TopologyPracticum

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1 Enzo Rodriguez

2 Lower Star Image Filtrations for Scanning

Here I take the notebook Lower Star Image Filtrations from ripser.py and adjust it to create a left-to-right scan on a letter from letters.csv and I would also perform a right-to-left scan on each letter. Then I plan on inverting the image and performing an up-to-down scan follwed by a down-to-up scan. First let's import everything they imported. I also import csv to read our csv file.

```
[1]: import numpy as np
  import matplotlib.pyplot as plt
  import scipy
  from scipy import ndimage
  import PIL
  from persim import plot_diagrams
  from ripser import ripser, lower_star_img
  import csv
```

2.1 Left-to-right Scanning Example

Our function on the image will be very simple: if the value we see in the corresponding row is 0 then the value on the pixel is 100, if the value is 1 then the value is the x1 coordinate. I'll be scanning the leters in sequential order from A-Z first left-to-right.

```
[2]: from numpy import genfromtxt import numpy as np # read in file of letters

# read in file of letters

letters = genfromtxt('letters.csv', delimiter=',') # take first letter letter_one_line=letters[0,:]

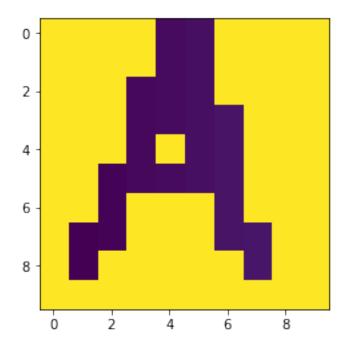
# initialize matrix of size 10x10 with all values 100 letter=np.full((10, 10), 100)

# convert one line letter to 10x10 matrix replacing zeros with 100 for k in range(1,101):
```

```
if letter_one_line[k] == 1.0:
    row = int((k-1)/10)
    column = (k-1)%10
    letter[row,column] = k%10
print(letter.shape)
print(letter)

plt.imshow(letter)
plt.show()
```

```
(10, 10)
[[100 100 100 100
                   6 100 100 100 100]
[100 100 100 100
                5
                   6 100 100 100 100]
[100 100 100 4 5
                   6 100 100 100 100]
[100 100 100 4 5
                       7 100 100 100]
[100 100 100
                     7 100 100 100]
           4 100
[100 100
             4 5
                       7 100 100 100]
                       7 100 100 100]
[100 100
         3 100 100 100
[100 2
         3 100 100 100
                          8 100 100]
[100
      2 100 100 100 100 100
                          8 100 100]
```

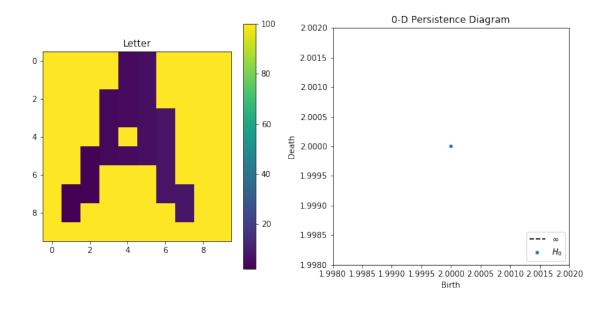


```
[3]: dgm = lower_star_img(letter)
print(dgm)
plt.figure(figsize=(10, 5))
plt.subplot(121)
```

```
plt.imshow(letter)
plt.colorbar()
plt.title("Letter")
plt.subplot(122)
plot_diagrams(dgm)
plt.title("0-D Persistence Diagram")
plt.tight_layout()
plt.show()
```

[[2. inf]]

/Users/enzo/anaconda2/lib/python2.7/site-packages/matplotlib/axes/_base.py:3152:
UserWarning: Attempting to set identical left==right results
in singular transformations; automatically expanding.
left=2.0, right=2.0
 'left=%s, right=%s') % (left, right))
/Users/enzo/anaconda2/lib/python2.7/site-packages/matplotlib/axes/_base.py:3471:
UserWarning: Attempting to set identical bottom==top results
in singular transformations; automatically expanding.
bottom=2.0, top=2.0
 'bottom=%s, top=%s') % (bottom, top))



[4]: help(lower_star_img)

Help on function lower_star_img in module ripser.ripser:

lower_star_img(img)

Construct a lower star filtration on an image

Parameters

img: ndarray (M, N)

An array of single channel image data

Returns

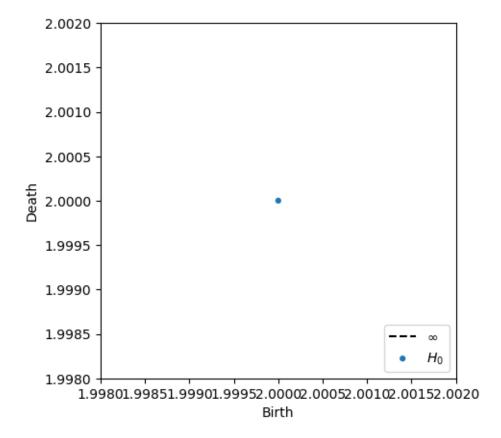
I: ndarray (K, 2)

