TRAJECTORIES with COSMOGRAVITY
TUTORIAL

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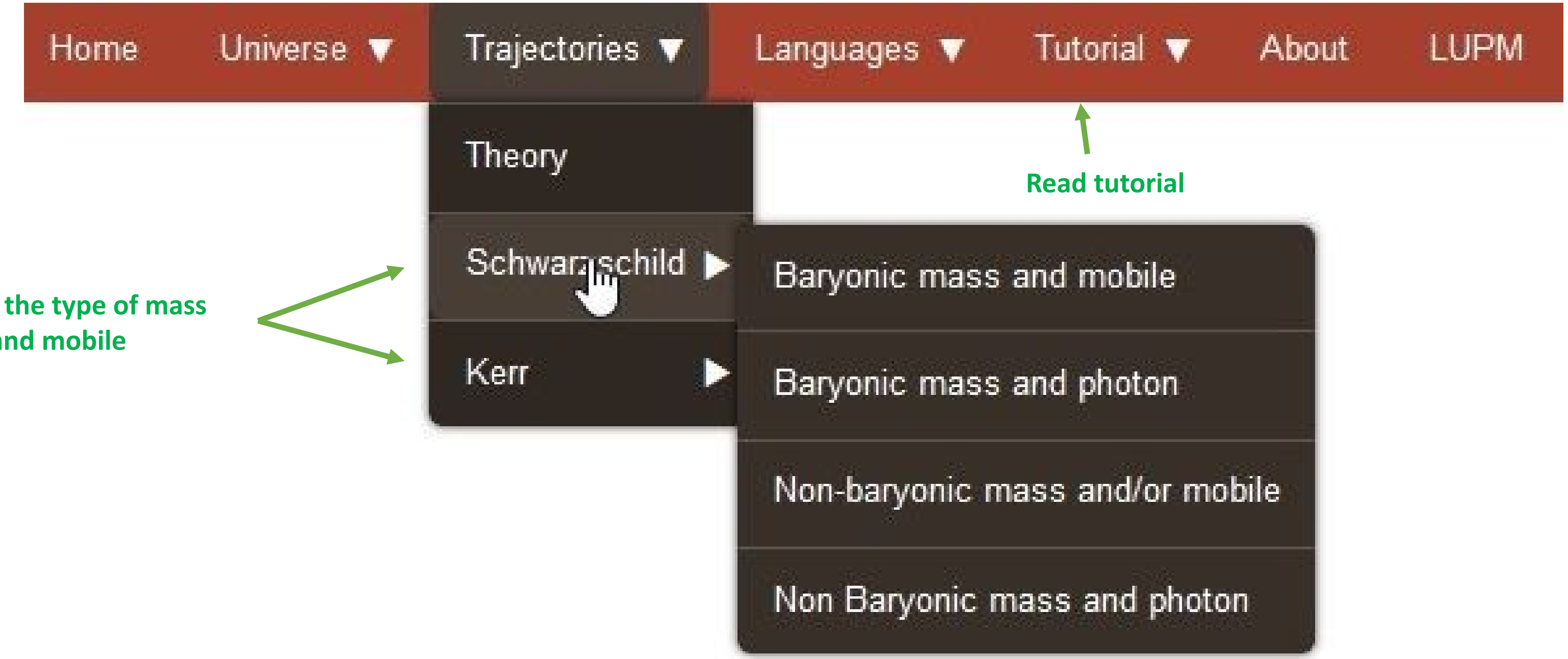
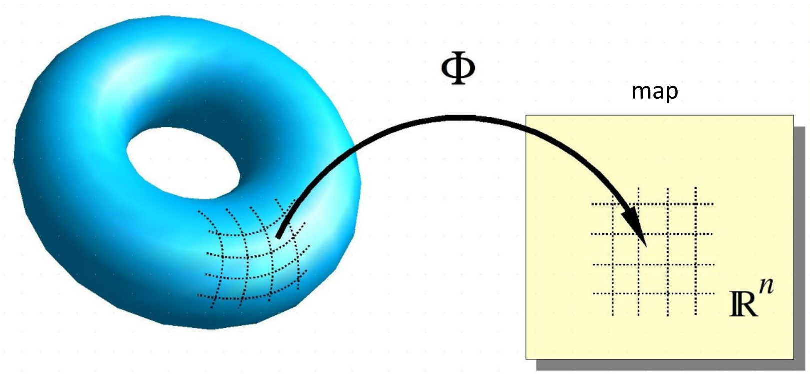


Read tutorial

Choose the type of mass and mobile

Geometric frame

Relativity has merged space and time, two notions that were completely distinct in Galilean mechanics. Four numbers are needed to determine an event in the space-time continuum: three for its spatial location (e.g. its Cartesian coordinates {x, y, z} or its spherical coordinates $\{r, \theta, \varphi\}$) and one for its date (t). The mathematical structure corresponding to this four-dimensional "continuum" is that of **variety**.



