

# **MARKING GUIDELINE**

## NATIONAL CERTIFICATE

# **NOVEMBER EXAMINATION**

### STRENGTH OF MATERIALS AND STRUCTURES N6

**28 NOVEMBER 2016** 

This marking guideline consists of 9 pages.

#### **QUESTION 1**

1.1 
$$\sigma_{Hmin} = \sigma_{Hmax} - \sigma_{R} = -130 + 80 = -50 MPa$$

at 250 mm: 
$$-50 \times 10^6 = a - \frac{b}{0.25^2} \dots \dots \dots \dots (1)$$

$$0 = a + \frac{b}{0.25^2} \dots \dots (2)$$

$$(1) + (2): -50 \times 10^6 = 2a$$

$$a = -25 \times 10^6 \checkmark$$

$$b = 1.5625 \times 10^6 \checkmark$$

at inner diameter:  $80 \times 10^6 = a + \frac{b}{d^2}$ 

$$80 \times 10^6 = -25 \times 10^6 + \frac{1,5625 \times 10^6}{d^2} \checkmark$$

$$d = 0.122 \, m\checkmark \tag{7}$$

1.2 
$$W = p \times A = 80 \times 10^6 \times \pi \times 0,061^2 \checkmark = 934,998 \, kN \checkmark$$
 (2)

1.3 
$$\sigma_L = \frac{pd^2}{D^2 - d^2} = \frac{80 \times 10^6 \times 0,122^2}{0,25^2 - 0,122^2} \checkmark = 25 MPa \checkmark$$
 (2)

1.4 
$$\delta d = \frac{d}{E} (\sigma_H - \nu \sigma_R) = \frac{0.122}{200 \times 10^9} (-50 \times 10^6 - 0.3 \times 0) \checkmark = -62.5 \times 10^{-6} \, \text{m} \, \checkmark$$
[2)

2.1  $deflection \ limit: \ y = \frac{wL^4}{8EI}$ 

$$12 \times 10^{-3} = \frac{4 \times 10^3 \times 5^4}{8 \times 200 \times 10^9 \times I} \checkmark$$

$$I = 130,208 \times 10^{-6} \, m^4 \checkmark$$

choose channel profile 381 x 102 x 55,1 kg/m√

stress limit:  $M = \frac{wL^2}{2} = \frac{4 \times 10^3 \times 5^2}{2} = 50 \text{ kNm}$ 

$$z = \frac{M}{\sigma} = \frac{50 \times 10^3}{120 \times 10^6} = 416,667 \times 10^{-6} \, m^3 \checkmark$$

choose channel profile 280 x 95 x 41,8 kg/m√

correct channel for both limits is  $381 \times 102 \times 55,1 \, kg/m \checkmark \checkmark$  (8)

2.2 
$$y = \frac{wL^4}{8EI} = \frac{4 \times 10^3 \times 5^4}{8 \times 200 \times 10^9 \times 149.1 \times 10^{-6}} = 10,48 \text{ mm}$$

$$\sigma = \frac{M}{z} = \frac{50 \times 10^3}{782,8 \times 10^{-6}} = 63,873 \, MPa \checkmark \tag{2}$$

2.3 
$$y_1 = \frac{wl_1^4}{8EI} = \frac{4 \times 10^3 \times 4^4}{8 \times 200 \times 10^9 \times 149, 1 \times 10^{-6}} = 4,292 \, mm\checkmark$$

$$y_2 = \frac{wl_1^3 \times l_2}{6EI} = \frac{4 \times 10^3 \times 4^3 \times 1}{6 \times 200 \times 10^9 \times 149, 1 \times 10^{-6}} = 1,431 \, mm\checkmark$$

$$y_T = y_1 + y_2 = 4,292 + 1,431 = 5,732 \, mm \checkmark$$
 (3) [13]

