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Exercise 11.1

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\begin{aligned}
(\neg((\neg B \wedge D) \wedge (C \vee D)) \wedge (B \vee A)) &\equiv (\neg(\neg B \wedge (D \wedge (C \vee D)) \wedge (B \vee A))) && \text{(Associativity)} \\
&\equiv (\neg(\neg B \wedge (D \wedge (D \vee C)) \wedge (B \vee A))) && \text{(Commutativity)} \\
&\equiv (\neg(\neg B \wedge D) \wedge (B \vee A))) && \text{(Absorption)} \\
&\equiv ((\neg\neg B \vee \neg D) \wedge (B \vee A)) && \text{(De Morgan)} \\
&\equiv ((B \vee \neg D) \wedge (B \vee A)) && \text{(Double Negation)} \\
&\equiv (B \vee (\neg D \wedge A)) && \text{(Distributivity)}
\end{aligned}
$$

Exercise 11.2

I will disprove the statement: Every single Literal is a clause and a monomial, because the terms clause and monomial are also used for the corner case with only one literal. Every literal is also a formula. Therefore there exists a formula which is both a monomial and a clause.

Exercise 11.3

(a)

(b)

Exercise 11.4

Exercise 11.5

(a) Because of distributivity we can transform the formula from CNF to DNF like this: $\phi_n = \bigwedge_{i=1}^{n} (a_0 \vee a_i) \equiv \bigvee_{S \in \mathcal{P}(\{1,\ldots,n\})} (a_0 \wedge \bigwedge_{i \in \{1,\ldots,n\}} a_i)$. Because the cardinality of the power set is $\mathcal{P}(1, \ldots, n) = 2^n$, we will have 2^n monomials.

(b) The equivalent form is $\phi_n = \bigwedge_{i=1}^{n} (a_0 \vee a_i) \equiv a_0 \vee \bigwedge_{i=1}^{n} a_i$ and has has size polynomial in n. The equivalence can be explained by the fact that if a_0 is true, the original formula is satisfied (since a_0 appears in every clause of the CNF), and if a_0 is false, then all a_i must be true for the original formula to be satisfied, which is exactly what the specified formula expresses.