 ${\tt A}$ O-dimensional persistence diagram corresponding to the sublevelset filtration

```
[5]: dgm = lower_star_img(letter)
  print(dgm.shape)
  print(dgm)
  plot_diagrams(dgm)

plt.show()
```

(1, 2) [[2. inf]]

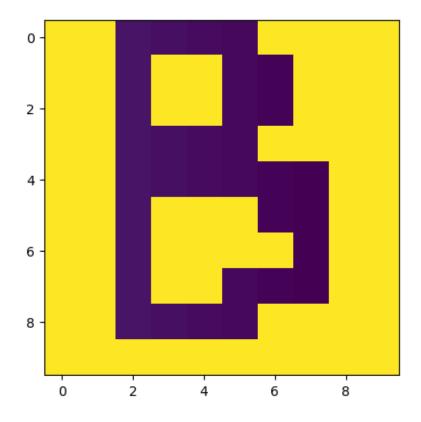


2.2 Right-to-left Scanning Example

Our function on the image will be very simple: if the value we see in the corresponding row is 0 then the value on the pixel is 100, if the value is 1 then the value is the x1 coordinate. I'll be scanning the leters in sequential order from A-Z right-to-left.

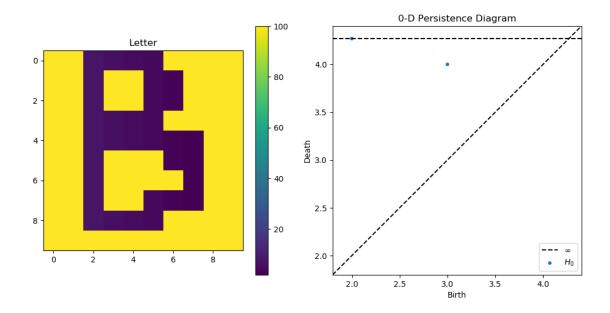
```
[6]: from numpy import genfromtxt
   import numpy as np
   # read in file of letters
    # read in file of letters
   letters = genfromtxt('letters.csv', delimiter=',') # take first letter
   letter_one_line=letters[1,:]
    # initialize matrix of size 10x10 with all values 100
   letter=np.full((10, 10), 100)
    # convert one line letter to 10x10 matrix replacing zeros with 100
   for k in range(1,101):
        if letter one line[k] == 1.0:
            row=int((k-1)/10)
            column=(k-1)\%10
            letter[row,column]=10-k%10 #this portion is how to scan the images_
    →right-to-left
   print(letter.shape)
   print(letter)
   plt.imshow(letter)
   plt.show()
```

```
(10, 10)
[[100 100
                      4 100 100 100 100]
              6
                  5
[100 100
          7 100 100
                         3 100 100 100]
[100 100
          7 100 100
                         3 100 100 100]
                      4
                      4 100 100 100 100]
Γ100 100
              6
                  5
[100 100
              6
                  5
                         3
                             2 100 100]
Γ100 100
          7 100 100 100
                             2 100 100]
Γ100 100
          7 100 100 100 100
                             2 100 1007
[100 100
          7 100 100
                             2 100 1007
                         3
[100 100
              6
                  5
                      4 100 100 100 100]
          7
```



```
[7]: dgm = lower_star_img(letter)
    print(dgm)
    plt.figure(figsize=(10, 5))
    plt.subplot(121)
    plt.imshow(letter)
    plt.colorbar()
    plt.title("Letter")
    plt.subplot(122)
    plot_diagrams(dgm)
    plt.title("0-D Persistence Diagram")
    plt.tight_layout()
    plt.show()
```

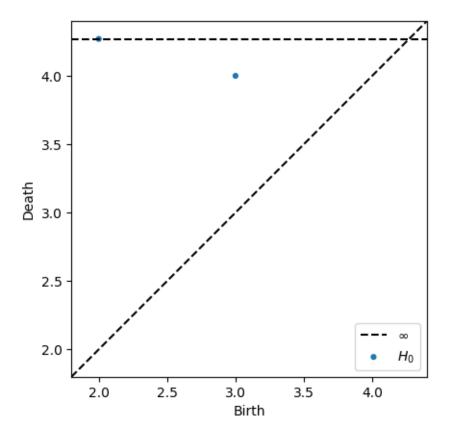
[[3. 4.] [2. inf]]



```
[8]: dgm = lower_star_img(letter)
  print(dgm.shape)
  print(dgm)
  plot_diagrams(dgm)

plt.show()
```

(2, 2) [[3. 4.] [2. inf]]



