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optimal Representation

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Setup

1. Suppose a physician can only see see ATE and some measure of representativeness. They have prior $\bar{\beta}$ and $\beta_{ATE} = (1/N)\sum \beta_i$.

2. need model for betas related to each other based on x's. WLOG, suppose

$$\beta(x_i) = x_i\gamma$$

   Where x_i is a vector of characteristics and γ is a vector of coefficients.
   If you know γ, then you know β for any given patient.

3. However, you don't observe γ, you instead observe: $\beta_{ATE} = \bar{x}\gamma$ where $\bar{x} = (\frac{1}{N})\sum x_i$

4. We know β_i for patients with characteristics $\bar{x}$ (it is β_{ATE}).

5. For other patients, need to solve

$$\beta_{i,post} = \mathrm{E}(x_i\gamma|\bar{x}\gamma = \beta_{ATE})$$

6. to solve

   (a)

$$\begin{aligned}
\beta_{i,post} &= \mathrm{E}(x_i\gamma|\bar{x}\gamma = \beta_{ATE}) \\
&= \mathrm{E}((x_i - c_i\bar{x})\gamma|\bar{x}\gamma = \beta_{ATE}) + c_i\mathrm{E}(\bar{x}\gamma|\bar{x}\gamma = \beta_{ATE}) \\
&= \mathrm{E}((x_i - c_i\bar{x})\gamma|\bar{x}\gamma = \beta_{ATE}) + c_i\beta_{ATE}
\end{aligned}$$

   For any constant c_i.
   Choose c_i so that

$$\mathrm{Cov}((x_i - c_i\bar{x})\gamma, \bar{x}\gamma) = 0$$

   maybe assume normality so that this guarantees independence. Then,

$$\mathrm{E}((x_i - c_i\bar{x})\gamma|\bar{x}\gamma = \beta_{ATE}) = (x_i - c_i\bar{x})\mathrm{E}(\gamma)$$

So then

$$(x_i - c_i\bar{x})\mathrm{E}(\gamma) + c_i\beta_{ATE} = x_i\mathrm{E}(\gamma) + c_i(\beta_{ATE} - \bar{x}\mathrm{E}(\gamma))$$

(c_i depends on x_i)
In other words, your belief is your prior, adjusted based on the difference between the observed ATE and your prior about the ATE. The key question is how much adjustment you do which depends on "c_i". We choose c_i to solve:

$$\mathrm{Cov}((x_i - c_i\bar{x})\gamma, \bar{x}\gamma) = 0$$

$$\iff \mathrm{Cov}(x_i\gamma, \bar{x}\gamma) - c_i\mathrm{Cov}(\bar{x}\gamma, \bar{x}\gamma) = 0$$

$$\iff \mathrm{Cov}(x_i\gamma, \bar{x}\gamma) = c_i\mathrm{Var}(\bar{x}\gamma)$$

$$\iff c_i = \frac{\mathrm{Cov}(\beta_i, \beta_{ATE})}{\mathrm{Var}(\beta_{ATE})}$$

The random variable in this context is γ (the coefficients on the x's) in this case $\mathrm{Var}(\beta_{ATE})$ is a measure of how uncertain one was about what β_{ATE} would be before doing the trial.
c_i is the equation for a regression of β_i on β_{ATE}. In other words, we take a bunch of patients with characteristics x_i and we keep redrawing the gammas from our prior distribution the we ask how correlated β_i and β_{ATE} are. If they are more correlated (as they would be for patients where the x_i are closer to $\bar{x}$ we update more.
To compute c_i, we just need to know x_i $\bar{x}$, and the distribution of γ.
Suppose we want to design the trial to minimize:

$$\min \mathrm{E}[(\beta_i - \beta_{i,post})^2]$$

Simple Cases

1. There is just one x and it is binary (old v young). Can it be solved analytically?
2. Can you solve a 2-dimensional case?

