GEODESIA I (Exercícios nº1)

8. a) Elipsóide WGS84: $a = 6378137{,}0\,m$
$e^2 = 0{,}00669437999 0$

VÉRTICE C. S. Jorge $\varphi = 38°42'49.''306$
$\lambda = -9°\ 7'\ 59'',386$
$h = 166{,}09\,m$

$$N = \frac{a}{\sqrt{1-e^2 \operatorname{sen}^2\varphi}}$$

$X = (N+h)\cos\varphi\cos\lambda = 4920218{,}058\,m$ $\quad N = 6386504{,}283\,m$

$y = (N+h)\cos\varphi\operatorname{sen}\lambda = -791012{,}402$ $\quad N+h = 6386670{,}373\,m$

$Z = [N(1-e^2)+h]\operatorname{sen}\varphi = 3967670{,}702\,m$ $\quad N(1-e^2) = 6343750{,}597\,m$

9. Datum Lisboa (Elipsóide Hayford): $a = 6378388{,}0\,m$
$e^2 = 0{,}00672267$

VÉRTICE C. S. Jorge $\varphi = 38°42'43'',64162$
$\lambda = -9°\ 7'\ 54{,}84465$
$h = 112{,}442\,m$

Azimute (α) = $10°21'21'',091$
Distância (S) = $20377{,}935\,m$

$$\varphi_{\text{Sernes}} = \varphi_{Lx} + \frac{V^3}{c}\cos\alpha \cdot S$$

$$\lambda_{\text{Sernes}} = \lambda_{Lx} + \frac{V\operatorname{sen}\alpha}{c\cdot\cos\varphi}\cdot S$$

$$V = \sqrt{1+e'^2\cos^2\varphi} = \frac{\sqrt{1-e^2\operatorname{sen}^2\varphi}}{\sqrt{1-e^2}}$$

$$c = \frac{a^2}{b} = \frac{a}{\sqrt{1-e^2}}$$

$V = 1{,}002058333$

$c = 6399936{,}608\,m$

$\varphi_2 = \varphi_1 + \Delta\varphi$

$\lambda_2 = \lambda_1 + \Delta\lambda$

$$\Delta\varphi = \frac{V^3}{c}\cos\alpha\cdot S = 0°{,}18057322 0 = 0°10'50'',06358$$

$$\Delta\lambda = \frac{V\operatorname{sen}\alpha}{c\cdot\cos\varphi}\cdot S = 0°{,}042114909 7 = 0°2'31'',61367$$

$\varphi_{\text{Sernes}} = 38°\ 53'\ 33'',7052$

$\lambda_{\text{Sernes}} = -9°\ 5'\ 23'',2310$

$$\alpha_{21} = \alpha_{12} + 180° + \frac{V}{c}\operatorname{sen}\alpha\cdot\operatorname{tg}\varphi\cdot S = 190°22'55'',911$$

8 - b) Coordenadas ITRF93

$$\text{Graz} = \begin{cases} X = 4194424{,}034\,m \\ Y = 1162702{,}490\,m \\ Z = 4647245{,}279\,m \end{cases}$$

Elipsoide GRS80: $a = 6378137{,}0\,m$

$e^2 = 0{,}00694380029$ $\quad N = \dfrac{a}{\sqrt{1-e^2 \operatorname{sen}^2\varphi}}$

$$\lambda = \operatorname{arctg}\left(\frac{Y}{X}\right) = 0{,}270412185^{rad} = 15°\ 29'\ 36{,}''517$$

$$\operatorname{tg}\varphi_0 = \frac{1}{1-e^2} \cdot \frac{Z}{\sqrt{X^2+Y^2}} ; \ \varphi_0 = 47{,}°0671444 \ ; \ N_0 = 6389611{,}876\,m$$

$$\operatorname{tg}\varphi_1 = \frac{Z + e^2 N_0 \operatorname{sen}\varphi_0}{\sqrt{X^2+Y^2}} , \ \varphi_1 = 47{,}°06712829 ; \ N_1 = 6389611{,}87\,m$$

$$\operatorname{tg}\varphi_2 = \frac{Z + e^2 N_1 \operatorname{sen}\varphi_1}{\sqrt{X^2+Y^2}} ; \ \varphi_2 = 47°06712814 ; \ N_2 = N_1$$

$$\varphi_2 = 47°\ 4'\ 1{,}''661$$

$$h = \frac{\sqrt{X^2+Y^2}}{\cos\varphi_2} - N_2 = 538{,}297\,m$$

10. Azimute de LISBOA - SERVES, $\alpha = ?$
comprimento " " , $S = ?$

Elipsoide Hayford:
$a = 6378388{,}0\,m$
$e^2 = 0{,}00672267$

$$c = \frac{a^2}{b} = \frac{a}{\sqrt{1-e^2}} = 6399936{,}608\,m$$

$$V_{Lx} = \frac{\sqrt{1-e^2\operatorname{sen}^2\varphi}}{\sqrt{1-e^2}} = 1{,}002058333$$

