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Exercise 3.1

(a)

(b)

Exercise 3.2

(a)

(b)

Exercise 3.3

We will refute the statement for all sets A, B it holds that $|A| < |A \cup B|$. Let $A = B$, then we have $A = A \cup B$. In this case it holds that $|A| = |A \cup B|$, which contradicts the statement.

Exercise 3.4

Let $g\colon \{n \in \mathbb{N} \mid n \bmod 2 = 0\} \to \mathbb{Z}\colon g(n) = -\frac{n}{2}$. The function $g(x)$ is a bijective function because it maps every even natural number to every negated natural number (which is injective and surjective). Let $s\colon \{n \in \mathbb{N} \mid n \bmod 2 \neq 0\} \to \mathbb{Z}\colon g(n) = \frac{n+1}{2}$. The function $s(x)$ is a bijective function because it maps every odd natural number to a natural number (it is a subset of the natural numbers). The union of two bijective functions is bijective, therefore

$$f(n) = \begin{cases} -\frac{n}{2}, & \text{if } n \text{ is even,} \\ \frac{n+1}{2}, & \text{otherwise,} \end{cases} \quad \text{for } n \in \mathbb{N},$$

is a bijective function from $\mathbb{N}$ to $\mathbb{Z}$ and thus $\mathbb{Z}$ is countable.