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Tags : Logic

Syntax of First Order Logic

Syntax

First Order Languages

To define the formulas of First Order Logic we first fix the underlying Language. A **First Order Language** is a triple $L = (R, F, C)$ where

- R is a countable set of *Relation Symbols*
- F is a countable set of *Function Symbols*
- C is a countable set of *Constants*
  We also have a countable set Var of variables.

Terms

We now build terms, that would then be connected to give formulas.
The set of **Terms** over a **First Order Language** L is the smallest set satisfying the following condition.

- Every constant $c \in C$ is a term
- Every variable $x \in Var$ is a term
- Let $t_1, t_2 \ldots, t_n$ be terms over L and let $f \in F$ be a function symbol of arity n. Then $f(t_1, t_2 \ldots t_n)$ is a term.
  A term which does not contain any variables is called *closed*.

Atomic Formulas

The atomic formulas of L are defined as follows.

- Let $r \in R$ be a relation symbol and $t_1 \ldots t_n$ are terms over L then $r(t_1, \ldots t_n)$ is an atomic formula
- Let t_1 and t_2 be terms, then $t_1 \equiv t_2$ is an atomic formulas

Formulas

Given the above set of atomic formulas, we can define the complete set of formulas Φ_L is the following way.

- Every atomic formula over L belongs to Φ_L
- If $\varphi \in \Phi_L$ then $\neg\varphi \in \Phi_L$
- If $\varphi, \psi \in \Phi_L$ then $\varphi \vee \psi \in \Phi_L$
- If $\varphi \in \Phi_L$ and $x \in Vars$, then $\exists x\, \varphi \in \Phi_L$
  And we use parentheses to disambiguate the formula.

> **Info**
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> The *Terms* and things related to it all, live inside the universe, Hence symbols from F are only used in terms. The *Formulas* are to be interpreted as true of false statements.

References

First Order Logic
Semantics of First Order Logic