$\Delta\varphi = 0{,}°180569008$
$\Delta\lambda = 0{,}°042101967$
$\Delta_1 = \Delta_2 = 0$

$$\alpha_{12} = \operatorname{arctg}\left[V_{Lx}^2 \cos\varphi_{Lx} \cdot \left(\frac{\Delta\lambda - \Delta_2}{\Delta\varphi - \Delta_1}\right)\right]$$

$$S = \frac{c \cdot (\Delta\varphi - \Delta_1)}{V_{Lx}^3 \cdot \cos\alpha_{12}}$$

$$\alpha_{12} = 0{,}180693607\ rad = 10°\ 21'\ 10{,}''7318$$

$$S = 20377.27\,m$$

GEODESIA I (Exercícios nº1)

11 • a)

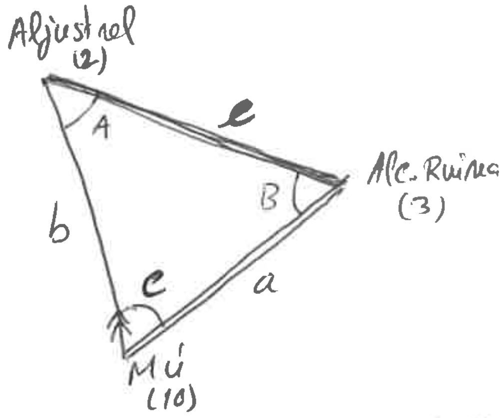
$\hat{A} = 53{,}^\circ0099639$

$\hat{B} = 81{,}^\circ7873083$

$\hat{C} = 45{,}^\circ2039528$

$\varphi_{Mú} = 37^\circ 22' 9{,}''81$

$\lambda_{Mú} = -8^\circ \cancel{4}' 50{,}''10$



Elipsoide $a = 6378388\,m$

$e^2 = 0{,}00672267$

$\alpha_{10-2} = 352{,}^\circ2910167 = 6{,}14638\,17\,rad$

$S_{10-3} = 46181{,}105$

$\bar{R} = \sqrt{N \cdot \rho}$

$N_{10} = 6\,386\,300{,}944\,m$

$\rho_{10} = 6359116{,}711\,m$

$\bar{R} = 6372694{,}33\,m$

$S^{rad}_{10-3} = \dfrac{S}{\bar{R}} \cdot \dfrac{180}{\pi} = 0{,}^\circ41520623$

$\dfrac{sen\,a}{sen\,A} = \dfrac{sen\,b}{sen\,B} = \dfrac{sen\,c}{sen\,C} \Rightarrow sen\,b = sen(S_{10-3}) = \dfrac{sen\,a}{sen\,A} \cdot sen\,B$

$S^{rad}_{10-3} = arc\,sen\left[\dfrac{sen(S_{10-3})}{sen\,\hat{A}} \cdot sen\,\hat{B}\right]$

$S_{10-3} = S^{rad}_{10-3} \times \bar{R} = 57\,224.761\,m$

$V_{10} = 1{,}0021351427$

$c = 6399936{,}608\,m$

$\eta^2 = 0{,}0042\,74844$

$\varphi_{(2)} = \varphi_{(10)} + d\varphi_{10-2} = 37{,}^\circ8802\,7417 = \underline{\underline{3\cancel{7}^\circ 52' 48{,}''9870}}$

$$d\varphi_{10-2} = \underbrace{\frac{V^3}{c}\cos\alpha_{10-2} \cdot S_{10-2}}_{0{,}^\circ5109\,3647} + \underbrace{\frac{-V^4}{c^2}\left(sen^2\alpha_{10-2} \cdot tg\,\varphi_{10} + 3\cos^2\alpha_{10-2}\,\eta^2 \cdot tg\,\varphi_{10}\right) \cdot \frac{S_{10-2}}{2}}_{-0{,}^\circ0000\,5396}$$

