

CA4012: Statistical Machine Translation

Assignment 2: EM Algorithm for Word-based Translation model

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Step 1 (Initialisation)

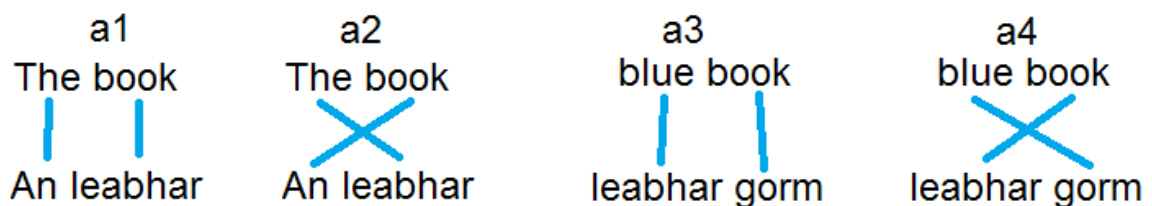
Input Words: {The, blue, book }

output Words: {leabhar, An, gorm}

$t(\text{leabhar} \mid \text{The})$	$= 1/3$
$t(\text{An} \mid \text{The})$	$= 1/3$
$t(\text{gorm} \mid \text{The})$	$= 1/3$
$t(\text{leabhar} \mid \text{book})$	$= 1/3$
$t(\text{An} \mid \text{book})$	$= 1/3$
$t(\text{gorm} \mid \text{book})$	$= 1/3$
$t(\text{leabhar} \mid \text{blue})$	$= 1/3$
$t(\text{An} \mid \text{blue})$	$= 1/3$
$t(\text{gorm} \mid \text{blue})$	$= 1/3$

Step 2(Expectation)

2-1: Calculate the probability of f and e under the alignment a : $p(f, a \mid e)$



$$p(a1, f \mid e) = p(a1, \text{An leabhar} \mid \text{The book}) = t(\text{An} \mid \text{The}) * t(\text{leabhar} \mid \text{book}) = \frac{1}{3} * \frac{1}{3} = \frac{1}{9}$$

$$p(a2, f \mid e) = p(a1, \text{An leabhar} \mid \text{The book}) = t(\text{leabhar} \mid \text{The}) * t(\text{An} \mid \text{book}) = \frac{1}{3} * \frac{1}{3} = \frac{1}{9}$$

$$p(a3, f \mid e) = p(a1, \text{leabhar gorm} \mid \text{blue book}) = t(\text{leabhar} \mid \text{blue}) * t(\text{gorm} \mid \text{book}) = \frac{1}{3} * \frac{1}{3} = \frac{1}{9}$$

$$p(a4, f \mid e) = p(a1, \text{leabhar gorm} \mid \text{blue book}) = t(\text{gorm} \mid \text{blue}) * t(\text{leabhar} \mid \text{book}) = \frac{1}{3} * \frac{1}{3} = \frac{1}{9}$$

2-2: Normalise for all alignments – probability distribution of the alignment a:

$$p(a | f, e)$$

$$p(a1 | f, e) = p(a1 | \text{An leabhar, The book}) = \frac{1}{9} \div \frac{2}{9} = \frac{1}{2}$$

$$p(a2 | f, e) = p(a2 | \text{An leabhar, The book}) = \frac{1}{9} \div \frac{2}{9} = \frac{1}{2}$$

$$p(a3 | f, e) = p(a3 | \text{leabhar gorm, blue book}) = \frac{1}{9} \div \frac{2}{9} = \frac{1}{2}$$

$$p(a4 | f, e) = p(a4 | \text{leabhar gorm, blue book}) = \frac{1}{9} \div \frac{2}{9} = \frac{1}{2}$$

Step 3 (maximisation)

3-1: Collect fractional counts c:

$$c(\text{An} | \text{The}) = \frac{1}{2} * 1 = \frac{1}{2}$$

$$c(\text{An} | \text{book}) = \frac{1}{2} * 1 = \frac{1}{2}$$

$$c(\text{leabhar} | \text{book}) = \frac{1}{2} * 1 + \frac{1}{2} * 1 = 1$$

$$c(\text{leabhar} | \text{The}) = \frac{1}{2} * 1 = \frac{1}{2}$$

$$c(\text{leabhar} | \text{blue}) = \frac{1}{2} * 1 = \frac{1}{2}$$

$$c(\text{gorm} | \text{book}) = \frac{1}{2} * 1 = \frac{1}{2}$$

$$c(\text{gorm} | \text{blue}) = \frac{1}{2} * 1 = \frac{1}{2}$$

3-2: Normalise fractional counts to yield revised parameter values

$$t(\text{An} | \text{The}) = \frac{1}{2} \div (\frac{1}{2} + \frac{1}{2}) = \frac{1}{2} \div 1 = \frac{1}{2}$$

$$t(\text{An} | \text{book}) = \frac{1}{2} \div (\frac{1}{2} + 1 + \frac{1}{2}) = \frac{1}{2} \div 2 = \frac{1}{4}$$

$$t(\text{leabhar} | \text{book}) = 1 \div (1 + \frac{1}{2} + \frac{1}{2}) = 1 \div 2 = \frac{1}{2}$$

$$t(\text{leabhar} | \text{The}) = \frac{1}{2} \div (\frac{1}{2} + \frac{1}{2}) = \frac{1}{2} \div 1 = \frac{1}{2}$$

$$t(\text{leabhar} | \text{blue}) = \frac{1}{2} \div (\frac{1}{2} + \frac{1}{2}) = \frac{1}{2} \div 1 = \frac{1}{2}$$

$$t(\text{gorm} | \text{book}) = \frac{1}{2} \div (\frac{1}{2} + \frac{1}{2} + 1) = 1 \div 2 = \frac{1}{4}$$

$$t(\text{gorm} | \text{blue}) = \frac{1}{2} \div (\frac{1}{2} + \frac{1}{2}) = \frac{1}{2} \div 1 = \frac{1}{2}$$

