that integrates deep learning with computational chess, using both the Mixture of Experts (MoE) method and Monte-Carlo Tree Search (MCTS). Our methodology employs a suite of specialized models, each designed to respond to specific changes in the game's input data. This results in a framework with sparsely activated models, which provides significant computational benefits. Our framework combines the MoE method with MCTS, in order to align it with the strategic phases of chess, thus departing from the conventional "one-for-all" model. Instead, we utilize distinct game phase definitions to effectively distribute computational tasks across multiple expert neural networks. Our empirical research shows a substantial improvement in playing strength, surpassing the traditional single-model framework. This validates the efficacy of our integrated approach and highlights the potential of incorporating expert knowledge and strategic principles into neural network design. The fusion of MoE and MCTS offers a promising avenue for advancing machine learning architectures.

#### 1 Introduction

Neural Networks (NNs) are crucial to the progress of machine learning and artificial intelligence. However, they face a significant challenge when applied to diverse and dynamic tasks. The traditional one-for-all network approach, while versatile, often fails to effectively address the complexities of varied applications [Riquelme *et al.*, 2021; Artetxe *et al.*, 2021].

This raises the question: should we continue to rely on monolithic network architectures, or is it time to pivot to more modular and adaptable structures? Recent explorations [Agarwala *et al.*, 2021] suggest that the latter may hold greater promise. The Mixture of Experts (MoE) paradigm, introduced in 1991 by Jacobs *et al.* [1991], has demonstrated remarkable versatility across multiple domains

of machine learning. Its significant role in shaping the development of Large Language Models (LLMs) [Fedus *et al.*, 2022; Du *et al.*, 2022] is particularly noteworthy. Building on this success, we propose reintegrating MoE into the domain of chess and other board games. MoE provide a strategic approach to breaking down complex tasks into smaller, more manageable segments, allowing a team of specialized experts to collaboratively tackle these challenges.

In the field of chess, a game with dynamic and strategic nuances, this approach takes on special significance. Traditional engines like Stockfish utilized phase-specific heuristics, but contemporary models such as AlphaZero [Silver *et al.*, 2018], MuZero [Schrittwieser *et al.*, 2020], and NNUE-enhanced Stockfish [Nasu, 2018] have moved towards a unified network approach for all game phases. We argue that taking a nuanced approach, which recognizes the distinct characteristics of each phase of chess (opening, middlegame, and endgame), can result in significant performance improvements.

Our goal is to create a chess agent that can adapt its strategy to the changing phases of the game, similar to skilled human players. We investigate the combination of MoE with Monte-Carlo Tree Search (MCTS) in modern chess engines, as shown in Figure 1.

Empirically, we demonstrate that our phase-specific architecture<sup>1</sup> outperforms traditional single-model approaches in the game of chess, achieving a substantial increase in playing strength by approximately 120 Elo points.

Our contributions can be summarized as follows:

- We investigate various training strategies for expert models and establish that a straightforward, separated learning regime is as effective, if not more so, than complex alternatives.
- We underscore the necessity and effectiveness of a MoE approach in MCTS for chess and other classical board games.
- Through rigorous testing, we confirm that incorporating a small number of out-of-distribution samples in batch-wise inference does not diminish an expert's performance.

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<sup>&</sup>lt;sup>1</sup>https://anonymous.4open.science/r/CrazyAra-3F10

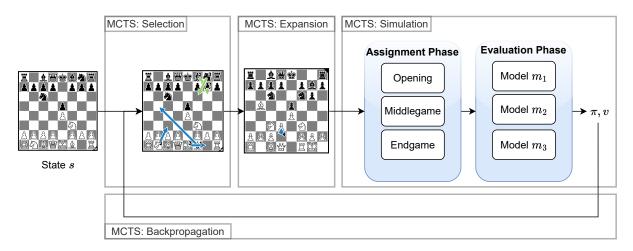


Figure 1: **Incorporating MoE into MCTS based on game phase selection.** First, a new sample is allocated in the MCTS selection phase. Next, a new leaf node of the search tree is expanded. In the simulation phase the game state is assigned to its designated phase. At last, the appropriate model is chosen to evaluate the sample and to backpropagate the evaluation result in the search tree.

We start by explaining our game-phase-specific MoE architecture. Subsequent sections present an extensive evaluation, including its performance against traditional single-network models. Before we conclude we discuss related work and limitations.

## 2 Combining MCTS and MoE

The MoE paradigm, an innovative approach in AI, consists of n specialized "expert networks"  $m_1, m_2, \ldots, m_n$  complemented by a strategic "gating network" G. This network outputs a sparse n-dimensional vector that effectively acts as a classifier. Each expert network is a distinct NN, with its own parameters and architecture, yet designed to have a uniform input and output shape across all networks.

The power of MoE lies in its ability to strategically synthesize the outputs of these experts. Several combination strategies are explored in Yang and Browne [2004], with averaging and the max operator being the most common:

$$y = \sum_{i=0}^{n} G_i(x)m_i(x) \quad \text{or} \quad y = \max_{G_i(x)}(m_i(x))$$
 (1)

In our work, we adopt the max operator for its clarity in selecting a single, relevant phase, thus eliminating the possibility of ambiguous phase assignments. This choice also brings significant computational efficiencies, as it requires inference from only one expert model  $m_i$  at a time.

Integrating MCTS with NNs typically involves using the model as an evaluator within MCTS, replacing the simulation phase with a forward pass. We innovate this process in MCTS by integrating both a gating mechanism G and a curated set of expert models. This approach, illustrated in Figure 1, not only preserves the core structure of MCTS, but enriches it with phase-specific evaluation capabilities.

## 2.1 Utilizing the Game Phase Definitions of Chess

Chess, a game known for its complexity and strategic depth, is traditionally divided into three central phases: the *opening*,

the *middlegame*, and the *endgame*. Each phase is distinct: the opening sets the strategic foundation of the game, where players compete for control of the board; the middlegame, characterized by intense piece interaction and tactical play, requires keen strategic insight; and the endgame focuses on exploiting earlier advantages to secure victory with fewer pieces in play. These phases are not just segments of a game, but represent critical strategic shifts that determine the course of the game.

In our approach, we adopt the Lichess game phase definitions (cf. Appendix B) as gating mechanism. Lichess<sup>2</sup>, an esteemed open-source chess server, provides a well-rounded and intuitive set of criteria for phase identification. This definition includes various factors such as board mixedness, back-rank sparseness, and piece count, offering a holistic view of the game's progression. By applying these phase conditions to any given game state, we can independently determine its phase without relying on previous positions, thus preserving the Markov property. This intuitive method allows us to return to previous phases if necessary, based on the current board position. For an in-depth exploration of Lichess phase definitions, we refer the reader to Appendix B. The distribution over the phases, using the mentioned phase definition is visualized in Figure 2.

#### 2.2 Training Multiple Experts

In this paper, we explore the effectiveness of integrating MoE within MCTS through three innovative training approaches:

**Separated Learning** This approach involves training each of the three expert networks exclusively on positions corresponding to their specific game phase. The objective is to simplify the learning process by narrowing the input space to a more manageable subspace. This focus enables each network to develop specialized expertise, enhancing the overall parameter count without compromising inference time. Implementation is straightforward and does not increase the

<sup>&</sup>lt;sup>2</sup>https://lichess.org/

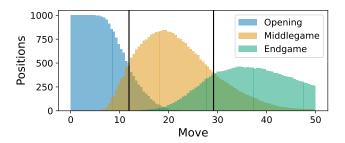


Figure 2: A common game phase definition in chess is to split states into opening, middlegame and endgame. This figure presents a detailed distribution of positions across the three game phases based on the Lichess phase definition, aggregated by move number. The average starting moves of the middlegame (11.89, std: 2.966) and the endgame (29.15, std: 7.441) are marked by vertical

training duration. In fact, the training process can be parallelized across experts and as such reduce training time significantly. While this method ensures depth in phase-specific knowledge, its limitation lies in a potential lack of understanding of transition nuances between phases. However, this is offset by the efficiency and specificity of the learning process, ensuring that training time remains on par with single network approaches.

