Information to Examinees Sitting for the Fundamentals of Surveying Examination

The Fundamentals of Surveying (FS) examination is a closed-book examination. Therefore, no reference material is allowed in the examination site. The formulas and information here are provided in both the morning and the afternoon examination booklets. This information is not intended to be an all-inclusive compilation of formulas required for the FS examination. Basic theories, conversions, formulas, and definitions that examinees are expected to know have not been included. As required, special material not included here will be provided in the question statement itself to assist the examinee in solving the question.

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CONVERSIONS AND OTHER USEFUL RELATIONSHIPS

* 1 U.S. survey foot = $\dfrac{12}{39.37}$ m

* 1 international foot = 0.3048 m

* 1 in. = 25.4 mm (international)

1 mile = 1.60935 km

* 1 acre = 43,560 ft^2 = 10 square chains

* 1 ha = 10,000 m^2 = 2.47104 acres

* 1 rad = $\dfrac{180^\circ}{\pi}$

1 kg = 2.2046 lb

1 L = 0.2624 gal

1 ft^3 = 7.481 gal

1 gal of water weighs 8.34 lb

1 ft^3 of water weighs 62.4 lb

1 atm = 29.92 in. Hg = 14.696 psi

Gravity acceleration (g) = 9.807 m/s^2 = 32.174 ft/sec^2

Speed of light in a vacuum (c) = 299,792,458 m/s = 186,282 miles/sec

°C = (°F – 32)/1.8

1 min of latitude (ϕ) ≅ 1 nautical mile

1 nautical mile = 6,076 ft

Mean radius of the earth ≅ 20,906,000 ft ≅ 6,372,000 m

* Denotes exact value. All others correct to figures shown.

METRIC PREFIXES		
Multiple	Prefix	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	**micro**	**μ**
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d

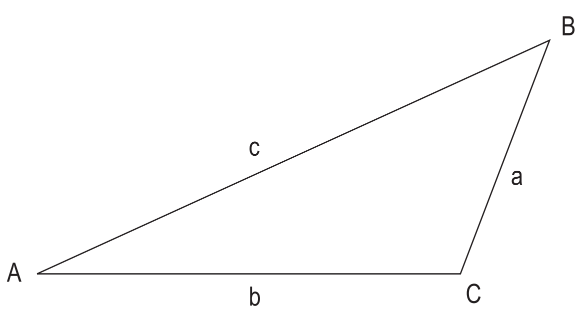
METRIC PREFIXES		
Multiple	Prefix	Symbol
10^{1}	deka	da
10^{2}	**hecto**	**h**
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	**peta**	**P**
10^{18}	exa	E

QUADRATIC EQUATION

$ax^2 + bx + c = 0$

$$\text{Roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

OBLIQUE TRIANGLES



Law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

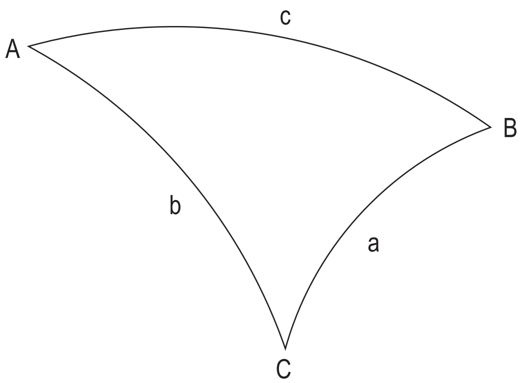
$$\text{Area} = \frac{ab\sin C}{2}$$

$$\text{Area} = \frac{a^2 \sin B \sin C}{2\sin A}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where s = (a + b + c)/2

SPHERICAL TRIANGLES



Law of sines

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Law of cosines

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\text{Area of sphere} = 4\pi R^2$$

$$\text{Volume of sphere} = \frac{4}{3}\pi R^3$$

$$\text{Spherical excess in sec.} = \frac{bc \sin A}{9.7 \times 10^{-6} R^2}$$

where R = mean radius of the earth

PROBABILITY AND STATISTICS

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\sum \frac{v^2}{n-1}}$$

where:

