Discrete Mathematics in Computer Science

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Exercise Sheet 9

Due: Monday, November 27, 2023, 4pm

Please carefully read the exercises FAQ on ADAM!

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file.

Exercise 9.1 (3 marks)
Are the following statements about induced subgraphs correct? Justify your answer.

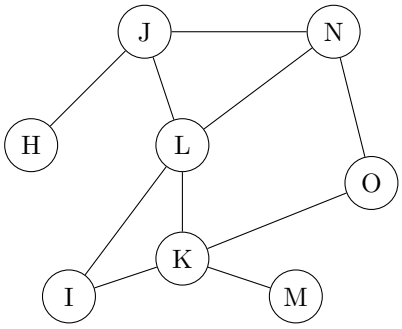
(a) If G is connected, then all its induced subgraphs are connected.

(b) If G is a forest, then all its induced subgraphs are forests.

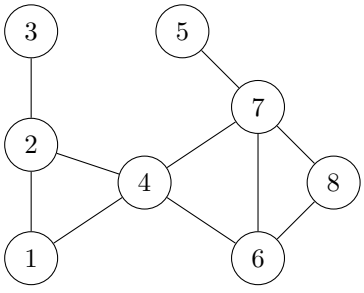
(c) If G is a tree, then all its induced subgraphs are trees.

Exercise 9.2 (1 mark)
Specify two graph invariants that were not mentioned in the lecture.

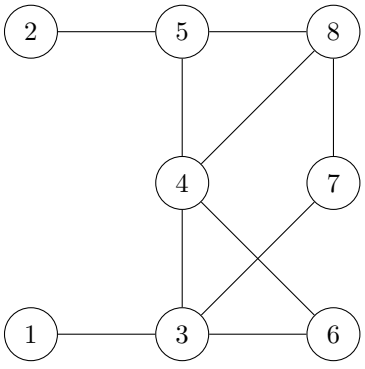
Exercise 9.3 (2 marks)
Consider the following graph G:



Are the following two graphs isomorphic to G? Justify your answer by either providing the bijective mapping of the vertices or by explaining why the graph cannot be isomorphic to G.

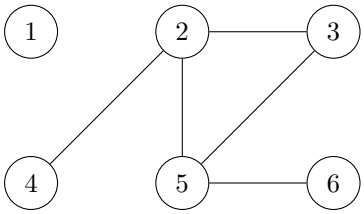


(a)



(b)

Exercise 9.4 (2 marks)
Consider the following graph $G = (V, E)$:



(a) Specify two sets of vertices $V_1, V_2 \subseteq V$ with $|V_1| = |V_2| = 4$ such that the graphs induced by V_1 and V_2 are isomorphic. Additionally provide the bijective mapping σ from V_1 to V_2.

(b) Specify a non-trivial symmetry σ'. Non-trivial means that it is not the identity function.

Exercise 9.5 (2 marks)
Consider a graph $G = (V, E)$ with n connected components, where the induced graph of each component G_i has s_i distinct symmetries.

(a) Justify that $\Pi_{i=1}^{n} s_i$ is a lower bound for the amount of distinct symmetries of G.

(b) Why is the lower bound from (a) not tight, that is why can there be more distinct symmetries?

Submission rules:
Upload a single PDF file (ending in .pdf). Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). There is a template that satisfies these requirements available on ADAM.