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Exercise 7.1

(a) I will disprove the statement with a counter example. For $a = b = c = 2$ we have that $a \mid c$ and $b \mid c$ is true because we have $ak = bk = 2k = c = 2$ for $k = 1$. But $ab \ \mid \ c$ which is equivalent to $4 \mid 2$ is not true because there exists no $k \in \mathbb{Z}$ such that $abk = 4k = 2 = c$. Therefore $a \mid c$ and $b \mid c$ does not imply $ab \ \mid c$.

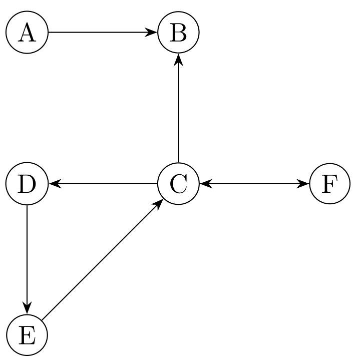
(b) If $a \mid b$ and $a \mid b - c$ there exist $k_1, k_2 \in \mathbb{Z}$ s.t. $ak_1 = b$ and $ak_2 = b - c$. By substitution we get $ak_2 = ak_1 - c \leftrightarrow ak_1 - ak_2 = c \leftrightarrow a(k_1 - k_2) = c$. We have that $k_1 - k_2 = k_3 \in \mathbb{Z}$. Therefore $ak_3 = c$ which implies $a \mid c$.

Exercise 7.2

(a)

(b)

Exercise 7.3



Exercise 7.4

The graph $G = (V, E)$ with
$V = \{H, I, J, K, L, M, N\}$ and
$E = \{\{K, J\}, \{K, M\}, \{K, N\}, \{K, H\}, \{K, L\}, \{J, M\}, \{M, N\}, \{M, L\}\}$
satisfies the properties.

Exercise 7.5

(a) The walk $\pi = \langle D, B, A, B \rangle$ from D to B is not a path.

(b) The set $P = \{\langle A, B, C \rangle, \langle C, B, A \rangle, \langle D, B, A \rangle, \langle D, B, C \rangle\}$ contains all paths with length 2.

(c) The tour $\langle A \rangle$ satisfies $v_0 = v_n = A$ and is not a cycle.

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(d) There are 6 cycles in the graph.