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Exercise 5.1

(a)

(b)

Exercise 5.2

(a)

(b)

Exercise 5.3

Exercise 5.4

(a) Let R be a total order over an arbitrary set S and let $x, y \in S$. If $x = y$, then the statements xRy and yRx are the same statements and thus count as only one statement. If $x \neq y$, then we have, because partial relations are antisymmetric, that if $xRy \in R$, then $yRx \notin R$ and vice versa. Therefore, either $xRy \in R$ xor $yRx \in R$ is true.

(b) I will disprove this statement by a counterexample. Let $S = \{a, b\}$. The only strict orders over S which exist are $R_1 = \{(a, b)\}$, $R_2 = \{(b, a)\}$.
Let's look at the strict order R_1. Here a is minimal, since there is no $y \in R_1$ with yR_1a. The element a is not maximal, since there exists $y \in R_1$ with aR_1y which is $y = b$.
An analogous argument can be made for R_2. Therefore, there does not exist a strict order over S where all $x \in S$ are both minimal and maximal. Thus the statement is false.

Exercise 5.5

(a)

(b)

(c)

(d)

(e)

(f)