Variety: seen closely, a variety looks like R^n (n = 2 on the figure), but this is not necessarily true at the global level.

It should be emphasized that the local similarity with R^4 stops at the labeling of the points and does not extend to the Euclidean space structure of R^4. In particular, the choice of coordinate system is completely free.

These notes are from [Gourgoulhon-Relativité Générale]

In the **Cosmogravity** software the "trajectories" are the geodesics followed by the different particles (baryonic, non-baryonic, photons) represented by their coordinates (r, φ) in R^2 as a function of the proper time (τ) of the particles or the time of the distant observer (t).

The distance that would be measured (using the scale of the simulation) between two positions of a particle is obviously not equal to the metric distance between these two positions.

Trajectory of a massive projectile with Schwarzschild metric

Enter the physical parameters of the trajectory

Warning ← **Read the warning**

Use tool tips → M (kg) = 2.6e30 | $r_{physical}$ (m) = 11000 | r_0 (m) = 21000 | V_0 (m/s) = 1e8 | φ_0° = 0 | Φ_0° = 90

Number of projectiles 1 | Show the potential's graph ☑

Complete trajectory | Simple trajectory | Distant observer | Space Walker | Bounce

Continuous or point-by-point plotting

Choose the reference frame

Resume | Reset | Save | Last values

Start to run the simulation
Resume after a pause
Reset to return to initial parameters
Save for an image of the simulation
Last values to return to user parameters

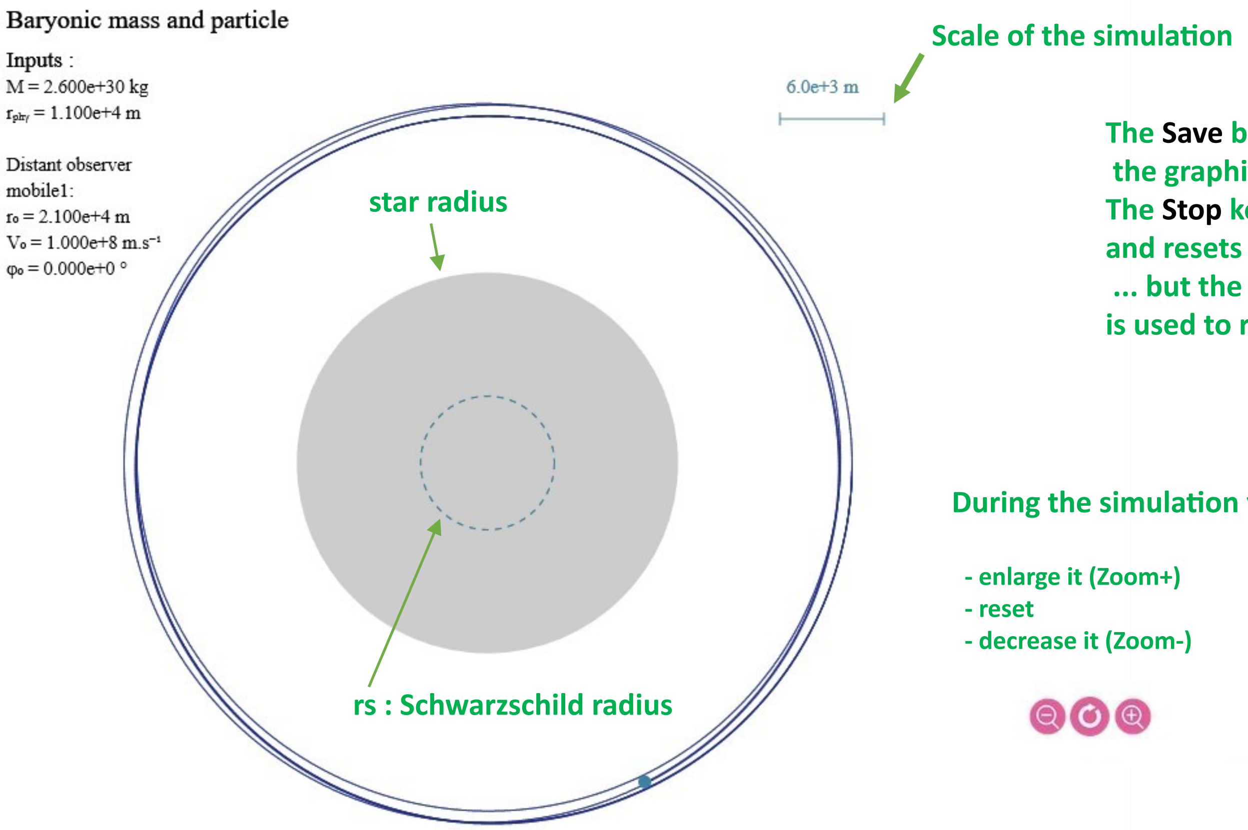
$L1(m)$	$E1$	$rs = \frac{2GM}{c^2}(m)$	$grav = \frac{GM}{R^2}\frac{1}{9.81}(g)$	$Vlib = c(\frac{rs}{R})^{1/2}(m.s^{-1})$	$T = 6.15*10^{-8}\frac{M_\odot}{M}(K)$	$t = 6.6*10^{74}(\frac{M}{M_\odot})^3(s)$
7.430e+3	9.583e-1	3.861e+3	1.462e+11	1.776e+8		

