Discrete Mathematics in Computer Science
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Exercise 3

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Exercise 3.1

(a)

(b)

Exercise 3.2

(a)

(b)

Exercise 3.3

We will refute the statement for all sets A, B it holds that $|A| < |A \cup B|$. Let $A = B$, then we have $A = A \cup B$. In this case it holds that $|A| = |A \cup B|$, which contradicts the statement.

Exercise 3.4

Let $g \colon \{n \in \mathbb{N} \mid n \bmod 2 = 0\} \to \mathbb{Z} \colon g(n) = -\frac{n}{2}$. The function $g(x)$ is a bijective function because it maps every even natural number to every negated natural number (which is injective and surjective). Let $s \colon \{n \in \mathbb{N} \mid n \bmod 2 \neq 0\} \to \mathbb{Z} \colon g(n) = \frac{n+1}{2}$. The function $s(x)$ is a bijective function because it maps every odd natural number to a natural number (it is a subset of the natural numbers). The union of two bijective functions is bijective, therefore

$$f(n) = \begin{cases} -\frac{n}{2}, & \text{if } n \text{ is even,} \\ \frac{n+1}{2}, & \text{otherwise,} \end{cases} \quad \text{for } n \in \mathbb{N},$$

is a bijective function from $\mathbb{N}$ to $\mathbb{Z}$ and thus $\mathbb{Z}$ is countable.

Exercise 3.5

Exercise 3.6

I will first proof that the set S containing all finite strings that consist only of symbols from the set $\Sigma = \{✈, ♣, ♥\}$ is countable.
For each $n \in \mathbb{N}$, there are 3^n strings of length n, which is finite. We can then enumerate all strings by length, starting with strings of length 0, then length 1, and so on. Within each length n, we enumerate the 3^n strings in some specific order. By mapping each string to a unique natural number based on its position in the complete enumeration, starting from 0, we establish a one-to-one correspondence between S and $\mathbb{N}$. Thus, the set S is countable.
Since all strings in the set of the tarradiddles T consist of symbols from Σ we have that $T \subseteq S$. Because the subset of a countable set is countable, T is countable.

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