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#### **QUESTION 3**

3.1 
$$A = \pi(R^{2} - r^{2}) = \pi(0,05^{2} - 0,025^{2}) = 5,89 \times 10^{-3} \, m^{2} \checkmark$$

$$\sigma_{Rmin} = \sigma_{d} - \sigma_{b} = 0$$

$$\sigma_{d} = \sigma_{b} \dots \dots \dots (1) \checkmark$$

$$\sigma_{Rmax} = \sigma_{d} + \sigma_{b} \dots \dots (2) \checkmark$$

$$(1) into (2): 80 = \sigma_{d} + \sigma_{d}$$

$$\sigma_{d} = 40 \, MPa \, \checkmark$$

$$F = \sigma_{d} \times A = 40 \times 10^{6} \times 5,89 \times 10^{-3} = 235,619 \, kN \checkmark$$
(5)

3.2 
$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(0.1^4 - 0.05^4) = 4.602 \times 10^{-6} \, m^4 \checkmark$$

$$\sigma_b = \sigma_d = 40 \, MPa \checkmark$$

$$M = \frac{\sigma_b \times I}{Y} = \frac{40 \times 10^6 \times 4,602 \times 10^{-6}}{0,05} = 3,682 \text{ kNm}$$

$$L = \sqrt{\frac{8M}{w}} = \sqrt{\frac{8 \times 3,682 \times 10^3}{500}} = 7,675 \, m\checkmark \tag{4}$$

4.1 
$$F_w = \frac{\rho g h^2}{2} = \frac{1000 \times 9.81 \times 4^2}{2} = 78.48 \, kN \checkmark$$

$$W_1 = \rho gAl = 2\ 100 \times 9.81 \times 0.5 \times 1 \times 4 = 41,202\ kN\checkmark$$

$$W_2 = \rho gAl = 2\,100 \times 9.81 \times 2 \times 4 = 164.808\,kN\checkmark$$

$$V = W_1 + W_2 = 41,202 + 164,808 = 206,01 \, kN \checkmark \tag{4}$$

4.2 
$$F \sim M = F_w \times \frac{h}{3} = 78,48 \times \frac{4}{3} = 104,64 \text{ kNm } \checkmark$$

$$W \sim M = W_1 x_1 + W_2 x_2 = 41,202 \times 0,667 + 104,808 \times 2 \checkmark = 357,084 \, kNm \checkmark \tag{3}$$

4.3  $Vx + F \sim M = W \sim M$ 

$$206,01x + 104,64 = 357,084$$

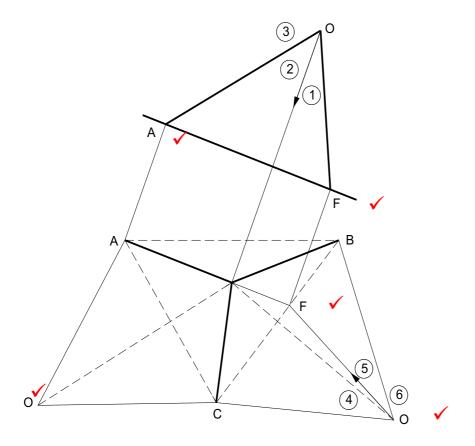
$$x = 1,225 m \text{ from toe}\checkmark$$

no tension because 
$$x > \frac{B}{3} \checkmark$$
 (3)

4.4 
$$e = 0.5B - x = 1.5 - 1.225 = 0.275 \,\text{m}$$

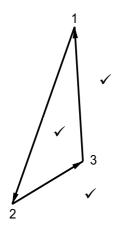
$$\sigma_{max} = \frac{V}{B} + \frac{6Ve}{B^2} = \frac{206,01}{3} + \frac{6 \times 206,01 \times 0,2275}{3^2} = 106,384 \, kPa \checkmark$$
[12]

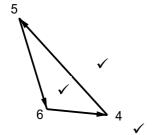
5.1



(6)

5.2





MEMBERS	MAGNITUDE	NATURE
OA (2-3)	22,8 kN√	Strut√
OB (5-6)	17 kN√	Strut√
OC (6-4)	25,3 kN√	Strut√

(9) **[15]** 

#### **QUESTION 6**

$$\frac{\sigma_s}{\sigma_c} = \frac{m(d-n)}{n}$$

$$\frac{140}{5} = \frac{15(0.8 - n)}{n} \checkmark$$

$$n = 0.279 \, m \, \checkmark$$
 (2)

6.2 
$$M_s = \sigma_s A_s (d-n) = 140 \times 10^6 \times 680 \times 10^{-6} (0.8 - 0.279) \checkmark = 49.593 \, kNm \checkmark$$

$$M_c = M - M_s = 150 - 49,593 = 100,407 \, kNm \checkmark$$
 (3)