$\eta^2 = \dfrac{e^2}{1-e^2}\cos^2\varphi$

$\lambda_{(2)} = \lambda_{(10)} + d\lambda_{10-2} = -8{,}^\circ1678\,2647 = \underline{\underline{-8^\circ 10' 4{,}''1753}}$

$$d\lambda_{10-2} = \underbrace{\frac{V}{c} \cdot \frac{sen\,\alpha_{10-2}}{\cos\varphi_{10}} \cdot S}_{-0{,}^\circ08665549} + \underbrace{\frac{2V^2}{c^2\cos\varphi_{10}} \cdot sen\,\alpha_{10-2} \cdot \cos\alpha_{10-2} \cdot tg\,\varphi_{10} \cdot \frac{S^2}{2}}_{-0{,}^\circ00058765}$$

$\alpha_{2-10} = \alpha_{10-2} + 180^\circ + d\alpha = 172{,}^\circ2481\,3902 = \underline{\underline{172^\circ 14' 53{,}''3005}}$

$$d\alpha = \underbrace{\frac{V}{c} \cdot sen\,\alpha \cdot tg\,\varphi \cdot S}_{-0{,}^\circ0424\,4537} + \underbrace{\frac{V^2}{c^2}\,sen\,\alpha \cdot \cos\alpha\,(1 + 2tg^2\varphi + \eta^2) \cdot \frac{S^2}{2}}_{-0{,}^\circ00043227}$$

$\varphi_2 = 37^\circ 52' 48{,}''9870$

$\lambda_2 = -8^\circ 10' 4{,}''1753$

$\alpha_{2-10} = 172^\circ 14' 53{,}''3005$ ///

11- b) Azimute Mú - Ale.Ruina - $\alpha_{10-3} = \alpha_{10-2} + \hat{C} = 37°\ 29'\ 41'',8900$

$S_{10-3} = 46181,105\ m$ $\quad V_{10} = 1,0021351427$

$\varphi_3 = \varphi_{10} + d\varphi_{10-3} = 37°,6990875 = \underline{37°\ 41'\ 56'',7150}$ $\quad c = 6399936,608m$

$\eta^2_{10} = 0,004273844$

$$d\varphi_{10-3} = \underbrace{\frac{V_{10}^3}{c}\cos\alpha_{10-3}\, S}_{0°,33013082} + \underbrace{\frac{-V^4}{c^2}\left(\text{sen}^2\alpha\ \text{tg}\,\varphi_{10} + 3\cos^2\alpha \cdot \eta^2\,\text{tg}\,\varphi_{10}\right)\cdot\frac{S^2}{2}}_{-0°,00139044}$$

$\lambda_3 = \lambda_{10} + d\lambda_{10-3} = -7°,7618634 = -7°\ \underline{45'\ 42'',7081}$

$$d\lambda_{10-3} = \underbrace{\frac{V\,\text{sen}\alpha_{10-3}}{c\cos\varphi_{10}}\cdot S}_{0°,31732953} + \underbrace{\frac{2V^2}{c^2\cos\varphi_{10}}\cdot \text{sen}\alpha \cdot \cos\alpha \cdot \text{tg}\varphi_{10}\cdot\frac{S^2}{2}}_{0°,00139044}$$

$\alpha_{3-10} = \alpha_{10-3} + 180 + d\alpha_{10-3} = 217°,6891435 = \underline{217°\ 41'\ 20'',9167}$

$$d\alpha_{10-3} = \underbrace{\frac{V}{c}\,\text{sen}\alpha\ \text{tg}\varphi_{10}\cdot S}_{0°,19260359} + \underbrace{\frac{V^2}{c^2}\,\text{sen}\alpha\cdot\cos\alpha\,(1 + 2\text{tg}^2\varphi + \eta^2)\frac{S^2}{2}}_{0°,00157048}$$

Azimute Ale.Ruina - Aljustrel - $\alpha_{3-2} = \alpha_{3-10} + \hat{B} = 299°\ 28'\ 35'',2267$

$$S_{3-2}^{rad} = \text{arcsen}\left[\frac{\text{sen}(S_{10-3}^{rad})}{\text{sen}\,\hat{A}}\cdot \text{sen}\,\hat{C}\right]$$

$S_{3-2} = S_{3-2}^{rad} \times \bar{R} = 41028,293\,m$ $\quad V_3 = 1,0021163673$