Calculated values during the simulation →

r(m)	Proper time	Gradient	V_r(m.s^{-1})	V_φ(m.s^{-1})	Distant observer time	Spectral shift / Energy expended	$V_{physique}$ (m.s^{-1})
2.054e+4	4.954e-3	8.718e+6	1.686e+6	1.020e+8	5.849e-3	1.801e-1	1.020e+8

Calculation on break

reference frame →

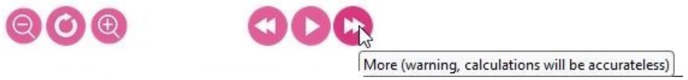


The **Save** button saves the graphic and the **Inputs**.
The **Stop** key ends the simulation and resets the inputs to the default values
... but the **Last values** key is used to recall the previous inputs.

During the simulation you can :

- enlarge it (Zoom+)
- reset
- decrease it (Zoom-)

- slow it down
- pause
- speed it up



Example : Neutron star

Trajectory of a massive projectile with Schwarzschild metric

2 mobiles around the asteroid

Warning

M (kg) = 2e13 | $r_{physical}$ (m) = 1000 | r_0 (m) = 3000 5000 | v_0 (m/s) = 0.4 0.5 | $\varphi°_0$ = 0 0 | $\alpha°$ = 65 230

Number of projectiles 2 Show the potential's graph

Complete trajectory | Simple trajectory | Distant observer | Space Walker | Bounce

Choose the impact absorption coefficient

Shock absorption : the bounce is limited to an impact speed of 0.3

Possibility of bounce

Stop | Reset | Save | Last values

$L1(m)$	$L2(m)$	$E1$	$E2$	$rs=\frac{2GM}{c^2}(m)$	$grav=\frac{GM}{R^2}\frac{1}{9.81}(g)$	$Vlib=c(\frac{rs}{R})^{1/2}$	$T=6.15*10^{-8}\frac{M_\odot}{M}(K)$	$t=6.6*10^{74}(\frac{M}{M_\odot})^3(s)$
3.628e-6	-6.388e-6	1.000e+0	1.000e+0	2.970e-14	1.361e-4	1.634e+0	6.464e+9	6.710e+23

r(m)	Proper time	Gradient	$V_r(m.s^{-1})$	$V_\varphi(m.s^{-1})$	Distant observer time	Spectral shift / Energy expended	$V_{physique}$ $(m.s^{-1})$
1.016e+3	1.829e+4	2.406e-6	0.000e+0	2.150e-1	1.829e+4		2.150e-1