6.3 
$$\sigma_{c1} = \frac{\sigma_c(n-t)}{n} = \frac{5(0,279 - 0,25)}{0,25} \checkmark = 0,521 \, MPa \checkmark \tag{2}$$

6.4 
$$M_c = \frac{1}{2}\sigma_c b n \frac{2}{3}n - \frac{1}{2}\sigma_{c1}(b-e)(n-t)\frac{2}{3}(n-t)$$

$$100\,407 = \frac{5\times10^6\times b\times0,279^2}{3} - \frac{0,521\times10^6\times (b-0,35)(0,279-0,25)^2}{3}\checkmark$$

 $100\,407 = 129\,779.8918b\checkmark - 146.71b\checkmark + 51.349\checkmark$ 

$$b = 0,774 \, m\checkmark \tag{5}$$

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$$7.1 F_H = \frac{wx_1^2}{2d}$$

$$1239 = \frac{5x_1^2}{2d} \checkmark$$

$$d = 2,0178 \times 10^{-3} \, x_1^2 \ldots \ldots (1) \checkmark$$

$$F_H = \frac{w(L - x_1)^2}{2(d + h)}$$

$$1239 = \frac{5(L - x_1)^2}{2(d+7)} \dots \dots (2) \checkmark$$

(1) into (2): 
$$1239 = \frac{5(L - x_1)^2}{2(2,0178 \times 10^{-3} x_1^2 + 7)} \checkmark$$

$$x_1^2 + 3469,2 = 12100 - 220x_1 + x_1^2 \checkmark$$

$$x_1 = 39,231 \, m \checkmark$$

$$d = 2,0178 \times 10^{-3} \times 39,231 = 3,105 \, m \checkmark \tag{7}$$

7.2 
$$x_2 = L - x_1 = 110 - 39,231 = 70,769 \, m\checkmark$$

$$F_{V2} = wx_2 = 5 \times 70,769 = 353,845 \, kN \checkmark$$

$$F_T = \sqrt{F_H^2 + F_V^2} = \sqrt{1239^2 + 353,845^2} = 1288,537 \, kN \checkmark \tag{3}$$

7.3 
$$A = \frac{F_T}{\sigma} = \frac{1288,537 \times 10^3}{180 \times 10^6} = 7,159 \times 10^{-3} \, m^2 \checkmark$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 7,159 \times 10^{-3}}{\pi}} = 95,47 \text{ mm}\checkmark$$
 (2)

7.4 
$$\theta = \tan^{-1} \frac{F_V}{F_H} = \tan^{-1} \frac{353,845}{1239} = 15,939$$
° (1)
[13]

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**QUESTION 8** 

8.1 
$$T_e = \sqrt{T^2 + M^2} = \sqrt{3^2 + 4^2} \checkmark = 5 \ kNm\checkmark$$
 (2)

8.2 
$$\tau = \frac{16T_e}{\pi d^3} = \frac{16 \times 5 \times 10^3}{\pi \times 0.1^3} \checkmark = 25,465 \, MPa\checkmark$$

8.3 
$$M_e = 0.5 \left( M + \sqrt{T^2 + M^2} \right) = 0.5(4+5) \checkmark = 4.5 \text{ kNm} \checkmark$$
 (2)

8.4 
$$\sigma_b = \frac{32M_e}{\pi d^3} = \frac{32 \times 4.5 \times 10^3}{\pi \times 0.1^3} \checkmark = 45.837 \, MPa\checkmark$$

8.5 
$$T_m = \frac{T}{1.15} = \frac{3}{1.15} = 2,609 \text{ kNm} \checkmark$$

$$P = 2\pi NT_m = 2\pi \times 10 \times 2,609 = 163,909 \, kW \checkmark \tag{2}$$

8.6 
$$\frac{D^4 - d^4}{D} = d^3$$

$$\frac{(2d)^4 - d^4}{2d} = 0.1^3 \checkmark$$

$$d = 51,0873 \, mm \checkmark$$

$$D = 102,1746 \, mm\checkmark \tag{3}$$
[13]

**TOTAL: 100** 

(2)

(2)