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# MERIT.jl: Julia's Version

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A Final Year Project submitted in partial fulfilment  
of the requirements for the degree of  
MAI (Electronic and Computer Engineering)

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# Abstract

Microwave imaging is being trialed as a safer and cheaper imaging modality over X-rays and ultrasound which is the current modality of choice in medical imaging. In the last 8 years, 10 clinical studies have been conducted that trialed novel microwave imaging systems, focused mainly in the area of breast tumor detection. The success of these trials has proven the viability of microwaves as an alternate imaging modality. MERIT.jl, then, aims to be a reliable, user-friendly and high-performance software framework that aims to increase the interoperability between microwave imaging systems, the data they produce and the software that analyzes that data. It contains helper functions as well as inverse scattering algorithms to aid in image reconstruction. The unique features in Julia, such as parametric polymorphism, multiple dispatch and type stability to name a few, have allowed MERIT.jl to possess unparalleled customizability and extensibility. The library also demonstrates a level of type safety by implementing a lightweight Points type that allows for the efficient processing of coordinate points. Unlike traditional methods that represent points as a matrix of numbers, the Points type allows the Julia compiler to recognize when an incorrect collection is passed in place of the points collection, throwing an error in the process. In this way, MERIT.jl can promise a level of safety without having to compromise on performance.

# Acknowledgements

Thanks, Everyone!

# List of Figures

1	Example of a Fully Multistatic Configuration (Top-Down) showing a circular antenna array with one transmitting antenna illuminating a point scatterer whose reflections are collected by 7 receiving antennas . . . . .	3
2	The MARIA M4 and M5 system. (a) The MARIA M4 multistatic antenna array. (b) The M4 system in a clinical setting. (c) The integrated M5 package [1]	3
3	The Wavelia System installed in a Hospital [2] . . . . .	4
4	The Leveled Multistatic Approach of Wavelia with a vertically moveable antenna array [2] . . . . .	5
5	The TSAR Prototype used in their clinical study [3] . . . . .	7
6	Phased antenna array demonstrating beamforming . . . . .	7
7	Point Emitter with waves incident on a phased array . . . . .	8
8	Comparason between DAS (a, c) and DMAS (b, d) Beamformers. The DMAS beamformer demonstrates a lower noise floor and reduced clutter . . . . .	10
9	Multiple Dispatch (top) vs Single Dispatch (bottom) demonstrating that in the multiple dispatch paradigm, the executed function also depends on the type and order of its inputs . . . . .	15
10	Type hierarchy for the Integer Type . . . . .	16
11	Current type hierarchy in MERIT.jl . . . . .	23
12	An overview of the current workflow used to image a scan in MERIT.jl . . . .	26
13	A comparison between Julia and MATLAB output, showing an averaged MSE of $8.4417 \times 10^{-7}$ . . . . .	27
14	Runtime for increasing Points . . . . .	28
15	Runtime for increasing Channels . . . . .	28
16	Runtime for increasing Frequency divisions . . . . .	28

# Contents

<b>Introduction</b>	<b>1</b>
<b>Background</b>	<b>2</b>
1.1 Imaging Systems . . . . .	2
1.1.1 MARIA M4 . . . . .	2
1.1.2 Wavelia . . . . .	4
1.1.3 TSAR . . . . .	6
1.2 Beamforming Algorithms . . . . .	7
1.2.1 Delay and Sum . . . . .	8
1.2.2 Weighted Delay and Sum Beamformer . . . . .	9
1.2.3 Delay Multiply and Sum . . . . .	10
1.3 Development Platforms . . . . .	11
1.3.1 Python . . . . .	11
1.3.2 MATLAB . . . . .	12
1.3.3 Julia . . . . .	12
<b>Programming Paradigms</b>	<b>13</b>
2.1 Multiple Dispatch . . . . .	13
2.2 Type Heirarchy . . . . .	15
2.3 Parametric Polymorphism . . . . .	17
2.4 Type Stability . . . . .	17
2.5 Closure . . . . .	18
2.6 Type Safety . . . . .	19
3.1 Julia For Scientific Computing . . . . .	21
3.2 Julia for MERIT.jl . . . . .	22
3.2.1 Multiple Dispatch in MERIT.jl . . . . .	22
3.2.2 Type Hierarchy in MERIT.jl . . . . .	22
3.2.3 Parametric Polymorphism in MERIT.jl . . . . .	23
3.2.4 Type Stability in MERIT.jl . . . . .	23
3.2.5 Closure in MERIT.jl . . . . .	24
3.2.6 Type Safety in MERIT.jl . . . . .	24
<b>Results</b>	<b>25</b>
4.1 Current Workflow . . . . .	25
4.2 Plotted Scans . . . . .	26
4.3 Performance of MERIT.jl . . . . .	27
4.4 MERIT.jl as a Library . . . . .	29

<b>Future Work</b>	<b>31</b>
5.1 Time Domain Implementation . . . . .	31
5.2 Implementation of More Beamformers . . . . .	31
5.3 Parallel Processing . . . . .	32
<b>Conclusions</b>	<b>33</b>

# Introduction

Microwave imaging has seen a rising interest in the medical field evidenced by the numerous clinical trials that are being conducted by research groups around the world [1,2,4]. A cursory search on GitHub for software around microwave imaging yielded few useful results, with many being specialized repositories for a particular task or performing some machine learning analysis on microwave data. Only one repository stood out as a generalized library that provides researchers with all the tools needed to easily test different algorithms; the MERIT toolbox developed by Dr. O'Loughlin, M. A. Elahi, E. Porter, et. al [5]. Other fields of research have seen numerous benefits from the introduction of comprehensive open-source libraries. Libraries such as PsychoPy and PsyToolkit have allowed psychology researchers to design and conduct experiments in a matter of hours by packaging common functions in an easy-to-use library [6]. With a rise in the number of systems that can perform microwave imaging and the vast amounts of data being generated from these systems, it is imperative that there are a variety of toolkits available to not only analyze data from these current systems but also from future systems. This thesis aims to consider the following questions:

- Improving the reliability of Microwave Imaging software
- Increasing the compatibility between systems, the data these systems generate and the software used to analyze this data
- Creating an intuitive, easily extensible and customizable library
- Leveraging the features of a coding language to create a performant library

The rest of the report will be divided up as follows:

- A literature review on existing microwave imaging systems
- A look into existing reconstruction algorithms
- A discussion about the design choices and Julia features that are included
- An examination of the results and possible future work

MERIT.jl being an open-source library has all its code available on GitHub for anyone to view and amend. It can be viewed at the following URL: <https://github.com/AaronDinesh/MERIT.jl>



# Background

## 1.1 Imaging Systems

The field of microwave imaging has seen intense research since E. Larsen and J. H. Jacobi released their paper on microwave imaging of an isolated canine kidney in 1979, showing that this new modality provided a viable alternative to established methods such as X-rays [7]. In recent years we have seen many clinical trials for imaging systems with various competing configurations and no clear benefits in choosing one configuration over another. Despite the lack of a prevailing paradigm, some patterns can be gleaned from these trials that are broadly representative of the direction that the field is moving towards. In all of these systems, the patient to be scanned would lie in the prone position on an imaging table. The patient would then pass their breast through a hole in the table into some type of imaging apparatus. The imaging apparatus varies slightly from system to system, but they all contain some type of antenna array that is used for the imaging process. Most systems will adopt one of the following antenna setups

1. Fully Multistatic
2. Leveled Multistatic
3. Monostatic

These terms relate to the position of antennas around the breast tissue and how the resulting scan data would be structured. The subsequent sections will consider an example of each of the above configurations.

### 1.1.1 MARIA M4

The first system to be reviewed is the MARIA M4 system developed by Preece *et al.* within the Electrical and Engineering Department of the University of Bristol [1]. This is the 4th iteration in a series of MARIA systems that evolved from a configuration of 16 UWB antennas to the current system which is equipped with 60 antennas. These all operate in a multistatic configuration, allowing any antenna in the array to transmit to and receive from any other antenna in the array, an example of which can be seen in Figure 1. This figure shows a top-down view, however, one can imagine this being generalized to a hemisphere of antennas around the breast.

As stated before, the MARIA M4 system makes use of the UWB spectrum over a frequency range of 3.0 to 8.0 GHz. A commercial-grade Vector Network Analyzer (VNA) was used as the system signal source. The VNA, operating in a stepped continuous wave mode, was used to both produce the transmitted sine waves and

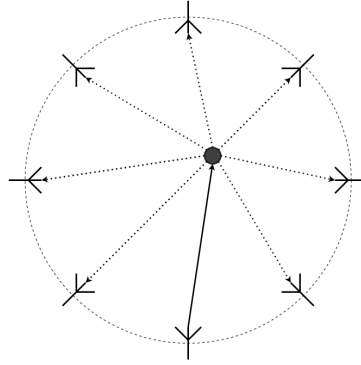


Figure 1: Example of a Fully Multistatic Configuration (Top-Down) showing a circular antenna array with one transmitting antenna illuminating a point scatterer whose reflections are collected by 7 receiving antennas

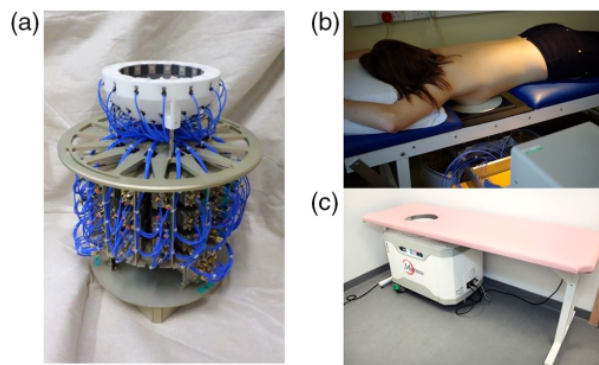


Figure 2: The MARIA M4 and M5 system. (a) The MARIA M4 multistatic antenna array. (b) The M4 system in a clinical setting. (c) The integrated M5 package [1]

record the frequency and phase of the reflected waves at the receiving antennas. The choice of a commercial-grade VNA is indicative of the prototype nature of the MARIA M4 system, where easy reconfigurability of the frequency imaging range was of higher importance than price. While not explicitly stated, one would presume that the M5 system would replace the VNA with some application specific hardware to reduce the overall cost of the system. The M4 system exploits the inherent symmetry in the antenna reciprocity to halve the number of channels (made of a transmitting and receiving antenna) collected, thereby speeding up the scan time. For the MARIA M4 system, this equates to a 1770 reduction in the number of channels collected. Figure 2, shows the antenna array used in the M4 system (a), as well as the M5 system (c) which is an integrated package. The team also conducted a clinical study in order to test the efficacy in women who already attend symptomatic breast care clinics. In total 86 patients were included with ages ranging from 24 – 78 years old; the mean age being 51.4. The inclusion criteria for the study required that possible participants:

- Be clinically symptomatic
- Be able to be imaged by ultrasounds and mammograms (these scans being the control)
- Be able to lie prone
- Have cup sizes between 310 and 850 mL.

