Jason Abaluck
optimal Representation
Ephraim Sutherland

Setup

1. Suppose a physician can only see see ATE and some measure of representativeness. They have prior $\bar{\beta}$ and $\beta_{ATE} = (1/N)\sum \beta_i$.

2. need model for betas related to each other based on x's. WLOG, suppose

$$\beta(x_i) = x_i\gamma$$

   Where x_i is a vector of characteristics and γ is a vector of coefficients.
   If you know γ, then you know β for any given patient.

3. However, you don't observe γ, you instead observe: $\beta_{ATE} = \bar{x}\gamma$ where $\bar{x} = (\frac{1}{N})\sum x_i$

4. We know β_i for patients with characteristics $\bar{x}$ (it is β_{ATE}).

5. For other patients, need to solve

$$\beta_{i,post} = \mathrm{E}(x_i\gamma|\bar{x}\gamma = \beta_{ATE})$$

6. to solve

   (a)

$$\beta_{i,post} = \mathrm{E}(x_i\gamma|\bar{x}\gamma = \beta_{ATE})$$

$$= \mathrm{E}((x_i - c_i\bar{x})\gamma|\bar{x}\gamma = \beta_{ATE}) + c_i\mathrm{E}(\bar{x}\gamma|\bar{x}\gamma = \beta_{ATE})$$

$$= \mathrm{E}((x_i - c_i\bar{x})\gamma|\bar{x}\gamma = \beta_{ATE}) + c_i\beta_{ATE}$$

   For any constant c_i.
   Choose c_i so that

$$\mathrm{Cov}((x_i - c_i\bar{x})\gamma, \bar{x}\gamma) = 0$$

   maybe assume normality so that this guarantees independence. Then,

$$\mathrm{E}((x_i - c_i\bar{x})\gamma|\bar{x}\gamma = \beta_{ATE}) = (x_i - c_i\bar{x})\mathrm{E}(\gamma)$$

   So then

$$(x_i - c_i\bar{x})\mathrm{E}(\gamma) + c_i\beta_{ATE} = x_i\mathrm{E}(\gamma) + c_i(\beta_{ATE} - \bar{x}\mathrm{E}(\gamma))$$

   (c_i depends on x_i)

In other words, your belief is your prior, adjusted based on the difference between the observed ATE and your prior about the ATE. The key question is how much adjustment you do which depends on "c_i". We choose c_i to solve:

$$
\begin{aligned}
& \mathrm{Cov}((x_i - c_i\bar{x})\gamma, \bar{x}\gamma) = 0 \\
\iff & \mathrm{Cov}(x_i\gamma, \bar{x}\gamma) - c_i\mathrm{Cov}(\bar{x}\gamma, \bar{x}\gamma) = 0 \\
\iff & \mathrm{Cov}(x_i\gamma, \bar{x}\gamma) = c_i\mathrm{Var}(\bar{x}\gamma) \\
\iff & c_i = \frac{\mathrm{Cov}(\beta_i, \beta_{ATE})}{\mathrm{Var}(\beta_{ATE})}
\end{aligned}
$$

The random variable in this context is γ (the coefficients on the x's) in this case $\mathrm{Var}(\beta_{ATE})$ is a measure of how uncertain one was about what β_{ATE} would be before doing the trial.

c_i is the equation for a regression of β_i on β_{ATE}. In other words, we take a bunch of patients with characteristics x_i and we keep redrawing the gammas from our prior distribution the we ask how correlated β_i and β_{ATE} are. If they are more correlated (as they would be for patients where the x_i are closer to $\bar{x}$ we update more.

To compute c_i, we just need to know x_i $\bar{x}$, and the distribution of γ.

Suppose we want to design the trial to minimize:

$$\min \mathrm{E}[(\beta_i - \beta_{i,post})^2]$$

Simple Cases

1. There is just one x and it is binary (old v young). Can it be solved analytically?
2. Can you solve a 2-dimensional case?