First observe that in our current setup, c does not depend on the γ's
We have that

$$c = \frac{\mathrm{Cov}(x\gamma, \bar{x}\gamma)}{\mathrm{Var}(\bar{x}\gamma)}$$

so for individual i, c reduces to

$$c = \frac{1^2\mathrm{Var}(\gamma_0) + x\bar{x}\mathrm{Var}(\gamma_1)}{1^2\mathrm{Var}(\gamma_0) + \bar{x}^2\mathrm{Var}(\gamma_1)} \quad \text{because } \mathrm{Var}(\gamma_0) = \mathrm{Var}(\gamma_1) = \mathrm{Var}(\gamma) = 1$$

$$= \frac{(1 + x\bar{x})\mathrm{Var}(\gamma)}{(1 + \bar{x}^2)\mathrm{Var}(\gamma)} = \frac{\vec{x}'\vec{\bar{x}}}{\vec{\bar{x}}'\vec{\bar{x}}}$$

1 Appendix

Derivations

Recall we want to minimize

$$\min \mathrm{E}_x \left(\mathrm{E}_{\gamma 0,1}[(\beta_i - \beta_{i,post})^2]\right)$$

One way we can rewrite these equations is as the effect of women vs men. let the squared error (SE) be

$$\sqrt{SE} = \beta_i - \beta_{i,post} = \underbrace{[(1-x)\gamma_0 + x\gamma_1]}_{\beta_i} - \underbrace{[((\vec{x} - c\vec{\bar{x}})\mathrm{E}(\gamma) + c\beta_{ATE})]}_{\beta_{i,post}}$$

$$= [(1-x)\gamma_0 + x\gamma_1] - [((1-x) - c(1-\bar{x}))\mathrm{E}(\gamma_0) + (x - c\bar{x})\mathrm{E}(\gamma_1) + c((1-\bar{x})\gamma_0 + \bar{x}\gamma_1)]$$

β_i $\beta_{i,post}$ and as a result, SE are all a function of x. We can then describe
$\beta_i^{men} = \beta_i(x = 1)$ and similarly for other terms to get
So

$$\beta_i^{men} = \gamma_1$$

$$\beta_i^{post,men} = -c_{men}(1-\bar{x})\bar{\gamma_0} + (1 - c_{men}\bar{x})\bar{\gamma_1} + c_{men}[(1-\bar{x})\gamma_0 + \bar{x}\gamma_1]$$

$$\beta_i^{women} = \gamma_0$$

$$\beta_i^{post,women} = [1 - c_{wom}(1-\bar{x})]\mathrm{E}(\gamma_0) - c_{wom}\bar{x}\mathrm{E}(\gamma_1) + c_{wom}(1-\bar{x})\gamma_0 + c_{wom}\bar{x}\gamma_1$$

this can be broken down into

$$\sqrt{SE} = (\beta_i^{men} - \beta_i^{post,men}) + (\beta_i^{wom} - \beta_i^{post,wom})$$

let

$$
\begin{aligned}
W^{men} &= (\beta_i^{men} - \beta_i^{post,men}) \\
&= (1 - \bar{x}c_{men})(\gamma_1 - \bar{\gamma}_1) - c_{men}(1 - \bar{x})(\gamma_0 - \bar{\gamma}_0) \\
&= (c_{wom}(1 - \bar{x}))(\gamma_1 - \bar{\gamma}_1) - c_{men}(1 - \bar{x})(\gamma_0 - \bar{\gamma}_0) \\
W^{wom} &= (\beta_i^{wom} - \beta_i^{post,wom}) \\
&= [1 - c_{wom}(1 - \bar{x})](\gamma_0 - \bar{\gamma}_0) - c_{wom}\bar{x}(\gamma_1 - \bar{\gamma}_1) \\
&= [c_{men}\bar{x}](\gamma_0 - \bar{\gamma}_0) - c_{wom}\bar{x}(\gamma_1 - \bar{\gamma}_1)
\end{aligned}
$$

and recall that

$$
\begin{aligned}
c_i &= \frac{(1-x)(1-\bar{x})\mathrm{Var}(\gamma_0) + x\bar{x}\mathrm{Var}(\gamma_1)}{(1-\bar{x})^2\mathrm{Var}(\gamma_0) + \bar{x}^2\mathrm{Var}(\gamma_1)} \\
&= \frac{(1-x)(1-\bar{x}) + x\bar{x}}{(1-\bar{x})^2 + \bar{x}^2}
\end{aligned}
$$