Staged Learning In this method, a network is initially trained on one phase, then its weights are used as a foundation for subsequent training on the next phase. This cycle repeats through all defined phases, with the final phase defining the network's expertise. This approach aligns with transfer learning principles, allowing knowledge transfer between phases while focusing on a single phase ultimately. Each training cycle involves resetting the optimizer state while preserving the network weights, ensuring consistent learning dynamics. The primary advantage is the imparted understanding of all phases, albeit with a focus on the final phase. However, this approach requires more training time and may encounter challenges like catastrophic forgetting.

**Weighted Learning** This strategy involves applying different weights to the loss function for training samples based on their game phase. It allows each expert to learn from the entire dataset while emphasizing its designated phase.

$$L_{\text{weighted}} = \frac{1}{B} \sum_{n=1}^{B} w_n L(x_n)$$
 (2)

Here, B is the batch size,  $w_n$  the weight of a specific sample, and  $L(x_n)$  the regular sample loss. To prioritize the main phase samples, we introduce the parameter  $a \geq 1.0$ , which balances the weights of the main phase samples  $(w_{\text{main}})$  and the ones of all other samples  $(w_{\text{other}})$ :  $w_{\text{main}} = a \cdot w_{\text{other}}$ . The approach ensures that the loss magnitude remains consistent across training and evaluation by normalizing the sample weights to keep the average sample weight close to 1.0. The test set is unweighted for fair performance comparison. While this method requires triple the GPU time, it offers comprehensive learning from the entire dataset, distinguished

by phase-specific emphases.

Each of these approaches is designed to enhance the MoE model's performance in MCTS. The detailed configurations and outcomes of these training strategies are further elaborated in Section 3.

## 2.3 Training MoE on Chess

In this study, we train individual "experts" to master 3 distinct sets of tasks in chess, corresponding to its well-defined game phases. This enables dynamic resource allocation and using specialized knowledge for each phase of a chess game. Implementing this concept, we use three distinct expert models, each fine-tuned to excel in one of the chess game phases: opening, middlegame, and endgame.

While traditional MoE systems typically use a gating network to select an expert for a given input, our approach differs by adopting a more direct method. Given the robustness and widespread acceptance of the game phase definitions used in this research (detailed in Section 2.1), we choose to use a handcrafted criterion for expert selection, bypassing the need for the additional complexity inherent in an additional gating network. This predefined input segmentation simplifies the learning process by ensuring that each expert is focused only on its respective phase. However, it assumes that the game phases are accurately identified and classifiable, an aspect that is not extensively explored in this paper. Also, for this work, we decided to assume that each state belongs to one exact phase, not a mixture of different phases.

By integrating this MoE into the MCTS within an AlphaZero-like approach, we aim to improve the performance of chess AI by exploiting phase-specific strategies to achieve a level of mastery unattainable with conventional methods.

#### 2.4 Training in Batches

While the ideal scenario is to query only the appropriate network aligned with the current phase of the MCTS position, this approach is not applicable to MCTS batch sizes larger than 1, as it is common to improve GPU utilization. Larger batch sizes therefore require buffering of multiple positions prior to neural network inference, thereby reducing the frequency of node statistics updates.

To address this, we have developed a strategic process for handling larger batches. When encountering a batch size larger than 1, we perform an analysis of all positions within the batch to identify the dominant game phase. The network of experts corresponding to this dominant phase is then applied to the entire batch. A detailed illustration of this batch processing method can be found in Figure 3. While this adaptation introduces the possibility of processing some instances with a non-ideal expert network, it is important to note that such instances are expected mainly during phase transitions. In these scenarios, the incorrectly processed positions are still close to their appropriate phase, ensuring minimal compromise in prediction accuracy. This modification is essential to maximize GPU efficiency and effectively handle larger batch sizes.

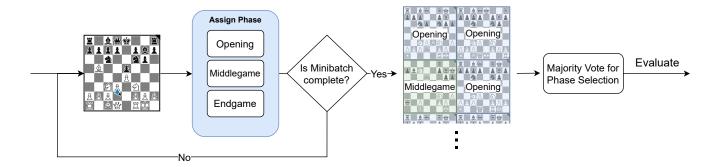


Figure 3: MCTS in combination with MoE can be used for batch sizes greater than 1. Each sample is assigned to its phase while allocating a new mini-batch during the MCTS search. When a mini-batch is complete, majority vote selection is used to select the model that contains the most samples of its associated phase. This model is then used to evaluate all samples from this mini-batch ensuring efficient and phase-focused processing.

## 3 Experiments

During the entire training phase, the model is subjected to an evaluation every 500 iterations. In addition to evaluating the currently trained expert on the dedicated evaluation dataset, assessments are extended to all four test sets (opening, middlegame, endgame, and no-phases). Checkpoints of the model state are saved if the evaluation loss surpasses the best recorded loss, and the final model is derived from the last saved checkpoint, representing optimal performance on the evaluation set.

The backbone of our NN architecture is RISEv3.3, an evolution of the RISEv2 architecture presented in the work by Czech *et al.* [2020], called *CrazyAra*. This architecture, a convolutional NN with residual blocks and dual output heads (value and policy), is further improved by incorporating  $5 \times 5$  convolutions and the Efficient Channel Attention module [Wang *et al.*, 2020]. For an in-depth understanding of our model's architecture, we refer readers to the CrazyAra wiki³. The input and output representation of our model is based on the work of Czech *et al.* [2023]. The training and test datasets are curated from the Kingbase Lite 2019 database<sup>4</sup>, which contains over one million games played by individuals rated at least 2200 Elo. We filter out games shorter than five moves to ensure the reliability of the data, as these are often either pre-arranged draws or database errors.

To increase the stability and reliability of the model, we have implemented a loss spike recovery mechanism. If there is a significant increase in the evaluation set loss from one cycle to the next, the model is reset to a previous state. This is controlled by the condition  $L_{\rm prev} \cdot s < L_{\rm curr}$ , where s=1.5,  $L_{\rm prev}$  is the previous evaluation loss, and  $L_{\rm curr}$  is the current loss

Our comprehensive evaluation includes a match of 1000 games that combines the trained model with MCTS, similar to AlphaZero's methodology. To objectively measure the strength of the individual approaches, we use the Elo rating system (detailed in Appendix C). In addition, we calculate 95% confidence limits for Elo differences based on the num-

ber of games played and their results. This calculation closely follows the method used in the Cutechess repository<sup>5</sup>, ensuring both accuracy and consistency in our ratings. More information on hyperparameters, datasets, and other technical details is provided in the supplementary materials.

## 3.1 Using MCTS with MoE

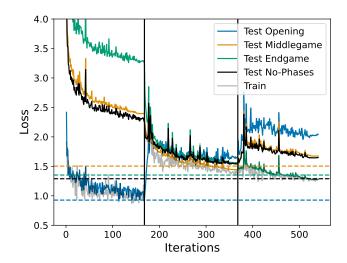


Figure 4: In staged learning, the endgame expert converges to a lower test loss compared to no-phases training. Shown is the endgame expert's loss over the course of the training process for the staged learning approach. The metrics are evaluated on the opening (blue), middlegame (orange), endgame (green) and no-phases (black) test set. The dashed lines describe the final loss of the no phases training setup and the vertical lines indicate the points at which the training on a new data set started (initialized with the parameters of the last model checkpoint of the previous training stage).

In our study, we exhaustively trained an expert model for each of the three game phases (cf. Section 2.1) using different training methodologies described in Section 2.2. We then evaluated the performance of these expert models, both

<sup>&</sup>lt;sup>3</sup>https://github.com/QueensGambit/CrazyAra/wiki/

<sup>4</sup>https://archive.org/details/KingBaseLite2019

<sup>&</sup>lt;sup>5</sup>https://github.com/cutechess/cutechess

Table 1: **MoE reduces overall loss in chess using game phase specific experts.** This table provides a comprehensive overview of the loss values for each expert's final model across different approaches. Performance is evaluated over four different test sets: opening, middlegame, endgame, and no-phases. In addition to the individual expert losses, we include the loss for the resulting mixture model that includes all three experts. This mixture model determines the appropriate expert based on the phase of the current position in the test set. The weighted learning approach was analyzed with different weighting factors to emphasize the main phase over others (see Section 2.2). The table uses color coding for each column, ranging from high loss (red) to low loss (blue). Bold text highlights the most effective model within each learning process for each test set, while the best performing model in the no-phases test set is underlined, highlighting its overall superiority.