2.3 Down-to-up Scanning Example

Our function on the image will be very simple: if the value we see in the corresponding row is 0 then the value on the pixel is 100, if the value is 1 then the value is the x1 coordinate. I'll be scanning the leters in sequential order from A-Z first left-to-right inverted, which is down-to-up.

```
[9]: from numpy import genfromtxt
import numpy as np
# read in file of letters

# read in file of letters
letters = genfromtxt('letters.csv', delimiter=',') # take first letter
letter_one_line=letters[2,:]

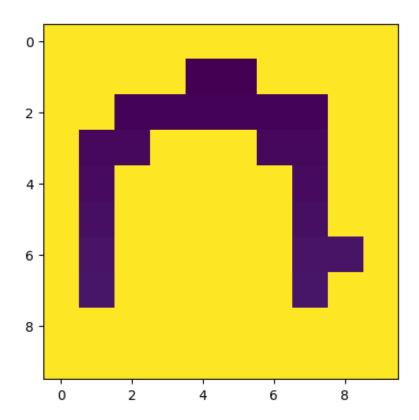
# initialize matrix of size 10x10 with all values 100
letter=np.full((10, 10), 100)

# convert one line letter to 10x10 matrix replacing zeros with 100
for k in range(1,101):
    if letter_one_line[k] == 1.0:
        row=int((k-1)%10)
```

```
column=(k-1)/10
    letter[row,column]=k%10
print(letter.shape)
print(letter)

plt.imshow(letter)
plt.show()
```

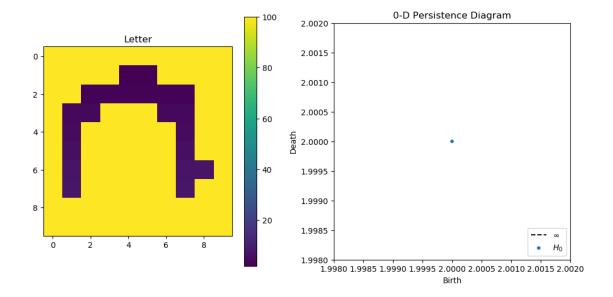
```
(10, 10)
[100 100 100 100
             2
               2 100 100 100 100]
[100 100
             3
                  3
                     3 100 100]
       3
          3
               3
                     4 100 100]
[100
     4
       4 100 100 100
                  4
[100
     5 100 100 100 100 100
                    5 100 100]
[100
     6 100 100 100 100 100
                     6 100 100]
[100
     7 100 100 100 100 100
                     7
                        7 100]
[100
     8 100 100 100 100 100
                     8 100 100]
```



```
[10]: dgm = lower_star_img(letter)
print(dgm)
```

```
plt.figure(figsize=(10, 5))
plt.subplot(121)
plt.imshow(letter)
plt.colorbar()
plt.title("Letter")
plt.subplot(122)
plot_diagrams(dgm)
plt.title("0-D Persistence Diagram")
plt.tight_layout()
plt.show()
```

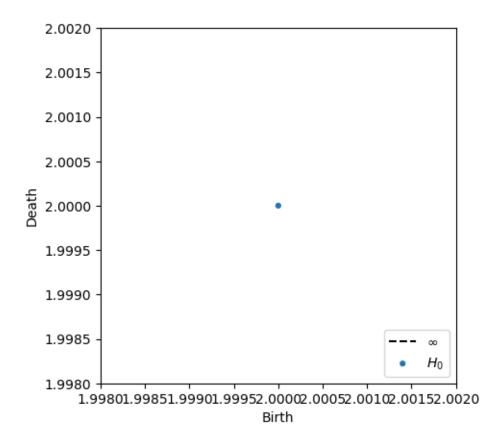
[[2. inf]]



```
[11]: dgm = lower_star_img(letter)
    print(dgm.shape)
    print(dgm)
    plot_diagrams(dgm)

plt.show()
```

(1, 2) [[2. inf]]



2.4 Up-to-down Inverted Scanning Example

Our function on the image will be very simple: if the value we see in the corresponding row is 0 then the value on the pixel is 100, if the value is 1 then the value is the x1 coordinate. I'll be scanning the leters in sequential order from A-Z right-to-left inverted, which is up-to-down.

```
[12]: from numpy import genfromtxt
import numpy as np
# read in file of letters

# read in file of letters
letters = genfromtxt('letters.csv', delimiter=',') # take first letter
letter_one_line=letters[3,:]

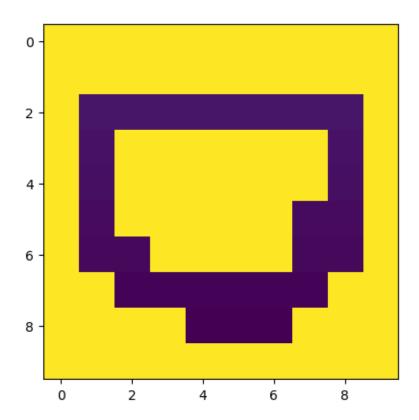
# initialize matrix of size 10x10 with all values 100
letter=np.full((10, 10), 100)

# convert one line letter to 10x10 matrix replacing zeros with 100
for k in range(1,101):
    if letter_one_line[k]==1.0:
        row=int((k-1)%10)
```

```
column=(k-1)/10
    letter[row,column]=10-k%10
print(letter.shape)
print(letter)

plt.imshow(letter)
plt.show()
```

```
(10, 10)
Γ100
                       7 100]
       7
          7
            7
               7
                       6 100]
[100
     6 100 100 100 100 100 100
[100
    5 100 100 100 100 100 100
                       5 100]
[100 4 100 100 100 100 100
                       4 100]
[100
       3 100 100 100 100
                       3 100]
                     3
[100 100
          2
                     2 100 100]
       2
             2
               2
                  2
[100 100 100 100
                  1 100 100 100]
             1
               1
```