The types of lesions included in the study were mostly cysts and cancers, but some other conditions such as hematoma, lipoma and fibroadenoma were also included. The goal of the study was to test the sensitivity of the M4 system; the sensitivity metric being determined based on the ability of the system to localize a lesion as it correlated with the location in the ultrasound or mammogram image. The M4 system showed a sensitivity of 74% (64/86) when compared with the “gold-standard” of an ultrasound. The research team also divided the group into pre-/peri- and post- menopausal women, discovering sensitivities of 75% and 73% respectively. However, the credibility of these results are questioned when considering the limited sample size of the study. Given a sample size of 86, assuming a normal distribution and that the results are statistically significant ( $p < 0.05$ ,  $Z = 1.96$ ), a 7.11% margin of error was calculated. While this may not be enough to conclusively prove that the M4 system is a viable alternative to mammograms, it is enough to show promise.

### 1.1.2 Wavelia

The second system considered was the Wavelia Microwave Breast Imaging System developed by MVG Industries [2]. The Wavelia system integrates the imaging system as well as the examination bed into one complete package (Figure 3). The integrated package makes the Wavelia system an appealing choice for some hospitals, however, its large size may be a barrier to adoption in some facilities where space is a premium.



Figure 3: The Wavelia System installed in a Hospital [2]

Like in the MARIA M4 system, patients lie prone on the examination table and place their breasts in the circular cutouts on the bed. Through the use of a stereoscopic camera, a 3D scan of the breast is collected allowing for an explicit calculation of the imaging domain, rather than the hemispherical approximation approach that needs to be taken with the MARIA system. The Wavelia system also makes use of the UWB spectrum while imaging, but opts to use a narrower part of the spectrum, 0.5 – 4.0 GHz compared to the 3.0 – 8.0 GHz range of the MARIA system. The antenna configuration, unlike the MARIA system, is an array of 18 Vivaldi-type probes arranged in a concyclic manner on a horizontal plane. These antennas operate in a Multistatic manner and image the breast in sections parallel to the coronal plane.

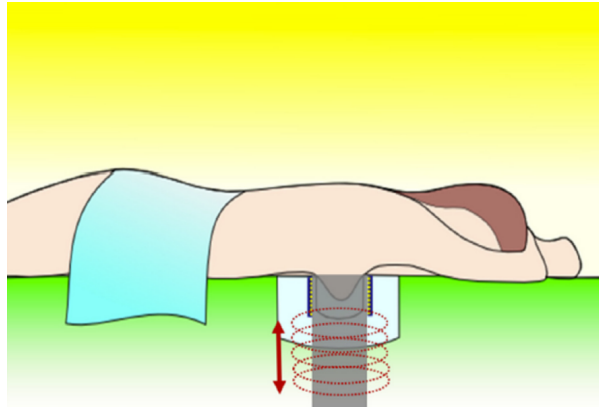


Figure 4: The Leveled Multistatic Approach of Wavelia with a vertically moveable antenna array [2]

The entire antenna assembly moves downwards in 5mm intervals to image the entire breast (Figure 4). This is a leveled multistatic system as opposed to the fully multistatic system in the MARIA M4. This approach has the benefit of a theoretically infinite vertical resolution. If the operator requires a finer resolution along the coronal plane, they would simply change the vertical step size of the array, rather than having to manufacture an entirely new antenna array, like in the MERIT system. This leveled approach allows for the parameters of the reconstruction algorithm to be tuned independently for each coronal slice. Whereas in the fully multistatic approach, significant logic would be required in the post-processing steps to determine which channels are coplanar with a particular slice.

The Wavelia paper [2] conducted a feasibility study on 25 female participants who were recruited after presenting with symptoms to the Symptomatic Breast Unit at Galway University Hospital, Ireland. Their inclusion criteria required that participants:

- Have a mammogram that was performed at the time of symptomatic presentation to the breast unit (these were used as controls)
- Be capable of assuming the prone position for a length of 15 minutes
- Have a bra size larger than a 32B and have a breast size equivalent to a B cup
- Have a breast such that when submerged, there would be a margin between the cylindrical container and the breast to accommodate the transition liquid

The suitability of the patient based on the final criterion was determined by a clinician at the time of the trial. Out of the 25 patients who presented with a palpable lump, one patient's scan had to be removed, due to the lump later being classified as normal tissue. 11 of the participants had a biopsy confirmed carcinoma and out of these, the Wavelia system detected 9 lesions, with 7 being located to the appropriate region. Overall the system detected an abnormality in 21 of the 24 participants, leading to a sensitivity of 87.5%. The researchers do note some limitations of the Wavelia system, namely that it cannot detect any lesions smaller than 10mm. This is significant since the size of the detected lesion plays a big factor when deciding whether it is cancerous or not. Another limitation of the system is that breast sizes that are too small cannot be scanned in any great detail by the Wavelia system. Due to the prone position assumed by the patients, their breast tissue needs to have a pendulous reach far enough such that multiple sections of the breast can be imaged by the antenna array. The

researchers are working on a subsequent system that should address all the aforementioned limitations. Overall the participants had a positive outlook on the system. 23 out of the 25 women said that they would recommend the procedure to other women and all of the women agreed that the information provided was clear and well understood.

### 1.1.3 TSAR

The third and final system considered for this thesis was the TSAR system [4]. Standing for Tissue Sensing Adaptive Radar, this system was developed by the University of Calgary to address some of the shortcomings of the aforementioned systems. In the MARIA and Wavelia systems, reflections from the skin can dominate in the received signals leading to artifacts in the final image. To combat this, both systems record an additional scan, where the antenna array is offset by a fixed rotational amount in the coronal plane. Any skin reflections that appear in the first scan would also appear in the subsequent scan with a similar intensity and timing, while the signals reflected from the tumor would appear at a different time position, provided that the tumor does not lie on the axis of rotation. The method aptly known as "Rotational Subtraction" involves subtracting the scans from each other, suppressing the skin reflections while preserving the tumor response. However, as noted by H. Benchakroun and Dr. O'Loughlin in their papers, Rotational Subtraction can introduce artifacts into the signal due to differences between the original and rotated antenna positions relative to the tumor. These artifacts tend to appear as "wave-like" inclusions on the generated image. Both studies also noted the presence of an "echo" in the final image that appears as a duplication of the tumor response close to the true location of the tumor. They observed that the degree of rotation has an impact on the relative amplitude of the response and its echo citing that for tumors greater than 3 cm away from the center, an increased rotation angle ( $> 20^\circ$ ) caused the tumor to go in and out of focus. For rotations less than  $15^\circ$  the tumor could not be identified in the image [8,9]. The TSAR system, on the other hand, makes use of an adaptive algorithm that estimates the skin response at an antenna as the weighted sum of responses from the neighboring antennas. This skin response can then be subtracted from the current antenna to remove the reflection artifacts [10].

Like the previous two systems, patients lie in the prone position on the examination table, with their breast submerged in an immersion liquid. However, unlike the previous two systems, TSAR operates in a monostatic configuration. In this setup, there is only one antenna that acts as both the transmitter and receiver. This antenna, usually fixed on some type of rotating apparatus, would move around the breast to image a section. It would then step vertically by a fixed amount and repeat the previous step, eventually imaging the entire breast. The TSAR system also operates on a much wider section of the UWB spectrum, from 50 MHz to 15 GHz, with the frequency data being collected by a commercial-grade VNA. To help with the reconstruction of the imaging domain during the post-processing step, TSAR makes use of a laser that explicitly 3D scans the breast. The prototype setup can be seen in Figure 5

One feature of a monostatic configuration is that the number of antenna locations per row and the number of rows are parameters that can be changed by the operator to balance the trade-off between speed and precision. The clinical trial conducted by this study was extremely limited, only including 8 successfully imaged patients. Due to the low sample size, one cannot reliably say whether this system and its adaptive reflection-suppressing algorithm can outperform the other two systems in terms of accuracy.



Figure 5: The TSAR Prototype used in their clinical study [3]

## 1.2 Beamforming Algorithms

Imagine an array of wave emitters. If all of these begin to emit at the same time, the individual waves would constructively and destructively interfere with each other as their peaks and troughs align and misalign. This property can be exploited in such a way that the waves constructively interfere in one direction and destructively interfere in all the other directions, effectively aiming the beam in a particular direction. A diagram of this can be seen in Figure 6.

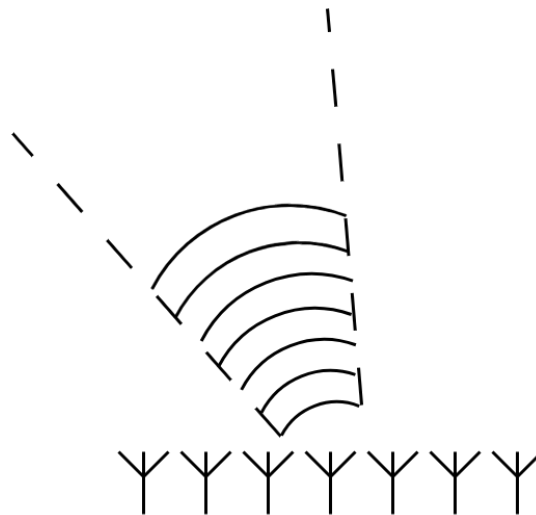


Figure 6: Phased antenna array demonstrating beamforming

This is the forward beamforming process, but one can also consider the reverse process. Considering Figure 7 imagine a point emitter embedded in a 2D plane, that radiates circular waves evenly in all directions, with the waves falling incident on an antenna array. Due to the diffusion of the wave through space, the same wavefront will appear at each antenna at

different times. This manifests in the recorded signals as similar amplitudes but shifted in time. Reverse beamforming then, is the process of varying the phase and amplitude of the received signals in order to estimate the intensity at the location where the wave originated. Inverse beamforming is a process used in many fields such as radio astronomy and seismic imaging, so there already is a wealth of research and plenty of algorithms to choose from. The rest of this section will talk about the various beamformers that are popular in the field of microwave imaging.