so if you are a man, then

$$
c_{men} = \frac{\bar{x}\mathrm{Var}(\gamma_1)}{(1-\bar{x})^2\mathrm{Var}(\gamma_0) + \bar{x}^2\mathrm{Var}(\gamma_1)}
$$

and likewise if you are a woman, then

$$
c_{woman} = \frac{(1-\bar{x})\mathrm{Var}(\gamma_0)}{(1-\bar{x})^2\mathrm{Var}(\gamma_0) + \bar{x}^2\mathrm{Var}(\gamma_1)}
$$

and

$$
\begin{aligned}
W^2_{men} &= \left(\frac{(1-\bar{x})^4}{((1-\bar{x})^2 + \bar{x}^2)^2}\right)\mathrm{Var}(\gamma_1) + \frac{\bar{x}^2(1-\bar{x})^2}{((1-\bar{x})^2 + \bar{x}^2)^2}\mathrm{Var}(\gamma_0) \\
&= (\frac{(1-\bar{x})^2\mathrm{Var}(\gamma_1)}{(1-\bar{x})^2 + \bar{x}^2} \\
W^2_{women} &= \left(\frac{(\bar{x})^4}{((1-\bar{x})^2 + \bar{x}^2)^2}\right)\mathrm{Var}(\gamma_0) + \frac{\bar{x}^2(1-\bar{x})^2}{((1-\bar{x})^2 + \bar{x}^2)^2}\mathrm{Var}(\gamma_1) \\
&= \frac{(\bar{x})^2\mathrm{Var}(\gamma_0)}{(1-\bar{x})^2 + \bar{x}^2}
\end{aligned}
$$

so then taking the mean (E_x) with respect to x, we can write the mean squared error (MSE) as

$$MSE = pW_{men}^2 + (1-p)W_{wom}^2$$

$$= p\left(\frac{(1-\bar{x})^4\text{Var}(\gamma_1) + (\bar{x}^2(1-\bar{x})^2)\text{Var}(\gamma_0)}{((1-\bar{x})^2+\bar{x}^2)^2}\right) + (1-p)\left(\frac{(\bar{x})^4\text{Var}(\gamma_0) + (\bar{x}^2(1-\bar{x})^2)\text{Var}(\gamma_1)}{((1-\bar{x})^2+\bar{x}^2)^2}\right)$$

so using the fact that $\text{Var}(\gamma_0) = \text{Var}(\gamma_1) = 1$ we get

$$MSE = p\left(\frac{(1-\bar{x})^4 + (\bar{x}^2(1-\bar{x})^2)}{((1-\bar{x})^2+\bar{x}^2)^2}\right) + (1-p)\left(\frac{(\bar{x})^4 + (\bar{x}^2(1-\bar{x})^2)}{((1-\bar{x})^2+\bar{x}^2)^2}\right)$$

$$MSE = p\left(\frac{(1-\bar{x})^2}{(1-\bar{x})^2+\bar{x}^2}\right) + (1-p)\left(\frac{(\bar{x})^2}{(1-\bar{x})^2+\bar{x}^2}\right)$$

$$= p\frac{(1-\bar{x})^2}{(1-\bar{x})^2+\bar{x}^2} + (1-p)\frac{\bar{x}^2}{(1-\bar{x})^2+\bar{x}^2}$$

In other words for individual i we have that

$$MSE = p(1 - c_{men}\bar{x}) + (1-p)(1 - c_{women}(1-\bar{x}))$$

$$MSE = p\left(1 - \frac{\bar{x}^2}{(1-\bar{x})^2+\bar{x}^2}\right) + (1-p)\left(1 - \frac{(1-\bar{x})^2}{(1-\bar{x})^2+\bar{x}^2}\right)$$

$$MSE = p\left(\frac{(1-\bar{x})^2}{(1-\bar{x})^2+\bar{x}^2}\right) + (1-p)\left(\frac{\bar{x}^2}{(1-\bar{x})^2+\bar{x}^2}\right)$$

