

Exercise 2.1

We examine the following theorem:

$$\sum_{i=0}^n i = \frac{n*(n+1)}{2} \quad \forall \quad n \in \mathbb{N}_0$$

Proof. Mathematical induction over n :

Basis $n - 1 = 0$:

$$0 = \frac{0 * 1}{2}$$

$$0 = 0$$

Induction Hypothesis:

$$\sum_{i=0}^k i = \frac{k*(k+1)}{2} \text{ for } k = n - 1$$

Inductive Step: $n - 1 \rightarrow n$

$$\begin{aligned} \sum_{i=0}^{n-1} i &= \sum_{i=0}^n i - n \stackrel{\text{IH}}{=} \frac{n*(n+1)}{2} - n \\ &\iff \frac{n*(n+1)}{2} - \frac{2n}{2} \\ &\iff \frac{n*(n+1) - 2n}{2} \\ &\iff \frac{n*(n+1-2)}{2} \\ &\iff \frac{n*(n-1)}{2} \\ &\iff \frac{n-1*(n-1+1)}{2} \end{aligned}$$

□

Exercise 2.2

Exercise 2.3

Exercise 2.4

- (a) The set builder notation is wrong because it does not specify that x is a natural number and it also does not define n . The correct notation would be $\{x \mid x \in \mathbb{N}, x < 20, x \bmod 2 = 1\}$.
- (b) The notation is wrong since x is undefined and the 6 alone is not a set. The correct notation would be $\{x \mid x \in \mathbb{N}, x \neq 6\}$.

Exercise 2.5

- (a) We can first find out what the union of A and B is: $A \cup B = U \setminus (A \cup B)^c = \{1, 3, 5, 6, 8, 9, 10\}$. With the latter and $A \cap B = \{1, 3\}$ it follows that $\{1, 3\} \subset A \subset A \cup B$ and $\{1, 3\} \subset B \subset A \cup B$. The following sets satisfy the properties: $A = \{1, 3, 5, 6, 8, 9\}, B = \{1, 3, 10\}$.
- (b) It is not possible to satisfy all conditions at the same time. If $A \cap B = \emptyset$, it means that A, B have no elements in common. However, if $A \subset B$, it means that every element of A is an element of B , which contradicts the first condition of A and B having no common elements. Therefore, A, B under the given conditions do not exist.
- (c) The following set A, B satisfy the required properties: $A = B = \{6, 7, 8\}$.