# CA4012: Statistical Machine Translation

# Assignment 3: EM Algorithm for Word-based Translation model

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## *Step 1 (Initialisation)*

Input Words: {The, blue, book }
output Words: {leabhar, An, gorm}

t(leabhar | The) = 1/3
t(An | The) = 1/3
t(gorm | The) = 1/3
t(leabhar | book) = 1/3
t(An | book) = 1/3
t(gorm | book) = 1/3
t(leabhar | blue) = 1/3
t(An | blue) = 1/3
t(gorm | blue) = 1/3

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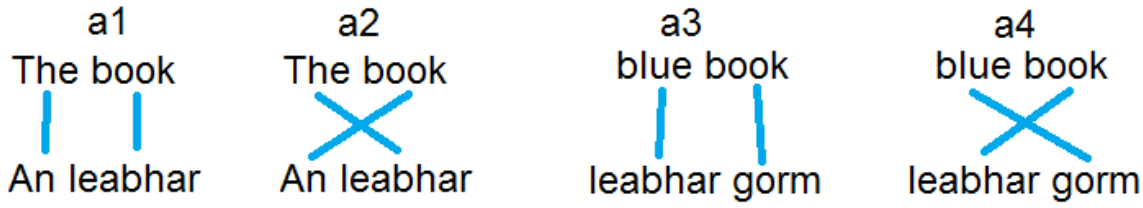
## *Step 2(Expectation)*

***2-1:Calculate the probability of f and e under the alignment a:*** $p(f, a|e)$




$$p(\text{a1}, f|e) = p(\text{a1}, \text{An leabhar} \mid \text{The book}) = t(\text{An} \mid \text{The}) * t(\text{leabhar} \mid \text{book}) = \frac{1}{3} * \frac{1}{3} = \frac{1}{9}$$

$$p(\text{a2}, f|e) = p(\text{a1}, \text{An leabhar} \mid \text{The book}) = t(\text{leabhar} \mid \text{The}) * t(\text{An} \mid \text{book}) = \frac{1}{3} * \frac{1}{3} = \frac{1}{9}$$

$$p(\text{a3}, f|e) = p(\text{a1}, \text{leabhar gorm} \mid \text{blue book}) = t(\text{leabhar} \mid \text{blue}) * t(\text{gorm} \mid \text{book}) = \frac{1}{3} * \frac{1}{3} = \frac{1}{9}$$

$$p(\text{a4}, f|e) = p(\text{a1}, \text{leabhar gorm} \mid \text{blue book}) = t(\text{gorm} \mid \text{blue}) * t(\text{leabhar} \mid \text{book}) = \frac{1}{3} * \frac{1}{3} = \frac{1}{9}$$

### 2-2: Normalise for all alignments – probability distribution of the alignment a:

$p(a \mid f, e)$

$$p(\text{a1} \mid f, e) = p(\text{a1} \mid \text{An leabhar, The book}) = \frac{1}{9} \div \frac{2}{9} = \frac{1}{2}$$

$$p(\text{a2} \mid f, e) = p(\text{a2} \mid \text{An leabhar, The book}) = \frac{1}{9} \div \frac{2}{9} = \frac{1}{2}$$

$$p(\text{a3} \mid f, e) = p(\text{a3} \mid \text{leabhar gorm, blue book}) = \frac{1}{9} \div \frac{2}{9} = \frac{1}{2}$$

$$p(\text{a4} \mid f, e) = p(\text{a4} \mid \text{leabhar gorm, blue book}) = \frac{1}{9} \div \frac{2}{9} = \frac{1}{2}$$

## Step 3 (maximisation)

### 3-1: Collect fractional counts c:

$$c(\text{An} \mid \text{The}) = \frac{1}{2} * 1 = \frac{1}{2}$$

$$c(\text{An} \mid \text{book}) = \frac{1}{2} * 1 = \frac{1}{2}$$

$$c(\text{leabhar} \mid \text{book}) = \frac{1}{2} * 1 + \frac{1}{2} * 1 = 1$$

$$c(\text{leabhar} \mid \text{The}) = \frac{1}{2} * 1 = \frac{1}{2}$$

$$c(\text{leabhar} \mid \text{blue}) = \frac{1}{2} * 1 = \frac{1}{2}$$

$$c(\text{gorm} \mid \text{book}) = \frac{1}{2} * 1 = \frac{1}{2}$$

$$c(\text{gorm} \mid \text{blue}) = \frac{1}{2} * 1 = \frac{1}{2}$$

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### 3-2: Normalise fractional counts to yield revised parameter values

$$t(\text{An} \mid \text{The}) = \frac{1}{2} \div (\frac{1}{2} + \frac{1}{2}) = \frac{1}{2} \div 1 = \frac{1}{2}$$

$$t(\text{An} \mid \text{book}) = \frac{1}{2} \div (\frac{1}{2} + 1 + \frac{1}{2}) = \frac{1}{2} \div 2 = \frac{1}{4}$$

$$t(\text{leabhar} \mid \text{book}) = 1 \div (1 + \frac{1}{2} + \frac{1}{2}) = 1 \div 2 = \frac{1}{2}$$

$$t(\text{leabhar} \mid \text{The}) = \frac{1}{2} \div (\frac{1}{2} + \frac{1}{2}) = \frac{1}{2} \div 1 = \frac{1}{2}$$

$$t(\text{leabhar} \mid \text{blue}) = \frac{1}{2} \div (\frac{1}{2} + \frac{1}{2}) = \frac{1}{2} \div 1 = \frac{1}{2}$$

$$t(\text{gorm} \mid \text{book}) = \frac{1}{2} \div (\frac{1}{2} + \frac{1}{2} + 1) = 1 \div 2 = \frac{1}{4}$$

$$t(\text{gorm} \mid \text{blue}) = \frac{1}{2} \div (\frac{1}{2} + \frac{1}{2}) = \frac{1}{2} \div 1 = \frac{1}{2}$$