σ	=	standard deviation (sometimes referred to as standard error)
Σv^2	=	sum of the squares of the residuals (deviation from the mean)
n	=	number of observations
$\bar{x}$	=	mean of the observations (individual measurements x_i)

$$\sigma_{sum} = \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2}$$

$$\sigma_{series} = \sigma\sqrt{n}$$

$$\sigma_{mean} = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_{product} = \sqrt{A^2\sigma_b^2 + B^2\sigma_a^2}$$

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2}$$ where θ = the counter clockwise angle from the x axis

Relative weights are inversely proportional to variances, or:

$$W_a \propto \frac{1}{\sigma_a^2}$$

Weighted mean:

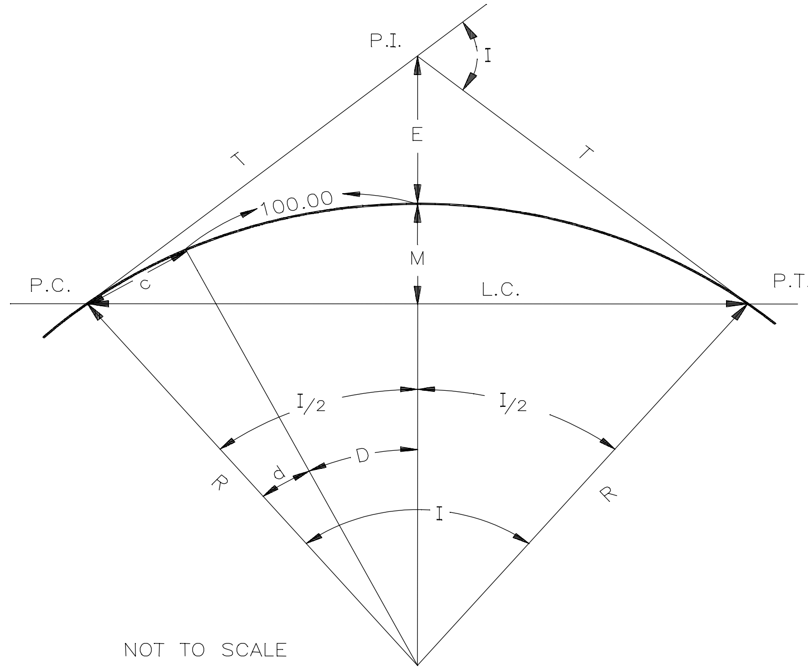
$$\overline{M}_w = \frac{\sum WM}{\sum W}$$

where:

M_w	=	weighted mean
ΣWM	=	sum of individual weights times their measurements
ΣW	=	sum of the weights

HORIZONTAL CIRCULAR CURVES

D = Degree of curve, arc definition
L = Length of curve from P.C. to P.T.
c = Length of sub-chord
ℓ = Length of arc for sub-chord
d = Central angle for sub-chord