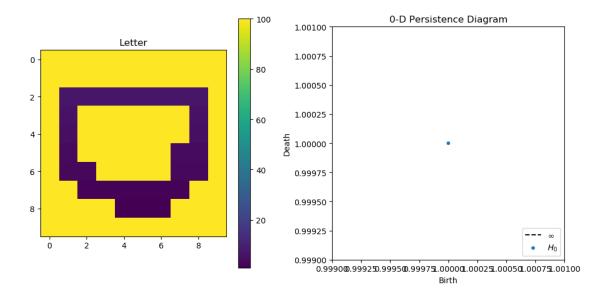


```
[13]: dgm = lower_star_img(letter)
print(dgm)
```

```
plt.figure(figsize=(10, 5))
plt.subplot(121)
plt.imshow(letter)
plt.colorbar()
plt.title("Letter")
plt.subplot(122)
plot_diagrams(dgm)
plt.title("0-D Persistence Diagram")
plt.tight_layout()
plt.show()
```

[[1. inf]]

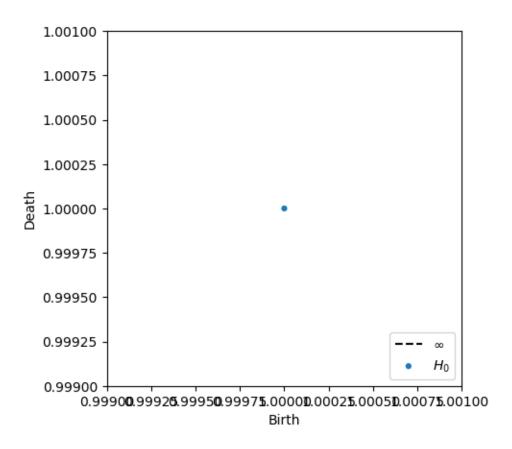
/Users/enzo/anaconda2/lib/python2.7/site-packages/matplotlib/axes/_base.py:3152:
UserWarning: Attempting to set identical left==right results
in singular transformations; automatically expanding.
left=1.0, right=1.0
 'left=%s, right=%s') % (left, right))
/Users/enzo/anaconda2/lib/python2.7/site-packages/matplotlib/axes/_base.py:3471:
UserWarning: Attempting to set identical bottom==top results
in singular transformations; automatically expanding.
bottom=1.0, top=1.0
 'bottom=%s, top=%s') % (bottom, top))



```
[14]: dgm = lower_star_img(letter)
    print(dgm.shape)
    print(dgm)
    plot_diagrams(dgm)
```

plt.show()

(1, 2) [[1. inf]]



2.5 Angle Scanning Example

```
[15]: from numpy import genfromtxt
import numpy as np
# read in file of letters

# read in file of letters
letters = genfromtxt('letters.csv', delimiter=',') # take first letter
letter_one_line=letters[4,:]

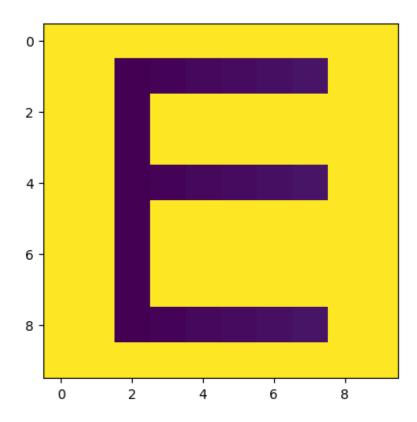
# initialize matrix of size 10x10 with all values 100
letter=np.full((10, 10), 100)

# convert one line letter to 10x10 matrix replacing zeros with 100
for k in range(1,101):
```

```
if letter_one_line[k] == 1.0:
    row = int((k-1)/10)
    column = (k-1)%10
    letter[row,column] = max(k%10,int(k-1)%10)
print(letter.shape)
print(letter)

plt.imshow(letter)
plt.show()
```

```
(10, 10)
[100 100
                        8 100 100]
            4
               5
                  6
                     7
[100 100
         3 100 100 100 100 100 100 100]
       3 100 100 100 100 100 100 100]
[100 100
[100 100
               5
                  6
                     7
                        8 100 100]
[100 100
       3 100 100 100 100 100 100 100]
[100 100
       3 100 100 100 100 100 100 100]
[100 100
        3 100 100 100 100 100 100 100]
[100 100
         3
            4
               5
                  6
                     7
                        8 100 100]
```



