Gegewensblad / Formula Sheet

Tydgebied ontwerp / Time domain design:

$$t_r \approx \frac{1.8}{\omega_n}, \quad t_p = \frac{\pi}{\omega_d}, \quad 1\% t_s = \frac{4.6}{\zeta\omega_n}, \quad 2\% t_s = \frac{4}{\zeta\omega_n}, \quad s = -\sigma \pm j\omega_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$e^{j\omega T} = \cos\omega T + j\sin\omega T, \qquad z = e^{sT}, \qquad G_{ho}G(z) = \frac{z-1}{z} Z\left\{\frac{G(s)}{s}\right\}$$

Z-transform:

$$F(z) = \sum_{k=-\infty}^{\infty} f(k) z^{-k}, \quad f(k+1) = zF(z) - z f(0), \quad f(k-1) = z^{-1}F(z), \quad \lim_{k\to\infty} f(k) = \lim_{z\to 1}(z-1)F(z)$$

$$e^{-at} \Leftrightarrow \frac{1}{s+a}$$

$$t\, e^{-at} \Leftrightarrow \frac{1}{(s+a)^2}$$

$F(s)$	$f(kT)$	$F(z)$
$\frac{1}{s}$	$1(kT)$	$\frac{z}{z-1}$
$\frac{1}{s+a}$	e^{-akT}	$\frac{z}{z-e^{-aT}}$
$\frac{a}{s(s+a)}$	$1-e^{-akT}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$\frac{1}{s^2}$	kT	$\frac{Tz}{(z-1)^2}$

Diskrete toestandsveranderlike model / Discrete state space model:

$$\mathbf{x}(k+1) = e^{\mathbf{A}T}\mathbf{x}(k) + \int_0^T e^{\mathbf{A}\theta}\mathbf{b}\, d\theta . u(k) \qquad \mathbf{F} = e^{\mathbf{A}T} = \mathbf{I} + \mathbf{A}T + (\mathbf{A}T)^2/2! + (\mathbf{A}T)^3/3! + \ldots.$$

$$\mathbf{x}(k+1) = \mathbf{F}\,\mathbf{x}(k) + \mathbf{g}\,u(k) \qquad \mathbf{g} = \int_0^T e^{\mathbf{A}\theta}\mathbf{b}\, d\theta = T\,\mathbf{\Psi}\,\mathbf{b}$$

$$\mathbf{\Phi}(t) = \ell^{-1}\left\{[s\mathbf{I} - \mathbf{A}]^{-1}\right\}$$

$$\mathbf{\Psi} = \mathbf{I} + \mathbf{A}T/2! + (\mathbf{A}T)^2/3! + \ldots.$$

TV na ODF (SS to TF): $G(z) = \dfrac{Y(z)}{U(z)} = \mathbf{c}(z\mathbf{I} - \mathbf{F})^{-1}\mathbf{g} + d = \dfrac{\mathbf{c}(z\mathbf{I} - \mathbf{F})^{+}\mathbf{g}}{|z\mathbf{I} - \mathbf{F}|} + d$

Beheerbaarheid / Controllability: $\mathbf{U} = \begin{bmatrix} \mathbf{g} & \mathbf{Fg} & \mathbf{F}^2\mathbf{g} & \ldots & \mathbf{F}^{n-1}\mathbf{g} \end{bmatrix}$

Waarneembaarheid / Observability: $\mathbf{V} = \begin{bmatrix} \mathbf{c} \\ \mathbf{cF} \\ \mathbf{cF}^2 \\ \vdots \\ \mathbf{cF}^{n-1} \end{bmatrix}$

Geslotelus KV / Closed loop CE: $\Delta_{cl}(z) = \left| z\mathbf{I} - \mathbf{F} + \mathbf{g}\,\mathbf{k} \right| = z^n + a_1 z^{n-1} + \ldots + a_{n-1}z + a_n = 0$

Waarnemer KV / Estimator CE: $\Delta_{est}(z) = \left| z\mathbf{I} - \mathbf{F} + \mathbf{m}_p\,\mathbf{c} \right| = z^n + a_1 z^{n-1} + \ldots + a_{n-1}z + a_n = 0$

Beheer Kanoniese Vorm / Control Canonical Form: $\mathbf{U}_c^{-1} = \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & 1 \\ \alpha_2 & \alpha_1 & 1 & 0 \\ \alpha_1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \mathbf{P} = \mathbf{U}\mathbf{U}_\mathbf{c}^{-1}$

Voorspellende waarnemer / Prediction observer:

$$\hat{\mathbf{x}}(k+1) = \mathbf{F}\,\hat{\mathbf{x}}(k) + \mathbf{g}\,u(k) + \mathbf{m}_p\left(y(k) - \mathbf{c}\,\hat{\mathbf{x}}(k)\right)$$