$$MSE = p\left[(1-\bar{x})c_{wom}\right] + (1-p)\left[\bar{x}c_{men}\right]$$

Interestingly this implies a FOC of

$$\frac{\partial MSE}{\partial \bar{x}} = p\left(\frac{2\bar{x}(1-\bar{x})}{((1-\bar{x})^2+\bar{x}^2)^2}\right) - (1-p)\left(\frac{2\bar{x}(1-\bar{x})}{((1-\bar{x})^2+\bar{x}^2)^2}\right) = 0$$

$$\frac{\partial MSE}{\partial \bar{x}} = (2p-1)\left(\frac{2\bar{x}(1-\bar{x})}{((1-\bar{x})^2+\bar{x}^2)^2}\right) = 0$$

meaning we've retrieved our original FOC.
In full generality we also can say

$$
\begin{aligned}
W_{men}^2 &= \left(\frac{(1-\bar{x})^4}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\right)\mathrm{Var}(\gamma_0)^2\mathrm{Var}(\gamma_1) \\
&- \frac{2(1-\bar{x})^2[\bar{x}(1-\bar{x})]}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\mathrm{Var}(\gamma_1)\mathrm{Var}(\gamma_0)\mathrm{Cov}(\gamma_0,\gamma_1) \\
&+ \frac{\bar{x}^2(1-\bar{x})^2}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\mathrm{Var}(\gamma_1)^2\mathrm{Var}(\gamma_0) \\
&= \mathrm{Var}(\gamma_0)\mathrm{Var}(\gamma_1)\Bigg(\frac{(1-\bar{x})^4}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\mathrm{Var}(\gamma_0) \\
&- \frac{2(1-\bar{x})^2[\bar{x}(1-\bar{x})]}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\mathrm{Cov}(\gamma_0,\gamma_1) \\
&+ \frac{\bar{x}^2(1-\bar{x})^2}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\mathrm{Var}(\gamma_1)\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
W_{women}^2 &= \left(\frac{(\bar{x})^4}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\right)\mathrm{Var}(\gamma_1)^2\mathrm{Var}(\gamma_0) \\
&- \frac{2\bar{x}^2[\bar{x}(1-\bar{x})]}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\mathrm{Var}(\gamma_1)\mathrm{Var}(\gamma_0)\mathrm{Cov}(\gamma_0,\gamma_1) \\
&+ \frac{\bar{x}^2(1-\bar{x})^2}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\mathrm{Var}(\gamma_0)^2\mathrm{Var}(\gamma_1) \\
&= \mathrm{Var}(\gamma_0)\mathrm{Var}(\gamma_1)\Bigg(\frac{(\bar{x})^4}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\mathrm{Var}(\gamma_1) \\
&- \frac{2\bar{x}^2[\bar{x}(1-\bar{x})]}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\mathrm{Cov}(\gamma_0,\gamma_1) \\
&+ \frac{\bar{x}^2(1-\bar{x})^2}{((1-\bar{x})^2\mathrm{Var}(\gamma_0)+\bar{x}^2\mathrm{Var}(\gamma_1))^2}\mathrm{Var}(\gamma_0)\Bigg)
\end{aligned}
$$

So letting $\mathrm{Cov}(\gamma_0,\gamma_1)=0$ we get

$$W_{men}^2 = \text{Var}(\gamma_0)\text{Var}(\gamma_1)\left(\frac{(1-\bar{x})^2}{(2-\bar{x})^2\text{Var}(\gamma_0)+\bar{x}^2\text{Var}(\gamma_1)}\right)$$

$$W_{women}^2 = \text{Var}(\gamma_0)\text{Var}(\gamma_1)\left(\frac{(\bar{x})^2}{(1-\bar{x})^2\text{Var}(\gamma_0)+\bar{x}^2\text{Var}(\gamma_1)}\right)$$

