

Exercise 9.1

1. This statement is false. This will be shown with a counterexample. Let $G = (\{a, b, c\}, \{\{a, b\}, \{b, c\}\})$. G is connected, because we can reach every vertex from every vertex. The Graph $G' = (\{a, c\}, \emptyset)$ is an induced subgraph of G , since it is formed from a subset of the vertices from G and the vertices a, c are not connected in G . G' is not connected. We have the case that G is connected but the induced graph G' is not connected. Therefore the statement is false.
2. This statement is correct. A forest is a graph without cycles, essentially a collection of trees. An induced subgraph of a forest will also be cycle-free, as removing vertices and their associated edges cannot create a cycle where there was none before. Therefore, any induced subgraph of a forest is also a forest.
3. If a graph is a tree, then it must also be connected. Because the statement in a) is false, this statement must also be false, i.e. it is not guaranteed that an induced graph of a connected graph is connected. But this would be required for the statement: "If G is a tree, then all its induced subgraphs are trees. "

Exercise 9.2

Exercise 9.3

Exercise 9.4

Exercise 9.5