



# Motion Control of a Hexapod Robot Over Uneven Terrain Using Signed Distance Fields

by

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*Thesis presented in partial fulfilment of the requirements for  
the degree of Master of Engineering (Electronic) in the  
Faculty of Engineering at Stellenbosch University*

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March 2023

# Declaration

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# Abstract

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Thesis: MEng (EE)

March 2023

In recent times great strides have been made in the field of autonomous robotics, especially with regards to autonomous navigation of wheeled and arial drones. Legged robotics however still face numerous problems before they can become practical to use, the most egregious of these problems being balancing of the robot and optimal foot placement.

This thesis focuses on providing a solution to the latter problem of foot placement. This is achieved by using an depth camera to, in real time, construct a localised map of the environment and subsequently analysing said map for optimal foot placement locations. The system is then tested using a hexapod robot both in simulation and on a physical robot.

# Acknowledgments

# Dedication

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# List of symbols

## Constants

$L_0 = 300 \text{ mm}$

## Variables

$Re_D$	Reynolds number (diameter) . . . . .	[ ]
$x$	Coordinate . . . . .	[ m ]
$\ddot{x}$	Acceleration . . . . .	[ m/s <sup>2</sup> ]
$\theta$	Rotation angle . . . . .	[ rad ]
$\tau$	Moment . . . . .	[ N·m ]

## Vectors and Tensors

$\vec{v}$  Physical vector, see equation ...

## Subscripts

$a$	Adiabatic
$a$	Coordinate

## Abbreviations

<b>IK</b>	Inverse Kinematics . . . . .	v
<b>SDF</b>	Signed Distance Field . . . . .	1
<b>MuJoCo</b>	Multi-Joint dynamics with Contact . . . . .	1

# Chapter 1

## Introduction

### 1.1 Background and Motivation

Starting from the big picture, gradually narrow focus down to this project and where this report fits in. Why this specific project/report is worthwhile.

### 1.2 Project Goal

Broken down into sub objectives

The objectives of the project (in some cases the objectives of the report). If necessary describe limitations to the scope.

### 1.3 Methodology

When deciding how to determine optimal foot placement various sensing methods were considered, such as using a depth camera to view the environment, placing force sensors on the robots feet or measuring servo torque to determine when the feet were in contact with a surface. A previous paper by ..... used a depth camera by storing past snapshots to adjust the feet to the optimal height, it was decided that the primary sensing method for this thesis would also be a depth camera but instead of storing snapshots, a Signed Distance Field (SDF) would be generated of the local environment. This would allow for more advanced methods of placement selection and preliminary collision checking for leg movements.

The first step in realising this system was to construct a accurate simulation of the hexapod. The primary simulation packages that were considered are Gazebo, PyBullet and Multi-Joint dynamics with Contact (MuJoCo). Gazebo was a appealing choice due to the easy integration with ROS, however it was

decided to use MuJoCo since it was found to have a far superior contact physics simulation.

Once the hexapod was adequately modelled in MuJoCo a tripod gait state machine, IK system and control interface was implement, at this stage the hexapod was capable of walking on flat terrain.

Next the the system to generate the SDF was implemented, this entailed sampling the depth camera, comparing cells in the SDF against the depth buffer and finally calculating the closest distance value of each SDF cell. Additionally a system to render the SDF using ray marching was also implemented, this was required because SDFs are otherwise difficult to visualise, more on this in chapter ....

Once the SDF was implemented it was possible to build the system responsible for foot placement this is discussed in detail in chapter .... , after which collision checking for the generated foot motion was implement, ensuring that the hexapod does not get stuck on pieces of terrain.

With this the system was realise in simulation, next the system was implemented and tested on the physical robot, discussed in detail in chapter ....

## 1.4 Thesis Outline

# Chapter 2

## Literature review

### 2.1 Discrete element method

The Discrete Element Method (DEM) analysis (Cundall and Strack, 1979) uses spherical objects. Lin and Ng (1997) developed a DEM model for ellipsoids.

# Chapter 3

## Content chapter

Unless the chapter heading already makes it clear, an introductory paragraph that explains how this chapter contributes to the objectives of the report/project.

### 3.1 Heading level 2

#### 3.1.1 Heading level 3

##### 3.1.1.1 Deepest heading, only if you cannot do without it

**Equations:** An equation must read like part of the text. The solution of the quadratic equation  $ax^2 + bx + c = 0$  given by the following expression (note the full stop after the equation to indicate the end of the sentence):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2b}. \quad (3.1)$$

In other cases the equation is in the middle of the sentence. Then the paragraph following the equation should start with a small letter. Euler's identity is

$$e^{i\pi} + 1 = 0, \quad (3.2)$$

where  $e$  is Euler's number, the base of natural logarithms.