Approach	Expert	Opening	Middlegame	Endgame	No-Phases
Regular Learning	-	0.9269	1.5043	1.3523	1.2897
Separated Learning	Opening	0.9169	2.4030	3.2433	2.2587
	Middlegame	1.6064	1.4371	1.5494	1.5260
	Endgame	2.0097	1.6605	1.2594	1.6293
	Mixture	0.9169	1.4371	1.2594	1.2245
Staged Learning	Opening	0.8786	1.8775	2.5501	1.8153
	Middlegame	1.5966	1.4437	1.5293	1.5203
	Endgame	2.0281	1.6748	1.2774	1.6458
	Mixture	0.8786	1.4437	1.2774	1.2218
Weighted Learning $(a = 4)$	Opening	0.8636	1.5125	1.3780	1.2820
	Middlegame	0.9661	1.5055	1.4207	1.3248
	Endgame	0.9339	1.5041	1.2864	1.2695
	Mixture	0.8636	1.5055	1.2864	1.2433
Weighted Learning ( $a = 10$ )	Opening	0.8887	1.6600	1.5389	1.3995
	Middlegame	0.9955	1.4834	1.4470	1.3346
	Endgame	1.0334	1.5547	1.3036	1.3241
	Mixture	0.8887	1.4834	1.3036	1.2483

individually and as part of the MoE ensemble, against our extensive test set. Our results, detailed in Table 1, reveal that the MoE approaches consistently outperform the baseline model — CrazyAra using RISEv3.3 [Czech et al., 2020] — in terms of combined losses. The baseline model was trained under similar conditions, but without phase-specific distinctions.

The separated learning and staged learning approaches show an average improvement of about 0.06 over our regular learning approach, while the weighted learning regime shows an improvement of about 0.04. This improvement translates into a significant increase in playing strength, ranging from 55 to 122 Elo points (cf. Table 2 and Figure 26) against the baseline. An exemplary training process for staged learning is shown in Figure 22.

The separated and staged learning methods significantly outperform our weighted learning approach. Interestingly, a higher factor (a=10) in our weighted learning strategy resulted in more pronounced Elo gains than a lower factor (a=4), despite similar overall loss values for both settings. It suggests that further increasing the specialization factor could potentially yield even more competitive results, similar to or better than other approaches. However, it is important to note that using all samples for training in the weighted and staged learning approach triples the training time compared to separated learning, where each expert is trained on only a third of the dataset.

In summary, our MoE methods clearly outperform the baseline model, not only in terms of training loss, but also in terms of actual playing strength, as evidenced by a substantial relative Elo gain of 122.20. To our knowledge, this is the first instance where such a combination has been successfully implemented and empirically validated.

#### 3.2 Comparing different Mixture Models

Our analysis, detailed in Table 1, indicates that the middlegame and endgame models contribute significantly to the overall effectiveness of our MoE framework. To quantify the individual impact of each expert model on the Elo gain of MoE, we organized a series of tournaments. In these tournaments, a single expert model was used for its respective phase, while the baseline model was used for the other phases. The results, illustrated in Figure 6, reveal that the middlegame and endgame networks contribute comparably to the overall performance improvement. However, the use of the opening net either slightly hinders performance or shows no significant improvement over the baseline model. Several factors may contribute to this observed behavior. First, the opening play in our expert dataset may lack sufficient variability, potentially leading the expert model to overfit specific lines rather than gaining a broad understanding of opening strategies. This narrow focus may limit the ability of the opening model to generalize across the wide range of opening scenarios encountered in play. Secondly, the opening expert's relative underperformance might stem from its limited exposure to terminal game states during training. This limited perspective could hinder its ability to evaluate opening positions in the broader context of the game's possible trajectories.

Finally, there is a notable difference between the opening

Table 2: **MoE methods outperform our baseline "one-for-all" approach in direct comparison.** This table presents an in-depth analysis of the relative Elo gains achieved by various MoE approaches over our baseline model across different batch sizes. Each value represents the average Elo gain across all configurations of nodes and move times for the specified batch size. The data is visually coded for clarity, with the best performances in blue and the least effective in red. The rightmost column aggregates the average Elo gain across all 55 experiments conducted for each method. Bold text highlights the most successful model within each approach.

	Batch Size					
Approach	1	8	16	32	64	Average
Separated Learning	106.89	123.45	126.81	124.36	129.49	122.20
Staged Learning	111.68	120.81	122.49	125.00	125.55	121.11
Weighted Learning $(a = 4)$	25.61	23.20	23.52	22.28	21.31	23.18
Weighted Learning ( $a = 10$ )	50.41	52.42	56.33	63.55	56.51	55.84

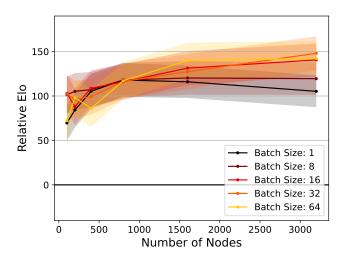


Figure 5: MCTS with MoE significantly outperforms standard MCTS. The MoE framework, utilizing the separated learning strategy outlined in Section 2.2, achieves up to 150 Elo points higher than a single 'one-for-all' model. This elevation in performance was consistently observed across a range of experiments, involving various batch sizes from 1 to 64, and different tree complexities, measured in terms of nodes per tree.

suites used in our validation tournaments and those in the training set. Our test tournament used a more randomized suite of openings generated by a computer algorithm, as opposed to the expert gameplay-based common openings in the training set. This mismatch between training and testing conditions may explain the reduced effectiveness of the opening network in real game scenarios. These findings not only highlight the nuanced contributions of each expert model to the MoE framework, but also underscore the importance of dataset diversity and training methodology in developing robust, stage-specific AI strategies for chess.

## 3.3 Influence of the Game Phase Definition

In our research, we initially adopted a traditional view of chess, dividing the game into three distinct phases with based on classical rules defining the game phases. This raises intriguing questions like how does varying the phase subdivision, as with that the number of experts, impact the overall

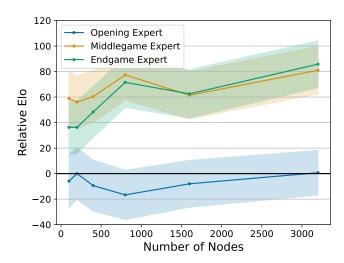


Figure 6: **Expert models are especially helpful in middle- and endgame.** This figure shows the relative Elo gains achieved by selectively deploying individual phase experts within Monte-Carlo Tree Search (MCTS), compared to using a baseline network for all phases. The experts, derived from our separated learning approach, exhibit notable strengths in both the middlegame and endgame phases of chess. The experiments were conducted using a consistent batch size of 64 for all matches.

result and can we simplify the phase definition using only the move counter. To explore these, we divided our dataset into 2, 3, 4, and 5 segments purely based on the move counter, generating similar sized phase datasets. The Elo comparison for different numbers of partitions is shown in Figure 7. It becomes apparent that using more phases does not guarantee better results and that the number of experts, i.e., phases, has to be adapted to the specific problem definition. Further, a logical semantic phase selection criterion is far superior to relying on a simple feature, like the move counter, when comparing the 3 phases Lichess agent with the 3 phases move counter agent. To conclude, simple features, like the move counter in chess, alone are not a sufficient game phase definition. To define a good gating mechanism, a clustering over concepts is necessary.

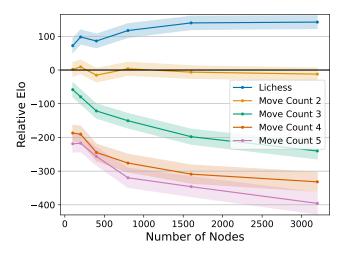


Figure 7: Game phase definitions strongly influence the MoE performance. Our analysis shows that simplistic approaches to phase classification, such as relying only on move counts, lead to suboptimal phase categorization and consequently to inferior performance. This effect increases with the number of experts. In contrast, adopting the Lichess phase definition leads to a significant performance improvement.

## 4 Limitations

In this study, we have advanced the use of the MoE framework in chess AI and demonstrated its significant potential. However, like any pioneering approach, it comes with its own set of challenges and future prospects.