## 2nd Iteration

## Step 2 (Expectation)

$$p(\text{a1}, f \mid e) = p(\text{a1}, \text{An leabhar} \mid \text{The book}) = t(\text{An} \mid \text{The}) * t(\text{leabhar} \mid \text{book}) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

$$p(\text{a2}, f \mid e) = p(\text{a1}, \text{An leabhar} \mid \text{The book}) = t(\text{leabhar} \mid \text{The}) * t(\text{An} \mid \text{book}) = \frac{1}{2} * \frac{1}{4} = \frac{1}{8}$$

$$p(\text{a3}, f \mid e) = p(\text{a1}, \text{leabhar gorm} \mid \text{blue book}) = t(\text{leabhar} \mid \text{blue}) * t(\text{gorm} \mid \text{book}) = \frac{1}{2} * \frac{1}{4} = \frac{1}{8}$$

$$p(\text{a4}, f \mid e) = p(\text{a1}, \text{leabhar gorm} \mid \text{blue book}) = t(\text{gorm} \mid \text{blue}) * t(\text{leabhar} \mid \text{book}) = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

### 2-2 Normalization

$$p(\text{a1} \mid f, e) = p(\text{a1} \mid \text{An leabhar, The book}) = \frac{1}{4} \div (\frac{1}{4} + \frac{1}{8}) = \frac{1}{4} \div \frac{3}{8} = \frac{2}{3}$$

$$p(\text{a2} \mid f, e) = p(\text{a2} \mid \text{An leabhar, The book}) = \frac{1}{8} \div (\frac{1}{8} + \frac{1}{4}) = \frac{1}{8} \div \frac{3}{8} = \frac{1}{3}$$

$$p(\text{a3} \mid f, e) = p(\text{a3} \mid \text{leabhar gorm, blue book}) = \frac{1}{8} \div (\frac{1}{8} + \frac{1}{4}) = \frac{1}{8} \div \frac{3}{8} = \frac{1}{3}$$

$$p(\text{a4} \mid f, e) = p(\text{a4} \mid \text{leabhar gorm, blue book}) = \frac{1}{4} \div (\frac{1}{4} + \frac{1}{8}) = \frac{1}{4} \div \frac{3}{8} = \frac{2}{3}$$

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## Step 3 (Maximisation)

$$c(\text{An} \mid \text{The}) = \frac{2}{3} * 1 = \frac{2}{3}$$

$$c(\text{An} \mid \text{book}) = \frac{1}{3} * 1 = \frac{1}{3}$$

$$c(\text{leabhar} \mid \text{book}) = \frac{2}{3} * 1 + \frac{2}{3} * 1 = \frac{4}{3}$$

$$c(\text{leabhar} \mid \text{The}) = \frac{1}{3} * 1 = \frac{1}{3}$$

$$c(\text{leabhar} \mid \text{blue}) = \frac{1}{3} * 1 = \frac{1}{3}$$

$$c(\text{gorm} \mid \text{book}) = \frac{1}{3} * 1 = \frac{1}{3}$$

$$c(\text{gorm} \mid \text{blue}) = \frac{2}{3} * 1 = \frac{2}{3}$$

### 3-2: Normalization

$$t(\text{An} \mid \text{The}) = \frac{2}{3} \div (\frac{2}{3} + \frac{1}{3}) = \frac{2}{3} \div 1 = \frac{2}{3} = 0.6666$$

$$t(\text{An} \mid \text{book}) = \frac{1}{3} \div (\frac{1}{3} + \frac{4}{3} + \frac{1}{3}) = \frac{1}{3} \div 2 = \frac{1}{6} = 0.1666$$

$$t(\text{leabhar} \mid \text{book}) = \frac{4}{3} \div (\frac{4}{3} + \frac{1}{3} + \frac{1}{3}) = \frac{4}{3} \div 2 = \frac{2}{3} = 0.6666$$

$$t(\text{leabhar} \mid \text{The}) = \frac{1}{3} \div (\frac{1}{3} + \frac{2}{3}) = \frac{1}{3} \div 1 = \frac{1}{3} = 0.3333$$

$$t(\text{leabhar} \mid \text{blue}) = \frac{1}{3} \div (\frac{1}{3} + \frac{2}{3}) = \frac{1}{3} \div 1 = \frac{1}{3} = 0.3333$$

$$t(\text{gorm} \mid \text{book}) = \frac{1}{3} \div (\frac{1}{3} + \frac{1}{3} + \frac{4}{3}) = \frac{1}{3} \div 2 = \frac{1}{6} = 0.1666$$

$$t(\text{gorm} \mid \text{blue}) = \frac{2}{3} \div (\frac{2}{3} + \frac{1}{3}) = \frac{2}{3} \div 1 = \frac{2}{3} = 0.6666$$

As the values after the 2nd iterations maximisation step have not converged we would continue to repeat steps 2 and 3 until convergence however in this question we are only asked to calculate the translation probabilities after the two iterations above