Huidige waarnemer / Current observer:

$$\bar{\mathbf{x}}(k+1) = \mathbf{F}\,\hat{\mathbf{x}}(k) + \mathbf{g}\,u(k)$$

$$\hat{\mathbf{x}}(k+1) = \bar{\mathbf{x}}(k+1) + \mathbf{m}_c\left(y(k+1) - \mathbf{c}\,\bar{\mathbf{x}}(k+1)\right) \text{ met/with } \mathbf{m}_p = \mathbf{F}\mathbf{m}_c \quad \text{or} \quad \mathbf{m}_c = \mathbf{F}^{-1}\mathbf{m}_p$$

Verwysingsintree / Reference input:

$$\begin{bmatrix} \mathbf{N}_x \\ N_u \end{bmatrix} = \begin{bmatrix} \mathbf{F}-\mathbf{I} & \mathbf{g} \\ \mathbf{c} & 0 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}, \qquad \overline{N} = \mathbf{k}\,\mathbf{N}_x + N_u$$

$$\overline{N} = 1/\mathbf{c}(\mathbf{I} - \mathbf{F} + \mathbf{gk})^{-\mathbf{1}}\mathbf{g}$$

Integrasiebeheer / Integral control:

$$u(k) = -\mathbf{k}_p \mathbf{x}(k) - k_i v(k) + \overline{N}\, r(k), \qquad z_i = \frac{\overline{N}}{k_i + \overline{N}} \quad \text{or} \quad \overline{N} = \frac{k_i z_i}{1 - z_i}$$

$$\begin{bmatrix} \mathbf{x}(k+1) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{F} & \mathbf{0} \\ \mathbf{cF} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ v(k) \end{bmatrix} + \begin{bmatrix} \mathbf{g} \\ \mathbf{cg} \end{bmatrix} u(k) + \begin{bmatrix} \mathbf{0} \\ -1 \end{bmatrix} r(k)$$

Optimale LQR beheer / Optimal LQR control:

$$J = \frac{1}{2} \sum_{k=0}^{N} \left[\mathbf{x}(k)^T \mathbf{Q}_1 \mathbf{x}(k) + \mathbf{u}(k)^T \mathbf{Q}_2 \mathbf{u}(k) \right]$$

$$\mathbf{u}(k) = -\mathbf{K}(k)\mathbf{x}(k), \qquad \mathbf{K}(k) = \left[\mathbf{Q}_2 + \mathbf{G}^T \mathbf{S}(k+1)\mathbf{G} \right]^{-1} \mathbf{G}^T \mathbf{S}(k+1)\mathbf{F}$$

$$\mathbf{S}(k) = \mathbf{F}^T \mathbf{M}(k+1)\mathbf{F} + \mathbf{Q}_1$$

$$\mathbf{M}(k+1) = \mathbf{S}(k+1) - \mathbf{S}(k+1)\mathbf{G}\left[\mathbf{Q}_2 + \mathbf{G}^T \mathbf{S}(k+1)\mathbf{G} \right]^{-1} \mathbf{G}^T \mathbf{S}(k+1)$$

Resiproke wortellokus / Reciprocal root locus:

$$J = \frac{1}{2} \sum_{k=0}^{N} \left[\mathbf{x}(k)^T \mathbf{C}^T \rho \mathbf{C}\, \mathbf{x}(k) + u(k)^2 \right] = \frac{1}{2} \sum_{k=0}^{N} \left[\rho\, y(k)^2 + u(k)^2 \right]$$

$$1 + \rho G_{ol}(z^{-1}) G_{ol}(z) = 0$$

Kleinste Kwadraat Afskatting / Least Squares Estimation:

$$J = \frac{1}{2} \sum_{k=0}^{N} e(k)^2 = \frac{1}{2} \mathbf{e}^T \mathbf{e} = \frac{1}{2} (\mathbf{y} - \mathbf{C}\hat{\mathbf{x}})^T (\mathbf{y} - \mathbf{C}\hat{\mathbf{x}})$$

$$\hat{\mathbf{x}} = \left[\mathbf{C}^T \mathbf{C} \right]^{-1} \mathbf{C}^T \mathbf{y} \qquad \text{of/or} \qquad \hat{\mathbf{x}} = \left[\mathbf{C}^T \mathbf{R}^{-1} \mathbf{C} \right]^{-1} \mathbf{C}^T \mathbf{R}^{-1} \mathbf{y}$$

Rekursiewe KK Afskatting / Recursive LS Estimation:

$$\hat{\mathbf{x}}_n = \hat{\mathbf{x}}_o + \mathbf{P}_n \mathbf{C}_n^T \mathbf{R}^{-1} (\mathbf{y}_n - \mathbf{C}_n \hat{\mathbf{x}}_o)$$

$$\mathbf{P}_n = \left[\mathbf{P}_o^{-1} + \mathbf{C}_n^T \mathbf{R}^{-1} \mathbf{C}_n \right]^{-1}$$

Optimale Afskatting / Optimal Estimation:

