Discrete Mathematics in Computer Science

M. Helmert, G. Röger
S. Eriksson
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University of Basel
Computer Science

Exercise Sheet 10
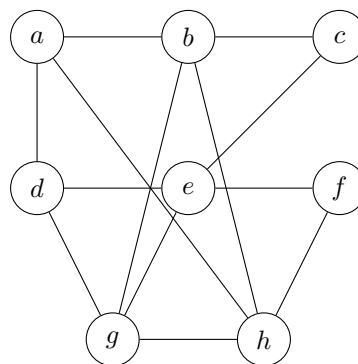### Due: Monday, December 4, 2023, 4pm

Please carefully read the exercises FAQ on ADAM!

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file.

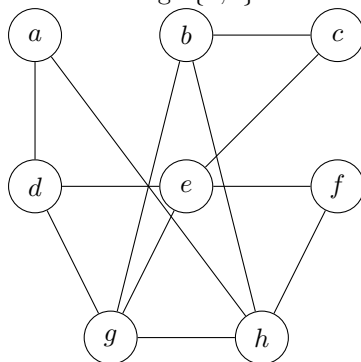**Exercise 10.1** (2 marks)

Consider the following graph G:

Prove that G is not planar by showing that K_5 is a minor of G. Only apply one transformation each step and provide both the transformed graph as well as a description what transformation you applied to which part of the graph for each step.

Hint: You only need three transformation steps.

Example:

1. remove edge $\{a, b\}$:

Exercise 10.2 (2 marks)

Consider the following formalizations of statements in natural language as formulas over the set of all finite strings. Explain all syntactic and semantic errors. With semantic errors, we mean that the formula does not match the meaning of the natural language sentence.

(a) Either we eat now, or we eat later and the food is cold: EatNow $\lor$ EatLater $\land$ FoodCold

(b) In general you should only go swimming if there is no storm: (Swimming $\leftrightarrow$ ¬(Storm))

Exercise 10.3 (3 marks)

Consider the formula $\varphi = \big((X \lor (Y \land \neg Z)) \land (\neg X \lor \neg Y)\big)$ over $\{X, Y, Z\}$.

(a) Specify a model $\mathcal{I}$ for φ.

(b) Prove that your interpretation $\mathcal{I}$ from (a) is indeed a model for φ, that is that $\mathcal{I} \models \varphi$ holds. In your solution, only use a single definition per step, and do not use a truth table.

(c) Specify an interpretation that is not a model of φ. You do not need to justify that your interpretation is not a model.

Exercise 10.4 (3 marks)

(a) Is formula $\varphi = \big(((Y \land Z) \to X) \lor \neg (X \lor \neg Z)\big)$ over $\{X, Y, Z\}$ satisfiable, unsatisfiable, falsifiable, valid? Justify your answer for each of the four properties without a truth table.

(b) Specify a forumla over $\{A, B\}$ that is unsatisfiable. Your formula must contain both A and B at least once. You do not need to justify your answer.

Submission rules:
Upload a single PDF file (ending in .pdf). Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). There is a template that satisfies these requirements available on ADAM.