A primary limitation of MoE is its susceptibility to overfitting, especially in scenarios with smaller datasets. This inherent risk requires careful data management and model training strategies. In addition, MoE has traditionally struggled to efficiently handle larger batch sizes due to the selective activation of experts. However, our framework proves resilient to this problem through the use of a majority voting mechanism for all samples in a mini-batch. By strategically mixing a few outlier samples from different phases, we have achieved a model that performs robustly across different batch sizes. In particular, our model maintained consistent performance with batch sizes ranging from 1 to 64, but it is unclear how well this strategy works for batch sizes beyond 512. Exploring more sophisticated methods, such as dividing the batch into multiple intermediate segments or even merging the insights of multiple experts, could potentially refine our decision-making process.

The requirement for increased VRAM, due to the necessity of loading all experts into memory, is a technical consideration with MoE. However, in the context of MCTS and with a limited number of experts, this has not posed a significant constraint in our experiments.

#### 5 Related Work

In advancing the application of MoE to chess, our work builds upon ongoing developments in both MCTS and RL techniques, particularly in the context of AlphaZero inspired methods. The integration of MoE into deep learning, as demonstrated by Shazeer *et al.* [2017], has shown promising results, especially in the context of LLMs and transformer-based architectures, driven by the growing complexity of modern datasets [Fedus *et al.*, 2022; Du *et al.*, 2022; Jiang *et al.*, 2024]. However, the combination of MoE with MCTS, in a manner similar to AlphaZero [Silver *et al.*, 2018], represents a novel approach in the field.

Our inspiration to adapt MoE to chess is further strengthened by the work of Dobre and Lascarides [2017] and McIlroy-Young *et al.* [2022]. The former demonstrates the utility of MoE in complex games such as Settlers of Catan, using expert models trained on diverse datasets. The latter demonstrates the potential of MoE to develop specialized chess models that can be fine-tuned to individual player behavior. These works underscore the diverse applicability of chess as a research domain, given its rich history, extensive data corpus, and active community engagement.

The shift from heuristic-based models to NN approaches, as exemplified by Silver et al. [2018]; Nasu [2018], has significantly improved performance, but at the cost of interpretability and adherence to human-centered chess theories [Pálsson and Björnsson, 2023; McGrath et al., 2022]. The use of game phase definitions in evaluation, a concept familiar to chess players and heuristic-based engines, found its place in early RL explorations in chess, such as the work of Block et al. [2008]. We also took inspiration to use game phase-specific NNs for chess by the work of dkappe<sup>6</sup>, showing this can have improved performance in endgame scenarios. His work has also been integrated into Daniel Shawul's NN based chess engine Scorpio<sup>7</sup>, demonstrating the practical applicability of such approaches. In contrast to our work, their implementation falls short in delivering a comprehensive experimental evaluation. Additionally, their approach relies on a rather simplified framework, employing materialbased criteria to ascertain the currently active phase. Based on the experiments in Section 3.3 we believe that improving the game phase definition will also improve this approach even further. Our research aims to bridge this gap by reintegrating phase-specific insights with advanced NN methods to enrich the strategic depth and performance of chess AI.

#### 6 Conclusion

Our exploration of modern chess AI with the MoE framework has resulted in significant progress, as evidenced by an impressive gain of approximately 120 Elo points. This achievement underscores the efficacy of employing phase-specific models tailored to the unique strategic requirements of each phase of a chess game. The MoE framework we developed and implemented is characterized by its ease of implementation and remarkable impact on performance. By separating the training process into distinct subsets, we have successfully cultivated specialized expertise within each expert model, thereby enhancing the overall performance of the system. In addition, our framework has demonstrated considerable robustness, effectively handling a range of minibatch sizes from 1 to 64 during the search process. This is achieved

<sup>&</sup>lt;sup>6</sup>https://github.com/dkappe/leela-chess-weights

<sup>&</sup>lt;sup>7</sup>https://github.com/dshawul/Scorpio/releases/tag/3.0

through our reliable majority voting mechanism, which selects the most appropriate expert for each game phase.

Looking ahead, there are exciting opportunities to further refine and develop our MoE framework. One particularly promising avenue is the development of a learnable gating network. This dynamic approach could replace our current method of using fixed, hand-crafted phase definitions, providing greater adaptability and precision in expert selection.

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#### **Ethics Statement**

In conducting this research, it is important to us to emphasize the ethical considerations associated with data collection, model development, and experimentation. Throughout the study, ethical guidelines were strictly adhered, ensuring the respectful use of data of any entities involved, like the usage of the Kingbase chess dataset, a collection of chess games played by human players, for the purpose of training and evaluating artificial intelligence models. We acknowledge the potential for biases in the dataset, such as variations in time periods, regions, or player demographics, however were not able to discover any in our research. We provide algorithm and models to make our results reproducible and transparent. We also encourage other researchers to use our framework in their work.

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## A Data Availability Statement

All data and code, needed to reproduce our experiments can be found online. The Kingbase Lite Dataset can be downloaded from https://archive.org/details/KingBaseLite2019. To recreate our dataset split, use the code provided by us in https://anonymous.4open.science/r/CrazyAra-3F10, following the instructions in the *readMe* file. To test our approach, we provide some of the models used in this paper anonymized<sup>8</sup>. These models will be added to the official github repository after acceptance.

## B Lichess game phase definitions

Lichess is an open-source chess server for online play, study, and analysis. State-of-the-art chess engines power their analysis section, which is why players of all skill levels, from beginner to master, use them on a daily basis. Submitting a game for engine analysis results in getting a report of the game development, including a separation of the game in the three typical phases to teach people where they went wrong and how they could improve. The site uses a more sophisticated system to determine game phases by incorporating several transition criteria.

**Endgame definition.** According to the Lichess implementation, a position belongs to the endgame if the total count of major and minor pieces (queens, rooks, bishops, and knights) is less than or equal to 6. Note that while this resembles a material count criterion, it does not use different relative values for the pieces and instead values them all equally as 1. A potential reason for this approach could be that the complexity of a position is not tied to the relative value of its pieces but rather to their total amount. Furthermore, the pawn count is irrelevant to the decision.

Middlegame definition. A position counts towards the middlegame if it does not qualify as an endgame position and if one of the following three criteria is fulfilled. The number of major and minor pieces is less than or equal to 10, the backrank of at least one player is sparse, or the total mixedness score of the position is bigger than 150. Here, backrank sparseness is defined as having less than four total pieces on rank 1 (for white) or 8 (for black), including the king. The mixedness score describes how close black and white pieces are to each other. It is calculated by going through all two by two squares of the chess board, starting from the square a1, b1, a2, b2 and ending at the square g7, h7, g8, h8. For each of those two by two squares, we count the number of black and white pieces inside it and assign a score based on the result and the square's location. We then sum up all square scores to get the final mixedness score of a position. The exact implementation can be found in the Lichess repository<sup>9</sup>. **Opening definition.** All remaining positions are classified as opening positions.

Using the previously described phase definitions may lead to transitions to previous phases (e.g., going back to the opening because the mixedness score has increased again). Therefore, to do a strict separation into three sections, Lichess forbids such transitions and only allows transitions to later phases.

## **B.1** Resulting Dataset Statistics

After defining the game phases as in Section B, we examine the distributions of input and output in the resulting datasets for each phase.

Figure 12 shows the distribution of game outcomes in our training set. Every game contributes to the opening phase data, with 34.42% games won by White, 40.36% draws, and 25.22% Black wins. Notably, games extending into the middlegame and endgame phases have a lower draw rate (38.51% and 36.09%, respectively) and a higher rate of decisive outcomes.

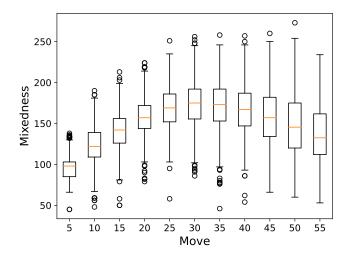


Figure 8: The mixedness scores (according to the definition in Section B) of chess game aggregated by move number. The data is based on all positions of the chess test set.

Figure 13 illustrates the distribution of game phases. While 96.81% of the games reach the middlegame, only 67.85% reach the endgame. Nevertheless, the number of endgame positions (31.63%) exceeds that of openings (28.67%), because endgames tend to last longer.

Table 3 gives a comprehensive overview of our datasets, the full dataset contains 1,112,647 games and 91,413,951 positions. Table 4 summarizes the number of games contributing to each phase, highlighting the variance in phase coverage.