2nd Iteration

Step 2 (Expectation)

$$p(a1, f | e) = p(a1, \text{An leabhar} | \text{The book}) = t(\text{An} | \text{The}) * t(\text{leabhar} | \text{book}) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

$$p(a2, f | e) = p(a1, \text{An leabhar} | \text{The book}) = t(\text{leabhar} | \text{The}) * t(\text{An} | \text{book}) = \frac{1}{2} * \frac{1}{4} = \frac{1}{8}$$

$$p(a3, f | e) = p(a1, \text{leabhar gorm} | \text{blue book}) = t(\text{leabhar} | \text{blue}) * t(\text{gorm} | \text{book}) = \frac{1}{2} * \frac{1}{4} = \frac{1}{8}$$

$$p(a4, f | e) = p(a1, \text{leabhar gorm} | \text{blue book}) = t(\text{gorm} | \text{blue}) * t(\text{leabhar} | \text{book}) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

2-2 Normalization

$$p(a1 | f, e) = p(a1 | \text{An leabhar}, \text{The book}) = \frac{1}{4} \div \left(\frac{1}{4} + \frac{1}{8}\right) = \frac{1}{4} \div \frac{3}{8} = \frac{2}{3}$$

$$p(a2 | f, e) = p(a2 | \text{An leabhar}, \text{The book}) = \frac{1}{8} \div \left(\frac{1}{8} + \frac{1}{4}\right) = \frac{1}{8} \div \frac{3}{8} = \frac{1}{3}$$

$$p(a3 | f, e) = p(a3 | \text{leabhar gorm}, \text{blue book}) = \frac{1}{8} \div \left(\frac{1}{8} + \frac{1}{4}\right) = \frac{1}{8} \div \frac{3}{8} = \frac{1}{3}$$

$$p(a4 | f, e) = p(a4 | \text{leabhar gorm}, \text{blue book}) = \frac{1}{4} \div \left(\frac{1}{4} + \frac{1}{8}\right) = \frac{1}{4} \div \frac{3}{8} = \frac{2}{3}$$

Step 3 (Maximisation)

$$c(\text{An} | \text{The}) = \frac{2}{3} * 1 = \frac{2}{3}$$

$$c(\text{An} | \text{book}) = \frac{1}{3} * 1 = \frac{1}{3}$$

$$c(\text{leabhar} | \text{book}) = \frac{2}{3} * 1 + \frac{2}{3} * 1 = \frac{4}{3}$$

$$c(\text{leabhar} | \text{The}) = \frac{1}{3} * 1 = \frac{1}{3}$$

$$c(\text{leabhar} | \text{blue}) = \frac{1}{3} * 1 = \frac{1}{3}$$

$$c(\text{gorm} | \text{book}) = \frac{1}{3} * 1 = \frac{1}{3}$$

$$c(\text{gorm} | \text{blue}) = \frac{2}{3} * 1 = \frac{2}{3}$$

3-2: Normalization

$$t(\text{An} | \text{The}) = \frac{2}{3} \div \left(\frac{2}{3} + \frac{1}{3}\right) = \frac{2}{3} \div 1 = \frac{2}{3} = 0.6666$$

$$t(\text{An} | \text{book}) = \frac{1}{3} \div \left(\frac{1}{3} + \frac{4}{3} + \frac{1}{3}\right) = \frac{1}{3} \div 2 = \frac{1}{6} = 0.1666$$

$$t(\text{leabhar} | \text{book}) = \frac{4}{3} \div \left(\frac{4}{3} + \frac{1}{3} + \frac{1}{3}\right) = \frac{4}{3} \div 2 = \frac{2}{3} = 0.6666$$

$$t(\text{leabhar} \mid \text{The}) = \frac{1}{3} \div \left(\frac{1}{3} + \frac{2}{3}\right) = \frac{1}{3} \div 1 = \frac{1}{3} = 0.3333$$

$$t(\text{leabhar} \mid \text{blue}) = \frac{1}{3} \div \left(\frac{1}{3} + \frac{2}{3}\right) = \frac{1}{3} \div 1 = \frac{1}{3} = 0.3333$$

$$t(\text{gorm} \mid \text{book}) = \frac{1}{3} \div \left(\frac{1}{3} + \frac{1}{3} + \frac{4}{3}\right) = \frac{1}{3} \div 2 = \frac{1}{6} = 0.1666$$

$$t(\text{gorm} \mid \text{blue}) = \frac{2}{3} \div \left(\frac{2}{3} + \frac{1}{3}\right) = \frac{2}{3} \div 1 = \frac{2}{3} = 0.6666$$

As the values after the 2nd iterations maximisation step have not converged we would continue to repeat steps 2 and 3 until convergence however in this question we are only asked to calculate the translation probabilities after the two iterations above