Alex Lutsch
Ephraim Siegfried

Exercise 4.1

(a) We have that $\{2,3\} \times \emptyset = \{\langle a,b \rangle \mid a \in \langle 2,3 \rangle \text{ and } b \in \emptyset\}$. But there exists no b for which $b \in \emptyset$ is true. Therefore the statement is wrong and the correct statement would be $\{2,3\} \times \emptyset = \emptyset$.

(b) With the definition of the cartesian product we have that $\{\langle 1,0 \rangle\} \times \{0\} = \{\langle \langle 1,0 \rangle, 0 \rangle\} =$ S. Neither of the elements in the right-hand side of the statement are elements of S, therefore the statement is wrong.

(c) The order matters in tuples, therefore $\langle 1,2 \rangle \neq \langle 2,1 \rangle$. Thus the set on the left-hand side is not a subset of the set in the right-hand side and vice versa, which means the two sets are not equal. Therefore the statement is wrong.

(d) The cartesian product of the two sets $C = \{0,1,2\} \times \{3,4,5\}$ results in a set of tuples. The set $\{2,4\}$ is not a tuple. Therefore it cannot be element of C, which means that the statement is wrong.

Exercise 4.2

(a) $A = \{2,3,5\} = B$

(b) $|A \cup B| = 4$ so $A \cup B = \{1,6,4,x\}$
$|A \times B| = 6$ so $|A| * |B| = 6$
$\langle \langle 1,6 \rangle, 4 \rangle \in (A \times A) \times B$ so $\{1,6\} \in A$ and $\{4\} \in B$
$A = \{1,6,3\}$ $B = \{3,4\}$

Exercise 4.3

(a) $R_1 = \{\langle a,a \rangle, \langle c,b \rangle, \langle b,c \rangle\}$

(b) This is not possible, which I will show by contradiction. Suppose it was possible, then per definition $\langle a,b \rangle \in R_2$. Because the relation should also be symmetric, we know that $\langle b,a \rangle \in R_2$. With the transitivity property, it must be the case that if $\langle a,b \rangle \in R_2$ and $\langle b,a \rangle \in R_2$, then $\langle a,a \rangle \in R_2$. But if $\langle a,a \rangle \in R_2$ then the relation is not irreflexive. This is a contradiction and therefore this relation cannot exist.

Exercise 4.4

We examine the following binary Relation: $R = \{\langle i, j * i \rangle | i, j \in \mathbb{N}_0\}$
Since $i, j \in \mathbb{N}_0, i = (j * i)$ because every number of i can be formed with setting j one or zero and the multiplication of two natural numbers is a natural number which is i.
It is reflexive since $i = (j * i)$.
It's not irreflexive since its reflexive.
It's symetric since any natural number appears on both sides of the tuple.
It's not asymmetric or antisymmetric since its symetric.
Since both elements of the tuple represent all possible natural numbers we can guarantee transitivity.