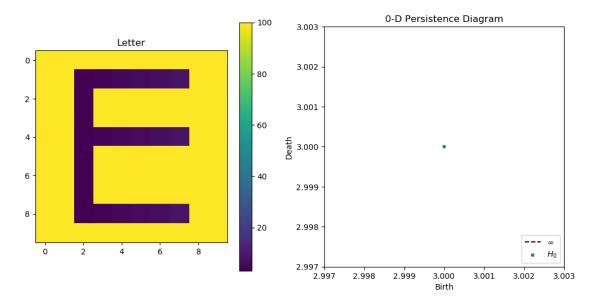
```
[16]: dgm = lower_star_img(letter)
    print(dgm)
    plt.figure(figsize=(10, 5))
    plt.subplot(121)
    plt.imshow(letter)
    plt.colorbar()
    plt.title("Letter")
    plt.subplot(122)
    plot_diagrams(dgm)
    plt.title("0-D Persistence Diagram")
    plt.tight_layout()
    plt.show()
```

[[3. inf]]

/Users/enzo/anaconda2/lib/python2.7/site-packages/matplotlib/axes/_base.py:3152:
UserWarning: Attempting to set identical left==right results
in singular transformations; automatically expanding.
left=3.0, right=3.0
 'left=%s, right=%s') % (left, right))
/Users/enzo/anaconda2/lib/python2.7/site-packages/matplotlib/axes/_base_py:3471:

/Users/enzo/anaconda2/lib/python2.7/site-packages/matplotlib/axes/_base.py:3471: UserWarning: Attempting to set identical bottom==top results in singular transformations; automatically expanding. bottom=3.0, top=3.0

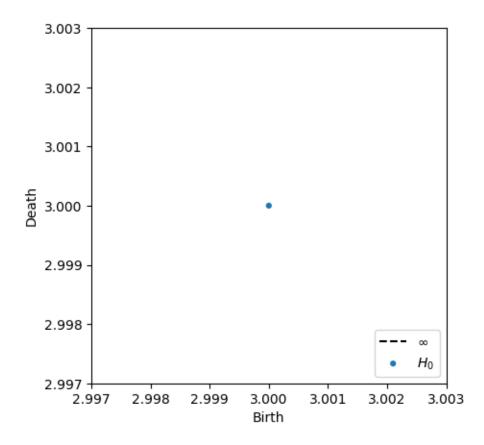
'bottom=%s, top=%s') % (bottom, top))



```
[17]: dgm = lower_star_img(letter)
    print(dgm.shape)
    print(dgm)
```

```
plot_diagrams(dgm)
plt.show()
```

```
(1, 2)
[[ 3. inf]]
```



2.6 Diagonal Scanning Example

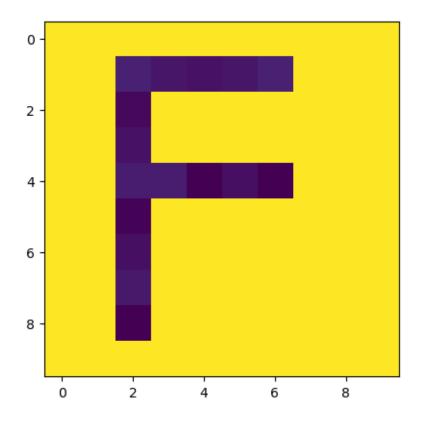
```
[18]: from numpy import genfromtxt
import numpy as np
# read in file of letters

# read in file of letters
letters = genfromtxt('letters.csv', delimiter=',') # take first letter
letter_one_line=letters[5,:]

# initialize matrix of size 10x10 with all values 100
letter=np.full((10, 10), 100)
```

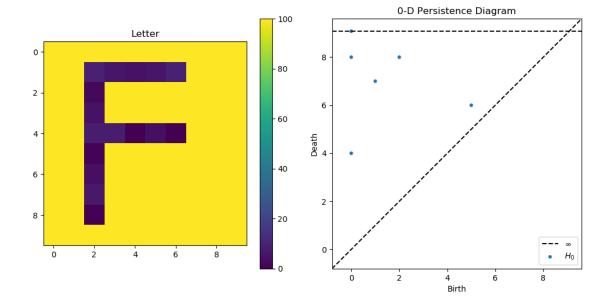
```
# convert one line letter to 10x10 matrix replacing zeros with 100
for k in range(1,101):
    if letter_one_line[k] == 1.0:
        row=int((k-1)/10)
        column=(k-1)%10
        letter[row,column] = (column+row)*k%10
print(letter.shape)
print(letter)
plt.imshow(letter)
plt.show()
```

```
(10, 10)
9 100 100 100]
Γ100 100
             5
                6
[100 100
       2 100 100 100 100 100 100 100]
Γ100 100
      5 100 100 100 100 100 100 100]
[100 100
             0
                4
                  0 100 100 100]
[100 100
      1 100 100 100 100 100 100 100]
[100 100
      4 100 100 100 100 100 100 100]
[100 100 7 100 100 100 100 100 100 100]
```



```
[19]: dgm = lower_star_img(letter)
    print(dgm)
    plt.figure(figsize=(10, 5))
    plt.subplot(121)
    plt.imshow(letter)
    plt.colorbar()
    plt.title("Letter")
    plt.subplot(122)
    plot_diagrams(dgm)
    plt.title("0-D Persistence Diagram")
    plt.tight_layout()
    plt.show()
```