## C Elo Ratings

The Elo rating system, invented by Arpad Elo, is used to measure relative skill differences between players of a game. Everyone starts with the same arbitrary starting value, and the

<sup>&</sup>lt;sup>8</sup>https://drive.google.com/drive/folders/ 1d8CoQBieNqeEomhbYfI\_TifZUbCFyS3O?usp=sharing

<sup>&</sup>lt;sup>9</sup>https://github.com/lichess-org/scalachess/blob/master/src/main/scala/Divider.scala

Dataset/Phase	No-Phases	Opening	Midgame	Endgame
Train Chess	91,413,951	26,212,273	36,287,651	28,914,027
Val Chess	79,042	24,566	29,333	25,143
Test Chess	85,114	24,938	31,364	28,812

Table 3: Overview of dataset sizes by phase, showing the number of positions.

Dataset/Phase	No-Phases	Opening	Midgame	Endgame
Train Chess	1,112,647	1,112,647	1,077,136	754,899

Table 4: Number of games contributing to each phase dataset, indicating phase coverage.

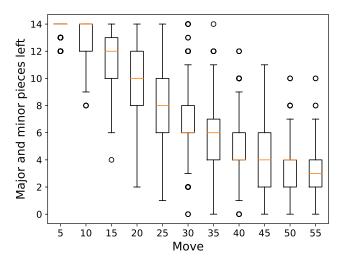


Figure 9: The average number of major and minor pieces left aggregated by move number. The data is based on all positions of the chess test set.

ratings are adjusted from that point on based on the outcome of finished games. Winning games increases the Elo rating, while losing decreases it. Using the rating of two players A and B, it is possible to calculate the expected score of player A (and player B by replacing  $R_{\rm B}-R_{\rm A}$  with  $R_{\rm A}-R_{\rm B}$ ):

$$E_{\rm A} = \frac{1}{1 + 10^{(R_{\rm B} - R_{\rm A})/400}}. (3)$$

# D Sample weights for the Weighted Learning Approach

We carry out experiments for two different a values  $(a \in \{4, 10\})$  to see whether a big weight difference (more expert specialization) or a small weight difference (more expert similarity) leads to better overall performance. The values of the resulting normalized sample weights can be found in Table 5.

## E Extended Experimental Setup

#### E.1 Datasets

The KingBase dataset is a comprehensive collection of chess games, renowned for featuring high-quality matches played by grandmasters and skilled players. Typically provided in Portable Game Notation (PGN) format, it encompasses detailed information on moves, players, dates, and tournament

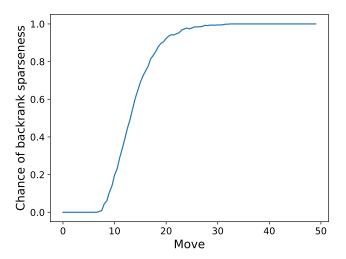


Figure 10: The percentage of positions with a sparse backrank by move number. The data is based on all positions of the chess test

specifics. Widely employed in computer chess research, machine learning, and artificial intelligence, the dataset is valued for studying advanced chess strategies, training and testing chess engines, and analyzing player performance.

Our chess datasets are built based on the Kingbase Lite 2019 database. This database includes over one million chess games from players rated at least 2200 Elo. We filter out all games shorter than five moves in total as these games are either quickly arranged draws or errors in the database and, therefore, unreliable sources. In order to create the training and evaluation data for our experiments, we build four datasets. The first dataset, which we will refer to as "no-phases", contains all positions left after filtering the database as described above. For the remaining three datasets ("opening", "midgame" and "endgame"), we strictly split the database into one part for each of the three phases. This splitting is carried out by going through every game in the dataset and removing all board positions that do not belong to the phase for which we are currently preparing the dataset. This procedure leads to some games being completely excluded for a specific part if they contain no position of the particular phase. For example, about 32 percent of games ended before the endgame began, therefore contributing no position for the endgame dataset.

Table 5:	Sample	weights for	the Weighted	Learning.	Approach

a	expert main phase	$w_{\mathrm{main}}$	$w_{\mathrm{other}}$	$w_{ m opening}$	$w_{ m midgame}$	$w_{ m endgame}$
	opening	2	0.5	2	0.5	0.5
4	midgame	2	0.5	0.5	2	0.5
	endgame	2	0.5	0.5	0.5	2
	opening	2.5	0.25	2.5	0.25	0.25
10	midgame	2.5	0.25	0.25	2.5	0.25
	endgame	2.5	0.25	0.25	0.25	2.5

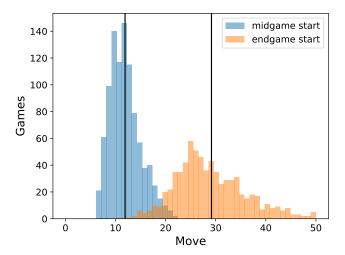


Figure 11: The distribution of starting moves of each phase aggregated by move number. The average starting moves of the middlegame (11.89 with an std of 2.966) and the endgame (29.15 with an std of 7.441) are added as vertical lines. The data is based on all games of the chess test set.

## **E.2** Neural Network Architecture and Input Representation

Since our approach is based on and compared to the CrazyAra approach, by Czech *et al.* [2020], we use the same model architecture called RISE in its current version (RISEv3.3). This architecture is in turn an improved ResNet architecture how they were used in AlphaZero. More information about the architecture can be found under https://github.com/QueensGambit/CrazyAra/wiki/Model-architecture. As loss function we used

$$\ell = -\alpha (\mathbf{W} \mathbf{D} \mathbf{L}_{\mathsf{t}}^{\top} \log \mathbf{W} \mathbf{D} \mathbf{L}_{\mathsf{p}}) - \boldsymbol{\pi}^{\top} \log \boldsymbol{p} + \beta (p l y_{\mathsf{t}} - p l y_{\mathsf{p}})^{2} + c \|\boldsymbol{\theta}\|_{2}^{2}$$

introduced in the work of Czech *et al.* [2023]. Within the approach our MCTS implementation follows the work by Kocsis and Szepesvári [2006]; Coulom [2006] with the adaptations, done by Silver *et al.* [2018], e.g., using the PUCT formula instead of UCT.

Similar to AlphaZero [Silver *et al.*, 2018] or CrazyAra [Czech *et al.*, 2020], we represent the game state in the form of a stack of so-called levels or planes. The complete stack of planes can be found in Table 6 and is taken from Czech *et al.* [2023]. Each layer or plane represents a channel describing one of the input features in the current state. Each plane is encoded as a map with  $8 \times 8$  bits. We hereby distinguish

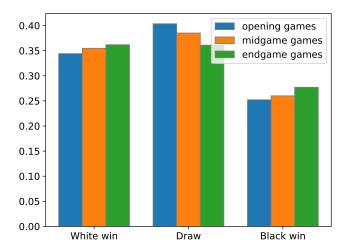


Figure 12: Game result distributions in the chess training set, calculated for games reaching each phase.

between two types of planes, bool and int. In a bool plane each bit is representing a different field of a chessboard, e.g., if a pawn is placed on this field. In some cases, like *repetitions*, a single information is stored in a plane, in these cases all bits of the plane show the same information. In Table 6 this is marked with \*. Integer or int planes have the same functionality but instead of 0 and 1, they store integer, like how many pawns are left on the board.

The output of our network, also follows Silver *et al.* [2018] and is described as the expected utility of the game position, represented by a numeric value in the range of [-1,1], often called value, and a distribution over all possible actions, called policy  $\pi$ .

## E.3 Reproducibility and Hyperparameters

Our hyperparameter setup is very similar to the settings proposed in Czech *et al.* [2020]. The list of hyperparameters can be seen in Table 7 and the schedules for learning rate and momentum are depicted in Figure 14. An exemplary command for running cutechess<sup>10</sup> matches with all used parameters can be found in Figure 15. In order to provide a diversified playing ground, we make use of an opening suite<sup>11</sup> featuring posi-

<sup>&</sup>lt;sup>10</sup>For detailed information about the cutchess-cli.https://manpages.ubuntu.com/manpages/xenial/en/man6/cutechess-cli.6.html

<sup>&</sup>lt;sup>11</sup>https://raw.githubusercontent.com/ianfab/books/master/chess.epd

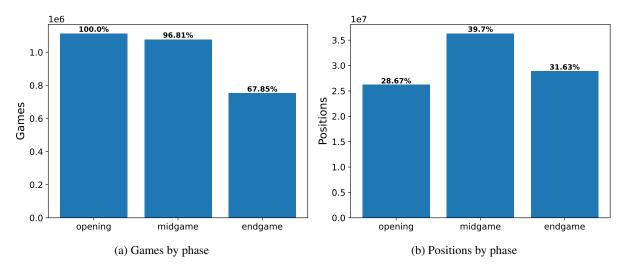


Figure 13: Distribution of games and positions across phases in the chess training set.

