Statistical Orbit Determination
B-plane Introduction for Project 2

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The B-plane is picture in its classical drawing representation in Fig. 1. The basics of the B-plane are derived in 2 papers by William Kizner that are on Canvas, although the basics are repeated here for your reference.

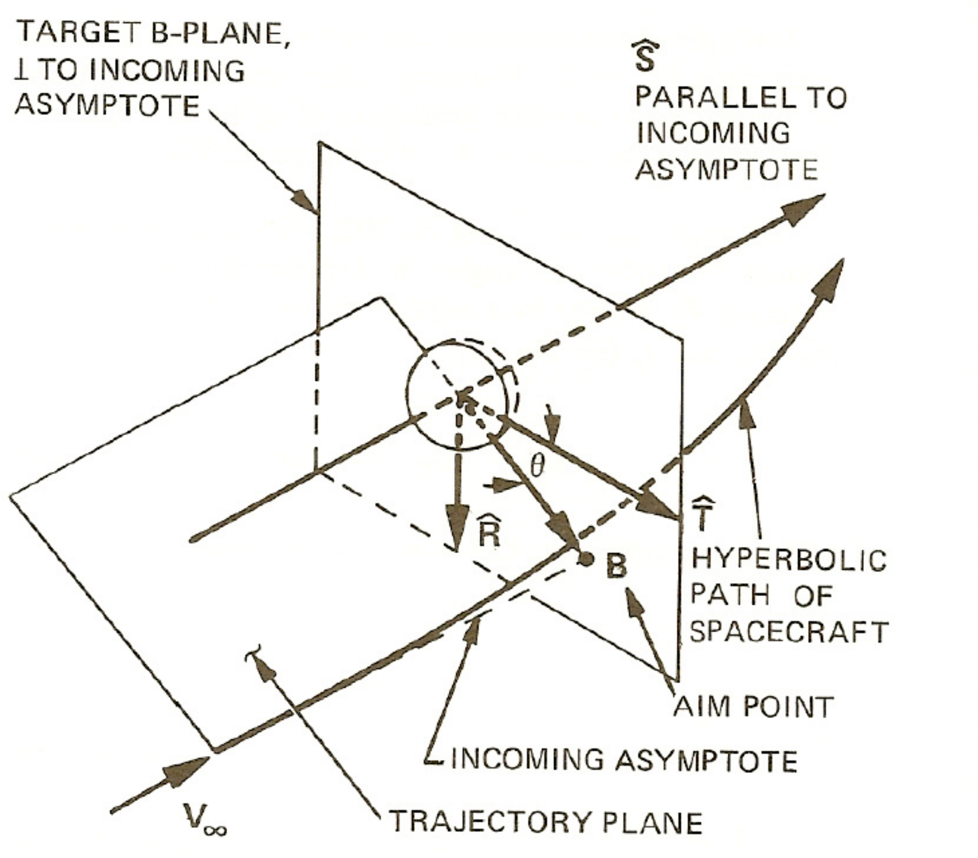

Figure 1: The standard B-plane illustration

Given the current state, $\mathbf{r}$ and $\mathbf{v}$ (note non-bolded means magnitude of the vector), first compute the perifocal frame, $\{\hat{\mathbf{P}}, \hat{\mathbf{Q}}, \hat{\mathbf{W}}, \}$ where $\hat{\mathbf{P}}$ points to periapse $\hat{\mathbf{W}}$ is along the orbit normal. Also compute the semimajor axis, a, and the eccentricity, e, as usual, using the following equations:

$$\mathbf{e} = \frac{1}{\mu}\left[(v^2 - \frac{\mu}{r})\mathbf{r} - (\mathbf{r}\cdot\mathbf{v})\mathbf{v}\right]; \tag{1}$$

$$\hat{\mathbf{P}} = \frac{\mathbf{e}}{e} \tag{2}$$

$$\mathbf{h} = \mathbf{r} \times \mathbf{v} \tag{3}$$

$$\hat{\mathbf{W}} = \frac{\mathbf{h}}{h} \tag{4}$$

$$\hat{\mathbf{Q}} = \hat{\mathbf{W}} \times \hat{\mathbf{P}} \tag{5}$$

Solve for the semimajor axis from the vis-viva equation (note for a hyperbolic orbit, semimajor axis is negative)

$$\frac{-\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r} \tag{6}$$

$$p = \frac{h^2}{\mu} \tag{7}$$

$$e = |\mathbf{e}| = 1 - \frac{p}{a} \tag{8}$$

The semiminor axis can be computed as

$$b = |a|\sqrt{e^2 - 1} \tag{9}$$

Recall that $\hat{\mathbf{S}}$ is in the direction of the incoming $\mathbf{v}_\infty$

$$\hat{\mathbf{S}} = \frac{\mathbf{v}_\infty}{v_\infty} \tag{10}$$

Take $\hat{\mathbf{N}} = [1 \quad 0 \quad 0]^T$ (in Earth centered coordinates, thus this points at the North pole). We then find

$$\hat{\mathbf{T}} = \frac{\hat{\mathbf{S}} \times \hat{\mathbf{N}}}{|\hat{\mathbf{S}} \times \hat{\mathbf{N}}|} \tag{11}$$

and

$$\hat{\mathbf{R}} = \hat{\mathbf{S}} \times \hat{\mathbf{T}} \tag{12}$$

The B-vector is then given by

$$\mathbf{B} = b\left(\hat{\mathbf{S}} \times \hat{\mathbf{W}}\right) \tag{13}$$

Use $\hat{\mathbf{S}}$, $\hat{\mathbf{T}}$ and $\hat{\mathbf{R}}$ to determine a DCM to rotate from your ECI frame to the B-plane frame.

The major outstanding question is when to call $\mathbf{v} = \mathbf{v}_\infty$? Theoretically, this is done at the end of the hyperbolic orbit, but that doesn't have a real meaning in reality. Thus it is typically done in some relation to the sphere of influence of the planet being flown by. In this case, we will declare the point at which we determine $\mathbf{v}_\infty$ when the distance from Earth, r, is equal to 3 RSOI of Earth (where 1 RSOI = 925,000 km).

Finally, you must consider the time at which you cross the B-plane in order to understand when to propagate your fit. Do this using the Linearized Time of Flight $LTOF$.

$$LTOF = \frac{\mu}{v_\infty^3}\left(\sinh(f) - f\right) \tag{14}$$

$$f = \text{arcCosh}\left(1 + \frac{v_\infty^2}{\mu}\frac{a(1-e^2)}{1+e\cos(\nu)}\right) \tag{15}$$

and the true anomaly is just the angle between periapse and the current position,

$$\cos(\nu) = \frac{\mathbf{r}}{r}\cdot\hat{\mathbf{P}} \tag{16}$$

The follow the steps below:

- Integrate to 3 RSOI and calculate $LTOF$. This determines the time of B-plane crossing (which is $LTOF$ seconds after the 3 RSOI time).
- Integrate from Data Cut Off (DCO) to B-plane crossing.
- Map final filter uncertainty using STM from DCO to B-plane crossing
- Either map state deviation to B-plane crossing or use Best Estimate of the state and integrate to B-plane crossing
- Rotate covariance into the STR frame
- Use the TR portion of the 3x3 position covariance and calculate the uncertainty ellipsoid parameters and orientation
- Plot the BdotR and BdotT location of the actual trajectory intersection
- Plot the uncertainty ellipse located around that location