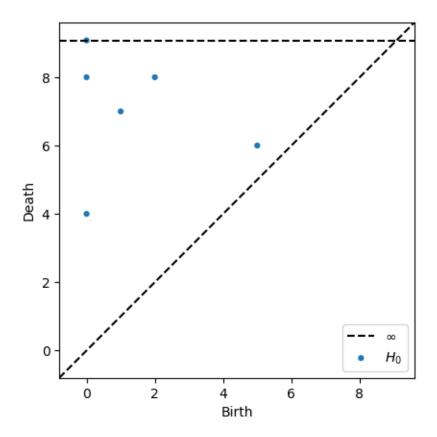
[[0. 4.] [5. 6.] [1. 7.] [0. 8.] [2. 8.] [0. inf]]



```
[20]: dgm = lower_star_img(letter)
print(dgm.shape)
print(dgm)
plot_diagrams(dgm)

plt.show()
```

```
(6, 2)
[[ 0. 4.]
[ 5. 6.]
[ 1. 7.]
[ 0. 8.]
[ 2. 8.]
[ 0. inf]]
```



3 Looping through all the letters in every direction

```
[21]: # Left-to-right scanning through loops
letters = genfromtxt('letters.csv', delimiter=',') # Upload the file

dgmLR = [None]*26 #Initialize an empty list
for i in range(26):
    letter_one_line=letters[i,:]

# initialize matrix of size 10x10 with all values 100
letter=np.full((10, 10), 100)
```

```
# convert one line letter to 10x10 matrix replacing zeros with 100
         for k in range(1,101):
             if letter_one_line[k] == 1.0:
                 row=int((k-1)/10)
                 column=(k-1)\%10
                 letter[row,column]=k%10
         dgmLR[i] = lower_star_img(letter)
[22]: # Print A-Z diagrams
     print(dgmLR[0:25])
    [array([[ 2., inf]]), array([[ 3., inf]]), array([[ 2., inf]]), array([[ 3.,
    inf]]), array([[ 3., inf]]), array([[ 3., inf]]), array([[ 2., inf]]), array([[
    3., inf]]), array([[ 4., 5.],
           [4., inf]]), array([[4., 6.],
           [ 4., inf]]), array([[ 3., inf]]), array([[ 3., inf]]), array([[ 2.,
    inf]]), array([[ 2., inf]]), array([[ 2., inf]]), array([[ 3., inf]]), array([[
    6., 7.],
           [2., inf]]), array([[3., inf]]), array([[3., 8.],
           [3., inf]]), array([[2., inf]]), array([[3., inf]]), array([[2.,
    inf]]), array([[ 2., inf]]), array([[ 3., 4.],
           [ 3., inf]]), array([[ 3., inf]])]
[23]: # Print A digram
     print(dgmLR[0])
    [[ 2. inf]]
[24]: # Print z diagram
     print(dgmLR[25])
    [[ 3. 7.]
     [ 2. inf]]
[25]: # Right-to-left scanning through loops
     letters = genfromtxt('letters.csv', delimiter=',') # Upload the file
     dgmRL = [None] *26 #Initialize an empty list
     for i in range(26):
         letter_one_line=letters[i,:]
         # initialize matrix of size 10x10 with all values 100
         letter=np.full((10, 10), 100)
         # convert one line letter to 10x10 matrix replacing zeros with 100
         for k in range(1,101):
             if letter_one_line[k] == 1.0:
```

```
row=int((k-1)/10)
                 column=(k-1)\%10
                 letter[row,column]=10-k%10
        dgmRL[i] = lower_star_img(letter)
[26]: # Print A-Z diagrams
    print(dgmRL[0:25])
    [array([[ 2., inf]]), array([[ 3., 4.],
           [2., inf]]), array([[2., 7.],
           [2., inf]]), array([[1., inf]]), array([[2., 7.],
           [2., 7.],
           [ 2., inf]]), array([[ 3., 7.],
           [3., inf]]), array([[3., 7.],
           [3., inf]]), array([[3., inf]]), array([[4., 5.],
           [4., inf]]), array([[4., inf]]), array([[3., 6.],
           [ 3., inf]]), array([[ 3., inf]]), array([[ 2., inf]]), array([[ 3.,
    inf]]), array([[ 2., inf]]), array([[ 2., inf]]), array([[ 2., inf]]), array([[
    3., 5.],
           [ 3., inf]]), array([[ 2., 7.],
           [2., inf]]), array([[2., inf]]), array([[2., inf]]), array([[2.,
    inf]]), array([[ 1., inf]]), array([[ 3., 5.],
           [ 3., inf]]), array([[ 3., inf]])]
[27]: # Print A diagram
    print(dgmRL[0])
    [[ 2. inf]]
[28]: # Print Z diagram
    print(dgmRL[25])
    [[ 2. 6.]
     [ 2. inf]]
[29]: # Down-to-up scanning through loops
    letters = genfromtxt('letters.csv', delimiter=',') # Upload the file
    dgmDU = [None]*26 #Initialize an empty list
    for i in range(26):
        letter_one_line=letters[i,:]
         # initialize matrix of size 10x10 with all values 100
        letter=np.full((10, 10), 100)
         # convert one line letter to 10x10 matrix replacing zeros with 100
        for k in range(1,101):
```

```
if letter_one_line[k] == 1.0:
                 row=int((k-1)\%10)
                 column=(k-1)/10
                 letter[row,column]=k%10
         dgmDU[i] = lower_star_img(letter)
[30]: # Print A-Z diagrams
     print(dgmDU[0:25])
    [array([[ 2., inf]]), array([[ 3., inf]]), array([[ 2., inf]]), array([[ 3.,
    inf]]), array([[ 3., inf]]), array([[ 3., inf]]), array([[ 2., inf]]), array([[
    3., inf]]), array([[ 4., 5.],
           [ 4., inf]]), array([[ 4., 6.],
