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Exercise 3.1

$A, B \subseteq \{1, ..., 10\}$
$|B|= 3$, so we choose arbitrarily $B = \{1, 2, 3\}$.
$|\mathcal{P}(A)|= 16 \iff |A|= \log_2(16) = 4$
$|A \cup B|= 5$ so to reach sum of 5, A has to share 2 Elements with B while having 4 in total.
$A = \{2, 3, 4, 5\}$ satisfies these conditions.

Exercise 3.2

(a) $\{e1, e4, e6, e7\} = 01001011$

(b) $01110110 = \{e1, e2, e3, e5, e6\}$

(c) $A \cup B \iff$ A bitwise OR B
$A \cap B \iff$ A bitwise AND B
$\neg A \iff$ bitwise NOT A

Exercise 3.3

We will refute the statement for all sets A, B it holds that $|A| < |A \cup B|$. Let $A = B$, then we have $A = A \cup B$. In this case it holds that $|A| = |A \cup B|$, which contradicts the statement.

Exercise 3.4

Let $g\colon \{n \in \mathbb{N} \mid n \bmod 2 = 0\} \to \mathbb{Z}\colon g(n) = -\frac{n}{2}$. The function $g(x)$ is a bijective function because it maps every even natural number to every negated natural number (which is injective and surjective).
Let $s\colon \{n \in \mathbb{N} \mid n \bmod 2 \neq 0\} \to \mathbb{Z}\colon g(n) = \frac{n+1}{2}$. The function $s(x)$ is a bijective function because it maps every odd natural number to a natural number (it is a subset of the natural numbers).
The union of two bijective functions is bijective, therefore

$$f(n) = \begin{cases} -\frac{n}{2}, & \text{if } n \text{ is even,} \\ \frac{n+1}{2}, & \text{otherwise,} \end{cases} \quad \text{for } n \in \mathbb{N},$$

is a bijective function from $\mathbb{N}$ to $\mathbb{Z}$ and thus $\mathbb{Z}$ is countable.

Exercise 3.5

Proof. We want to show that $\mathbb{Q}$ is countable. We already know that $\mathbb{Q}_+$ is countable.
We want to form a bijection from $\mathbb{Q}_+$ to $\mathbb{Q}_-$ to show that $|\mathbb{Q}_+| = |\mathbb{Q}_-|$ and hence $\mathbb{Q}_-$ is countable too. We define $f : \mathbb{Q}_+ \to \mathbb{Q}_-$ as $f(x) = -x$.
We can define $\mathbb{Q} = \mathbb{Q}_+ \cup \mathbb{Q}_-$, and we know that the union of two countable sets is countable. $\square$

Exercise 3.6