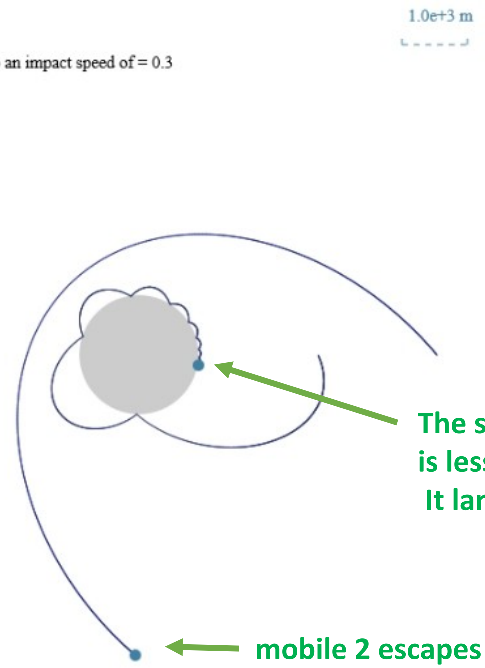
r(m)	Proper time	Gradient	$V_r(m.s^{-1})$	$V_\varphi(m.s^{-1})$	Distant observer time	Spectral shift / Energy expended	$V_{physique}$ $(m.s^{-1})$
5.048e+3	1.829e+4	3.809e-9	0.000e+0	-3.794e-1	1.829e+4		3.794e-1

Calculation on break

Baryonic mass and particle

Inputs :
M = 2.000e+13 kg
r_{phy} = 1.000e+3 m
Shock absorption : the bounce is limited to an impact speed of = 0.3
Space Walker
mobile1:
r_0 = 3.000e+3 m
V_0 = 4.000e-1 $m.s^{-1}$
φ = 0.000e+0
mobile2:
r_0 = 5.000e+3 m
V_0 = 5.000e-1 $m.s^{-1}$
φ= 0.000e+0

reference frame



The speed impact of mobile 1 is less than 300 m/s :
It lands on the asteroid

mobile 2 escapes

Example : Small asteroid

Example : Photons trajectories

Trajectory of a photon with Schwarzschild metric

Warning

M (kg) = 2e30 | $r_{physical}$ (m) = 0 | r_0 (m) = 10000 | 8000 | $\varphi°_0$ = 50 | 210 | α° = 135 | 135

Number of projectiles 2 | Show the potential's graph

Complete trajectory | Simple trajectory | Distant observer | Photon | Bounce

Stop | Reset | Save | Last values

$L1(m)$	$L2(m)$	$E1$	$E2$	$rs = \frac{2GM}{c^2}(m)$	$grav = \frac{GM}{R^2}\frac{1}{9.81}(g)$	$Vlib = c(\frac{rs}{R})^{1/2}$	$T = 6.15 * 10^{-8}\frac{M_\odot}{M}(K)$	$t = 6.6 * 10^{74}(\frac{M}{M_\odot})^3(s)$
8.434e+3	7.134e+3	1.000e+0	1.000e+0	2.970e+3			6.464e-8	6.710e+74

r(m)	Proper time	Gradient	$V_r(m.s^{-1})$	$V_\varphi(m.s^{-1})$	Distant observer time	$V_{physique}$ $(m.s^{-1})$
9.912e+3	0.000e+0		2.105e+8	2.135e+8	6.735e-5	2.99792458e+8
r(m)	**Proper time**	**Gradient**	**$V_r(m.s^{-1})$**	**$V_\varphi(m.s^{-1})$**	**Distant observer time**	**$V_{physique}$ $(m.s^{-1})$**
0.000e+0	0.000e+0				Infinity	

Calculation on break

The speed of the photon is identical in all reference frames

The proper time of a photon is always zero.

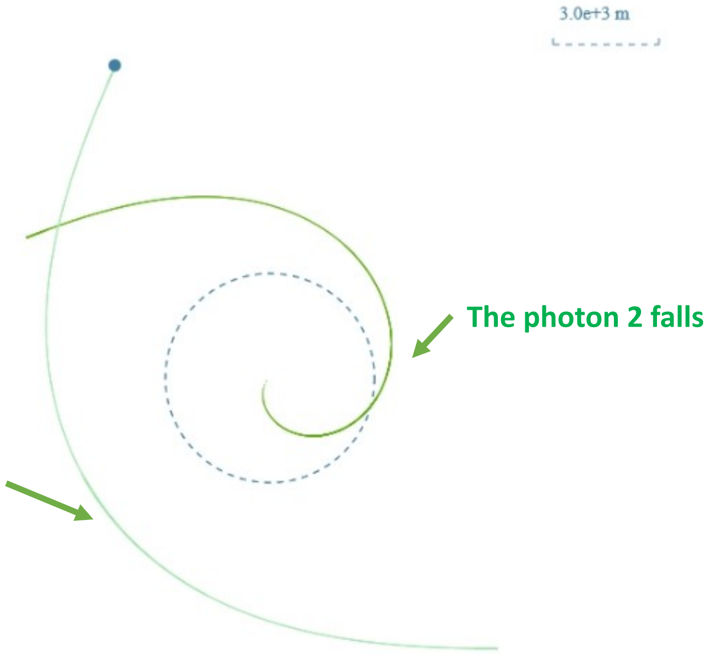
For the distant observer the mobile takes an infinite time to reach rs

