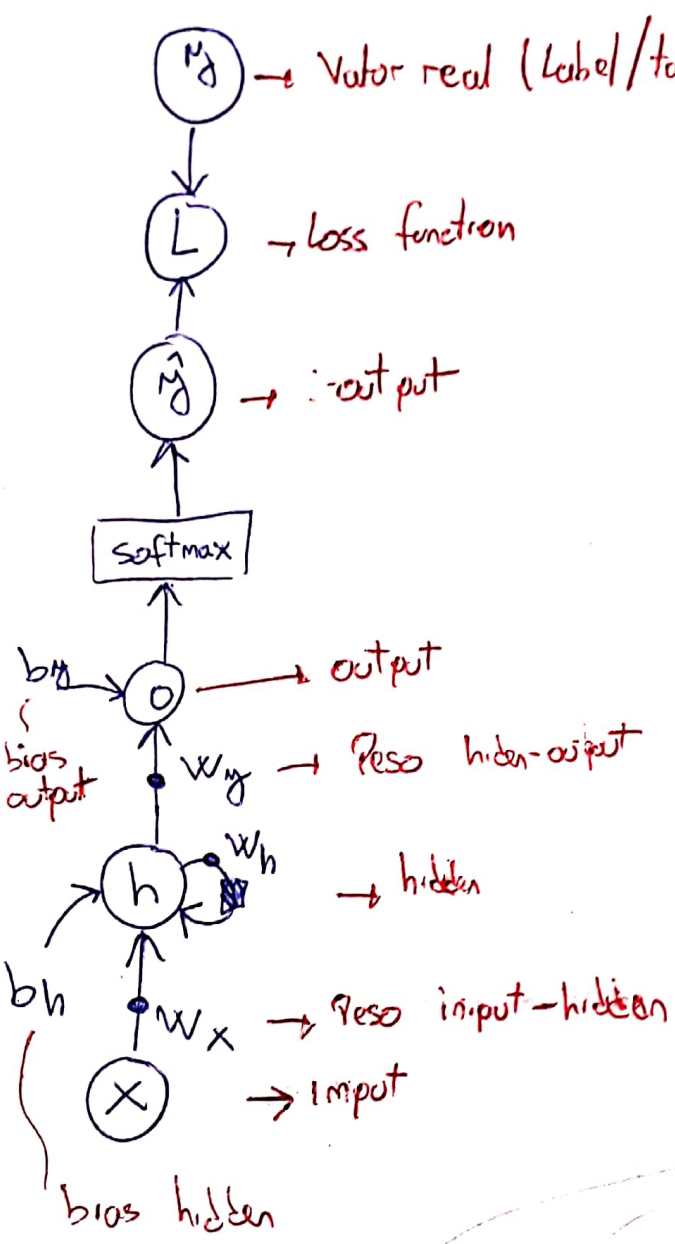
Back propagation Through Time (BPTT)