$$D = \frac{5,729.58}{R}$$

$$T = R \tan(I/2)$$

$$L = RI\frac{\pi}{180} = \frac{I}{D}(100)$$

$$LC = 2R \sin(I/2)$$

$$c = 2R \sin(d/2)$$

$$d = \ell D/100$$

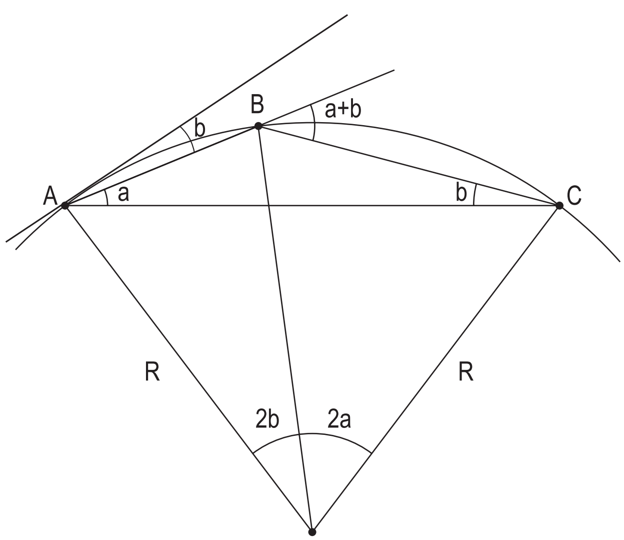
$$M = R\left[1 - \cos(I/2)\right]$$

$$E = R\left[\frac{1}{\cos(I/2)} - 1\right]$$

$$\text{Area of sector} = \frac{RL}{2} = \frac{\pi R^2 I}{360}$$

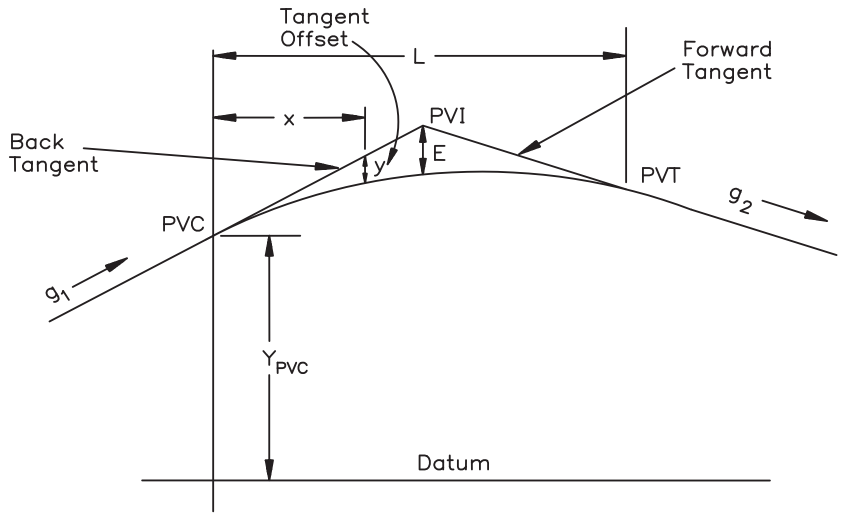
$$\text{Area of segment} = \frac{\pi R^2 I}{360} - \frac{R^2 \sin I}{2}$$

$$\text{Area between curve and tangents} = R(T - L/2)$$



$$R = \frac{AC}{2 \sin(a+b)}$$

VERTICAL CURVE FORMULAS



VERTICAL CURVE FORMULAS
NOT TO SCALE

- L = Length of curve (horizontal)
- PVC = Point of vertical curvature
- PVI = Point of vertical intersection
- PVT = Point of vertical tangency
- g_1 = Grade of back tangent
- g_2 = Grade of forward tangent
- x = Horizontal distance from PVC (or point of tangency) to point on curve
- a = Parabola constant
- y = Tangent offset
- E = Tangent offset at PVI
- r = Rate of change of grade

Tangent elevation = $Y_{PVC} + g_1 x$
and = $Y_{PVI} + g_2 (x - L/2)$

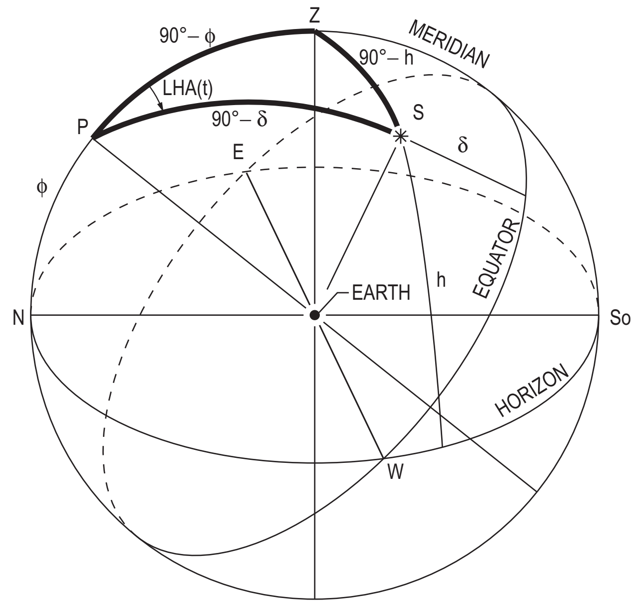
Curve elevation = $Y_{PVC} + g_1 x + ax^2$
= $Y_{PVC} + g_1 x + [(g_2 - g_1)/(2L)]x^2$

$$y = ax^2; \qquad a = \frac{g_2 - g_1}{2L};$$

$$E = a\left(\frac{L}{2}\right)^2; \quad r = \frac{g_2 - g_1}{L}$$

Horizontal distance to min/max elevation on curve, $x_m = -\dfrac{g_1}{2a} = \dfrac{g_1 L}{g_1 - g_2}$

