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Tags : Logic

First Order Logic

Motivation

Consider the typical structures in Maths and CS. Groups, Rings, Monoids etc etc. All of them are sets, equipped with *functions* and *relations* on them and sometimes they have *special promoted terms*, like units in groups.

First Order Logic gives a natural frameword to talk about these thingies.
The idea is to fix symbols to denote functions, relations and constants and combine the with the standard $\neg$ and $\vee$ operator. In addtion to all that we have operators to quantify over all elements $\exists$ and $\forall$, and a $\equiv$ operator to check equality of primitive constructs.

The syntax and semantics are a bit more involved for first order logic, than in *Propositional Logic*.

Groups are a really good example to get introduced to first order logic

A Group is a structure $(G, +, 0)$ where G is a set, $+$ is a binary operation on it, a special element 0 and the following properties hold:

- $+$ is associative
- 0 is the right identity for the operation $+$
- For every element in G, it has a right inverse, such applying the operator on those two elements will yield 0

To formalize this we have the following statements in **First Order Logic** where op represents the operator and ϵ represents the identity.

- $\forall x\, \forall y\, \forall z\; op(op(x, y), z) \equiv op(x, op(y, z))$
- $\forall x\; op(x, \epsilon) \equiv x$
- $\forall x \exists y\; op(x, y) \equiv \epsilon$

To give this meaning, we fix a set S which is the set we are working with, make op a binary relation on the set, $\equiv$ equivalence on it and ϵ an element from S which is treated in a special way. so S is the same as the set G, op represents $+$ and ϵ is the identity here.

Goal

The goal of first order logic is to capture the properties of mathematical structures.

Todo

Revisit Groups after finishing First Order Logic Syntax and Semantics

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References

Isomorphism Between First Order Interpretations