           [4., inf]]), array([[3., inf]]), array([[3., inf]]), array([[2.,
    inf]]), array([[ 2., inf]]), array([[ 2., inf]]), array([[ 3., inf]]), array([[
    6., 7.],
           [2., inf]]), array([[3., inf]]), array([[3., 8.],
           [3., inf]]), array([[2., inf]]), array([[3., inf]]), array([[2.,
    inf]]), array([[ 2., inf]]), array([[ 3., 4.],
           [ 3., inf]]), array([[ 3., inf]])]
[31]: # Print A diagram
     print(dgmDU[0])
    [[ 2. inf]]
[32]: # Print Z diagram
     print(dgmDU[25])
    [[ 3. 7.]
     [ 2. inf]]
[33]: # Up-to-down scanning through loops
     letters = genfromtxt('letters.csv', delimiter=',') # Upload the file
     dgmUD = [None]*26 #Initialize an empty list
     for i in range(26):
         letter_one_line=letters[i,:]
         # initialize matrix of size 10x10 with all values 100
         letter=np.full((10, 10), 100)
         # convert one line letter to 10x10 matrix replacing zeros with 100
         for k in range(1,101):
             if letter_one_line[k] == 1.0:
                 row=int((k-1)\%10)
                 column=(k-1)/10
```

```
letter[row,column]=10-k%10
        dgmUD[i] = lower_star_img(letter)
[34]: # Print A-Z diagrams
     print(dgmUD[0:25])
    [array([[ 2., inf]]), array([[ 3., 4.],
           [ 2., inf]]), array([[ 2., 7.],
           [ 2., inf]]), array([[ 1., inf]]), array([[ 2., 7.],
           [2., 7.],
           [ 2., inf]]), array([[ 3., 7.],
           [3., inf]]), array([[3., 7.],
           [3., inf]]), array([[3., inf]]), array([[4., 5.],
           [4., inf]]), array([[4., inf]]), array([[3., 6.],
           [ 3., inf]]), array([[ 3., inf]]), array([[ 2., inf]]), array([[ 3.,
    inf]]), array([[ 2., inf]]), array([[ 2., inf]]), array([[ 2., inf]]), array([[
    3., 5.],
           [ 3., inf]]), array([[ 2., 7.],
           [2., inf]]), array([[2., inf]]), array([[2., inf]]), array([[2.,
    inf]]), array([[ 1., inf]]), array([[ 3., 5.],
           [ 3., inf]]), array([[ 3., inf]])]
[35]: # Print A diagram
     print(dgmUD[0])
    [[ 2. inf]]
[36]: # Print Z diagram
     print(dgmUD[25])
    [[ 2. 6.]
     [ 2. inf]]
[37]: # Angle scanning through loops
     letters = genfromtxt('letters.csv', delimiter=',') # Upload the file
     dgmAngle = [None] *26 #Initialize an empty list
     for i in range(26):
        letter_one_line=letters[i,:]
         # initialize matrix of size 10x10 with all values 100
        letter=np.full((10, 10), 100)
        # convert one line letter to 10x10 matrix replacing zeros with 100
        for k in range(1,101):
             if letter_one_line[k] == 1.0:
                 row=int((k-1)/10)
```

```
column=(k-1)\%10
                 letter[row,column]=\max(k\%10,int(k-1)\%10)
         dgmAngle[i] = lower_star_img(letter)
[38]: # Print A-Z diagrams
     print(dgmAngle[0:25])
    [array([[ 2., inf]]), array([[ 3., inf]]), array([[ 2., inf]]), array([[ 3.,
    inf]]), array([[ 3., inf]]), array([[ 3., inf]]), array([[ 2., inf]]), array([[
    3., inf]]), array([[ 4., 5.],
           [4., inf]]), array([[4., 6.],
           [ 4., inf]]), array([[ 3., inf]]), array([[ 3., inf]]), array([[ 2.,
    inf]]), array([[ 2., inf]]), array([[ 2., inf]]), array([[ 3., inf]]), array([[
    6., 7.],
           [2., inf]]), array([[3., inf]]), array([[3., 8.],
           [ 3., inf]]), array([[ 2., inf]]), array([[ 3., inf]]), array([[ 2.,
    inf]]), array([[ 2., inf]]), array([[ 3., 4.],
           [ 3., inf]]), array([[ 3., inf]])]
[39]: # Print A digram
     print(dgmAngle[0])
    [[ 2. inf]]
[40]: # Print z diagram
     print(dgmAngle[25])
    [[ 3. 7.]
     [ 2. inf]]
[41]: # Diagonal scanning through loops
     letters = genfromtxt('letters.csv', delimiter=',') # Upload the file
     dgmDiagonal = [None]*26 #Initialize an empty list
     for i in range(26):
         letter_one_line=letters[i,:]
         # initialize matrix of size 10x10 with all values 100
         letter=np.full((10, 10), 100)
         # convert one line letter to 10x10 matrix replacing zeros with 100
         for k in range(1,101):
             if letter_one_line[k] ==1.0:
                 row=int((k-1)/10)
                 column=(k-1)\%10
                 letter[row,column]=(column+row)*k%10
         dgmDiagonal[i] = lower_star_img(letter)
```