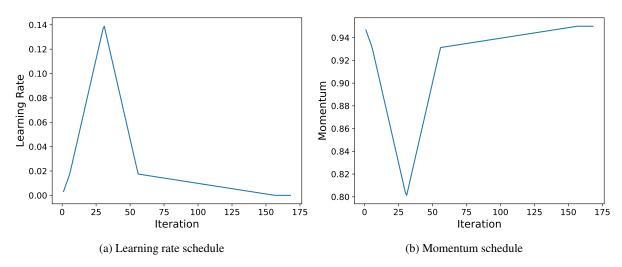


Figure 14: Learning rate and momentum schedules over the course of a training run of a single expert model.

Table 6: Input representation, taken from Czech *et al.* [2023]. Features are encoded as binary maps, and specific features are indicated with \* as single values applied across the entire  $8 \times 8$  plane. \* represent scalar planes only saving one value over all 64 bits of the plane. The historical context is captured as a trajectory spanning the last eight moves. Overall the input representation consists of 52 planes.

Feature	Planes	Type	Comment
P1 pieces	6	bool	order: {PAWN, KNIGHT, BISHOP, ROOK, QUEEN, KING}
P2 pieces	6	bool	order: {PAWN, KNIGHT, BISHOP, ROOK, QUEEN, KING}
Repetitions*	2	bool	how often the board positions has occurred
En-passant square	1	bool	the square where en-passant capture is possible
P1 castling*	2	bool	<pre>binary plane, order: {KING_SIDE, QUEEN_SIDE}</pre>
P2 castling*	2	bool	binary plane, order: {KING_SIDE, QUEEN_SIDE}
No-progress count*	1	int	sets the no progress counter (FEN halfmove clock)
Last Moves	16	bool	origin and target squares of the last eight moves
is960*	1	bool	if the 960 variant is active
P1 pieces	1	bool	grouped mask of all P1 pieces
P2 pieces	1	bool	grouped mask of all P2 pieces
Checkerboard	1	bool	chess board pattern
P1 Material difference*	5	int	order: {PAWN, KNIGHT, BISHOP, ROOK, QUEEN}
Opposite color bishops*	1	bool	if they are only two bishops of opposite color
Checkers	1	bool	all pieces giving check
P1 material count*	5	int	order: {PAWN, KNIGHT, BISHOP, ROOK, QUEEN}
Total	52		

Table 7: Hyperparameter Configuration for Experimental Settings. This table provides a comprehensive overview of the essential hyperparameters utilised in our experimental design.

Hyperparameter	Value	Hyperparameter	Value
max learning rate	0.14	value loss factor	0.01
min learning rate	0.00001	policy loss factor	0.988
batch size	2048	wdl loss factor	0.01
max momentum	0.95	plys to end loss factor	0.002
min momentum	0.8	stochastic depth probability	0.05
epochs	7	pytorch version	1.12.0
optimizer	NAG	spike thresh	1.5
weight decay (wd)	0.0001	dropout rate	0.0
seed	9	•	

Table 8: Hard- and software configuration for our experimental section.

Hardware/Software	Description
GPU	8 × NVIDIA® Tesla V100
NGC Container	pytorch:22.05
GPU-Driver	CUDA 11.4
CPU	Dual Intel Xeon Platinum 8168
Operating System	Ubuntu 20.04 LTS
CrazyAra	Release 1.0.4
Backend	TensorRT-8.4.1

tions appearing after the first several moves have already been played. These positions are intentionally chosen in a way that each position is imbalanced, with a slight edge for either side to reduce the amount of resulting draws. For each match, we randomly sample 500 opening positions from this opening suite and let our agents play against each other starting from there. In order to take the imbalances of the positions

into account, we use each starting position twice so that each agent is playing once as white and once as black.

Ensuring reproducibility is paramount in scientific research, as it establishes the foundation for the reliability and credibility of study findings. For this, we provide an anonymous version of our code, with a short manual how to run the code in the *ReadMe* as well as our trained models used in the

```
./cutechess-cli -variant standard -openings file=chess.epd format=epd order=random -pgnout /data/cutechess_res.pgn -resign movecount=5 score=600 -draw movenumber=30 movecount=4 score=20 -concurrency 1 -engine name=ClassicAra_correct_phases cmd=./ClassicAradir=~/CrazyAra/engine/build option.Model_Directory=/data/model/ClassicAra/chess/correct_phases proto=uci -engine name=ClassicAra_no_phases cmd=./ClassicAra dir=~/CrazyAra/engine/build option.Model_Directory=/data/model/ClassicAra/chess/no_phases proto=uci -each option.First_Device_ID=0 option.Batch_Size=64 option.Fixed_Movetime=0 tc=0/6000+0.1 option.Nodes=100 option.Simulations=200 option.Search_Type=mcts -games 2 -rounds 500 -repeat
```

Figure 15: Exemplary command for running a cutechess match between two approaches. The *Batch\_Size* is set to 1, 8, 16, 32, and 64. The *Nodes* parameter (values of 0, 100, 200, 400, 800, 1600, 3200) limits the number of nodes that the MCTS can visit per move during its search. The *Simulations* parameter describes the number of simulations per move and is set to double the *Nodes* value. When the *Nodes* parameter is 0, we instead limit the extent of the search by a fixed move time in milliseconds (*Fixed\_Movetime* values of 100, 200, 400, 800, 1600).

experimental section. We used seeds where it was applicable, e.g., in the training process and rerun our experiments to test the robustness of our results. The experiments were run on a setup, described in Table 8, using the NVIDIA GPU Cloud (NGC) docker container for pytorch<sup>12</sup>. As stated before, all needed data is openly available.

#### F Additional Results

In this section, we present supplementary findings and extended analyses that complement and enrich the core results discussed in the paper, providing a more comprehensive understanding of the investigated approach.

**Training Process** From Figure 16 till Figure 25 we show the training process of our three learning approaches for all three experts on all four test sets (opening, midgame, endgame and no phases). It can be seen that the experts outperform other models in their designated phase.

Elo Development over Training Figures 26, 27 and 28 show the result of test matches between the model in training and a baseline model for different restricting factors in the search and evaluation of game states. On the left we used the number of iterations, i.e., the number of explored nodes within MCTS as restricting factor. On the right, instead of restricting the search by the number of iterations, we chose to restrict it, using a time limit, a common pratice in, e.g., computational chess. As baseline model in all these figures the regular learning approach, i.e., the single model approach, was taken.

**Extension to Figure 6** Since we did all our experiments using two restricting factors, i.e., using the number of nodes and the move time as restrictions, we also evaluated Figure 6 twice. The evaluation using time as the restricting factor, can be found in Figure 29.

<sup>&</sup>lt;sup>12</sup>https://catalog.ngc.nvidia.com/orgs/nvidia/containers/pytorch

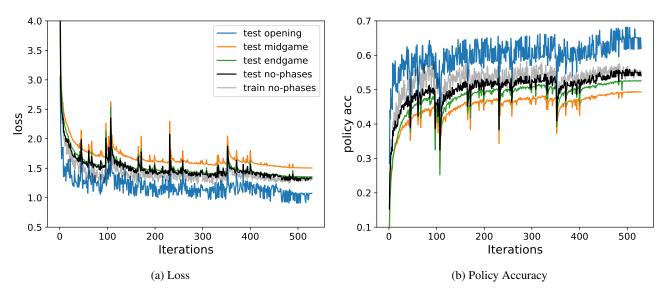


Figure 16: The loss (Fig. 16a) and policy accuracy (Fig. 16b) over the course of the training process for the **Regular Learning** approach. The metrics were evaluated on the opening (blue), midgame (orange), endgame (green) and no-phases (black) test set. The evaluation on the train set is shown in the background in grey. The values on the x-axis represent iterations, where one iteration is defined as doing backpropagation on one batch of training data.