Ruiserige Toestandsveranderlike Model / Noisy State Space Model:

$$\mathbf{x}(k+1) = \mathbf{F}\,\mathbf{x}(k) + \mathbf{G}\,\mathbf{u}(k) + \mathbf{H}\,\mathbf{w}(k)$$

$$\mathbf{y}(k) = \mathbf{C}\,\mathbf{x}(k) + \mathbf{v}(k)$$

$$E\{\mathbf{w}(k)\} = E\{\mathbf{v}(k)\} = \mathbf{0} \qquad \text{and} \qquad E\{\mathbf{w}(i)\mathbf{w}(j)^T\} = E\{\mathbf{v}(i)\mathbf{v}(j)^T\} = \mathbf{0}$$

$$E\{\mathbf{w}(k)\mathbf{w}(k)^T\} = \mathbf{R}_w \qquad \text{and} \qquad E\{\mathbf{v}(k)\mathbf{v}(k)^T\} = \mathbf{R}_v$$

Meting opdatering / Measurement update:

$$\hat{\mathbf{x}}(k) = \bar{\mathbf{x}}(k) + \mathbf{P}(k)\mathbf{C}^T \mathbf{R}_v^{-1}\left(\mathbf{y}(k) - \mathbf{C}\,\bar{\mathbf{x}}(k)\right)$$

$$\mathbf{P}(k) = \left[\mathbf{M}(k)^{-1} + \mathbf{C}^T \mathbf{R}_v^{-1}\mathbf{C}\right]^{-1} = \mathbf{M}(k) - \mathbf{M}(k)\mathbf{C}^T\left[\mathbf{C}\,\mathbf{M}(k)\mathbf{C}^T + \mathbf{R}_v\right]^{-1}\mathbf{C}\,\mathbf{M}(k)$$

Voorspelling opdatering / Prediction update:

$$\bar{\mathbf{x}}(k+1) = \mathbf{F}\,\hat{\mathbf{x}}(k) + \mathbf{G}\,\mathbf{u}(k)$$

$$\mathbf{M}(k+1) = \mathbf{F}\,\mathbf{P}(k)\mathbf{F}^T + \mathbf{H}\,\mathbf{R}_w\mathbf{H}^T$$

Resiproke wortellokus / Reciprocal root locus:

$$1 + (R_w/R_v)G_e\left(z^{-1}\right)G_e(z) = 0 \quad \text{with} \quad G_e(z) = \mathbf{c}(z\mathbf{I} - \mathbf{F})^{-1}\mathbf{h}$$

<u>Standaard Integrale / Standard Integrals:</u>

$$\int \sin ax\,dx = -\frac{\cos ax}{a} \qquad \int \cos ax\,dx = \frac{\sin ax}{a}$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \sin^3 ax\,dx = -\frac{\cos ax}{a} + \frac{\cos^3 ax}{3a} \qquad \int \cos^3 ax\,dx = \frac{\sin ax}{a} - \frac{\sin^3 ax}{3a}$$

$$\int \sin^4 ax\,dx = \frac{3x}{8} - \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a} \qquad \int \cos^4 ax\,dx = \frac{3x}{8} + \frac{\sin 2ax}{4a} + \frac{\sin 4ax}{32a}$$

$$\int x\sin ax\,dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a} \qquad \int x\cos ax\,dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a}$$

$$\int \sin^n ax\cos ax\,dx = \frac{\sin^{n+1} ax}{(n+1)a} \qquad \int \cos^n ax\sin ax\,dx = -\frac{\cos^{n+1} ax}{(n+1)a}$$

$$\int \sin ax\cos ax\,dx = \frac{\sin^2 ax}{2a} \qquad \int \frac{x\,dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b)$$

<u>Beskrywingsfunksies / Describing functions:</u>

$$A_1 = \frac{1}{\pi}\int_0^{2\pi} y(t)\cos\omega t\,d\omega t \qquad x(t) = X\sin\omega t$$

$$B_1 = \frac{1}{\pi}\int_0^{2\pi} y(t)\sin\omega t\,d\omega t \qquad N(X) = \frac{B_1 + jA_1}{X}$$

$$1 + N(X)G(j\omega) = 0$$

$$\sin 2\theta = 2\sin\theta\cos\theta\,, \qquad \cos 2\theta = \cos\theta^2 - \sin\theta^2$$

$$\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta\,, \qquad \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

<u>Krasovskii metode / Krasovskii method:</u>

$$V(\boldsymbol{x}) = \boldsymbol{f}^T(\boldsymbol{x})\boldsymbol{P}\boldsymbol{f}(\boldsymbol{x})$$

$$\dot{V}(\boldsymbol{x}) = -\boldsymbol{f}^T(\boldsymbol{x})\boldsymbol{Q}\boldsymbol{f}(\boldsymbol{x})$$

$$\boldsymbol{Q} = -[\boldsymbol{J}^T(\boldsymbol{x})\boldsymbol{P} + \boldsymbol{P}\boldsymbol{J}(\boldsymbol{x})]$$

$$\boldsymbol{J}(\boldsymbol{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$