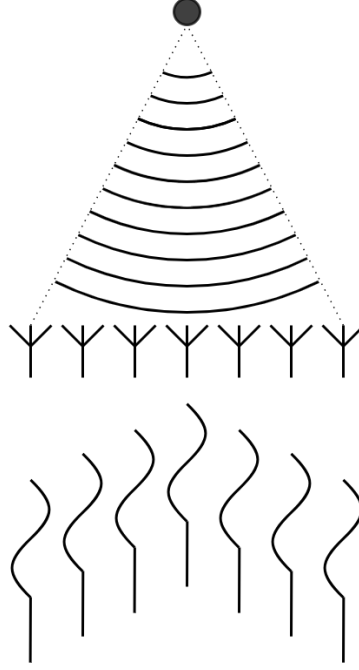


Figure 7: Point Emitter with waves incident on a phased array

### 1.2.1 Delay and Sum

The DAS beamformer posits that every antenna has recorded the same source and that the delay in each signal is due to the relative distance between the receiving antenna ( $\mathbf{a}' \in \mathcal{A}'$ ) and the transmitting antenna ( $\mathbf{a} \in \mathcal{A}$ ). As such, if one was to delay the signals by their respective path delay, and sum over all the received signals, one would be able to estimate the energy at the source. This same idea can be applied to signals received from the aforementioned imaging systems. Since these work in the frequency domain, the following explanations and equations will work on this assumption, but these equations can easily be converted to the relative time domain functions, by replacing the multiplication by a complex exponential with the equivalent time delay. As stated before a point from the imaging domain is chosen ( $\mathbf{r}$ ). The path delay of the wave from  $\mathbf{a}$  to  $\mathbf{a}'$  via  $\mathbf{r}$  is estimated via the following equation:

$$\tau_{\varepsilon_i, \mathbf{a}', \mathbf{a}_j}(r) = \frac{\sqrt{\varepsilon}}{c_0} [\|\mathbf{a}' - \mathbf{r}\| + \|\mathbf{r} - \mathbf{a}\|] \quad (1.1)$$

with  $\varepsilon$  being the relative permittivity of the medium. In reality,  $\varepsilon$  changes from patient to patient and even varies within the breast of each patient depending on the path taken. However, to achieve a practical beamformer, one must estimate an averaged relative permittivity value for the entire breast, henceforth this will be labeled as  $\varepsilon_i$  and it will parametrize the DAS beamformer. Equation 1.1 also assumes a straight-line path between  $\mathbf{r}$ ,  $\mathbf{a}$  and  $\mathbf{A}'$ , even though in most cases, this assumption does not hold. However, as found by Dr. O'Loughlin, B. L. Oliveria and M. A. Elahi *et al.*, this assumption yields a maximal error of 3 mm in position, while greatly simplifying the delay calculations [11]. Using this, the received signals are delayed, summed across all stepped input frequencies and finally squared to yield the energy at the chosen point. Overall the DAS beamforming equation can be represented as such:

$$I_{\varepsilon_i}(\mathbf{r}) = \left[ \sum_{\Omega} \sum_{\mathcal{A}'} \sum_{\mathcal{A}} S_{\mathbf{a},\mathbf{a}'}[\omega] e^{j\omega\tau_{\varepsilon_i,\mathbf{a},\mathbf{a}'}(\mathbf{r})} \right]^2 \quad (1.2)$$

The DAS algorithm was implemented as the beamformer of choice due to its simplicity and time constraints placed on the thesis. Two other well known beamformers will be discussed for completeness and comparison with the DAS beamformer.

## 1.2.2 Weighted Delay and Sum Beamformer

The Weighted Delay and Sum (WDAS) Beamformer can be considered as a further generalization of the DAS beamformer. In equation 1.2 all channels contribute equally to the final result due to the implicit unit weighting factor. The DAS equation also assumes that all signals travel along similar paths and also have a constant speed, due to the fixed  $\varepsilon_i$ . In reality, this is only true for antennas that are closer together. The greater the distance between  $\mathbf{a}$  and  $\mathbf{a}'$ , the more likely it is that the waves deviate from the straight-line path assumption, ergo the estimation of speed and subsequently the delay along that path will be wrong. S. A Shah Karam, et al. proposed a solution to this issue in their 2021 paper "Weighted delay-and-sum beamformer for breast cancer detection using microwave imaging" [12]. They suggested a weighting factor based on the transmitter-receiver distance ( $TRD_i$ ) for the  $t^{th}$  observation:

$$w_i = \alpha - |\mathbf{r}_{T_{r_i}} - \mathbf{r}_{R_{c_i}}| \quad (1.3)$$

$\alpha$  is a positive parameter that allows the above weighting factor to reward signals from nearby antennas while penalizing signals from antennas that are far away. This parameter is patient-specific and must be changed based on the homo- or heterogeneity of the breast tissue, with the paper using a value of 20 cm for their tests. The paper also noted that since  $w_i$  is data-independent and focal point independent, a set of normalized weighting factors can be computed before hand and applied to the signals at collection time rather than processing time. The normalized weighting factor  $\hat{w}_i$  is calculated as follows, where  $M$  is the number of



channels:

$$\hat{w}_i = \frac{w_i}{\sum_{i=0}^M w_i} \quad (1.4)$$

### 1.2.3 Delay Multiply and Sum

The Delay Multiply and Sum (DMAS) algorithm, first proposed by Hooi Been Lim *et al.* in 2008 [13], suggests a pair-wise multiplication of the delayed signals before summing across the frequency range. For a focal point  $r$  the equation would be as follows:

$$I_{\varepsilon_i}(\mathbf{r}) = \sum_{\Omega} \sum_{\mathcal{A}'} \sum_{\mathcal{A}} \sum_{\mathcal{B}'} \sum_{\mathcal{B}} S_{\mathbf{a},\mathbf{a}'}[\omega] e^{j\omega\tau_{\varepsilon_i,\mathbf{a},\mathbf{a}'}(\mathbf{r})} S_{\mathbf{b},\mathbf{b}'}[\omega] e^{j\omega\tau_{\varepsilon_i,\mathbf{b},\mathbf{b}'}(\mathbf{r})} \quad (1.5)$$

Here  $\mathcal{A}' = \mathcal{B}'$  are the receiving antennas and  $\mathcal{A} = \mathcal{B}$  are the transmitting antennas. Upon comparison with the traditional DAS algorithm, the researchers found that the pair-wise multiplication before summation provided greater contrast and sharper results, greatly increasing the Signal-to-Clutter ratio of the image. Figure 8 shows the differences between the DAS and DMAS algorithms and most importantly, how the DMAS beamformer provides a greatly reduced noise floor. The researchers never provided an explanation for the effectiveness of

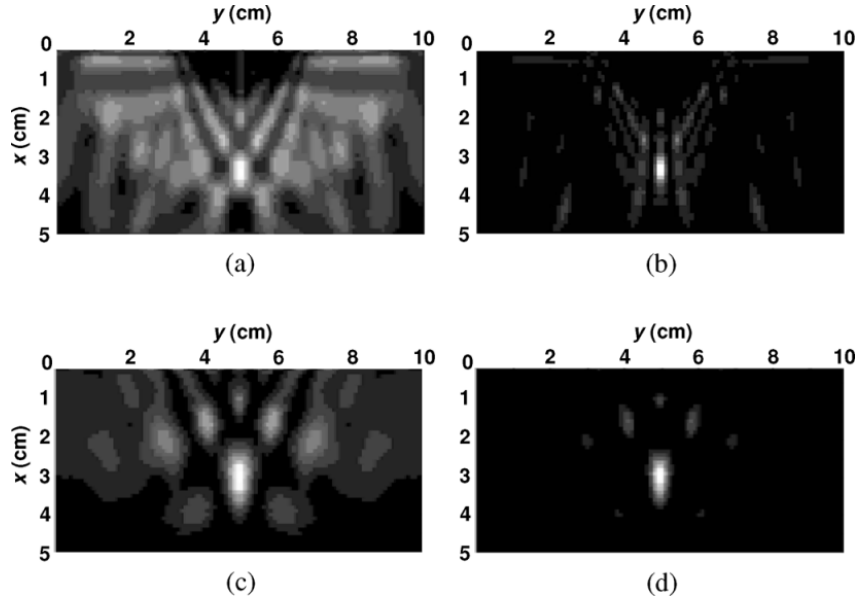


Figure 8: Comparason between DAS (a, c) and DMAS (b, d) Beamformers. The DMAS beamformer demonstrates a lower noise floor and reduced clutter

the algorithm, leaving it for a future paper that they never wrote. However, this did not stop the wider research community from providing possible explanations as to how the DMAS algorithm provides such a low noise floor. An explanation provided by Dr. O'Loughlin posits that the pair-wise multiplication rewards signals that have a high degree of coherency while disproportionately penalizing incoherent signals. In essence, the coherent signals get amplified a lot more than incoherent signals [14]. G. Matrone *et al.* further reinforces this hypothesis by noting that the DMAS beamformer, after the signals are time aligned like in DAS, computes the autocorrelation of the receiver antenna with the auto terms removed [15].

## 1.3 Development Platforms

Data science libraries in the last 20 years have seen a gradual migration from languages such as C++ and Fortran towards higher level languages such as Python, MATLAB and R, attracted by their relatively easy-to-use syntax and comprehensive package managers. As of 2024, Python boasts 10,000 different libraries that are related to science and engineering, with MATLAB and R claiming to have 3,094 and 16,444 packages in their respective package managers. Their high-level syntax makes them an attractive language to create and share libraries. This is backed up by statistics collected by JetBrains in 2022 which showed that out of the 23,000 people surveyed for their Python usage, 53% of these people used Python for some form of data science [16]. MATLAB also reports similar statistics, mentioning that their software suite is used by over 6500+ universities around the world for various purposes [17]. No usage statistics could be found for R. Julia is another programming language that has been gaining interest in research communities due to having high-level syntax like Python and MATLAB but also the performance that comes with low-level languages such as C, as evidenced by the 10,760 packages cataloged by JuliaHub. The Julia team conducted their own study of 1,329 individuals over a 3 month period in 2023 and found that 84% of people surveyed used Julia for research or teaching [18]. For this thesis, the scope was narrowed down to Python, MATLAB and Julia programming languages, with the following sections presenting the advantages and disadvantages of using each language.

### 1.3.1 Python

Python's philosophy was code readability over all and this fact is complimented by its high-level syntax and indentation requirements. It was first introduced towards the end of the 1980's by Guido van Rossum and has had an expansive community ever since. The introduction of the PyPI Python Package Index in the late 2000's catapulted the new 20 year old language into the limelight by greatly simplifying the process of distributing libraries. Its success is evident with over 20.4TB worth of Python libraries being hosted by PyPI [19]. However, several drawbacks severely limit the usability of Python for high performance applications. Firstly, Python is an interpreted language, meaning that at runtime, the Python interpreter reads the Python file line by line and calls the relevant machine code. Due to this interpretation step, raw Python code is slow to run [20]. For this reason, many libraries that require high performance in Python have a significant portion of their code written in C or C++, as demonstrated by their language breakdown in their respective GitHub repositories [21, 22]. So developers who are concerned about speed have to be proficient in C and C++ as well as Python, creating a high barrier of entry which goes against MERIT.jl's goal of "easily extensibility".

Secondly, the Python Interpreter makes use of a runtime lock known as the Global Interpreter Lock (GIL) which makes parallel processing in Python difficult [23]. Python implemented the GIL as a consequence of their use of reference counting for memory management. Reference counting is a method where each object in Python gets assigned a reference variable that keeps track of the number of references that point to that object. As references to the object are created, the count goes up, as references are deleted or reach the end of their scope, the count goes down. When this variable reaches zero, it becomes a flag for the object to be deleted. In some threaded applications, references can get created or deleted by multiple threads, causing this counter to be updated simultaneously. In some cases, this can cause the reference count to never reach zero leading to a memory leak, or reaching zero too early,

prematurely deleting the object. To combat this, Python created the GIL to act as a lock on the interpreter. Any Python bytecode needs to first acquire a lock on the interpreter before it can be executed. This makes multithreading in Python difficult and slower than it would be in other languages. So for these reasons, Python was rejected as the software of choice.

### 1.3.2 MATLAB

MATLAB was another language that was considered. Developed in 1984, it became the goto software for many research purposes due to its ease of use and intuitive operations on matrices and its multi-dimensional analogs. MATLAB also has seen success in industry with one company estimating that it is used in over 57,811 companies [24]. However, MATLAB is an interpreted language like Python, meaning that it is slower than compiled code. A study conducted by Aruoba and Fernández-Villaverde found that their MATLAB code ran about 3x slower than the same code written in C++, highlighting just how big of a difference a compiler can make [25]. It should be noted that the same MATLAB code ran about 30.26x faster than their Python implementation, implying that even though both are interpreted languages, the MATLAB interpreter is much more optimized than the Python interpreter. But one of the biggest drawbacks by far, is the fact that MATLAB is a license-based language. In order to use MATLAB, one must pay a yearly subscription of anywhere from €120 to €3,650 depending on the purpose for which it is used [26]. This goes against the open-source nature of MERIT.jl. The yearly licensing cost can be expensive for small research teams, barring them from contributing to the library. Octave was briefly considered as it is a free and open-source competitor to MATLAB, but this idea was quickly dropped when it became clear that Octave's main goal was compatibility with MATLAB scripts rather than absolute performance. For these reasons, MATLAB was also rejected as the software of choice.

### 1.3.3 Julia

Julia was developed by Jeff Bezanson, Stefan Karpinski, Viral B. Shah and Alan Edelman, and was first released to the public in 2012 [27]. The creators wanted a language that was as fast as C while also retaining the dynamism of high-level languages such as Ruby and Python. The Just In Time (JIT) compiler employed by the Julia runtime compiles the high-level language syntax into machine code allowing for large performance gains over other high-level languages. The aforementioned study found that Julia was only 1.47x slower than the comparable C++ implementation [25]. Added to this speed, Julia offers features that are not available in Python or MATLAB such as parametric polymorphism, multiple dispatch and efficient garbage collection. Julia also has native support for multiple parallel processing paradigms such as GPU programming and multithreading, making it an appealing choice for people working on high-performance compute clusters. One other feature that made Julia wildly popular was the native ability to call C and Fortran libraries without having to create any special wrappers. These features allow Julia to have an “It Just Works!” ideology, where libraries written in other supported languages as well as in Julia can all work together without significant additions of “glue-code”. As stated before Julia has already garnered an interest in academia, this coupled with its performance made it a clear choice for MERIT.jl.

# Programming Paradigms

As mentioned in the section before, Julia's development was centered around the philosophy of creating a dynamic yet high performing language by combining features like Multiple Dispatch and Parametric Polymorphism among others. The following sections will describe a small subset of these features.

## 2.1 Multiple Dispatch

Multiple Dispatch is a programming paradigm in which a function can be invoked based not only on the function name but also on the type and order of input arguments. This is opposed to the single dispatch programming paradigm where the function dispatched depends on a special argument placed before the function name. In almost all programming languages this is the variable name for that class. For example, consider the following code:

```
1 # Defined in one library
2 Class Dog:
3     name::string
4     function says(a::string):
5         print("The dog, $self.name says $a")
6     end
7 end
8
9 billy = Dog("Billy")
10
11 # Defined in another library
12 Class Cat:
13     name::string
14     function says(a::string):
15         print("The cat, $self.name says $a")
16     end
17 end
18
19 kate = Cat("Kate")
```

If one wanted to call the `says` from the `Cat` or `Dog` class in the single dispatch paradigm, one would have to write `billy.says("Hello World")` or `kate.says("Hello World")`. This concept fits well with object-oriented programming languages where classes are used to encapsulate concepts. However, a drawback of this system is that the compiler relies on the user to remember which methods belong to the class and which methods are callable. Also

since both the Cat and Dog classes are defined in different libraries it is not clear how either developer could create a function where these classes interact with each other without having to create a separate third library that implements compatibility code.

Multiple and its particular implementation in Julia solves some of these issues. In Julia, methods no longer belong to classes, instead like in C they simply belong to a particular library, also known as a “Module” in Julia. Functions defined in modules are usually exported such that they are available in the namespace of any other module that uses it. This allows modules to use and overload functions and structures from other modules in a way that is completely transparent to the end user.

```
1 # Defined in library A
2 struct Dog{
3     name::string
4 }
5
6 function says(pet::Dog, a::string)
7     print("The dog, $pet.name says $a")
8 end
9
10 # Defined in library B
11 struct Cat{
12     name::string
13 }
14
15 function encounters(petA, petB)
16     print("$petA.name encounters $petB.name and $meets(petA,
17         petB)")
18 end
19
20 function meets(petA::Cat, petB::Dog)
21     return "hisses"
22 end
23
24 function meets(petA::Dog, petB::Cat)
25     return "barks"
26 end
27
28 # Overloading function from library A
29 function says(pet::Cat, a::string)
30     print("The cat, $pet.name says $a")
31 end
32
33 # Someone else using both libraries
34 billy = Dog("Billy")
35 kate = Cat("Kate")
36
37 says(billy, "Hello World")
```

```

38 says(kate, "Hello World")
39
40 encounters(kate, billy)
41 encounters(billy, kate)

```

In the above code block, library B overloads the `says` function to accept a struct of type `Cat`. In addition to this, it also creates two new `meets` functions that handle the interaction between the `Cat` and the `Dog` types, as well as an `encounters` function. A third user can make use of both libraries as they did in the single dispatch case, but now they only have to call the function name, rather than the function name prepended with the class variable name. In the case of the `says` function call, the Julia compiler automatically dispatches the correct implementation based on the type of the input argument. This is further exemplified in the final two function calls. As stated before multiple dispatch is sensitive to both the type and order of the inputs. In the first call to the `encounters` function, the `meets` function on line 19 will be dispatched as the argument order was of type `Cat` and then `Dog`. Whereas in the second call to the `encounters` function, the `meets` function defined on line 23 will be dispatched due to the reverse ordering of the types.

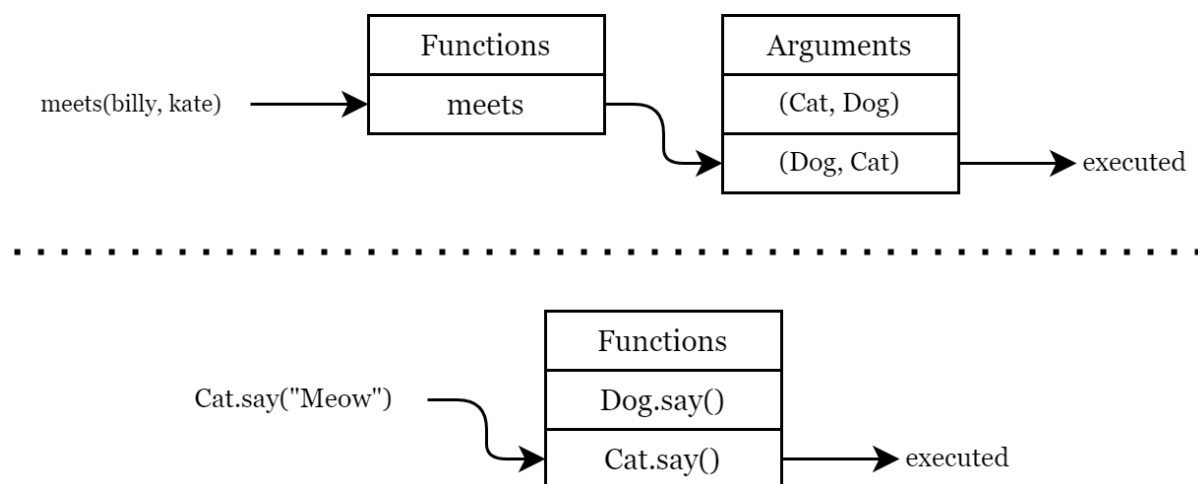


Figure 9: Multiple Dispatch (top) vs Single Dispatch (bottom) demonstrating that in the multiple dispatch paradigm, the executed function also depends on the type and order of its inputs

## 2.2 Type Heirarchy

In Julia, all types are arranged in a tree-like structure and can be broadly classified into two categories, an Abstract Type or a Concrete Type. Abstract types are the internal nodes of the type tree, having both parents and children, while concrete types are the “leaves” of the tree, having parents but no children. Another notable difference between abstract types and concrete types is that abstract types cannot be instantiated, they serve only as nodes in the type graph. Shown in Figure 10, is the type hierarchy for the Integer type, abstract types are highlighted in red, whereas concrete types are highlighted in blue.

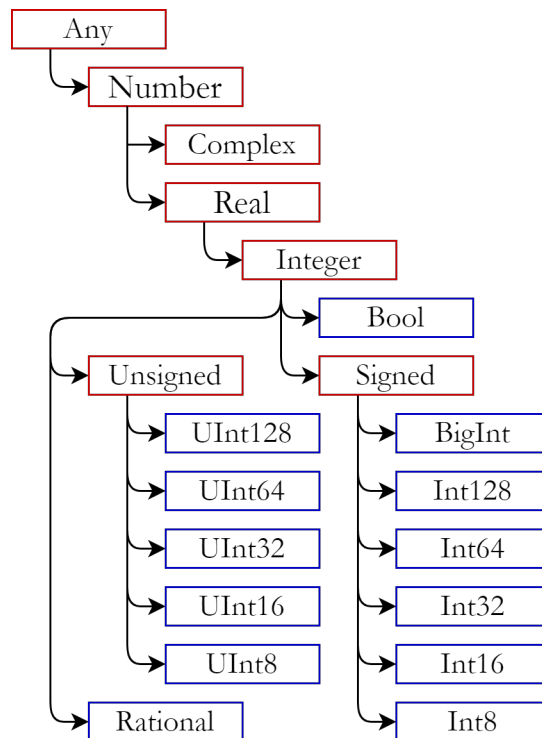


Figure 10: Type hierarchy for the Integer Type

The type hierarchy not only provides a way to logically organize types but also tightly integrates with the multiple dispatch system mentioned before. It is this pairing that allows Julia to fulfill its promise of easy and powerful extensibility. The following example helps illustrate this further.

```

1 # Defined in library A
2 abstract type Animal end
3
4 struct Cat <: Animal
5     name::string
6 end
7
8 struct Dog <: Animal
9     name::string
10 end
11
12 function encounters(petA::Animal, petB::Animal)
13     verb = meets(petA, petB)
14     println("$(petA.name) meets $(petB.name) and $(verb)")
15 end
16
17 meets(petA::Animal, petB::Animal) = "passes by."
18 meets(petA::Cat, petB::Dog) = "hisses"
19 meets(petA::Dog, petB::Cat) = "barks"
20
21 # Defined in library B

```

```

22 struct Rabbit <: Animal
23     name::String
24 end
25
26 whiskers = Cat("Whiskers")
27 chomper = Rabbit("Chomper")
28 encounters(whiskers, chomper)

```

Library A defines a type hierarchy with Cat and Dog being a subtype of the abstract Animal type. The library then implements a series of `meets` functions and an `encounters` function, similar to the example in the previous section. However, unlike the example before, there is a new `meets` which accepts an argument of type Animal. Library B, importing library A, defines a Rabbit type that is a subtype of the Animal abstract type and then calls the `encounters` function from library A. The dispatch system cannot execute the `meets` function from lines 18, since `chompers` is not of type Dog. Instead, it executes the `meets` function on line 17, since both `whiskers` and `chompers` are valid Animal types. This highlights an important interplay between the type hierarchy and the multiple dispatch system, in that when dispatching, the Julia compiler will select the function that is most specific across all its input arguments. The developer of library A need not worry about all the other possible Animal types, or what their fields may contain, they can create a generic `meets` function that accepts any subtype of the Animal class and will execute successfully provided the subtyped class contains the fields being accessed. Neither developer has to worry about the completeness of the other's implementation, provided developer B follows the standards set by developer A, they can develop an extension to library A and trust that the two libraries will be comparable.

## 2.3 Parametric Polymorphism

Parametric polymorphism refers to the programming paradigm where a function can be made generic to the input argument type. At compilation time, the compiler can strongly type the function based on the types of the input arguments. Consider the simple example of an `addtwo(x)` function.

```

1 function addTwo(x::T) where {T <: Real}
2     return x + T(2)
3 end

```

Instead of specifying a concrete type for `x`, it is parametrized by the variable `T` which has an abstract type restriction on it. This implies that `T`, and by extension `x`, can be any type that is a subtype of the Real abstract type. If parametric polymorphism was not a Julia feature, this function would have to be duplicated 16 times to account for the various concrete types that are a subtype of Real (see Figure 10).

## 2.4 Type Stability

Type Stability is a coding discipline that Julia recommends all code use. A function whose contained variables have a consistent type for its lifetime is considered to be “type stable” or to have “type stability”. Another way to think about this, is that if the output type



depends only on the input types, then the function is type stable. Consider the following two functions:

```
1 function unstable()
2     x = 1
3     for i = 1:10
4         x /= rand()
5     end
6     return x
7 end
8
9 function stable()
10    x = 1.0
11    for i = 1:10
12        x /= rand()
13    end
14    return x
15 end
```

Both functions have a variable, `x` which is divided 10 times by random floats and is returned at the end of the function. However, based on the description above, the first function is considered to be “Type Unstable” whereas the second function is considered to be “Type Stable”. This is because in the first function the type of `x` changes in the lifetime of the function (turns from `Int64` to `Float64`). However in the second function, the data type of `x` stays as a `Float64` throughout the lifetime of the function. The notion of type stability is important as it allows the Julia compiler to perform optimizations based on the known data type of the variables at compile time. For type unstable functions, their unstable variables default to type `Any`, ergo forgoing any optimizations. These optimizations are evident when benchmarking the functions above. This was performed using the `@benchmark` macro from the `BenchmarkTools` module. Both functions were run multiple times to ensure that only the compiled versions of the functions were executed rather than the interpreted version. These benchmarks showed that on average the type unstable function ran 1.87x slower than the type stable function (573.355 ns and 306.178 ns respectively).

## 2.5 Closure

Closure in programming refers to the practice of calling a function `A` that returns a function `B` which has some information about the variables contained in the scope of function `A`. Consider the following simple example:

```
1 function addX(x)
2     scalar = x
3     function calc_(a)
4         return a + scalar
5     end
6 end
7
8 add5 = addX(5)
9 add7 = addX(7)
```

```

10
11 add5(2)    # Will return 7
12 add7(10)   # Will return 17

```

Defined above is a function called `addX` which accepts an input `x` and assigns it to a variable `scalar`, before constructing and returning the function `calc_`. This can be beneficial as it allows developers to create a family of functions that are parametrized by their input variable. It must be noted that the captured variable, in the above case `scalar`, must never be reassigned, otherwise, the compiler will not be able to infer the data types leading to type unstable code, and by extension all the negative consequences mentioned in the previous section.

## 2.6 Type Safety

Type Safety refers to a program or language's ability to detect and discourage errors that arise from performing operations on the wrong data type. For example in Julia, it is simple to add two numeric types, but performing the same operation on two strings would yield an error, since the summation operator is not defined for two string types. This offers some level of protection against illegal operations, but there are many cases where the type of the variables agree with the operation being performed, but their semantic meaning disagrees. The example below will illustrate this idea.

```

1 function calcSpeed(dist::Vector{T}, t::Vector{T}) where {T <:
    Real}
2     return dist ./ t
3 end
4
5 #####
6 distance = rand(1, 100)
7 time = rand(1, 100)
8 calcSpeed(distance, time)    # Will compute the speed
9
10 #####
11 distance = rand(1, 100)
12 time = rand(ComplexF64,1, 100)
13
14 # Will throw an error since there is a type mismatch
15 calcSpeed(distance, time)
16
17 #####
18 distance = rand(1, 100)
19 time = rand(1, 100)
20
21 # Will compute the wrong answer, but no error gets thrown
22 calcSpeed(time, distance)

```

In the code block above `calcSpeed` accepts a vector of distance and time and returns the associated speeds. Executing line 8, as expected returns the calculated speeds correctly.

Executing line 12 will throw a type error since `ComplexF64` is not a subtype of `Real`. Line 22 however, will execute and return successfully even though the wrong answer is returned, as the arguments were provided in the wrong positional order. This is because compilers can only make inferences regarding the legality of a statement based solely on the concrete types of its arguments rather than their semantic meaning in the context of the function being executed. However, the chance of variables being incorrectly passed positionally can be negated by creating lightweight types that encode this semantic information in their type name. In the above case, it would mean creating a “distance” type and a “time” type that act as wrappers around a numerical concrete type. This way when line 22 is executed, instead of returning an incorrect answer, the compiler will raise an error as the “time” and “distance” types were provided in the wrong order.

# Julia For Development

The features mentioned in the previous chapter are extensively used in the 10,760 packages currently being hosted on JuliaHub and the Julia Package Manager. The following sections will contextualize the role that Julia and its features play in the broader field of scientific computing as well as its role in MERIT.jl.

## 3.1 Julia For Scientific Computing

Julia and its libraries are an attractive choice for researchers in the fields of numerical analysis, data science and engineering. Consider the common task of solving differential equations. These equations, defined in terms of derivatives, underpin every field of engineering from the analysis of complex electronic systems to thermodynamical systems. The Julia package DifferentialEquations.jl [28] allows users to define and solve differential equations in as little as six lines of code. Consider the example below:

```
1 using DifferentialEquations
2 f(u, p, t) = 1.01 * u
3 u0 = 1 / 2
4 tspan = (0.0, 1.0)
5 prob = ODEProblem(f, u0, tspan)
6 solution = solve(prob)
```

After defining the differential equation, initial conditions and the solution time length, the example defines an ODEProblem which is a subtype of SciMLBase.AbstractODEProblem which itself is subtyped from SciMLBase.AbstractDEProblem and this pattern continues. Notice that the abstract type for ODEProblem was defined in the SciMLBase Module, not in the DifferentialEquations Module. SciMLBase is a separate library that DifferentialEquations extends in a way that is completely transparent to the end user. Any problems created using the ODEProblem type from DifferentialEquations can be used in any function from any library that can accept the SciMLBase.AbstractODEProblem or any of its supertypes. This highlights how easy it is for new libraries to extend older libraries in a way that is mutually beneficial for both libraries. The developer of the original library no longer has to worry about implementing functions for every type of differential equation, and the developer of the new library does not have to invest time in creating the complex type hierarchy or the helper functions from the original library. Once passed to the solve function, the library can automatically determine the best solver based on the problem and dispatch the relevant methods accordingly exemplifying the use of multiple dispatch, all while boasting C and Fortran-like speeds. Their webpage demonstrates them solving the Lorenz ODE in 789.794  $\mu$ s. Julia also has libraries for the field of Biology such as, but not limited to, JWAS.jl for whole genome analysis, BioStructures.jl

for manipulating macromolecules, `VariantVisualization.jl` for visualizing genomic variation data and `Gillespie.jl` for Gillespie-type simulations in Julia. In the field of Machine Learning, there are various libraries such as `Flux.jl` and `TensorFlow.jl` as generic machine learning libraries, `BrainFlow.jl` for EEG, EMG and ECG data, and `NeuralNetDiffEq.jl` for physics-informed neural networks, to name a few. The easy extensibility and intuitive interfaces that can be created using the language features in Julia make it an attractive language for developers creating libraries and researchers who want quick answers to their questions.

## 3.2 Julia for MERIT.jl

MERIT.jl also makes full use of the aforementioned features in order to create a library that is reliable, performant and easily extensible. The sections below will outline how each feature was used to attain the goals stated at the beginning.

### 3.2.1 Multiple Dispatch in MERIT.jl

MERIT.jl makes use of multiple dispatch in the beamforming algorithm implementations. As mentioned before, the DAS beamformer can be considered as a specialized case of the WDAS algorithm with a constant unit weighting factor. Due to both being effectively the same algorithm, it was decided to combine both of these under the DAS nomenclature and use multiple dispatch to select the correct function based on the presence of weighting factors in the argument list. This was also used when defining the addition, subtraction and exponentiation operators so that the users can make use of the inbuilt mathematical operators when working with the `Point3` and `Point2` datatypes. Multiple dispatch could also be used in the implementation of the time domain version of the DAS beamformer, the specifics of which will be detailed in section 5.1.

### 3.2.2 Type Hierarchy in MERIT.jl

The concept of abstract types and subtypes is used heavily in MERIT.jl due to the constant pace of progression in the field. Currently, most of the research in microwave imaging is centered around breast imaging and breast cancer, however in 2013, a pilot study published in the *International Journal of Biomedical Imaging* found the use of microwave imaging to be beneficial when imaging transverse sections of the forearm [29]. As such the library needs to be flexible enough to adapt to novel imaging domains. The type hierarchy in MERIT.jl is centered around the `Scan` abstract type, from this, the `BreastScan` type is subtyped which holds all the information regarding the scan from a particular breast. In order to incorporate the aforementioned study, one would have to create a `ForearmScan` type, subtyped from `Scan`, which has a similar field name structure to the `BreastScan` type. In this way, MERIT.jl achieves extensive flexibility and expandability which would not be possible in other languages.

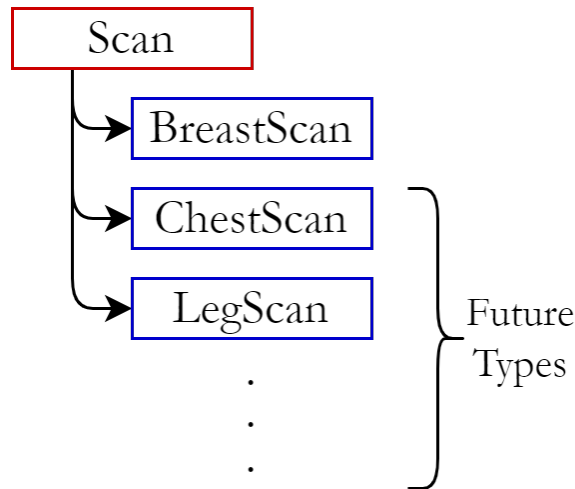


Figure 11: Current type hierarchy in MERIT.jl

This type hierarchy also synergizes with the multiple dispatch feature in Julia. Since the algorithms in MERIT.jl are not specific to a particular body part, the one-call processing pipeline demonstrated in section 4.1 could easily be extended to work with any type that subtypes the Scan type, provided that the new type has a similar field name structure to the current BreastScan struct. Full details surrounding the structure of the BreastScan struct can be found in the Scans.jl file.

### 3.2.3 Parametric Polymorphism in MERIT.jl

MERIT.jl makes use of parametric polymorphism so that the functions it implements can be agnostic to the data type of its inputs. When instantiating the BreastScan struct the user can provide the types of the data that will be loaded and can therefore set the internal data type of the struct. It would be impossible to support all 4,176 different permutations of every type pairing, however, with parametric polymorphism, each function only needs to be defined once and the Julia compiler will handle the rest. This means that researchers and developers don't have to worry about whether the library functions can support the data type of their data, so long as it follows the type restriction set on the function, it will produce an output, creating an intuitive and easy coding experience.

### 3.2.4 Type Stability in MERIT.jl

As shown in section 2.4, the performance gained by writing type stable code can be significant. These gains become important when considering the many millions of function calls required to generate an output in MERIT.jl. Consider just the distance calculations required between an 8 cm radius breast whose domain is discretized with a resolution of 0.25 cm and an antenna array made of 60 antennas; this equates to roughly 4.2 million calculations alone for each image. Performing this on a type unstable function would take far too long to compute, rendering the library unusable. Herein lies one of the drawbacks of writing Julia code, namely, the ease at which an inexperienced developer can write type unstable functions. In the simple case in section 2.4, the cause for the type instability is evident, however, with larger more complex functions it can be hard to narrow down the cause for the instability. The language does however, offer the `@code_warntype` macro that can analyze sections of

code and indicate, but not fix, places of instability where the compiler fails to infer the data type of the variable. This tool can considerably benefit developers of MERIT.jl to ensure that any extensions written for the language align with its philosophy of performance.

### 3.2.5 Closure in MERIT.jl

Closure, in MERIT.jl, was the programming paradigm that was used in order to achieve some customizability for the functions that required it. Currently, it is being used in the `get_delays` in the `Breamform.jl` file. Earlier sections showed how  $\varepsilon$  is a free parameter in the beamforming equations, one that researchers would be changing frequently to find the optimal parameter for each particular scan. This allows researchers to quickly create a set of parametrized delay functions that can be easily interpolated into the `BreastScan` struct to be used in the one-call processing pipeline, allowing them to answer questions regarding the optimal range for  $\varepsilon$  depending on the body part or tissue composition.

### 3.2.6 Type Safety in MERIT.jl

The MERIT.jl library exemplifies the idea of “strong” type safety through its implementation of the `Point` data type. The `Point` type is an abstract type from which the `Point3` and `Point2` concrete type subsets. These are lightweight wrappers around a grouping of 3 and 2 numerical types respectively and serve the purpose of being a 3D and 2D point.

```
1 abstract type Point end
2
3 # xyz can be any data type that is a subset of Real
4 mutable struct Point3{T <: Real} <: Point
5     x::T
6     y::T
7     z::T
8 end
9
10 mutable struct Point2{T <: Real} <: Point
11     x::T
12     y::T
13 end
```

Due to the custom nature of these data types, the inbuilt operators could not be used on them. So in addition, MERIT.jl had to extend the in-built operators using the concepts of multiple dispatch and parametric polymorphism as mentioned before such that these types could be useful. The full suite of implemented operators can be found in the GitHub repository. Every function in the library that needs to work with points accepts a collection of a `Points` subtype rather than a collection of numbers. This way no other collection of numbers can be erroneously passed in place of the `Points` subtypes. Some edge cases still exist however, there is nothing stopping a user from incorrectly passing a collection of points describing antenna locations to an argument which is for points from the imaging domain. Even though this issue still exists, clear and easy to understand documentation should make this a nonissue. It should be noted that in its current state, MERIT.jl is not completely strongly typed, this is an area of the library which can be improved upon in the future.

# Results

The results of the MERIT.jl library will be discussed in this chapter. The library will be judged on qualitative metrics such as ease of use, customizability and extensibility as well as quantitative metrics such as performance and its “Big  $\mathcal{O}$ ” notation. The chapter will conclude with a brief section that evaluates MERIT.jl in the context of current microwave imaging libraries and research libraries in general.

## 4.1 Current Workflow

The workflow in MERIT.jl was designed with speed and ease of use in mind. This is exemplified in the one-call processing pipeline that is currently implemented in the library and is the recommended workflow for researchers who want to process their scans while rapidly iterating through different  $\varepsilon$ 's and beamformers. Shown below is the full workflow needed to generate a plot from the scan data:

```
1 using MERIT
2 using Plots
3
4
5 plotlyjs()
6 scan = BreastScan{Float32, ComplexF32, UInt32}()
7 domain_hemisphere!(scan, 2.5e-3, 7e-2+5e-3)
8 load_scans!(scan, "data/B0_P3_p000.csv" , "data/B0_P3_p036.csv"
9               ", ',')
10 load_frequencies!(scan, "data/frequencies.csv", ',')
11 load_antennas!(scan, "data/antenna_locations.csv", ',')
12 load_channels!(scan, "data/channel_names.csv", ',')
13 scan.delayFunc = get_delays(Float32(8.0))
14 scan.beamformerFunc = DAS
15 image = abs.(beamform(scan))
16 imageSlice = get_slice(image, scan, 35e-3)
17 graphHandle = heatmap(scan.axes[1], scan.axes[2], imageSlice,
18                       colorscale="Viridis")
19 savefig(graphHandle, "GettingStarted.png")
```

A user would first instantiate the BrestScan struct and assign the data types that would be used throughout the processing pipeline. The first type sets the data type and thereby the precision of the points composing the imaging domain, the antenna locations and the frequency divisions. This can be any datatype that is a subset of type Real. The second type



controls the data type of the signal matrix containing the data collected from each antenna. This can be any type that is a subtype of Number allowing for time-domain (Real) signals or frequency-domain (Complex) signals. The third type controls the data type of the channels, and it can be any type that is a subtype of Integer. The choice of data type here has no accuracy impact on the final result and it is recommended to choose an Unsigned Integer data type that is big enough to index all the antennas. The next step would be to generate the imaging domain, this is accomplished using the `domain_hemisphere!` function. This accepts the resolution and the assumed or calculated radius of the breast. The user would then have to make use of the `load_XXX!` functions to load the data into the relevant fields of the struct. These functions assume the data is contained in a CSV file and contain no headers. The user then populates the relevant fields with their chosen beamformer and delay function. At this stage, the user can then pass the whole struct to the `beamform` which will beamform the provided signals into a set of data that can then be visualized as demonstrated towards the end of the code block above. Overall the entire workflow can be seen in Figure 12.