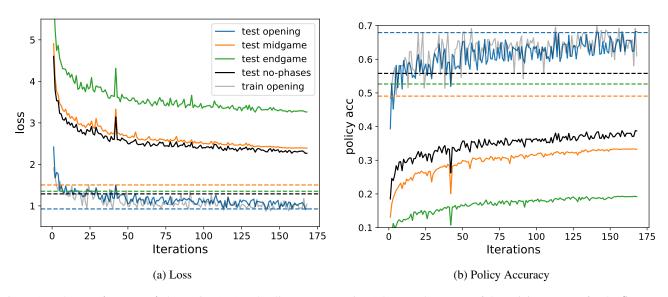


Figure 17: The **opening expert's** loss (Fig. 17a) and policy accuracy (Fig. 17b) over the course of the training process for the **Separated Learning** approach. The metrics were evaluated on the opening (blue), midgame (orange), endgame (green) and no-phases (black) test set. The evaluation on the train set is shown in the background in grey. The values on the x-axis represent iterations, where one iteration is defined as doing backpropagation on one batch of training data. The metric values of the final model checkpoint of the Regular Learning approach are added as dashed reference lines (same colors represent the same test set).

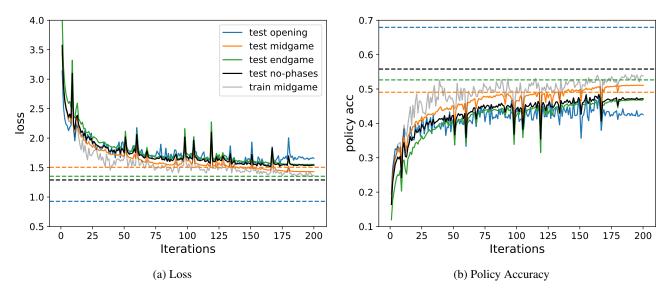


Figure 18: The **midgame expert's** loss (Fig. 18a) and policy accuracy (Fig. 18b) over the course of the training process for the **Separated Learning** approach. The metrics were evaluated on the opening (blue), midgame (orange), endgame (green) and no-phases (black) test set. The evaluation on the train set is shown in the background in grey. The values on the x-axis represent iterations, where one iteration is defined as doing backpropagation on one batch of training data. The metric values of the final model checkpoint of the Regular Learning approach are added as dashed reference lines (same colors represent the same test set).

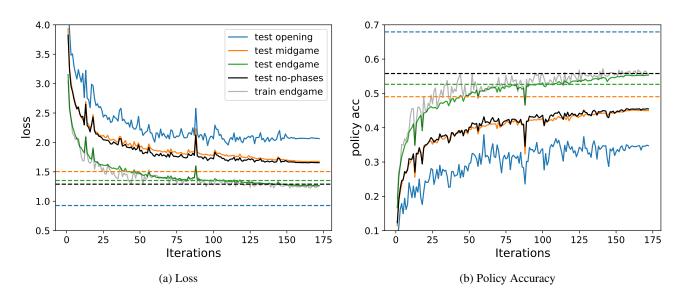


Figure 19: The **endgame expert's** loss (Fig. 19a) and policy accuracy (Fig. 19b) over the course of the training process for the **Separated Learning** approach. The metrics were evaluated on the opening (blue), midgame (orange), endgame (green) and no-phases (black) test set. The evaluation on the train set is shown in the background in grey. The values on the x-axis represent iterations, where one iteration is defined as doing backpropagation on one batch of training data. The metric values of the final model checkpoint of the Regular Learning approach are added as dashed reference lines (same colors represent the same test set).

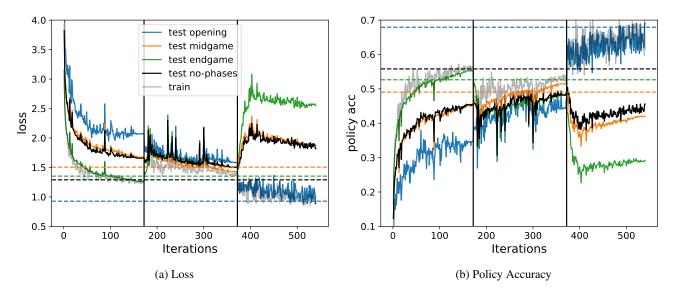


Figure 20: The **opening expert's** loss (Fig. 21a) and policy accuracy (Fig. 21b) over the course of the training process for the **Staged Learning** approach. The metrics were evaluated on the opening (blue), midgame (orange), endgame (green) and no-phases (black) test set. The evaluation on the train set is shown in the background. The values on the x-axis represent iterations, where one iteration is defined as doing backpropagation on one batch of training data. The vertical lines indicate the points at which the training on a new dataset started (initialized with the parameters of the last model checkpoint of the previous training stage).

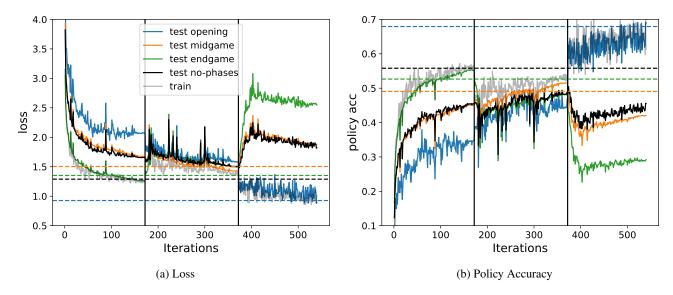


Figure 21: The **opening expert's** loss (Fig. 21a) and policy accuracy (Fig. 21b) over the course of the training process for the **Staged Learning** approach. The metrics were evaluated on the opening (blue), midgame (orange), endgame (green) and no-phases (black) test set. The evaluation on the train set is shown in the background. The values on the x-axis represent iterations, where one iteration is defined as doing backpropagation on one batch of training data. The vertical lines indicate the points at which the training on a new dataset started (initialized with the parameters of the last model checkpoint of the previous training stage).

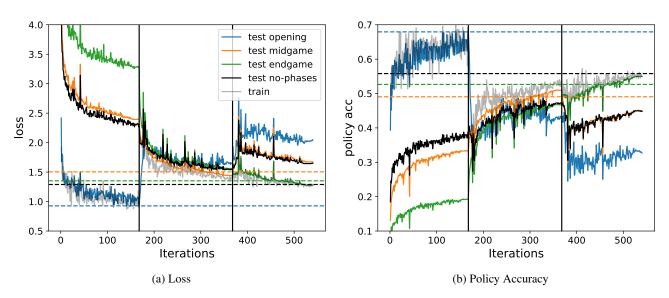


Figure 22: The **endgame expert's** loss (Fig. 22a) and policy accuracy (Fig. 22b) over the course of the training process for the **Staged Learning** approach. The metrics were evaluated on the opening (blue), midgame (orange), endgame (green) and no-phases (black) test set. The evaluation on the train set is shown in the background. The values on the x-axis represent iterations, where one iteration is defined as doing backpropagation on one batch of training data. The vertical lines indicate the points at which the training on a new dataset started (initialized with the parameters of the last model checkpoint of the previous training stage).

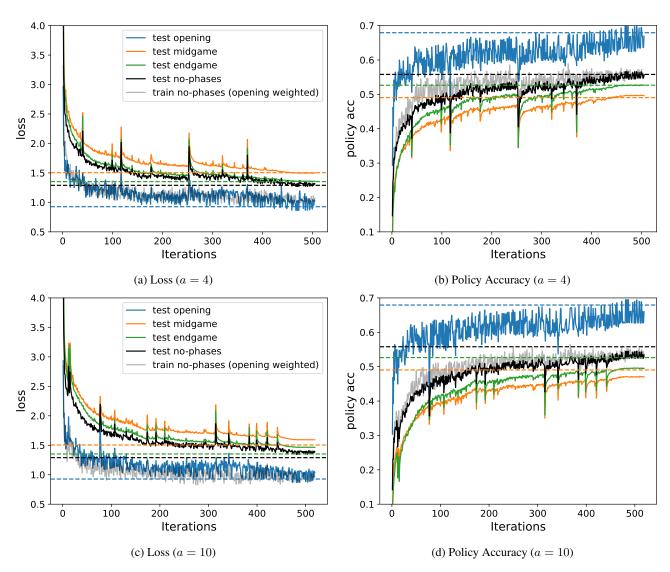


Figure 23: The **opening expert's** loss (Fig. 23a and Fig. 23c) and policy accuracy (Fig. 23b and Fig. 23d) over the course of the training process for the **Weighted Learning** approach with different *a* values. The metrics were evaluated on the unweighted opening (blue), midgame (orange), endgame (green) and no-phases (black) test set. The evaluation on the train set (weighted) is shown in the background. The values on the x-axis represent iterations, where one iteration is defined as doing backpropagation on one batch of training data. The metric values of the final model checkpoint of the Regular Learning approach are added as dashed reference lines (same colors represent the same test set).