$\varphi_2 = \varphi_3 + d\varphi_{3-2} = 37°,8802882 = \underline{37°\ 52'\ 49'',0375}$ $\quad c = 6399936,608m$

$\eta_3^2 = 0,004237214$

$$d\varphi_{3-2} = \underbrace{\frac{V_3^3}{c}\cos\alpha_{3-2}\cdot S}_{0°,18188968} + \underbrace{\frac{-V^4}{c^2}\left(\text{sen}^2\alpha_{3-2}\cdot\text{tg}\,\varphi_3 + 3\cos^2\alpha\cdot\eta_3^2\,\text{tg}\,\varphi_3\right)\frac{S^2}{2}}_{-0,00068900}$$

$\lambda_2 = \lambda_3 + d\varphi_{3-2} = -7°,7618634 + (-0°,40498761 - 0°,00098545) = -8°,1677404$

$\lambda_2 = -8°\ \underline{10'\ 4'',2255}$

$\alpha_{2-3} = \alpha_{3-2} + 180 + d\alpha_{32} = 299°,47645 + 180° + (-0°,24765577 - 0°,00113369)$

$\alpha_{2-3} = 119°,2276824 = 119°\ 13'\ 39'',6566$

$\alpha_{2-10} = \alpha_{2-3} + \hat{A} = \underline{172°\ 14'\ 15'',5266}$ (?) ($\neq 53'',3005$)

$\therefore$ φ e λ c/ diferenças de apenas décimas de segundo o azimute foi obtido c/ grande diferença!

GEODESIA I (Exercícios n.º 1)

12. a) Vértices geodésicos

$X = (N+h)\cos\varphi \cos\lambda$

$Y = (N+h)\cos\varphi \operatorname{sen}\lambda$

$Z = [N(1-e^2)+h]\operatorname{sen}\varphi$

MONGE: $\begin{cases} \varphi = 38°\,46'\,27'',568 \\ \lambda = -9°\,26'\,29'',356 \\ h = 554,5\text{ m} \end{cases}$

SERVES: $\begin{cases} \varphi = 38°\,53'\,39'',251 \\ \lambda = -9°\,5'\,27'',667 \\ h = 405,53\text{ m} \end{cases}$

ETRS89

$a = 6378137,0\text{ m}$

$e^2 = 0,00669438$

Monge: $N = \dfrac{a}{\sqrt{1-e^2\operatorname{sen}^2\varphi}} = 6386526,42\text{ m}$

Serves: $N = 6386540,246\text{ m}$

Monge:

$X = 4912033,676\text{ m}$

$Y = -816836,615\text{ m}$

$Z = 3973163,448\text{ m}$

Serves:

$X = 4908580,492\text{ m}$

$Y = -785438,047\text{ m}$

$Z = 3983440,137\text{ m}$

$\vec{\Delta r}^{G}_{M-S} = \begin{bmatrix} \Delta X = -3453,185\text{ m} \\ \Delta Y = 31398,57\text{ m} \\ \Delta Z = 10276,69\text{ m} \end{bmatrix}$

$\vec{\Delta r}^{GL} = P_2 \cdot R_2(\varphi - \tfrac{\pi}{2}) \cdot R_3(\lambda - \pi) \cdot \vec{\Delta r}^{G}$

$\varphi_M - \tfrac{\pi}{2} = -0,8940567 \text{ rad}$

$\lambda_M - \pi = 2,976807606$

$P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2(\theta) = \begin{bmatrix} \cos\theta & 0 & -\operatorname{sen}\theta \\ 0 & 1 & 0 \\ \operatorname{sen}\theta & 0 & \cos\theta \end{bmatrix}$

$R_3(\theta) = \begin{bmatrix} \cos\theta & -\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2(\varphi_M - \tfrac{\pi}{2}) = \begin{bmatrix} 0,626254509 & 0 & 0,7796187 \\ 0 & 1 & 0 \\ -0,779618682 & 0 & 0,6262545 \end{bmatrix}$

$R_3(\lambda_M - \pi) = \begin{bmatrix} -0,98645364 & -0,16404029 & 0 \\ 0,16404029 & -0,98645364 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\vec{\Delta r}^{GL}_{M-S} = \begin{bmatrix} \Delta X = 6919,571\text{ m} \\ \Delta Y = 31539,693\text{ m} \\ \Delta Z = 7795,653\text{ m} \end{bmatrix}$

distância espacial $= \|\Delta r\| = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} = 33217,539\text{ m}$

