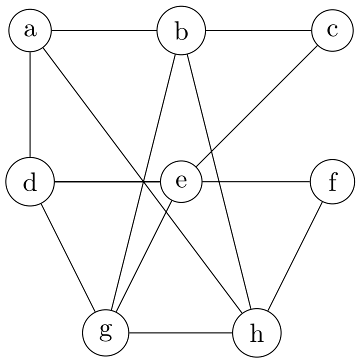
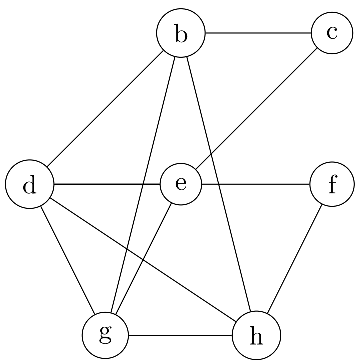
Exercise 10.1

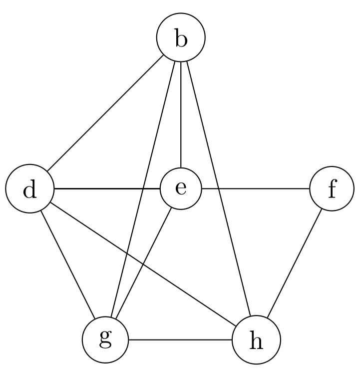
We will do three transformations to G to show that it has K_5 as a minor. This is the original graph:



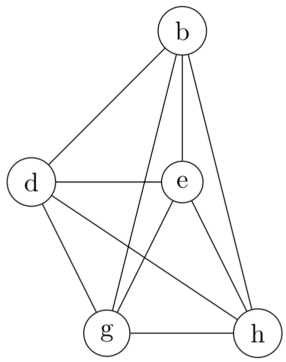
1. Contract $\{a, d\}$



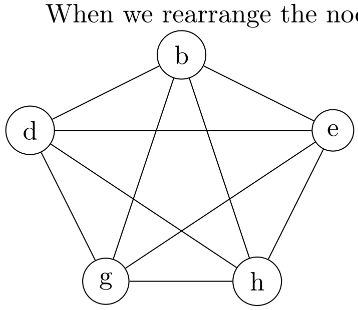
2. Contract $\{c, e\}$



3. Contract $\{f, e\}$



When we rearrange the nodes we can clearly see that it's a K_5 graph. Thus G is not planar.



Exercise 10.2

(a)

(b)

Exercise 10.3

(a)

(b)

Exercise 10.4

(a) We will analyze the statement $\phi = (((Y \land Z) \rightarrow X) \lor \neg(X \lor \neg Z))$ over $\{X, Y, Z\}$.
Satisfiability: Let $\mathcal{I} = \{X \mapsto 0, Y \mapsto 1, Z \mapsto 1\}$. We have $\mathcal{I} \models X$ and $\mathcal{I} \not\models Z$. From the definition of negation we have that $\mathcal{I} \models \neg Z$. Therefore, with the definition of disjuntion, we get $\mathcal{I} \not\models (X \lor \neg Z)$. With the definition of negation we have $\mathcal{I} \models \neg(X \lor \neg Z)$. With the definition of disjuntion we have $\mathcal{I} \models \psi \lor \neg(X \lor \neg Z)$ for all formulas ψ, in particular $\mathcal{I} \models (((Y \land Z) \rightarrow X) \lor \neg(X \lor \neg Z))$. Therefore ϕ is satisfiable.
Falsifiability: The statement ϕ is by the definition of disjuntion and negation false, iff both

$((Y \land Z) \rightarrow X)$ and $\neg(X \lor \neg Z)$ are false. The formula $((Y \land Z) \rightarrow X)$ is only false, if $\mathcal{I} = \{X \mapsto 0, Y \mapsto 1, Z \mapsto 1\}$. But under this interpretation, the formula $\neg(X \lor \neg Z)$ is true. Therefore ϕ can never be false.
Validness The formula ϕ is valid, since it can never be false.
Unsatisfiability The formula is not unsatisfiable because it's satisfiable.

(b) $\phi = (A \land \neg A) \land (B \land \neg B)$