Exercise 11.1

$$
\begin{aligned}
(\neg((\neg B \land D) \land (C \lor D)) \land (B \lor A)) &\equiv (\neg(\neg B \land (D \land (C \lor D)) \land (B \lor A))) && \text{(Associativity)}\\
&\equiv (\neg(\neg B \land (D \land (D \lor C)) \land (B \lor A))) && \text{(Commutativity)}\\
&\equiv (\neg(\neg B \land D) \land (B \lor A))) && \text{(Absorption)}\\
&\equiv ((\neg\neg B \lor \neg D) \land (B \lor A)) && \text{(De Morgan)}\\
&\equiv ((B \lor \neg D) \land (B \lor A)) && \text{(Double Negation)}\\
&\equiv (B \lor (\neg D \land A)) && \text{(Distributivity)}
\end{aligned}
$$

Exercise 11.2

I will disprove the statement: Every single Literal is a clause and a monomial, because the terms clause and monomial are also used for the corner case with only one literal. Every literal is also a formula. Therefore there exists a formula which is both a monomial and a clause.

Exercise 11.3

(a) This formula describes a conjunction of disjunctions over all combinations of visited lectures. If two or more lectures have been missed overall, then the conjunction is false.

(b) This formula describes a conjuction of the first formula in a) and a second formula which is a disjunction over all missed lectures. The second formula is only false if all lectures have been visited. So overall theres only one possbility for the formula to be true: If only one lecture out of all has been missed.

Exercise 11.4

Step(1)
- $B \lor ((\neg A \lor C) \leftrightarrow B)$
- $B \lor (((\neg A \lor C) \rightarrow B) \land (B \rightarrow (\neg A \lor C)))$
- $B \lor (((A \land \neg C) \lor B) \land (\neg B \lor (\neg A \lor C)))$

Step(2) Already done.

Step(3)
- $(B \lor ((A \land \neg C) \lor B)) \land (B \lor (\neg B \lor (\neg A \lor C)))$
- $(B \lor ((B \lor A) \land (B \lor \neg C))) \land (B \lor (B \lor (\neg B \lor \neg A) \land (\neg B \lor C)))$
- $((B \lor B \lor A) \land (B \lor (B \lor \neg C))) \land ((B \lor (\neg B \lor \neg A)) \land (B \lor (\neg B \lor C)))$

Step(4) $((B \lor A) \land (B \lor \neg C)) \land (\neg A \land C)$

Exercise 11.5

(a) Because of distributivity we can transform the formula from CNF to DNF like this: $\phi_n = \bigwedge_{i=1}^{n} (a_0 \vee a_i) \equiv \bigvee_{S \in \mathcal{P}(\{1,\ldots,n\})} (a_0 \wedge \bigwedge_{i \in \{1,\ldots,n\}} a_i)$. Because the cardinality of the power set is $\mathcal{P}(1,\ldots,n) = 2^n$, we will have 2^n monomials.

(b) The equivalent form is $\phi_n = \bigwedge_{i=1}^{n} (a_0 \vee a_i) \equiv a_0 \vee \bigwedge_{i=1}^{n} a_i$ and has has size polynomial in n. The equivalence can be explained by the fact that if a_0 is true, the original formula is satisfied (since a_0 appears in every clause of the CNF), and if a_0 is false, then all a_i must be true for the original formula to be satisfied, which is exactly what the specified formula expresses.