Ecuaciones

$$L_T = \sum_{t=0}^{T} L_t \quad (1)$$

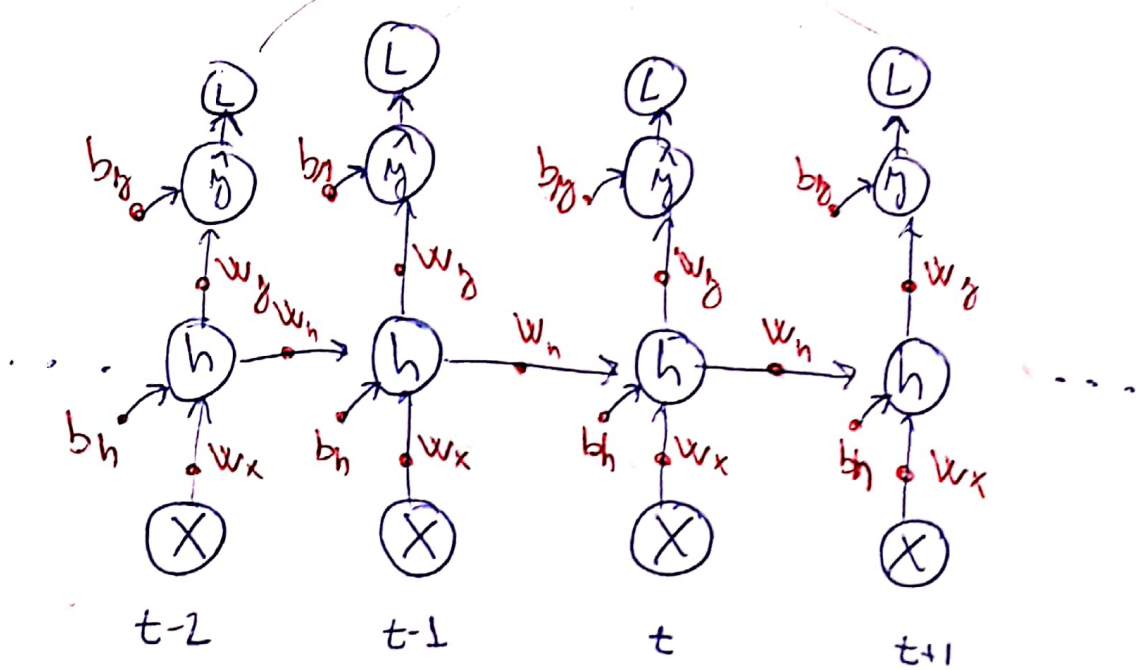
$$L_t = -y_t \log \hat{y}_t \quad (2)$$

$$\hat{y}_t = \text{softmax}(o_t) = \quad (3)$$

$$o_t = h_t \cdot W_y + b_y \quad (4)$$

$$h_t = \tanh\left[X_t W_x + h_{t-1} W_h + b_h\right] \quad (5)$$

$$L_T = \sum_{t=0}^{T} L_t$$



Parámetros a optimizar.

1° W_y → Peso hidden - output

2° b_y → bios output

3° W_h → Peso hiden - hiden

4° W_x → Peso input - hidden

5° b_h → bios hidden

¿Como vara mi L_T respecto de ellos?

$$1^{\circ}\ \frac{\partial L_T}{\partial W_y} = \frac{\partial}{\partial W_y} \sum_{t=0}^{T} L_t = \sum_{t=0}^{T} \frac{\partial L_t}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial o_t} \cdot \frac{\partial o_t}{\partial W_y}$$

Los pesos no varian con los Δt

sumo todos lo t para tener una L global

derivada de Loss function respecto de una softmax

$\Downarrow$

Ya fué demostrado $\leftarrow (\hat{y}_t - y_t)$

de ecuacion (4) en hoja (I)

$$\frac{\partial o_t}{\partial W_y} = h_t$$

- entonces llegamos a que $\frac{\partial L_t}{\partial W_y} = (\hat{y}_t - y_t)\, h_t \Rightarrow$

$$\boxed{\frac{\partial L_T}{\partial W_y} = \sum_{t=0}^{T} (\hat{y}_t - y_t)\, h_t} \quad (6)$$

2° $\frac{\partial L}{\partial b_y}$ se trabaja igual q 1°. reemplazando la última $\frac{\partial o_t}{\partial w_y}$ por $\frac{\partial' o_t}{\partial b_y}$

$$\frac{\partial L_T}{\partial b_y} = \sum_{t=0}^{T} (\hat{y}_t - y_t) \cdot 1 \quad (7)$$

$\rightarrow \frac{\partial o_t}{\partial b_y}$

3° $\frac{\partial L_T}{\partial w_h}$ ¿Cómo varía L cuando varío w_h? ojo! $h_t = \tanh(x w_x + h w_h + b_h)$

$$\frac{\partial L_{t+1}}{\partial w_h} = \frac{\partial L_{t+1}}{\partial \hat{y}_{t+1}} \cdot \frac{\partial \hat{y}_{t+1}}{\partial o_{t+1}} \cdot \frac{\partial o_{t+1}}{\partial h_{t+1}} \cdot \frac{\partial h_{t+1}}{\partial w_h} \quad (8)$$

ya lo vieron $\leftarrow (\hat{y}_{t+1} - y_{t+1})$

de ecuación (4) en hoja (I) $= w_y$

a resolver...

$$h_{t+1} = \tanh(x_{t+1} w_x + h_t w_h + b_h)$$

$\frac{\partial h_{t+1}}{\partial w_h}$

h_{t+1} es función de w_h y h_t

q es función de w_h y h_{t-1} ...

