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Exercise 3.1

$A, B \subseteq \{1, \dots, 10\}$

$|B| = 3$, so we choose arbitrarily $B = \{1, 2, 3\}$.

$|\mathcal{P}(A)| = 16 \iff |A| = \log_2(16) = 4$

$|A \cup B| = 5$ so to reach sum of 5, A has to share 2 Elements with B while having 4 in total.

$A = \{2, 3, 4, 5\}$ satisfies these conditions.

Exercise 3.2

(a) $\{e1, e4, e6, e7\} = 01001011$

(b) $01110110 = \{e1, e2, e3, e5, e6\}$

(c) $A \cup B \iff$ A bitwise OR B
 $A \cap B \iff$ A bitwise AND B
 $\neg A \iff$ bitwise NOT A

Exercise 3.3

We will refute the statement for all sets A, B it holds that $|A| < |A \cup B|$. Let $A = B$, then we have $A = A \cup B$. In this case it holds that $|A| = |A \cup B|$, which contradicts the statement.

Exercise 3.4

Let $g: \{n \in \mathbb{N} \mid n \bmod 2 = 0\} \rightarrow \mathbb{Z}: g(n) = -\frac{n}{2}$. The function $g(x)$ is a bijective function because it maps every even natural number to every negated natural number (which is injective and surjective).

Let $s: \{n \in \mathbb{N} \mid n \bmod 2 \neq 0\} \rightarrow \mathbb{Z}: s(n) = \frac{n+1}{2}$. The function $s(x)$ is a bijective function because it maps every odd natural number to a natural number (it is a subset of the natural numbers).

The union of two bijective functions is bijective, therefore

$$f(n) = \begin{cases} -\frac{n}{2}, & \text{if } n \text{ is even,} \\ \frac{n+1}{2}, & \text{otherwise,} \end{cases} \quad \text{for } n \in \mathbb{N},$$

is a bijective function from \mathbb{N} to \mathbb{Z} and thus \mathbb{Z} is countable.

Exercise 3.5

Exercise 3.6

I will first proof that the set S containing all finite strings that consist only of symbols from the set $\Sigma = \{\rightarrow, \clubsuit, \heartsuit\}$ is countable.

For each $n \in \mathbb{N}$, there are 3^n strings of length n , which is finite. We can then enumerate all strings by length, starting with strings of length 0, then length 1, and so on. Within each length n , we enumerate the 3^n strings in some specific order. By mapping each string to a unique natural number based on its position in the complete enumeration, starting from 0, we establish a one-to-one correspondence between S and \mathbb{N} . Thus, the set S is countable.

Since all strings in the set of the tarradiddles T consist of symbols from Σ we have that $T \subseteq S$. Because the subset of a countable set is countable, T is countable.

Exercise 3.5

Proof. We want to show that \mathbb{Q} is countable. We already know that \mathbb{Q}_+ is countable