The `amsmath` has a wealth of structure and information on formatting of mathematical equations.

**Symbols and numbers:** Symbols that represent values of properties should be printed in italics, but SI units and names of functions (e.g.  $\sin$ ,  $\cos$  and  $\tan$ ) must not be printed in italics. There must be a small hard space between a number and its unit, e.g. 120 km. Use the `siunitx` package to typeset numbers, angles and quantities with units:

$$\begin{aligned}\backslash\mathrm{num}\{1.23\mathrm{e}3\} &\rightarrow 1.23\times 10^3 \\ \backslash\mathrm{ang}\{30\} &\rightarrow 30^\circ \\ \backslash\mathrm{qty}\{20\}\{\mathrm{N.m}\} &\rightarrow 20\,\mathrm{N.m}\end{aligned}$$

**Figures and tables:** The `graphicx` package can import PDF, PNG and JPG graphic files.

Table 3.1: Standard ISO paper sizes

Paper	Sizes	
	$W$	$H$
	[mm]	[mm]
A0	841	1189
A1	594	841
A2	420	594
A3	297	420
A4	210	297
A5	148	210

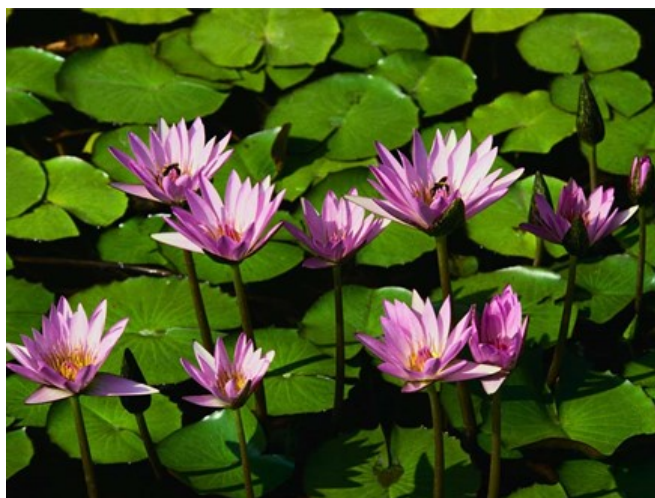


Figure 3.1: Water plants

# Chapter 4

## Modelling

This chapter covers the simulation environment and modelling of the hexapod in MuJoCo.

### 4.1 Hexapod Construction

### 4.2 Servo Modelling

### 4.3 Simulation Terrain



# Chapter 5

## Mapping

This chapter covers what a SDF is and how it is implemented.

### 5.1 Overview

### 5.2 Implementation

### 5.3 Generation

#### 5.3.1 Depth Step

#### 5.3.2 Distance Step

# Chapter 6

## Motion

This chapter describes the systems governing the motion of the robot, such as leg motion planning and gait generation.

### 6.1 Gait State Machine

### 6.2 Inverse Kinematics (IK)

### 6.3 Foot Motion

### 6.4 Foot Placement

# Chapter 7

## Physical Implementation

This chapter describes the process of implementing the system built in previous chapters on the physical robot.

# Chapter 8

## Testing

This chapter covers all tests performed to validate performance of the system.

## Chapter 9

## Conclusions

# Appendix A

## Mathematical proofs

### A.1 Euler's equation

Euler's equation gives the relationship between the trigonometric functions and the complex exponential function.

$$e^{i\theta} = \cos \theta + i \sin \theta \quad (\text{A.1})$$

Inserting  $\theta = \pi$  in (A.1) results in Euler's identity

$$e^{i\pi} + 1 = 0 \quad (\text{A.2})$$

### A.2 Navier Stokes equation

The Navier–Stokes equations mathematically express momentum balance and conservation of mass for Newtonian fluids. Navier-Stokes equations using tensor notation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} [\rho u_j] = 0 \quad (\text{A.3a})$$

$$\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} [\rho u_i u_j + p \delta_{ij} - \tau_{ji}] = 0, \quad i = 1, 2, 3 \quad (\text{A.3b})$$

$$\frac{\partial}{\partial t} (\rho e_0) + \frac{\partial}{\partial x_j} [\rho u_j e_0 + u_j p + q_j - u_i \tau_{ij}] = 0 \quad (\text{A.3c})$$

## Appendix B

### Experimental results

# List of references

- Cundall, P.A. and Strack, O.D.L. (1979). A discrete numerical model for granular assemblies. *Géotechnique*, vol. 29, no. 1, pp. 47–65.
- Lin, X. and Ng, T.T. (1997). A three-dimensional discrete element model using arrays of ellipsoids. *Géotechnique*, vol. 47, no. 2, pp. 319–329.