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Type : #Note
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Church Booleans

By convention, the defintions of `TRUE` and `FALSE` used (also called **Church Booleans**) is

$$\text{TRUE} := \lambda xy.\, x$$
$$\text{FALSE} := \lambda xy.\, y$$

and with these two defintions, we can definte the following logical opeators easily

$$\text{AND} := \lambda ab.\, aba$$
$$\text{OR} := \lambda ab.\, aab$$
$$\text{NOT} := \lambda p.\, p\,(\text{FALSE})\,(\text{TRUE})$$

This can now be used in defining *predicates*, which are functions that return a boolean value. For example the ISZERO predicate, which returns TRUE if an input number is then Church Numerals 0 and returns FALSE otherwise, can be written as

$$\text{ISZERO} := \lambda n.\, n\,(\lambda x.\, \text{FALSE}\,)\ \text{TRUE}$$

and the *predicate* LEQ

$$\text{LEQ} := \lambda mn.\, \text{ISZERO}\,(\,\text{SUB}\ m\ n)$$

where SUB is defined in Church Numerals

References

Church Numerals