[42]: # Print A-Z diagrams print(dgmDiagonal[0:25])

```
[array([[ 0., 3.],
      [ 0., 4.],
      [1., 5.],
      [ 0., 5.],
      [ 0., inf]]), array([[ 1., 2.],
      [ 0.,
            4.],
      [ 0.,
            4.],
      [ 2.,
            4.],
      [ 2.,
            5.],
      [0., 6.],
      [0., 6.],
      [1., 7.],
      [ 0., inf]]), array([[ 0., 4.],
      [ 1., 5.],
      [0., 5.],
      [4., 9.],
      [ 0., inf]]), array([[ 1., 2.],
      [ 0., 4.],
      [5., 6.],
      [2., 6.],
      [1., 7.],
      [0., 8.],
      [ 0., inf]]), array([[ 0., 4.],
      [0., 4.],
      [5., 6.],
      [1., 7.],
      [ 0., 8.],
      [0., 8.],
      [ 2.,
            8.],
      [4., 9.],
      [ 0., inf]]), array([[ 0., 4.],
      [5., 6.],
      [1., 7.],
      [0., 8.],
      [2., 8.],
      [ 0., inf]]), array([[ 1., 4.],
      [0., 4.],
      [0., 5.],
      [2., 5.],
      [5., 6.],
      [ 0., inf]]), array([[ 1., 7.],
      [2., 8.],
      [1., 8.],
      [ 0., inf]]), array([[ 0., 5.],
```

```
[ 0., 5.],
[ 0., 5.],
[ 0., inf]]), array([[ 2., 5.],
[ 0., 6.],
[2., 8.],
[ 0., inf]]), array([[ 1., 2.],
[1., 2.],
[2., 5.],
[0., 7.],
[2., 8.],
[ 0., inf]]), array([[ 0., 4.],
[1., 7.],
[2., 8.],
[ 0., inf]]), array([[ 0., 2.],
[2., 4.],
[0., 5.],
[2., 6.],
[2., 8.],
[0., 8.],
[ 0., inf]]), array([[ 1., 4.],
[2., 4.],
[ 0., 5.],
[ 0., inf]]), array([[ 0., 4.],
[0., 6.],
[5., 6.],
[2., 6.],
[1., 8.],
[0., 8.],
[ 0., inf]]), array([[ 1., 5.],
[5., 6.],
[2., 6.],
[ 0., 7.],
[ 0., inf]]), array([[ 0., 4.],
[0., 6.],
[5., 6.],
[0., 8.],
[0., 8.],
[2., 8.],
[ 0., inf]]), array([[ 1., 2.],
[1., 2.],
[0., 2.],
[2., 4.],
[2., 6.],
[0., 7.],
[ 0., inf]]), array([[ 0., 4.],
[2., 5.],
[5., 6.],
[0., 7.],
```

```
[2., 8.],
           [4., 9.],
           [ 0., inf]]), array([[ 0., 5.],
           [ 0., 5.],
           [ 0.,
                 5.],
           [ 4.,
                 9.],
           [4., 9.],
           [ 0., inf]]), array([[ 0., 4.],
           [ 1., 7.],
           [ 0., 8.],
           [0., 8.],
           [ 2.,
                 8.],
           [ 0., inf]]), array([[ 0., 2.],
           [2., 3.],
           [2., 8.],
           [ 0., inf]]), array([[ 1., 2.],
                 2.],
           [ 0.,
           [0., 3.],
           [ 0.,
                 5.],
           [ 0.,
                 6.],
           [5., 6.],
           [4., 8.],
           [0., 9.],
           [ 0., inf]]), array([[ 1., 2.],
           [0., 2.],
           [2., 4.],
           [ 0., 4.],
           [ 0., inf]]), array([[ 0., 5.],
           [0., 8.],
           [2., 8.],
           [ 0., inf]])]
[43]: # Print A digram
    print(dgmDiagonal[0])
    [[ 0. 3.]
     [ 0. 4.]
     [ 1. 5.]
     [ 0. 5.]
     [ 0. inf]]
[44]: # Print z diagram
    print(dgmDiagonal[25])
    [[ 2. 4.]
     [ 0. 5.]
```

[0., 8.],

[5. 6.] [0. 8.] [4. 9.] [0. inf]]

[]: