Cosmological Parameters and Conversions Memo

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1 Parallel to the Line-of-Sight

From Morales and Hewitt 2004, frequency Δf is related to the co-moving line-of-sight distance Δr_z by

$$\Delta f \approx \frac{f_{21} E(z)}{D_H (1+z)^2} \Delta r_z \tag{1}$$

The Fourier inverse of Δf is η and the Fourier inverse of Δr_z is k_z. These are related by

$$\eta \approx \frac{D_H (1+z)^2}{2\pi f_{21} E(z)} k_z \tag{2}$$

f_{21} is the frequency of the 21-cm signal, so $f_{21} = \frac{c}{0.21\mathrm{m}}$. D_H is the Hubble distance. From Hogg 2000,

$$D_H = \frac{c}{H_0} = 3000 \frac{\mathrm{Mpc}}{h} \tag{3}$$

where H_0 is the Hubble constant in the present epoch and h is a dimensionless quantity that is thought to be $0.6 < h < 0.9$ (the *IDL* function `cosmology_measures` defaults $h = 0.71$). H_0 is related to h by

$$H_0 = 100 \frac{h\mathrm{km}}{\mathrm{s\,Mpc}} \tag{4}$$

Again from Morales and Hewitt 2004,

$$E(z) = \sqrt{\Omega_M (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda} \tag{5}$$

where Ω_M is the matter constant, Ω_k is the curvature constant, and Ω_Λ is the "lambda" constant (whatever that is).

The *IDL* function `cosmology_measures` defaults these parameters to $\Omega_M = 0.27$, $\Omega_k = 0$, and $\Omega_\Lambda = 0.73$. Plugging these to $E(z)$ for $z = 7$ in gives $E(7) \approx 11.79$. Therefore at $z = 7$, the conversion between η and k_z is given by

$$k_z \approx \left(5.51 \times 10^5 \frac{h}{\mathrm{s\,Mpc}} \right) \eta \tag{6}$$

2 Perpendicular to the Line-of-Sight

From Morales and Hewitt 2004, sky positions θ_x and θ_y are related to co-moving distances r_x and r_y according to

$$\theta_x = \frac{r_x}{D_M(z)}, \quad \theta_y = \frac{r_y}{D_M(z)} \tag{7}$$

The Fourier inverses of θ_x and θ_y are u and v and the Fourier inverses of r_x and r_y are k_x and k_y. These are related by

$$u = \frac{k_x D_M(z)}{2\pi}, \quad v = \frac{k_y D_M(z)}{2\pi} \tag{8}$$

D_M is the "transverse co-moving distance."

From Hogg 2000,

$$D_M = \begin{cases} D_H \frac{1}{\sqrt{\Omega_k}} \sinh(\sqrt{\Omega_k} D_C/D_H), & \text{for } \Omega_k > 0 \\ D_C, & \text{for } \Omega_k = 0 \\ D_H \frac{1}{\sqrt{\Omega_k}} \sin(\sqrt{\Omega_k} D_C/D_H), & \text{for } \Omega_k < 0 \end{cases} \tag{9}$$

D_C is the co-moving distance and is defined as

$$D_C = D_H \int_0^z \frac{dz'}{E(z')} = D_H \int_0^z \frac{dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_k(1+z')^2 + \Omega_\Lambda}} \tag{10}$$

This integral must be evaluated numerically. With $z = 7$, $\Omega_M = 0.27$, $\Omega_k = 0$, and $\Omega_\Lambda = 0.73$ as above and evaluating the integral with *Mathematica*, we get that

$$D_C \approx D_H(2.09) = 6.27 \times 10^3 \frac{\text{Mpc}}{h} \tag{11}$$

This integral is also evaluated by *IDL*'s `cosmology_measures` function. Since we are assuming $\Omega_k = 0$, $D_M = D_C$ and

$$D_M \approx 6.27 \times 10^3 \frac{\text{Mpc}}{h} \tag{12}$$

To convert from (u, v) coordinates to (k_x, k_y) coordinates, we therefore get

$$k_x \approx \left(1.00 \times 10^{-3} \frac{h}{\text{Mpc}}\right) u, \quad k_y \approx \left(1.00 \times 10^{-3} \frac{h}{\text{Mpc}}\right) v \tag{13}$$