ASTRONOMY



Cos (Az) = (sin δ – sin ϕ sin h)/(cos ϕ cos h) (altitude method)

Tan (Az) = –sin (LHA)/(cos ϕ tan δ – sin ϕ cos LHA) (hour angle method)

Sin h = sin ϕ sin δ + cos ϕ cos δ cos LHA

t = LHA or 360° – LHA

Horizontal circle correction for sun's semi-diameter = SD/cos h

Equations accurate for Polaris only:

$$h = \phi + p \cos LHA$$

$$Az = -(p \sin LHA)/\cos h$$

where:

Az	=	Azimuth (from north) to sun/star
δ	=	Declination
ϕ	=	Latitude
h	=	Altitude of sun/star
LHA	=	Local hour angle (sometimes referred to as "t" or "hour angle")
SD	=	Arc length of sun's semi-diameter
p	=	Polar distance of Polaris

PHOTOGRAMMETRY

$$\text{Scale} = \frac{ab}{AB} = \frac{f}{H-h}$$
(vertical photograph)

$$\text{Relief displacement} = \frac{rh}{H}$$
(vertical photograph)

Parallax equations:

$$p = x - x'$$

$$X = \frac{xB}{p}$$

$$Y = \frac{yB}{p}$$

$$h = H - \frac{fB}{p}$$

$$h_2 = h_1 + \frac{(p_2 - p_1)}{p_2}(H - h_1)$$

where:

f = Focal length

h = Height above datum

H = Flying height above datum

r = Radial distance from principal point

p = Parallax measured on stereo pair

B = Airbase of stereo pair

x, y = Coordinates measured on left photo

x′ = Coordinate measured on right photo

X, Y = Ground coordinates

Lens equation:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

where:

o = Object distance
i = Image distance
f = Focal length

Snell laws:

$$n \sin \phi = n' \sin \phi'$$

where:

n = Refractive index
ϕ = Angle of incidence

Curvature and refraction:

$$(c + r) = 0.0206M^2$$

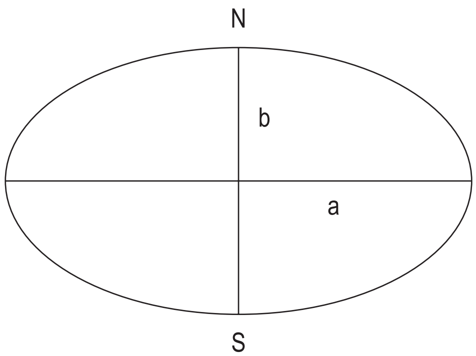
where:

(c + r) = Combined effect of curvature and refraction in feet

M = Distance in thousands of feet

GEODESY

Ellipsoid



a = semi-major axis

b = semi-minor axis

$$\text{Flattening, } f = \frac{a-b}{a}$$

$$\left(\text{usually published as } 1/f\right)$$

$$\text{Eccentricity, } e^2 = \frac{a^2 - b^2}{a^2}$$

$$\text{Radius in meridian, } M = \frac{a\left(1-e^2\right)}{\left(1-e^2 \sin\phi\right)^{3/2}}$$

$$\text{Radius in prime vertical, } N = \frac{a}{\left(1-e^2 \sin^2\phi\right)^{1/2}}$$

Angular convergence of meridians

$$\theta_{rad} = \frac{d \tan\phi\left(1-e^2 \sin^2\phi\right)^{1/2}}{a}$$

Linear convergence of meridians

$$= \frac{\ell d \tan\left(1-e^2 \sin^2\phi\right)^{1/2}}{a}$$

where:

- ϕ = Latitude
- d = Distance along parallel at latitude ϕ
- ℓ = Length along meridians separated by d

Ellipsoid definitions:

GRS80: a = 6,378,137.0 m
1/f = 298.25722101

Clark 1866: a = 6,378,206.4 m
1/f = 294.97869821

Orthometric correction:

Correction = $-0.005288 \sin 2\phi h \Delta\phi \,\text{arc}1'$

where:
- ϕ = latitude at starting point
- h = datum elevation in meters or feet at starting point
- $\Delta\phi$ = change in latitude in minutes between the two points (+ in the direction of increasing latitude or towards the pole)

STATE PLANE COORDINATES

Scale factor = Grid distance/geodetic (ellipsoidal) distance

Elevation factor = R/(R + H +N)

where:
- R = Ellipsoid radius
- H = Orthometric height
- N = Geoid height

For precision less than 1/200,000:
- R = 20,906,000 ft
- H = Elevation above sea level
- N = 0

ELECTRONIC DISTANCE MEASUREMENT

V = c/n

λ = V/f

$$D = \frac{\left(m\lambda + d\right)}{2}$$

where:
- V = Velocity of light through the atmosphere (m/s)
- c = Velocity of light in a vacuum
- n = Index of refraction
- λ = Wave length (m)
- f = Modulated frequency in hertz (cycles/sec)

D = Distance measured

m = Integer number of full wavelengths

d = Fractional part of the wavelength

ATMOSPHERIC CORRECTION

A 10°C temperature change or a pressure difference of 1 in. of mercury produces a distance correction of approximately 10 parts per million (ppm).

AREA FORMULAS

Area by coordinates where i is point order in a closed polygon.

$$\text{Area} = \frac{1}{2}\left[\sum_{i=1}^{n} X_i Y_{i+1} - \sum_{i=1}^{n} X_i Y_{i-1}\right]$$

Trapezoidal Rule

$$\text{Area} = w\left(\frac{h_1 + h_n}{2} + h_2 + h_3 + h_4 + \ldots + h_{n-1}\right)$$

Simpson's 1/3 Rule

$$\text{Area} = w\left[h_1 + 2\left(\sum h_{\text{odds}}\right) + 4\left(\sum h_{\text{evens}}\right) + h_n\right]/3$$

EARTHWORK FORMULAS

Average end area formula
volume = $L(A_1 + A_2)/2$

Prismoidal formula
volume = $L(A_1 + 4A_m + A_2)/6$

Pyramid or cone
volume = h(Area of Base)/3

TAPE CORRECTION FORMULAS

Correction for temperature
$C_t = 6.5 \times 10^{-6}\,(T - T_s)L$

Correction for tension
$C_p = (P - P_s)L/(AE)$

Correction for sag
$C_s = (w^2 l^3)/(24P^2)$

where:

- T = Temperature of tape during measurement, °F
- T_s = Temperature of tape during calibration, °F
- L = Distance measured, ft
- P = Pull applied during measurement, lb
- P_s = Pull applied during calibration, lb
- A = Cross-sectional area of tape, in^2
- E = Modulus of elasticity of tape, psi
- w = Weight of tape, lb/ft
- l = Length of unsupported span, ft

STADIA

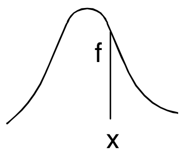
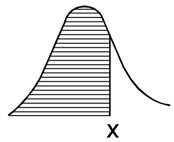
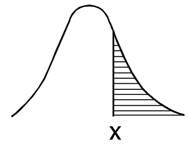
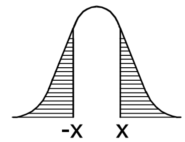
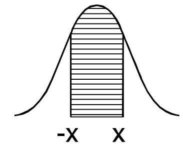
Horizontal distance = $KS \cos^2 \alpha$

Vertical distance = $KS \sin \alpha \cos \alpha$

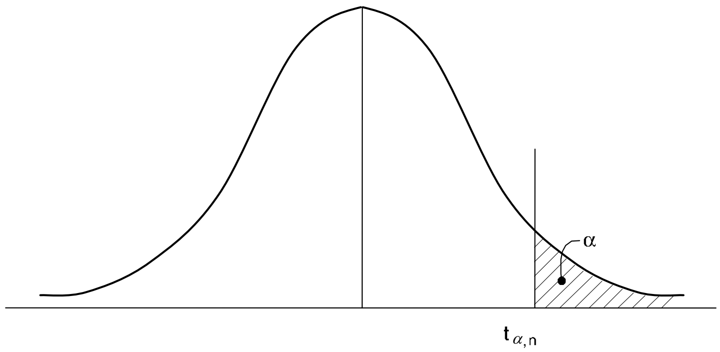
where:

