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Exercise 1.1

We will proof the following statement with a direct proof:

For all sets A, B, C it holds that $(A \cap B) \cup (A \cap C) \subset A \cap (B \cup C)$

Proof. Let A, B, C be arbitrary sets. We will show that $x \in (A \cap B) \cup (A \cap C)$ implies $x \in A \cap (B \cup C)$.

Consider any $x \in (A \cap B) \cup (A \cap C)$. By the definition of the union it holds that $x \in (A \cap B)$ or $x \in (A \cap C)$. With the definition of the intersection this means that ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$). In both cases of the disjunction we have that $x \in A$, therefore this implies that $x \in A$ and ($x \in A$ or $x \in C$), which is equivalent to $x \in A \cap (A \cup B)$. □

Exercise 1.2

Proof by contradiction to establish the following theorem:

For all sets A and B: If $(A \cap B) = \emptyset$, then $(A \setminus B) = A$.

Proof. Assume if $A \cap B = \emptyset$, then $A \setminus B \neq A$
Since $(A \setminus B) \neq A$ there is a $x \in B$ that's also $x \in A$. But since $A \cap B = \emptyset$ this is impossible. Hence the original statement must hold. □

Exercise 1.3

We will proof the following statement by contrapositive:

For all sets A, B we have if $A \cup B = B$ then $A \subseteq B$

Proof. Let A, B be arbitrary sets. We will show that if $A \nsubseteq B$ then $A \cup B \neq B$. Since $A \nsubseteq B$ there exists at least one x with $x \in A$ and $x \notin B$. Note that $A \subseteq A \cup B$ since all elements of A are elements of A or B. This means that $A \cup B$ contains at least one element which is not in B. Therefore not all elements of $A \cup B$ are elements of B, thus $A \cup B \nsubseteq B$ and by definition $A \cup B \neq B$. □

Exercise 1.4

We examine the following statement:

For all sets A, B and C: if $A \subseteq (B \cup C)$, then $A \subseteq B$ or $A \subseteq C$.

We choose a set $A = B \cup C$ where $B \neq C$ and $B, C \neq \emptyset$. It follows that $A \nsubseteq B$ and $A \nsubseteq C$ since B and C are now strict subsets of A. Since $A \subseteq (B \cap C)$ holds this contradicts the statement.