Aerodynamics Computational Assignment #2: Flow Over Thin Airfoils

Assigned Date: February 14, 2023
Due Date: March 3, 2023

Collaboration Policy:

Collaboration is permitted on the computational labs. You may discuss the means and methods for formulating and solving problems and even compare answers, but you are not free to copy someone else's work. *Copying material from any resource (including solutions manuals) and submitting it as one's own is considered plagiarism and is an Honor Code violation.*

Matlab Code Policy:

Computational codes must be written individually and are expected to be written in MATLAB. If you have collaborated with others while writing your code be sure to acknowledge them in the header of your code, otherwise you may receive a zero for plagiarism. All code files required to successfully run the computational assignment driver script should be submitted via the course website by 11:59pm on the due date. Code files will not be accepted after the given due date.

Reflection Questions:

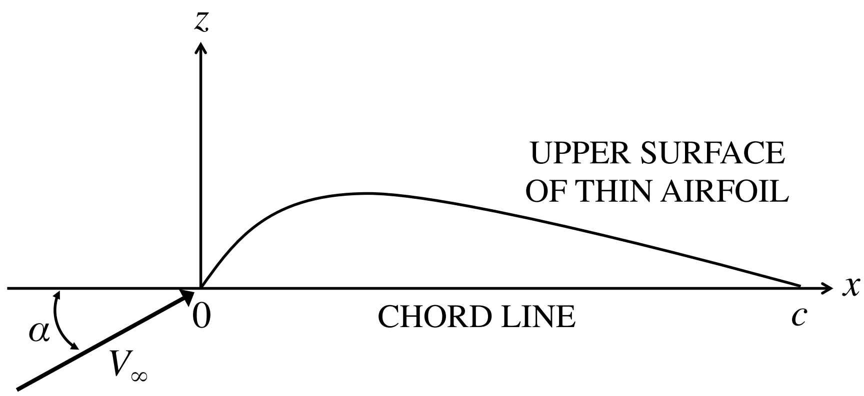
In this assignment, there are multiple reflection questions. These reflection questions are provided to help you review the functionality of your code, help you analyze and understand your results, and to test your understanding of the concepts being studied.

Learning Outcomes:

1. Understand how thin airfoil theory is used to approximate aerodynamic forces.
2. Practice using the superposition of elementary flows to complete analysis of an airfoil.
3. Understand the effect of flow parameters on streamlines, equipotential lines, and pressure contours.

Problem Description:

The flow about a thin symmetric airfoil can be approximated by potential flow theory, as elaborated in Chapter 4 of Anderson. Suppose that the chord of the airfoil extends along the x-axis from $x = 0$ to $x = c$ as illustrated in the below figure:

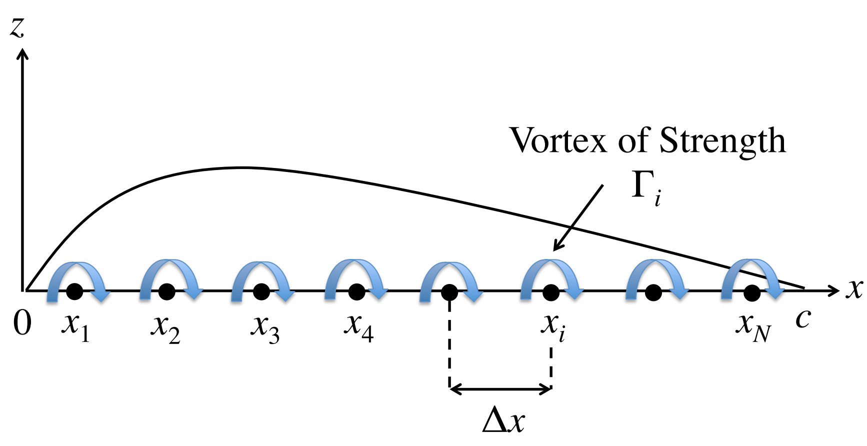


Then, the flow is represented by a vortex sheet whose strength $\gamma(x)$ is given by:

$$\gamma(x) = 2\alpha V_\infty \sqrt{\frac{1-\frac{x}{c}}{\frac{x}{c}}}$$

where α is the angle of attack of the incoming flow relative to the x-axis and V_∞ is the free-stream flow speed.

Numerically, one can approximate the vortex sheet by a set of N discrete vortices separated by a distance $\Delta x = c/N$ where the i^{th} vortex has strength $\Gamma_i = \gamma(x_i)\Delta x$. This is depicted in the below figure:



Write a MATLAB function which plots the stream lines, equipotential lines, and pressure contours for flow about a thin symmetric airfoil using the approximations detailed above.

Your function should take the form:

```
function Plot_Airfoil_Flow(c,alpha,V_inf,p_inf,rho_inf,N)
```

where `c` is the chord length c (in meters), `alpha` is the angle of attack α (in degrees), `V_inf` is the free-stream flow speed V_∞ (in meters per second), `p_inf` is the free-stream pressure p_∞ (in Pascals), `rho_inf` is the free-stream density ρ_∞ (in kilograms per meter cubed), and `N` is the number of discrete vortices N employed to approximate the vortex sheet.

Using your MATLAB function,

- Visualize or generate plots of the stream lines, equipotential lines, and pressure contours for flow about a thin symmetric airfoil with $c = 5$ m, $\alpha = 15°$, $V_\infty = 34$ m/s, $p_\infty = 101.3 \times 10^3$ Pa, and $\rho_\infty = 1.225$ kg/m^3.
- Conduct a study of the effect of the number of discrete vortices N on the resulting flow and pressure field accuracy for the aforementioned values. The assessment of accuracy should not only be qualitative in nature but also quantitative. Namely, a study of error in velocity and pressure as a function of N should be conducted. It is up to the student to define a measure (or measures) of error for this problem. Generate a plot of the convergence of the chosen error assessment versus the the number of discrete vortices N.
- Conduct a study on how stream lines and equipotential lines are affected by changes in: 1) chord length, 2) angle of attack, and 3) free-stream flow speed. For this study, it is recommended that you graphically compare these variables using a figure with sub-plots so that the changes in each variable can be compared side-by-side. To best facilitate this comparison ensure that your plots are consistently scaled and rendered.

Reflection: Evaluate the pressure contours, and identify the locations of minimum and maximum pressure. How do these change as the above conditions are altered? Consider the streamlines and equipotential lines; are the fields continuous? What does this imply?

Hint: You will need to use the principle of superposition within your MATLAB function. This principle is illustrated in the MATLAB file `Lifting_Cylinder.m` located on the course website, wherein the stream lines for the flow around a cylinder with circulation are plotted by superposing flow from a uniform flow, a dipole, and a vortex. It is recommended that you follow the logic of `Lifting_Cylinder.m` when building your function `Plot_Airfoil_Flow`.