- K = Stadia interval factor (usually 100)
- S = Rod intercept
- α = Slope angle measured from horizontal

UNIT NORMAL DISTRIBUTION TABLE

					
x	**f(x)**	**F(x)**	**R(x)**	**2R(x)**	**W(x)**
0.0	0.3989	0.5000	0.5000	1.0000	0.0000
0.1	0.3970	0.5398	0.4602	0.9203	0.0797
0.2	0.3910	0.5793	0.4207	0.8415	0.1585
0.3	0.3814	0.6179	0.3821	0.7642	0.2358
0.4	0.3683	0.6554	0.3446	0.6892	0.3108
0.5	0.3521	0.6915	0.3085	0.6171	0.3829
0.6	0.3332	0.7257	0.2743	0.5485	0.4515
0.7	0.3123	0.7580	0.2420	0.4839	0.5161
0.8	0.2897	0.7881	0.2119	0.4237	0.5763
0.9	0.2661	0.8159	0.1841	0.3681	0.6319
1.0	0.2420	0.8413	0.1587	0.3173	0.6827
1.1	0.2179	0.8643	0.1357	0.2713	0.7287
1.2	0.1942	0.8849	0.1151	0.2301	0.7699
1.3	0.1714	0.9032	0.0968	0.1936	0.8064
1.4	0.1497	0.9192	0.0808	0.1615	0.8385
1.5	0.1295	0.9332	0.0668	0.1336	0.8664
1.6	0.1109	0.9452	0.0548	0.1096	0.8904
1.7	0.0940	0.9554	0.0446	0.0891	0.9109
1.8	0.0790	0.9641	0.0359	0.0719	0.9281
1.9	0.0656	0.9713	0.0287	0.0574	0.9426
2.0	0.0540	0.9772	0.0228	0.0455	0.9545
2.1	0.0440	0.9821	0.0179	0.0357	0.9643
2.2	0.0355	0.9861	0.0139	0.0278	0.9722
2.3	0.0283	0.9893	0.0107	0.0214	0.9786
2.4	0.0224	0.9918	0.0082	0.0164	0.9836
2.5	0.0175	0.9938	0.0062	0.0124	0.9876
2.6	0.0136	0.9953	0.0047	0.0093	0.9907
2.7	0.0104	0.9965	0.0035	0.0069	0.9931
2.8	0.0079	0.9974	0.0026	0.0051	0.9949
2.9	0.0060	0.9981	0.0019	0.0037	0.9963
3.0	0.0044	0.9987	0.0013	0.0027	0.9973
Fractiles					
1.2816	0.1755	0.9000	0.1000	0.2000	0.8000
1.6449	0.1031	0.9500	0.0500	0.1000	0.9000
1.9600	0.0584	0.9750	0.0250	0.0500	0.9500
2.0537	0.0484	0.9800	0.0200	0.0400	0.9600
2.3263	0.0267	0.9900	0.0100	0.0200	0.9800
2.5758	0.0145	0.9950	0.0050	0.0100	0.9900

t-DISTRIBUTION TABLE

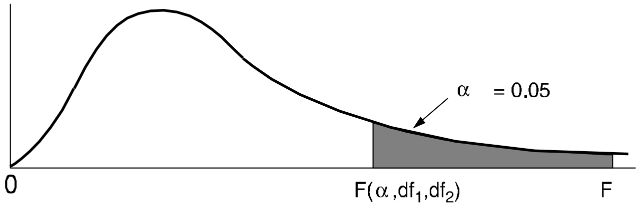


VALUES OF $t_{\alpha,n}$

n	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$	n
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
∞	1.282	1.645	1.960	2.326	2.576	∞

CRITICAL VALUES OF THE *F* DISTRIBUTION − TABLE

For a particular combination of numerator and denominator degrees of freedom, entry represents the critical values of *F* corresponding to a specified upper tail area (α).