Now observe that if $\alpha = \text{Var}(\gamma_1)$ and $\beta = \text{Var}(\gamma_0)$ then

$$\alpha W_{women}^2 + \beta W_{men}^2 = \text{Var}(\gamma_0)\text{Var}(\gamma_1)\left[\frac{\text{Var}(\gamma_1)\bar{x}^2 + \text{Var}(\gamma_0)(1-\bar{x})^2}{(1-\bar{x})^2\text{Var}(\gamma_0)+\bar{x}^2\text{Var}(\gamma_1)}\right]$$

$$= \text{Var}(\gamma_0)\text{Var}(\gamma_1)$$

So we can write

$$\alpha W_{women}^2 = \text{Var}(\gamma_0)\text{Var}(\gamma_1) - \beta W_{men}^2$$

$$W_{women}^2 = \frac{\text{Var}(\gamma_0)\text{Var}(\gamma_1) - \beta W_{men}^2}{\alpha}$$

$$W_{women}^2 = \frac{\text{Var}(\gamma_0)\text{Var}(\gamma_1) - \text{Var}(\gamma_0) W_{men}^2}{\text{Var}(\gamma_1)}$$

So we can conclude

$$MSE = (1-p)W_{women}^2 + pW_{men}^2$$

$$MSE = pW_{men}^2 + (1-p)\left(\frac{\text{Var}(\gamma_0)\text{Var}(\gamma_1) - \text{Var}(\gamma_0) W_{men}^2}{\text{Var}(\gamma_1)}\right)$$

$$MSE = p\text{Var}(\gamma_1)W_{men}^2 + (1-p)\left(\frac{\text{Var}(\gamma_0)\text{Var}(\gamma_1) - \text{Var}(\gamma_0) W_{men}^2}{\text{Var}(\gamma_1)}\right)$$

$$= \frac{\left(p[\text{Var}(\gamma_0)+\text{Var}(\gamma_1)] - \text{Var}(\gamma_0)\right)W_{men}^2 + (1-p)\text{Var}(\gamma_0)\text{Var}(\gamma_1)}{\text{Var}(\gamma_1)}$$

Thus when $p > \frac{1}{2}$ we can clearly see that MSE is minimized when W_{men}^2 is minimized (when $\bar{x} = 1$. And inversely when $p < \frac{1}{2}$ MSE is minimized when W_{men}^2 is maximaximized (when

$\bar{x} = 0$. In other words, when the proportion of men is $p > \frac{1}{2}$ it is optimal to have only men ($\bar{x}$) in the trial, and vice versa. And when there are eqal number of men and women in the population, MSE does not depend on W^2_{men} and thus has equal error of $\frac{1}{2}$ for any $\bar{x}$.

Note this also gives us a generalized error of

$$MSE = (1-p)W^2_{women} + pW^2_{men} = \text{Var}(\gamma_0)\text{Var}(\gamma_1)\big[\frac{(1-p)\bar{x}^2 + p(1-\bar{x})^2}{(1-\bar{x})^2\text{Var}(\gamma_0) + \bar{x}^2\text{Var}(\gamma_1)}\big]$$

Which gives a derivative of

$$\frac{MSE}{d\bar{x}} = -\frac{2\text{Var}(\gamma_0)\text{Var}(\gamma_1)(1-\bar{x})\bar{x}\big[\text{Var}(\gamma_0)(p-1) + \text{Var}(\gamma_1)p\big]}{(\text{Var}(\gamma_0)(\bar{x}-1)^2 + \text{Var}(\gamma_1)\bar{x}^2)^2}$$

Here we can see that we still have roots $\bar{x} = 0$ and $\bar{x} = 1$ and all that changes is we get a variances-weighted edge-case whenever $(\text{Var}(\gamma_0) + \text{Var}(\gamma_1))p - \text{Var}(\gamma_0) = 0$

And to get which is the minizing solution we can observe the SOC

$$\frac{MSE}{d\bar{x}^2} = -\frac{2\text{Var}(\gamma_0)\text{Var}(\gamma_1)[\text{Var}(\gamma_0)(p-1) + \text{Var}(\gamma_1)p][\text{Var}(\gamma_0)(2x+1)(x-1)^2 + \text{Var}(\gamma_1)x^2(2x-3)]}{[\text{Var}(\gamma_0)(x-1)^2 + \text{Var}(\gamma_1)x^2]^3}$$

From the second order condition, we can look at the two roots. We can see that the numerator reduces to two cases. When $p > \frac{\text{Var}(\gamma_0)}{(\text{Var}(\gamma_0)+\text{Var}(\gamma_1))}$, then we can observe that the numerator is positive for the root $\bar{x} = 1$. Conversely, the numerator is positive when $\bar{x} = 0$