The speed of the photon is meaningless inside the black hole horizon

Baryonic mass and photon

Inputs :
M = 2.000e+30 kg
r_{phy} = 0.000e+0 m

Photon
mobile1:
r_0 = 1.000e+4 m
V_0 = 2.998e+8 $m.s^{-1}$
φ = 8.727e-1
mobile2:
r_0 = 8.000e+3 m
V_0 = 2.998e+8 $m.s^{-1}$
φ= 3.665e+0



The photon 2 falls into the black hole

The trajectory of photon 1 is deviated in the gravitational field of the black hole

Trajectory of a massive projectile with Schwarzschild metric
(non baryonic case)

Warning

M (kg) = 2e30 | $r_{physical}$ (m) = 7e8 | r_0 (m) = 9e8 | 4e8 | 6e8 | v_0(m.s^{-1}) = 3e5 | 3e5 | 3e5 | φ °$_0$ = 0 | 90 | 180 | α° = 90 | 90 | 90

Number of projectiles 3 | Show the potential's graph

Complete trajectory | Simple trajectory | Distant observer | Space Walker

Stop | Reset | Save | Last values

$L1(m)$	$L2(m)$	$L3(m)$	$E1$	$E2$	$E3$	$rs = \frac{2GM}{c^2}(m)$	$grav = \frac{GM}{R^2}\frac{1}{9.81}(g)$	$Vlib = c(\frac{rs}{R})^{1/2}$	$T = 6.15 * 10^{-8}\frac{M\odot}{M}(K)$	$t = 6.6 * 10^{74}(\frac{M}{M\odot})^3(s)$
9.006e+5	4.003e+5	6.004e+5	1.000e+0	1.000e+0	1.000e+0	2.970e+3	2.777e+1	6.176e+5	6.464e-8	6.710e+74

r(m)	Proper time	Gradient	V_r(m.s^{-1})	V_φ(m.s^{-1})	Distant observer time	Spectral shift / Energy expended	$V_{physique}$ (m.s^{-1})
8.546e+8	2.176e+4	1.773e-8	7.706e+4	3.159e+5	2.176e+4	2.326e-6	3.252e+5
r(m)	Proper time	Gradient	V_r(m.s^{-1})	V_φ(m.s^{-1})	Distant observer time	Spectral shift / Energy expended	$V_{physique}$ (m.s^{-1})
4.593e+8	2.176e+4	1.360e-6	4.372e+4	2.613e+5	2.176e+4	3.116e-6	2.649e+5
r(m)	Proper time	Gradient	V_r(m.s^{-1})	V_φ(m.s^{-1})	Distant observer time	Spectral shift / Energy expended	$V_{physique}$ (m.s^{-1})
5.177e+8	2.176e+4	1.742e-6	-7.006e+4	3.477e+5	2.176e+4	3.302e-6	3.547e+5

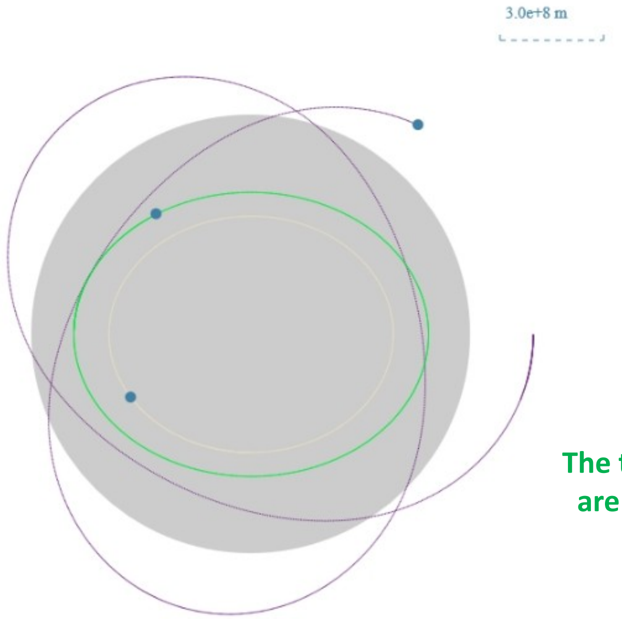
Calculation on break

For the calculation of the spectral shift the observer is assumed to be very far (non-cosmological distance) in a direction perpendicular to the trajectory plane