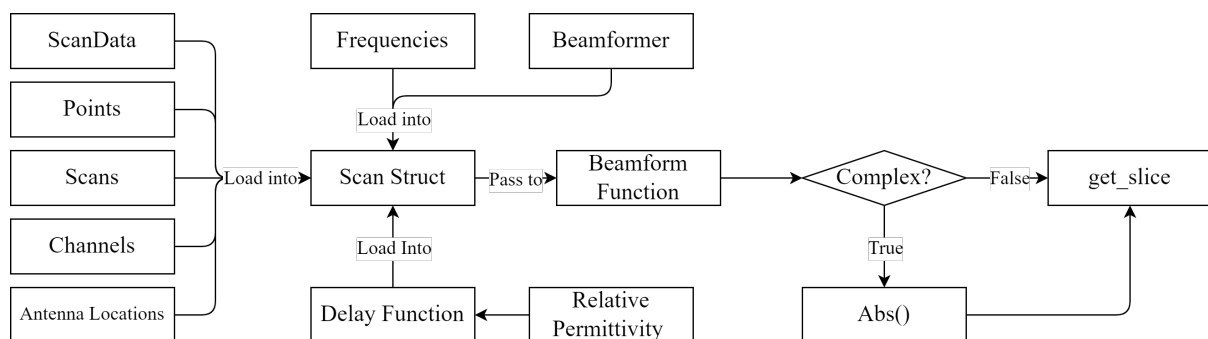


Figure 12: An overview of the current workflow used to image a scan in MERIT.jl

This workflow exemplifies how easy it is to use the MERIT.jl library. The function names were deliberately chosen to be verbose and conform to established nomenclature in the field so that anyone who wants to use the library can understand what each function does without having to delve into the source code. In this way, the MERIT.jl library is somewhat self documenting. Additional information is provided above each function in the form of docstrings which can be used with the `help` function built into the Julia REPL.

## 4.2 Plotted Scans

The library was benchmarked against the MATLAB implementation created by Dr. O'Loughlin *et al.* to ensure that the results provided by MERIT.jl are provably correct. Both implementations were given the same data, B0\_P3\_p000.csv and B0\_P3\_p036.csv. Rotational subtraction was performed on these in both libraries to reduce the presence of skin reflections in the data. The data was then processed according to the processing pipeline recommended by both libraries, the result of which can be seen below in Figure 13. It should be noted that the `imshow` function was used in MATLAB to plot the image. This had the effect of reflecting across the x-axis, it also slightly dilated the image along the x-axis, however since both image matrices share the same layout, a numerical comparison between the two matrices is possible. The averaged MSE was chosen as the numerical comparator due to the error squared term in the equation. This causes any small difference to be magnified in the error, which is desirable when the goal is absolute similarity between the MERIT.jl and its MATLAB counterpart.

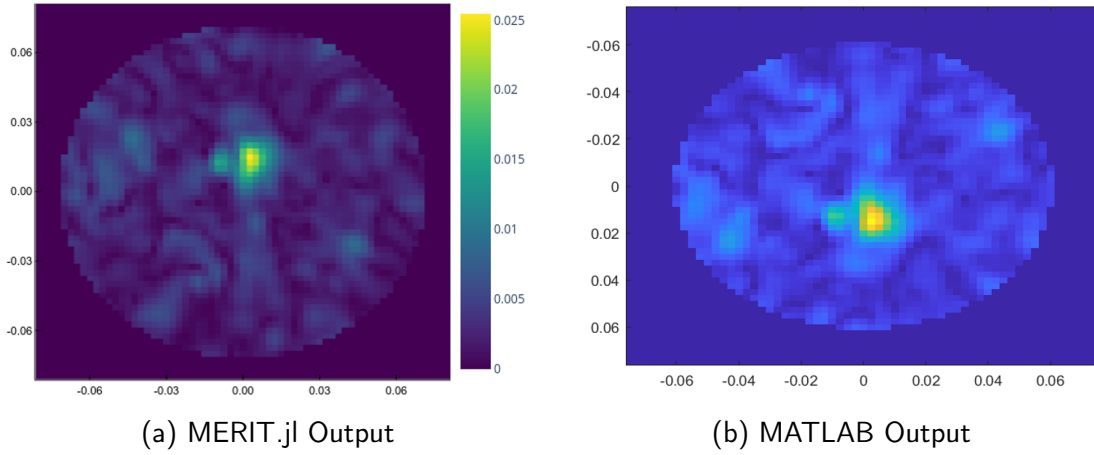


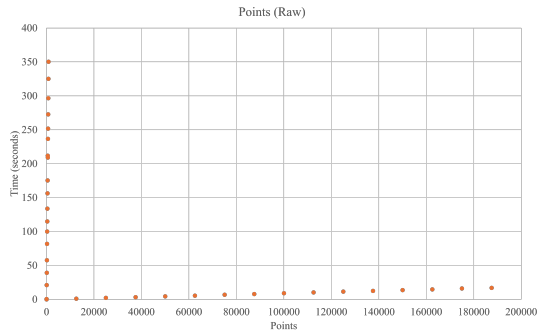
Figure 13: A comparison between Julia and MATLAB output, showing an averaged MSE of  $8.4417 \times 10^{-7}$

Computing the averaged MSE between the two images yielded an error of  $8.4417 \times 10^{-7}$  which is within the accuracy of a float, making it effectively identical to the images produced by MATLAB. This shows that the Julia library in its current state provides a viable alternative to the MATLAB implementation for frequency domain analysis.

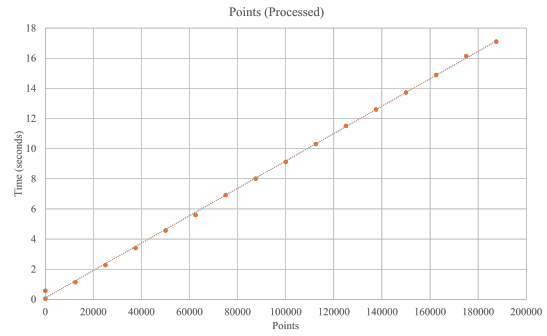
### 4.3 Performance of MERIT.jl

One of the requirements for MERIT.jl was performance. Through the use of type stability, SIMD optimized for loops and the disabling of array bounds checking, the library could process the provided data in 8 seconds. However, after the addition of the Points data type for type safety, the processing time climbed to 12 seconds. While the overall runtime is still acceptable, an increase of 3 seconds is less than ideal. An analysis to narrow down the cause of the increased runtime could not be conducted, however, unoptimized functions from the Points library may be the culprit. To capture the full performance characteristics of the library, a scalability test was performed, in which the number of points, channels and frequency divisions were progressively increased. This was performed in an automated manner using the functions provided by BenchmarkTools.jl [30]. The results from the benchmark suites were exported to CSV files and analyzed in Excel to judge the “Big O” notation of MERIT.jl.

Figures 14, 15, 16 show the results from BenchmarkTools. A large number of outliers were found as can be seen in the raw data. Subsequent analysis revealed that these outliers were the product of excessive usage of the swap file on the test system; a consequence of the insufficient amounts of available RAM and the large overhead introduced by the BenchmarkTools library. Since these data points are an artifact caused by the BenchmarkTools library, they were removed from the data set. A curve was fitted to the remaining points, to indicate the time complexity. From this analysis, it was concluded that MERIT.jl scales linearly with the number of points ( $\mathcal{O}(n)$ ) as can be seen in Figure 14, and quadratically with the number of channels ( $\mathcal{O}(n^2)$ ) as seen by the slightly quadratic increase in Figure 15. This aligns with expectations based on an algorithm analysis of the DAS beamformer. The main loop consists of a single for loop iterating over the points as such, the algorithm is expected to behave linearly with



(a) Raw Points



(b) Processed Points

Figure 14: Runtime for increasing Points

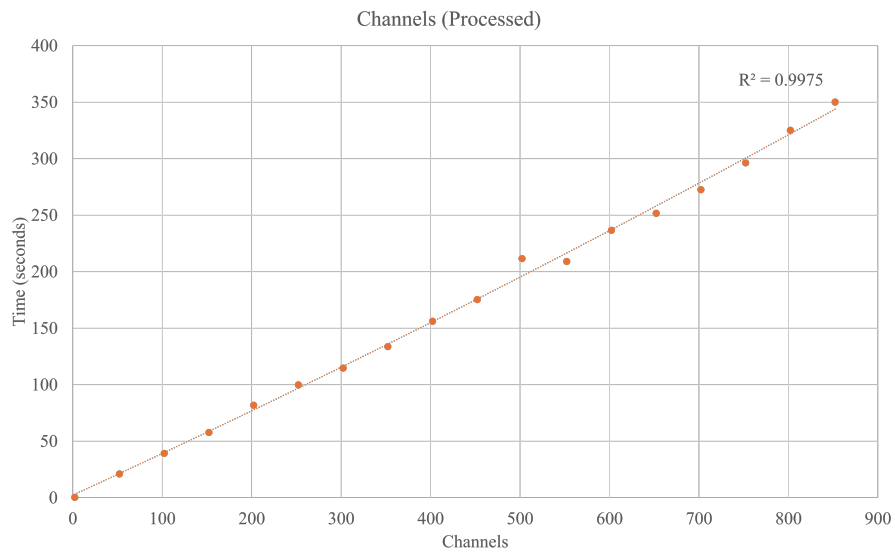
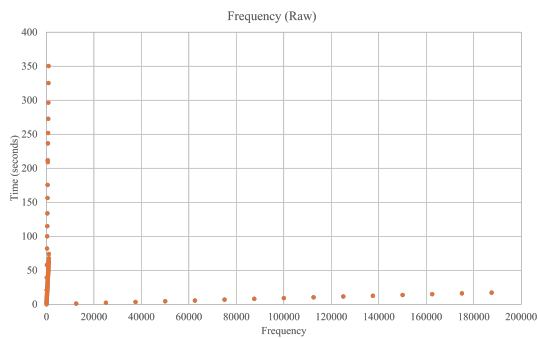
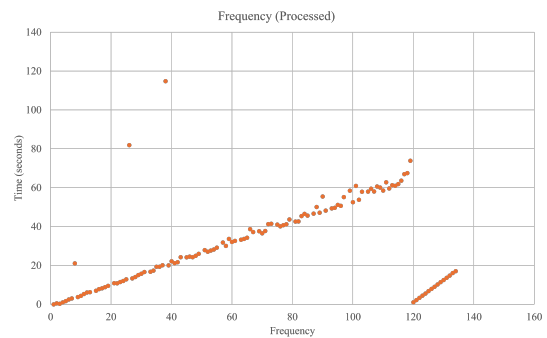


Figure 15: Runtime for increasing Channels



(a) Raw Frequencies



(b) Processed Frequencies

Figure 16: Runtime for increasing Frequency divisions

an increase in points. With channels, every additional antenna corresponds to a quadratic increase in channels since every antenna can send to and receive from any other antenna in

a fully multistatic system. The algorithm is expected to have  $\mathcal{O}(n)$  growth in the number of frequency divisions since they are only used once in the algorithm process to delay each signal. This is evidenced by the linear growth trend that can be seen in Figure 16. Effectively, MERIT works very well with a large number of points and with relatively few channels. This in turn affected the layout of the matrices in the library. Since data to do with points would be accessed most frequently, it was decided to place this data along the columns of the matrices. Since Julia is a column major language, this would provide the quickest access to this data. Data that relate to the channels were placed along the second dimension since this would be accessed less frequently than data related to the points. Finally, data relating to the frequency were placed along the third dimension, since this data is accessed very infrequently, it was decided that the latency required to load this data from memory would be acceptable provided it allowed us easy access to data relating to points and channels. The above graphs demonstrate the performance of the MERIT.jl library. From limited testing on my laptop with an Intel i7-1185G7 CPU, the Julia library executed in the same amount of time as its MATLAB counterpart. However, further testing to accurately quantify the runtime of both libraries is needed.

