In Optimisation we described how we are interested in finding elements of the semantic cone $\mathcal{C}_{\text{sem}}$, and that there is a duality correspondence between these positive elements and constraints on the space of limit functionals.

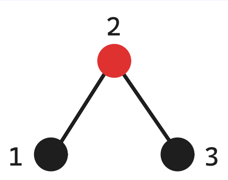
Assume $\mathcal{G}$ is some class of (c_e, c_v)-graphs where each graph is regular. Then we can derive some constraints on the space of limit functionals.

Extensions

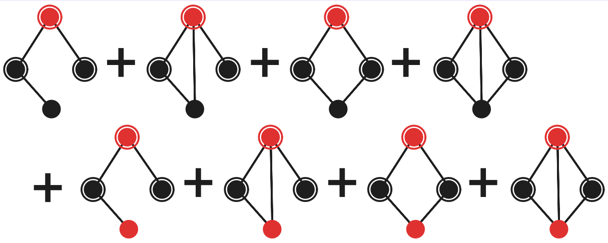
Let σ be some type of size k. Let ext_i^σ be the sum of all σ-flags of size $k+1$ which have an edge between the unlabelled vertex and the vertex labelled i.

Example

Let $\mathcal{G}$ be red-black vertex coloured regular graphs. Let σ be the following type



Then ext_1^σ is the following sum of σ-flags.



Lemma: *For any* σ *and* $G \in \mathcal{G}^\sigma$ *we have* $\rho(\text{ext}_i^\sigma; G) = 1$ *for all* $i \in [|\sigma|]$.

Todo

Proof.

Corollary: *For any limit functional* $\phi \in \Phi^\sigma$ *we have* $\phi(\text{ext}_i^\sigma) = 1$.

This actually only requires that $\lim \rho(\text{ext}^\sigma; G_k) = 1$ so we get this so long as $\mathcal{G}$ has graphs which are "asymptotically regular", meaning we can have some small constant divergence from regularity without effect.

Corollary: *For* $\phi \in \Phi^\sigma$ *we have:*

- $\phi(\text{ext}_i^\sigma - \text{ext}_j^\sigma) = 0$ *for all* $i, j \in [|\sigma|]$.
- $\phi(f \cdot \text{ext}_i^\sigma) = \phi(f)$ *for all* $f \in \mathcal{L}^\sigma$.

Corollary: *If* σ *is a local type then for any* $\phi \in \Phi^\emptyset$ *we have:*

- $\phi([\![\text{ext}_i^\sigma - \text{ext}_j^\sigma]\!]) = 0$ *for all* $i, j \in [|\sigma|]$.
- $\phi([\![f \cdot \text{ext}_i^\sigma]\!]) = \phi([\![f]\!])$ *for all* $f \in \mathcal{L}^\sigma$.

Todo

Proof.

Important

This property allows us to express some $f \in \mathcal{L}^\sigma$ as a sum of flags of larger size; Multiplying by ext_σ^i always increases the number of vertices in any given flag by 1.