Azimute $= \alpha = 2\operatorname{arctg}\left[\dfrac{\Delta Y}{\Delta X + \sqrt{\Delta X^2 + \Delta Y^2}}\right] = 77,°6257881$

dist. zenital $= Z = \operatorname{arcsen}\left(\dfrac{\Delta z}{|\Delta r|}\right) - \tfrac{\pi}{2} = 76,°42695482$

12. b) Vértice VALE DE ÁGUA

$\varphi = 39°21'58",511$ *

$\lambda = -8°0'44",335$

$h = 294,05$ m

Datum 73

$a = 6378388,0$ m

$e^2 = 0,00672267$

$N = 6387030,916$ m

Dist. = S = 2194,568 m

Az = α = 124° 23' 51",56

Dist. Zenital = Z = 92° 12' 24"

$$\vec{\Delta r}^{GL} = S \cdot \begin{bmatrix} \operatorname{sen} Z \cdot \cos Az \\ \operatorname{sen} Z \cdot \operatorname{sen} Az \\ \cos Z \end{bmatrix} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

$$\vec{\Delta r}^{GL} = \begin{bmatrix} \Delta x = -1238,865\text{ m} \\ \Delta y = 1809,478\text{ m} \\ \Delta z = -84,500\text{ m} \end{bmatrix}$$

$\pi - \lambda_{VA} = -3°,0017514$

$\frac{\pi}{2} - \varphi_{VA} = 0°,883725597$

$$\vec{\Delta r}^{G} = R_3(\pi - \lambda) \cdot R_2(\tfrac{\pi}{2} - \varphi) \cdot P_2 \cdot \vec{\Delta r}^{GL}$$

$$R_3 = \begin{bmatrix} -0,990238 & 0,139386 & 0 \\ -0,139386 & -0,990238 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 = \begin{bmatrix} 0,63425267 & 0 & -0,773107 \\ 0 & 1 & 0 \\ +0,773107 & 0 & 0,63425267 \end{bmatrix} \quad P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{\Delta r}^{G} = \begin{bmatrix} \Delta X = 461,206\text{ m} \\ \Delta Y = 1892,233\text{ m} \\ \Delta Z = -1011,372\text{ m} \end{bmatrix}$$

$X_{VA} = 4889882,553$ m

$Y_{VA} = -688300,01$ m

$Z_{VA} = 4024087,798$ m

$X_{3E} = X_{VA} + \Delta X = 4890343,758$ m

$Y_{3E} = Y_{VA} + \Delta Y = -686407,777$ m

$Z_{3E} = Z_{VA} + \Delta Z = 4023076,426$ m

Parâmetros transf. Helmert de Datum 73 – ETRS89

$\Delta X = -230,994$ m

$\Delta Y = 102,591$ m

$\Delta Z = 25,199$ m

$d\alpha = 1,95$ ppm

$\theta_x = -0",633$

$\theta_y = 0",239$

$\theta_z = -0",900$

$\alpha = 1 + d\alpha = 1,00000195$

$\theta_x = -1,5344 \times 10^{-7}$ rad

$\theta_y = 5,79352 \times 10^{-8}$ rad

$\theta_z = -2,1817 \times 10^{-7}$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ETRS89} = \begin{bmatrix} \Delta X + \alpha X + \alpha\theta_z Y - \alpha\theta_y Z \\ \Delta Y - \alpha\theta_z X + \alpha Y + \alpha\theta_x Z \\ \Delta Z + \alpha\theta_y X - \alpha\theta_x Y + \alpha Z \end{bmatrix} = \begin{bmatrix} 4890122,217\text{ m} \\ -686306,075\text{ m} \\ 4023109,648\text{ m} \end{bmatrix}$$

$\lambda = \operatorname{arctg} \frac{Y}{X} = \underline{-7°0'39",535}$

$\varphi_0 = \frac{1}{1-e^2} \cdot \frac{Z}{\sqrt{X^2+Y^2}} = 39°,3588603$

$\varphi_1 = \frac{Z + e^2 N \operatorname{sen}\varphi_0}{\sqrt{X^2+Y^2}} = \underline{39°21'31",912}$

$N_0 = 6386740,421$

$N_1 = 6386740,42$ m

$h = \frac{\sqrt{X^2+Y^2}}{\cos\varphi} - N_1 = \underline{-143,073\text{ m}}$