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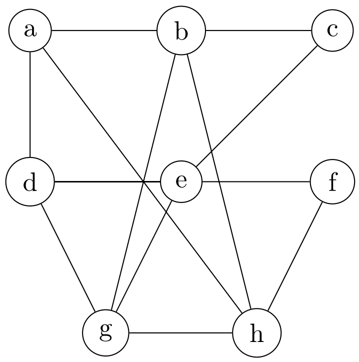
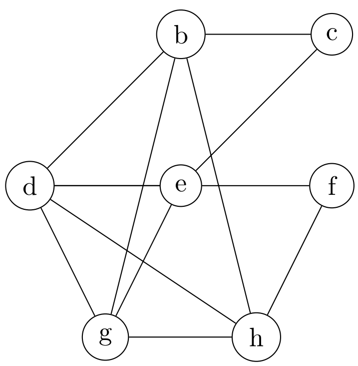
Exercise 10.1

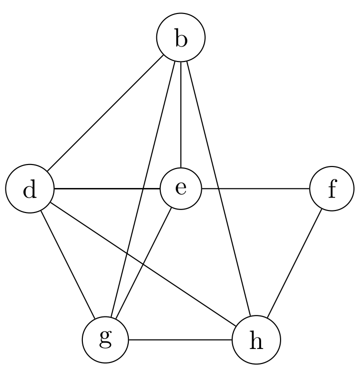
We will do three transformations to G to show that it has K_5 as a minor. This is the original graph:



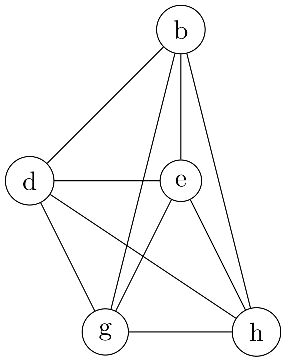
1. Contract $\{a, d\}$



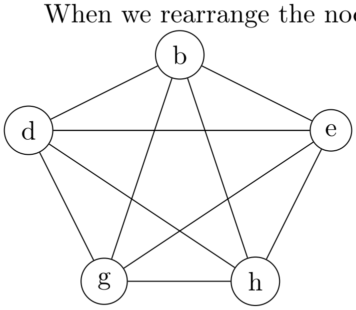
2. Contract $\{c, e\}$



3. Contract $\{f, e\}$



When we rearrange the nodes we can clearly see that it's a K_5 graph. Thus G is not planar.



Exercise 10.2

(a) The syntactic interpretation is correct.
Semantically what the natural language means to say is: Eat now or (eat later then food cold).
Which would be: Eat Now $\vee$(eat later $\rightarrow$ food cold).

(b) The syntactic interpretation is incorrect. What it expresses is Swimming $\rightarrow \neg$ Storm.
That is also the semantic meaning.

Exercise 10.3

(a) $\mathcal{I} = \{X \mapsto 0, Y \mapsto 1, Z \mapsto 0\}$.

(b) 1) $\mathcal{I} \models Y$
2) $\mathcal{I} \not\models Z$
3) $\mathcal{I} \models \neg Z$
4) $\mathcal{I} \models Y \wedge \neg Z$
5) $\mathcal{I} \models \psi \vee (Y \wedge \neg Z)$ for all formulas ψ.
6) $\mathcal{I} \not\models Y$
7) $\mathcal{I} \models \neg Y$

8) $\mathcal{I} \models \neg Y \vee \psi$ for all formulas ψ.
9) From 5) and 8) we get $\mathcal{I} \models (\psi \vee (Y \wedge \neg Z)) \wedge (\neg Y \vee \psi)$ for all formulas ψ.
   In particular $(X \vee (Y \wedge \neg Z)) \wedge (\neg Y \vee \neg X)$.

(c) $\mathcal{I} = \{X \mapsto 0, Y \mapsto 0, Z \mapsto 0\}$.

Exercise 10.4

(a) We will analyze the statement $\phi = (((Y \wedge Z) \rightarrow X) \vee \neg(X \vee \neg Z))$ over $\{X, Y, Z\}$.
Satisfiability: Let $\mathcal{I} = \{X \mapsto 0, Y \mapsto 1, Z \mapsto 1\}$. We have $\mathcal{I} \models X$ and $\mathcal{I} \not\models Z$. From the definition of negation we have that $\mathcal{I} \models \neg Z$. Therefore, with the definition of disjuntion, we get $\mathcal{I} \not\models (X \vee \neg Z)$. With the definition of negation we have $\mathcal{I} \models \neg(X \vee \neg Z)$. With the definition of disjuntion we have $\mathcal{I} \models \psi \vee \neg(X \vee \neg Z)$ for all formulas ψ, in particular $\mathcal{I} \models (((Y \wedge Z) \rightarrow X) \vee \neg(X \vee \neg Z))$. Therefore ϕ is satisfiable.
Falsifiability: The statement ϕ is by the definition of disjuntion and negation false, iff both $((Y \wedge Z) \rightarrow X)$ and $\neg(X \vee \neg Z)$ are false. The formula $((Y \wedge Z) \rightarrow X)$ is only false, if $\mathcal{I} = \{X \mapsto 0, Y \mapsto 1, Z \mapsto 1\}$. But under this interpretation, the formula $\neg(X \vee \neg Z)$ is true. Therefore ϕ can never be false.
Validness The formula ϕ is valid, since it can never be false.
Unsatisfiability The formula is not unsatisfiable because it's satisfiable.

(b) $\phi = (A \wedge \neg A) \wedge (B \wedge \neg B)$