Just Men

$$\begin{aligned}
\beta_i &= \gamma_0 + x\gamma_1 \\
\beta_i^{post} &= \text{E}(\gamma_0) + \text{E}(\gamma_1) + c_i(\gamma_0 + \bar{x}\gamma_1 - (\text{E}(\gamma_0) + \bar{x}\text{E}(\gamma_1))) \\
\beta_i^{women} &= \gamma_0 \\
\beta_i^{men} &= \gamma_0 + \gamma_1 \\
\beta_i^{post,men} &= \text{E}(\gamma_0) + \text{E}(\gamma_1) + c_{men}(\gamma_0 + \bar{x}\gamma_1 - (\text{E}(\gamma_0) + \bar{x}\text{E}(\gamma_1))) \\
\beta_i^{post,women} &= \text{E}(\gamma_0) + c_{women}(\gamma_0 + \bar{x}\gamma_1 - (\text{E}(\gamma_0) + \bar{x}\text{E}(\gamma_1)))
\end{aligned}$$

And we have

$$C_i = \frac{\text{Var}(\gamma_0) + x\bar{x}\text{Var}(\gamma_1)}{\text{Var}(\gamma_0) + \bar{x}^2\text{Var}\gamma_1)}$$

$$C_{women} = \frac{\text{Var}(\gamma_0)}{\text{Var}(\gamma_0) + \bar{x}^2\text{Var}(\gamma_1)}$$

$$C_{men} = \frac{\text{Var}(\gamma_0) + \bar{x}\text{Var}(\gamma_1)}{\text{Var}(\gamma_0) + \bar{x}^2\text{Var}\gamma_1)}$$

$$
\begin{aligned}
W_{women} &= \beta_i^{women} - \beta_i^{post,women} \\
&= (\gamma_0 - \mathrm{E}(\gamma_0)) - C_{men}\big[(\gamma_0 - \mathrm{E}(\gamma_0)) + \bar{x}(\gamma_1 - \mathrm{E}(\gamma_1))\big] \\
&= (1 - c_{women})(\gamma_0 - \mathrm{E}(\gamma_0)) - C_{women}\bar{x}(\gamma_1 - \mathrm{E}(\gamma_1)) \\
W_{men} &= \beta_i^{men} - \beta_i^{post,men} \\
&= (\gamma_0 - \mathrm{E}(\gamma_0)) + (\gamma_1 - \mathrm{E}(\gamma_1) - C_{men}\big[(\gamma_0 - \mathrm{E}(\gamma_0)) + \bar{x}(\gamma_1 - \mathrm{E}(\gamma_1))\big] \\
&= (1 - c_{men})(\gamma_0 - \mathrm{E}(\gamma_0)) + (1 - C_{men}\bar{x})(\gamma_1 - \mathrm{E}(\gamma_1))
\end{aligned}
$$

Squaring

$$
\begin{aligned}
W_{women}^2 &= (1 - c_{wom})^2\mathrm{Var}(\gamma_0) - 2(1 - c_{wom})c_{wom}\bar{x}\mathrm{Cov}(\gamma_0, \gamma_1) + c_{wom}^2\bar{x}\mathrm{Var}(\gamma_1) \\
W_{men}^2 &= (1 - c_{men})^2\mathrm{Var}(\gamma_0) + 2(1 - c_{men})(1 - c_{men}\bar{x})\mathrm{Cov}(\gamma_0, \gamma_1) + (1 - c_{men}\bar{x})^2\mathrm{Var}(\gamma_1)
\end{aligned}
$$

And assuming $\mathrm{Cov}(\gamma_0, \gamma_1) = 0$ we get

$$
\begin{aligned}
W_{women}^2 &= (1 - c_{wom})^2\mathrm{Var}(\gamma_0) + c_{wom}^2\bar{x}^2\mathrm{Var}(\gamma_1) \\
W_{men}^2 &= (1 - c_{men})^2\mathrm{Var}(\gamma_0) + (1 - c_{men}\bar{x})^2\mathrm{Var}(\gamma_1)
\end{aligned}
$$