Discrete Mathematics in Computer Science

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Exercise Sheet 11

Due: Monday, December 11, 2023, 4pm

Please carefully read the exercises FAQ on ADAM!

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file.

Exercise 11.1 (3 marks)

Show that $\varphi = \Big(\neg\big((\neg B \wedge D) \wedge (C \vee D)\big) \wedge (B \vee A)\Big)$ is equivalent to $(B \vee (\neg D \wedge A))$ using *only* the equivalence rules from slides 17 to 19 of chapter D2. Apply only one equivalence rule in each step, and specify which rule you applied. Do not use any rules implicitly.
Hint: Our model solution consists of 6 steps, using the following rules (not in this order): absorption, associativity, commutativity, de Morgan, distributivity, double negation.

Exercise 11.2 (1 mark)

Prove or disprove: There is no formula that is both a clause and a monomial.

Exercise 11.3 (2 marks)

Let L be the set of all Discrete Mathematics lectures this semester, and $\texttt{missed-}l$ for all $l \in L$ denote that you missed lecture l. Describe the following formulas *concisely* in natural language.

(a) $\bigwedge_{l,l' \in L, l \neq l'} (\neg\texttt{missed-}l \vee \neg\texttt{missed-}l')$

(b) $(\bigwedge_{l,l' \in L, l \neq l'} (\neg\texttt{missed-}l \vee \neg\texttt{missed-}l') \wedge \bigvee_{l \in L} \texttt{missed-}l)$

Exercise 11.4 (2 marks)

Transform $\chi = \Big(\neg B \rightarrow \big((\neg A \vee C) \leftrightarrow B\big)\Big)$ into CNF by applying the algorithm on slide 17 of chapter D3. At the least you must show the formula after each step of the algorithm; we would however advise an even more fine-grained solution since according to experience smaller steps help to reduce the amount of errors.
You are allowed to abbreviate parentheses for conjunctions of conjunctions and disjunctions of disjunctions; for example you can write $(A_1 \wedge A_2 \wedge A_3)$.

Exercise 11.5 (2 marks)

Consider the CNF formula $\varphi_n = \bigwedge_{i=1}^{n} (a_0 \vee a_i)$ parameterized by $n \in \mathbb{N}_1$.

(a) If we would transform the formula into DNF using the algorithm from slide 17 of chapter D3 *without simplifying* the formula at any time, how many monomials would the resulting formula have? Specify your answer in relation to n and briefly justify your answer.

Hint: The formula would look similar to the example on slide 21 of chapter D3.

(b) Specify a DNF formula that is equivalent to φ but has size polynomial in n, and briefly explain why the formula is equivalent.

Hint: There exists an equivalent DNF formula with just 2 monomials.

Submission rules:
Upload a single PDF file (ending in .pdf). Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). There is a template that satisfies these requirements available on ADAM.