Nonbaryonic mass and or particle

Inputs :
M = 2.000e+30 kg
r_{phy} = 7.000e+8 m
Distant observer
mobile1:
r_0 = 9.000e+8 m
V_0 = 3.000e+5 m.s^{-1}
φ = 0.000e+0
mobile2:
r_0 = 4.000e+8 m

V_0 = 3.000e+5 m.s^{-1}
φ= 1.571e+0



The three particles of baryonic matter are subject only to the gravitational field of the central mass

Example : Matter of the Sun supposed to be non-baryonic with constant mass density

Example :
photon and massive rotating black hole

Trajectory of a photon with Kerr metric

Warning

M (kg) = 2e39 | r_0 (m) = 5e12 | J ($kg.m^2.s^{-1}$)= 8.4e59 | φ_0 (°) = 0 | φ_D (°) = 138 | nzoom = -5 | Show the potential's graph ☑

Choose a decrease - or an increase + of scale before the plot

Complete trajectory | Simple trajectory | Distant observer | Photon

Stop | Reset | Save | Last values | Pre-zoom

$L(m)$	E	$rs = \frac{2GM}{c^2}(m)$	$a = \frac{J}{cM}(m)$	$Rh+(m)$	$Rh-(m)$	$g = \frac{c^2}{2Rh+}\frac{(Rh+^2-a^2)}{(Rh+^2+a^2)}(m.s^{-2})$
3.686e+12	1.000e+0	2.970e+12	1.401e+12	1.978e+12	9.921e+11	1.492e+16

r(m)	Proper time	Acceleration gradient	V_r ($m.s^{-1}$)	V_φ ($m.s^{-1}$)	$V_{physique}$ ($m.s^{-1}$)	Distant observer time
1.97829545e+12	0.00000000e+0					2.11961102e+5

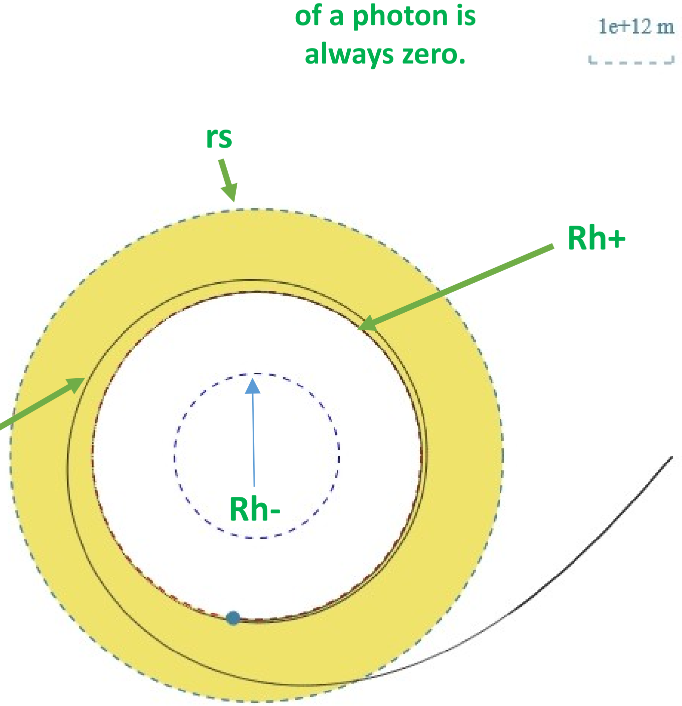
Calculation on break

The proper time of a photon is always zero.

Trajectory of a photon with Kerr metric

Inputs :
M = 2.000e+39 kg
r_0 = 5.000e+12 m
a = 1.401e+12 m
φ = 1.380e+2 °
Distant observer

Reference frame



In the reference frame of the distant observer, the photon wraps itself indefinitely around the event horizon Rh+

Potential graph (see **Theory**)