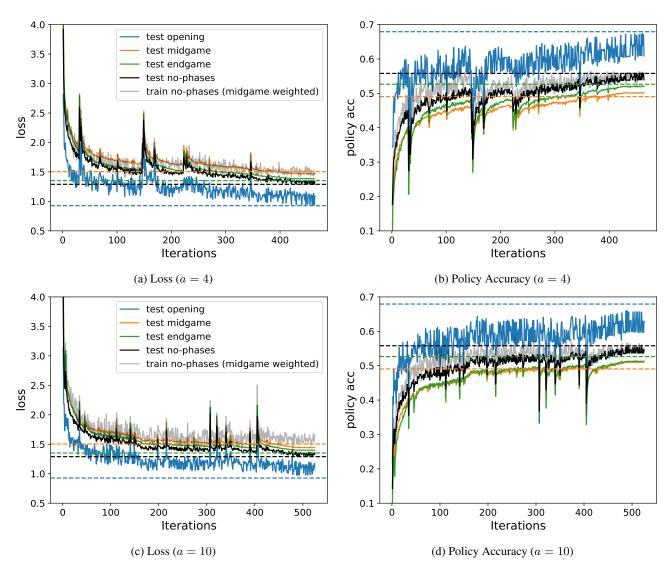


Figure 24: The **midgame expert's** loss (Fig. 24a and Fig. 24c) and policy accuracy (Fig. 24b and Fig. 24d) over the course of the training process for the **Weighted Learning** approach with different *a* values. The metrics were evaluated on the unweighted opening (blue), midgame (orange), endgame (green) and no-phases (black) test set. The evaluation on the train set (weighted) is shown in the background. The values on the x-axis represent iterations, where one iteration is defined as doing backpropagation on one batch of training data. The metric values of the final model checkpoint of the Regular Learning approach are added as dashed reference lines (same colors represent the same test set).

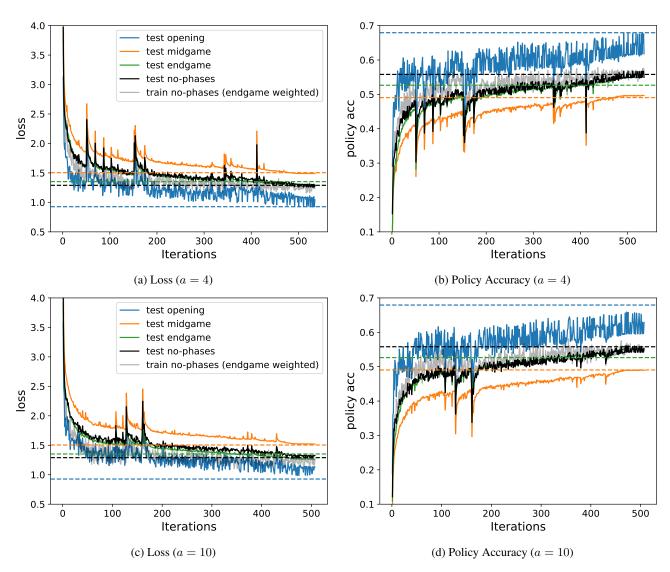


Figure 25: The **endgame expert's** loss (Fig. 25a and Fig. 25c) and policy accuracy (Fig. 25b and 25d) over the course of the training process for the **Weighted Learning** approach with different *a* values. The metrics were evaluated on the unweighted opening (blue), midgame (orange), endgame (green) and no-phases (black) test set. The evaluation on the train set (weighted) is shown in the background. The values on the x-axis represent iterations, where one iteration is defined as doing backpropagation on one batch of training data. The metric values of the final model checkpoint of the Regular Learning approach are added as dashed reference lines (same colors represent the same test set).

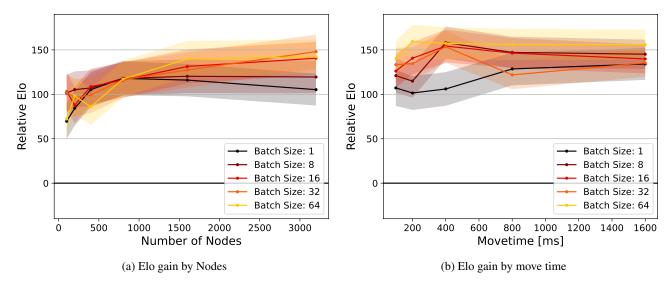


Figure 26: Relative Elo gain of the **Separated Learning** approach for different batch sizes. Node values of 100, 200, 400, 800, 1600 and 3200 in Fig. 26a. Move time values of 100, 200, 400, 800 and 1600 in Fig. 26b

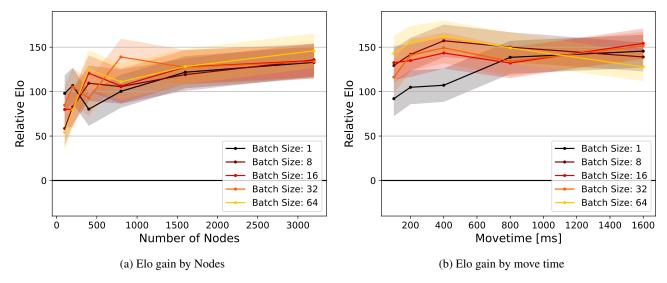


Figure 27: Relative Elo gain of the **Staged Learning** approach for different batch sizes. Node values of 100, 200, 400, 800, 1600 and 3200 in Fig. 27a. Move time values of 100, 200, 400, 800 and 1600 in Fig. 27b

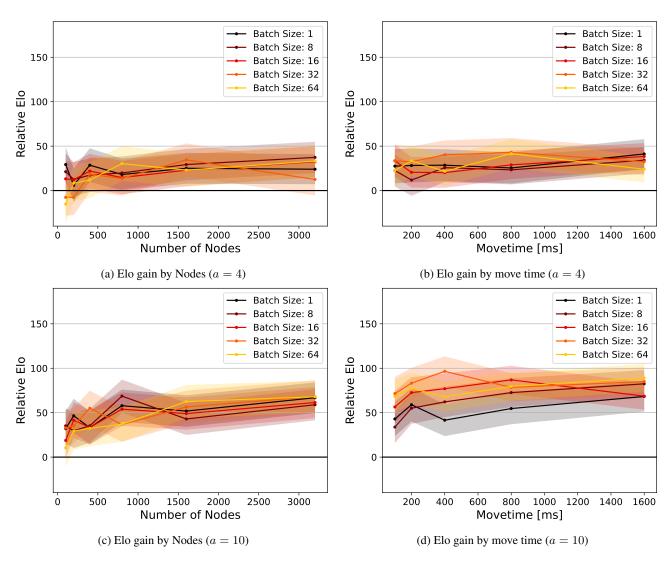


Figure 28: Relative Elo gain of the **Weighted Learning** approach for different batch sizes and a values of 4 and 10. Node values of 100, 200, 400, 800, 1600 and 3200 in Fig. 28a and Fig. 28c. Move time values of 100, 200, 400, 800 and 1600 in Fig. 28b and 28d

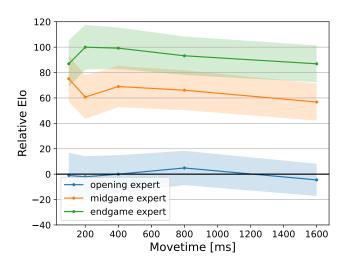


Figure 29: Relative Elo gain of using only one specific phase expert in the MCTS and using the baseline network for all remaining predictions. The used experts are taken from the separated learning approach for **chess** and the batch size was set to 64 for all matches.