## 4.4 MERIT.jl as a Library

Creating generalized software for research is a challenge as most researchers create software and libraries that are specific to the question they are trying to answer. Conducting a search for microwave imaging libraries on Github yields 23 repositories coded in either MATLAB, Python, C, C++ or R which upon further inspection were all created to solve a particular research question. For example, DLMiMed was created to investigate the use of deep learning for breast tumor classification in medical microwave images [31]. Other researchers who might want to answer other questions relating to deep learning in microwave imaging cannot make use of this repository due to its fixed neural network and lack of generalizable code. Another example is the “Microwave-Imaging” repository created by André and Lucas Batista and Ricardo Adriano for their implementation of a quadratic programming approach to microwave breast imaging [32]. Again, this repository is specific to their research question and is only useful to researchers who want to investigate similar questions. Also, it is not clear how one could extend this repository to answer other questions in the field of microwave imaging in a way that is transparent to the end user. Only the MERIT library created by Dr. O’Loughlin stood out as a truly generalized library by providing researchers 3 well known beamforming algorithms as well as helper functions that would allow them to visualize the provided scan data. This is the gap that MERIT.jl aims to fill. Section 4.1 shows how quick and easy it would be for a researcher to ingest their data into the BreastScan struct and to visualize the output. Through the use of Closure as mentioned in section 3.2.5, MERIT.jl allows for extensive flexibility for the choice of  $\varepsilon$ , which is a parameter that is often changed by researchers. MERIT.jl also provides researchers with flexibility in the choice of delay and beamforming function used.

```

1 function delay_template(relative_permiativity)
2     # Capture the relatively_permiativity
3
4     # Create a delay function
5     function calc_(channels, antenna, domain_points):
6         #calculate and return a time matrix

```

```

7         #Size = (1 x #Channels x #Points)
8     end
9 end
10
11 function beamformer(delayed_signals)
12     #Do some processing
13     #return should be of size (1 x 1 x #Points)
14 end

```

By creating a delay and beamformer function that follows the template provided above, researchers can substitute their own functions into the BreastScan struct and the one-call processing pipeline will work as normal. Section 3.1 showcased how easy it was for the DifferentialEquations.jl module to extend the classes and functions already provided in SciMLBase so that structures from one library could be used with functions from another library in a way that was completely transparent to the end users, by leveraging the aforementioned features in Julia. Since MERIT.jl also makes use of these features, it also benefits from the easy extensibility that comes as a consequence.

# Future Work

MERIT.jl in its current state already provides researchers with the tools to analyze and visualize scans and through the integration of the aforementioned programming features, it achieves flexibility and extensibility. This chapter will consider the scope of future work and how it can be integrated with MERIT.jl

## 5.1 Time Domain Implementation

While this library focused on the frequency domain implementations of the beamformers, equivalent time domain representations do exist, the main difference being the method in which the signals are delayed. One could extend the `delay_signal!` function in the `Process.jl` file to accept signals that are subtyped from the `Real` abstract type. This way when working with time domain signals, the Julia compiler, based on the principals of multiple dispatch, will select the correct implementation of the `delay_signal!` function in a way that is fully transparent to the user. One could also extend the `get_delay` function in `Beamform.jl` to accept a sampling rate as well as  $\varepsilon$ , using the concept of closure, these variables would be captured and could be used to return the delay in terms of samples instead of seconds. The functional implementations of the beamformers can stay as they are, as they sum over the already delayed signals, they are indifferent to the numerical type of the input.

## 5.2 Implementation of More Beamformers

The implementation of further beamformers can be an area for future work. Section 3.1 showed how the structures and functions from multiple libraries could be combined behind the scenes to provide a more streamlined experience for the end user. A similar idea could be implemented in MERIT.jl where the beamformers could be defined in another library that gets used in MERIT.jl. The main benefit of this approach is that users are given the option to download a “lightweight” version of the library with one beamformer if the particular choice of beamformer does not matter, or they could install the secondary module and have access to a larger suite of beamformers. Another benefit is that the developers of MERIT.jl need not concern themselves with having to implement all current and future beamformers. This can be handled by a separate team thereby dividing the workload. Provided this team follows the template of the DAS beamformer, they can have the guarantee that their function will also work in MERIT.jl.

## 5.3 Parallel Processing

Parallel processing was not a feature that was explored as it was outside the scope of this thesis. However, there are areas of the code that have been identified as “embarrassingly parallel”. These are sections of code that are amenable to significant acceleration through the use of multithreading and parallel processing. Consider the beamforming equations in section 1.2. For all of these, the response at each point is calculated independently of the other points. As such this operation can be easily split across all available threads or even all available GPU cores, providing exponential increases to the performance of the library overall. The Julia language provides native support for threaded for-loops through the use of the `Threads.@threads` macro, which will evenly split the for-loop range across the threads available to the Julia runtime. However, the onus still lies on the user to ensure that no data race conditions can occur. Julia also supports GPU programming natively through the use of `CUDA.jl` for Nvidia GPUs, `AMDGPU.jl` for AMD GPUs, `oneAPI.jl` for Intel GPUs as well as `Metal.jl` for the current Apple integrated GPUs [33]. However, one drawback to parallel processing is the increased logic required to collect all the answers at the end. To prevent race conditions when writing to the output array, each spawned thread would either have to acquire a lock on the final array in order to write to it or pass its calculated answer to another thread that would sequentially write to the output array. In both cases, the performance provided by parallelizing the code would be hampered by the overhead introduced by collecting the results.

# Conclusions

The success of the MARIA, Wavelia and TSAR clinical trials indicate that microwave imaging can provide a viable alternative to traditional X-ray and ultrasound modalities. Aside from quantitative metrics, the subjective experiences of women also play a significant role in the success of an imaging modality. One qualitative study on women's satisfaction after receiving a mammogram found that they felt pain and discomfort during the imaging process, with one woman stating that they never showed up to their second appointment due to the pain experienced in the first one [34]. Whereas a survey conducted on the participants of the Wavelia clinical trial showed that 92% of the women found the bed comfortable and would recommend the procedure to other women. Indicating that women feel safe and comfortable with this new imaging modality. While the hardware has progressed, generalized software that can analyze the data from these systems has been lacking. Before the MERIT library created by Dr. O'Loughlin for the MATLAB programming language, most available software was specific to a particular system or created to answer a specific research question as was discussed at length in section 4.4. MERIT.jl was motivated by this lack of software and was created to be an alternative for researchers who want to use the Julia programming language in their studies.

MERIT was used to benchmark the MERIT.jl library in terms of performance and results. By making use of features such as type stability, MERIT.jl can process the data from the MERIT tutorial in under 12 seconds when running on a single thread on an i7-1185G7. MERIT.jl also demonstrates a similar level of customizability and flexibility as the MATLAB library by allowing for custom delay and beamforming functions, as well as allowing researchers to easily change the  $\varepsilon$  parameter in the default delay function. MERIT.jl also offers a form of reliability that is not seen in the MATLAB library through the introduction of a Point data type. This Point type allowed for a distinction to be created between a collection of numbers that represent a coordinate point, and a general collection of numbers. If the user was to incorrectly pass a collection of the point type to a function in place of another argument, the Julia compiler would throw an error, thereby creating a level of reliability. MERIT.jl also makes use of the multiple dispatch feature in Julia to hide implementation-specific details from the end users. This is heavily used in the Point type to allow the inbuilt mathematical operators to work on the Point type. This feature could also be used when the library needs to be extended to include different body parts. If the library needed to accommodate the data from the study conducted by Zhao *et al.* [29], one would only have to create a ForearmScan struct subtyped from the Scan abstract struct, and then overload the domain generation function. Users following the current pipeline in MERIT.jl would not notice any difference as the compiler will handle dispatching the correct functions. This demonstrates another one of the benefits that MERIT.jl has over the MATLAB version. The only way in MATLAB to achieve this would be to create long chains of if-else statements that check the types



of the provided input. Whereas in Julia, each specific implementation of a function can be grouped under one name. In this way, functions in MERIT.jl can be considered as concepts that encapsulate functional implementations. As was seen in section 4.2, the MERIT.jl library provides numerically identical outputs to the MATLAB implementation with an averaged MSE of  $8.4417 \times 10^{-7}$ , showing that the MERIT.jl library can be a viable alternative to the MATLAB library.

The Julia programming language has proved to be an attractive choice for the developers of scientific libraries with 10,760 packages being tracked by JuliaHub relating to some field of science, engineering or mathematics. Its popularity among these developers is a direct cause of the programming paradigm adopted by Julia, as was extensively discussed in chapters 2 and 3. Julia allows developers to code with a high level, easy to learn, language while also using a JIT compiler to generate highly optimized machine code at run time, even offering machine specific optimizations. This level of optimization and performance is critical for tasks requiring large amounts of numerical computation. Moreover, Julia offers native support for creating and working with matrices and their higher dimensional analogs, with operations that are reminiscent of MATLAB-like syntax. This allows developers may have a large scientific code base in MATLAB to easily port their matrix operations to Julia while availing of the optimized linear algebra operations provided by the Julia implementation of BLAS. Additionally features such as multiple dispatch and the type hierarchy allow developers to package multiple separate implementations of a particular function under one function name, whereas in other languages such as MATLAB, this could only be achieved by long if-else blocks checking the type of the input arguments. The benefit being code that is easily maintainable and easily extensible. As was seen in section 3.1, the multiple dispatch system allows other developers to create libraries that extend existing libraries in a way that is transparent to the end user. This is demonstrated by the interplay between the DifferentialEquations.jl library and the SciMLBase.jl library; users can use structures and functions from both libraries, which work together as if they were part of the same library. This transparent integration is what allows packages in Julia to work seamlessly together, meaning that communities of developers can create comprehensive collections of libraries around a particular topic provided they agree on a common interface and standard.

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