$$\frac{\partial h_{t+1}}{\partial w_h} = \frac{\partial h_{t+1}}{\partial h_t} \frac{\partial h_t}{\partial w_h} \Rightarrow \text{Podemos escribir:}$$

$$\frac{\partial h_{t+1}}{\partial w_h} = \sum_{k=0}^{t+1} \frac{\partial h_{t+1}}{\partial h_k} \frac{\partial h_k}{\partial w_h}$$ → veamos que da eso

$$\frac{\partial h_3}{\partial w_h} = \sum_{k=1}^{3} \frac{\partial h_3}{\partial h_k} \frac{\partial h_k}{\partial w_h}$$

$$= \underbrace{\frac{\partial h_3}{\partial h_1} \frac{\partial h_1}{\partial w_h}}_{h_3 \text{ varia porque } h_1 \text{ varia por que } w_h \text{ varia}} + \underbrace{\frac{\partial h_3}{\partial h_2} \frac{\partial h_2}{\partial w_h}}_{h_3 \text{ varia xq } h_2 \text{ varia xq } w_h \text{ varia}} + \underbrace{\frac{\partial h_3}{\partial h_3} \frac{\partial h_3}{\partial w_h}}_{h_3 \text{ varia xq } w_h \text{ varia}}$$

Recapitulamos ecuacion (8) de hoja (III)

$$\frac{\partial L_{t+1}}{\partial w_h} = \frac{\partial L_{t+1}}{\partial \hat{y}_{t+1}} \cdot \frac{\partial \hat{y}_{t+1}}{\partial o_{t+1}} \cdot \frac{\partial o_{t+1}}{\partial h_{t+1}} \cdot \underbrace{\sum_{k=0}^{t+1} \frac{\partial h_{t+1}}{\partial h_k} \cdot \frac{\partial h_k}{\partial w_h}}_{\frac{\partial h_{t+1}}{\partial w_h}}$$

Esto es otra regla de la cadena mas!!.

$$\frac{\partial h_{t+1}}{\partial h_k} = \prod_{j=k}^{t} \frac{\partial h_{j+1}}{\partial h_j} \rightarrow \text{algunos terminos} \rightarrow \frac{\partial h_{k+1}}{\partial h_k} \cdot \ldots \cdot \frac{\partial h_{t+1}}{\partial h_t}$$

el 1º el último

Ahora si unimos todo

$$\frac{\partial L_{t+1}}{\partial w_h} = \frac{\partial L_{t+1}}{\partial \hat{y}_{t+1}} \cdot \frac{\partial \hat{y}_{t+1}}{\partial o_{t+1}} \cdot \frac{\partial o_{t+1}}{\partial h_{t+1}} \sum_{k=1}^{t+1} \left[\prod_{j=k}^{t} \left(\frac{\partial h_{j+1}}{\partial h_j} \right) \cdot \frac{\partial h_k}{\partial w_h} \right]$$

obs! es para 1 solo tiempo.

$$\frac{\partial L_T}{\partial W_h} = \sum_t^T \frac{\partial L_t}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial O_t} \cdot \frac{\partial O_t}{\partial h_t} \sum_{k=1}^{t} \left[\prod_{j=k}^{t} \left(\frac{\partial h_j}{\partial h_{j-1}} \right) \frac{\partial h_k}{\partial W_h} \right]$$

(I)

(9)

4° Para $\frac{\partial L_T}{\partial W_x}$ se computa igual que 3°$\left(\frac{\partial L_T}{\partial W_h}\right)$

Ya que cada hidden state h_t es función de W_x y h_{t-1}

$$\frac{\partial L_T}{\partial W_x} = \sum_t^T \frac{\partial L_t}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial O_t} \cdot \frac{\partial O_t}{\partial h_t} \cdot \sum_{k=1}^{t} \left[\prod_{j=k}^{t} \left(\frac{\partial h_j}{\partial h_{j-1}} \right) \cdot \frac{\partial h_k}{\partial W_x} \right]$$

↑ dif. acá cambia

5° Igual procedimiento

$$\frac{\partial L_T}{\partial b_h} = \sum_t^T \frac{\partial L_t}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial O_t} \cdot \frac{\partial O_t}{\partial h_t} \sum_{k=1}^{t} \left[\prod_{j=k}^{t} \left(\frac{\partial h_j}{\partial h_{j-1}} \right) \cdot \frac{\partial h_k}{\partial b_x} \right]$$

↓

$= 1$ de ecuación (5) hoja (I)