Denominator df_2	Numerator df_1: 1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	**6.61**	**5.79**	**5.41**	**5.19**	**5.05**	**4.95**	**4.88**	**4.82**	**4.77**	**4.74**	**4.68**	**4.62**	**4.56**	**4.53**	**4.50**	**4.46**	**4.43**	**4.40**	**4.36**
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	**4.96**	**4.10**	**3.71**	**3.48**	**3.33**	**3.22**	**3.14**	**3.07**	**3.02**	**2.98**	**2.91**	**2.85**	**2.77**	**2.74**	**2.70**	**2.66**	**2.62**	**2.58**	**2.54**
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	**4.54**	**3.68**	**3.29**	**3.06**	**2.90**	**2.79**	**2.71**	**2.64**	**2.59**	**2.54**	**2.48**	**2.40**	**2.33**	**2.29**	**2.25**	**2.20**	**2.16**	**2.11**	**2.07**
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	**4.35**	**3.49**	**3.10**	**2.87**	**2.71**	**2.60**	**2.51**	**2.45**	**2.39**	**2.35**	**2.28**	**2.20**	**2.12**	**2.08**	**2.04**	**1.99**	**1.95**	**1.90**	**1.84**
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	**4.24**	**3.39**	**2.99**	**2.76**	**2.60**	**2.49**	**2.40**	**2.34**	**2.28**	**2.24**	**2.16**	**2.09**	**2.01**	**1.96**	**1.92**	**1.87**	**1.82**	**1.77**	**1.71**
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	**4.17**	**3.32**	**2.92**	**2.69**	**2.53**	**2.42**	**2.33**	**2.27**	**2.21**	**2.16**	**2.09**	**2.01**	**1.93**	**1.89**	**1.84**	**1.79**	**1.74**	**1.68**	**1.62**
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

ECONOMICS

Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$(F/P, i\%, n)$	$(1+i)^n$
Single Payment Present Worth	to P given F	$(P/F, i\%, n)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F	$(A/F, i\%, n)$	$\frac{i}{(1+i)^n-1}$
Capital Recovery	to A given P	$(A/P, i\%, n)$	$\frac{i(1+i)^n}{(1+i)^n-1}$
Uniform Series Compound Amount	to F given A	$(F/A, i\%, n)$	$\frac{(1+i)^n-1}{i}$
Uniform Series Present Worth	to P given A	$(P/A, i\%, n)$	$\frac{(1+i)^n-1}{i(1+i)^n}$
Uniform Gradient Present Worth	to P given G	$(P/G, i\%, n)$	$\frac{(1+i)^n-1}{i^2(1+i)^n}-\frac{n}{i(1+i)^n}$
Uniform Gradient † Future Worth	to F given G	$(F/G, i\%, n)$	$\frac{(1+i)^n-1}{i^2}-\frac{n}{i}$
Uniform Gradient Uniform Series	to A given G	$(A/G, i\%, n)$	$\frac{1}{i}-\frac{n}{(1+i)^n-1}$

Nomenclature and Definitions

A Uniform amount per interest period
B Benefit
BV Book Value
C Cost
d Combined interest rate per interest period
D_j Depreciation in year j
F Future worth, value, or amount
f General inflation rate per interest period
G Uniform gradient amount per interest period
i Interest rate per interest period
i_e Annual effective interest rate
m Number of compounding periods per year
n Number of compounding periods; or the expected life of an asset
P Present worth, value, or amount
r Nominal annual interest rate
S_n Expected salvage value in year n

Subscripts

j at time j
n at time n
† $F/G = (F/A - n)/i = (F/A) \times (A/G)$

Nonannual Compounding

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Book Value

BV = Initial cost – ΣD_j

Depreciation

Straight line $D_j = \dfrac{C - S_n}{n}$

Accelerated Cost Recovery System (ACRS)

D_j = (factor from table below) C

MODIFIED ACRS FACTORS				
Year	**Recovery Period (Years)**			
	3	**5**	**7**	**10**
	Recovery Rate (%)			
1	33.3	20.0	14.3	10.0
2	44.5	32.0	24.5	18.0
3	14.8	19.2	17.5	14.4
4	7.4	11.5	12.5	11.5
5		**11.5**	**8.9**	**9.2**
6		5.8	8.9	7.4
7			8.9	6.6
8			4.5	6.6
9				6.5
10				**6.5**
11				3.3

Capitalized Costs

Capitalized costs are present worth values using an assumed perpetual period of time.

Capitalized costs = $P = \dfrac{A}{i}$