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Pairs in Lambda Calculus

Pair are a collection of two values considered together.
To construct a useful definition of pairs, we require a method to pull out components of a pair to use them separately.
A good candidate for a choosing function would be somewhat similar to the Church Booleans as $TRUE$ takes in two inputs and returns the first one and $FALSE$ takes in two inputs and returns the second one.
This idea is actually enough to provide a working description of pairs as

$$PAIR := \lambda ab.\,(\lambda v.\,vab)$$

Where v is supposed to be a **Church Boolean**.

The pair function take in two arguments are returns a function which in a sense stores the two inputs, it wait for a selector function when it will later return one of the values

So while using the booleans as selector functions, we can use the following aliases

$$FIRST := TRUE, \quad SECOND := FALSE$$

infix notation

we can write $PAIR(a,b)$ as (a,b)
and we can write the first and second function in following way
$(a,b).1 = a$
$(a,b).2 = b$

References

Church Numerals