# Aerodynamics Computational Assignment #1: Computation of Lift and Drag

**Assigned Date:** Tuesday, January 17, 2023
**Due Date:** Friday, February 10, 2023

**Collaboration Policy:**

Collaboration is permitted on the computational labs. You may discuss the means and methods for formulating and solving problems and even compare answers, but you are not free to copy someone else's work. *Copying material from any resource (including solutions manuals) and submitting it as one's own is considered plagiarism and is an Honor Code violation.*

**Matlab Code Policy:**

Computational codes must be written individually and are expected to be written in MATLAB. If you have collaborated with others while writing your code be sure to acknowledge them in the header of your code, otherwise you may receive a zero for plagiarism. All code files required to successfully run the computational assignment driver script should be submitted via the course website in a single .zip file by 11:59pm on the due date. Code files will not be accepted after the given due date.

**Reflection Questions:**

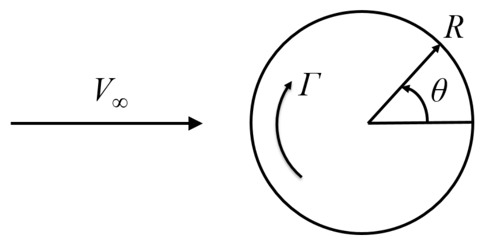
In this assignment, there are multiple reflection questions. These reflection questions are provided to help you review the functionality of your code, help you analyze and understand your results, and to test your understanding of the concepts being studied.

**Learning Outcomes:**

1. Understand how to integrate pressure values to calculate lift and drag.
2. Understand different numerical integration techniques and when to use them.
3. Understand the effect of the number of pressure sensors and their placement on the accuracy of calculated lift and drag.
4. Understand the effects of different models on lift and drag.

**Problem #1:**

Consider ideal (incompressible and inviscid) flow over a rotating cylinder of radius $R$ as depicted in the figure below:




The coefficient of pressure, defined as:

$$C_p \equiv \frac{p - p_\infty}{q_\infty}$$

where $p$ is the static pressure, $p_\infty$ is the freestream static pressure, $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$ is the freestream dynamic pressure, $\rho_\infty$ is the freestream density, and $V_\infty$ is the freestream airspeed, is known to be:

$$C_p(\theta) = 1 - \left[4\sin^2(\theta) + \frac{2\Gamma\sin(\theta)}{\pi R V_\infty} + \left(\frac{\Gamma}{2\pi R V_\infty}\right)^2\right]$$

where $\Gamma$ represents the circulation about the cylinder, and the sectional coefficients of lift and drag, defined as:

$$c_l \equiv \frac{L'}{2q_\infty R} \quad \text{and} \quad c_d \equiv \frac{D'}{2q_\infty R}$$

where $L'$ and $D'$ are the lift and drag per unit span, are known to be

$$c_l = -\frac{1}{2}\int_0^{2\pi} C_p(\theta)\sin(\theta)d\theta \quad \text{and} \quad c_d = -\frac{1}{2}\int_0^{2\pi} C_p(\theta)\cos(\theta)d\theta$$

For $\Gamma = 2\pi R V_\infty$, complete the following tasks:

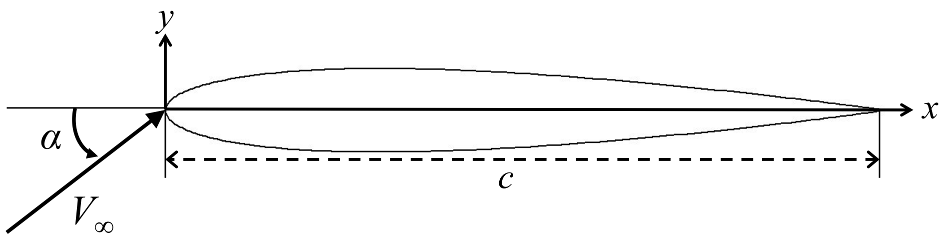
- Analytically determine the sectional lift and drag coefficients and print these values to the command window.
- Produce plots of the sectional lift and drag coefficients predicted by the composite trapezoidal rule versus the number of panels, N, used to discretize the surface of the cylinder.
- Produce plots of the sectional lift and drag coefficients predicted by the composite Simpson's rule versus the number of panels, N, used to discretize the surface of the cylinder.
- Print to the command window the number of panels, N, required to achieve a predicted sectional lift coefficient to within one percent relative error using the composite trapezoidal rule.

- Print to the command window the number of panels, N, required to achieve a predicted sectional lift coefficient to within one percent relative error using the composite Simpson's rule.

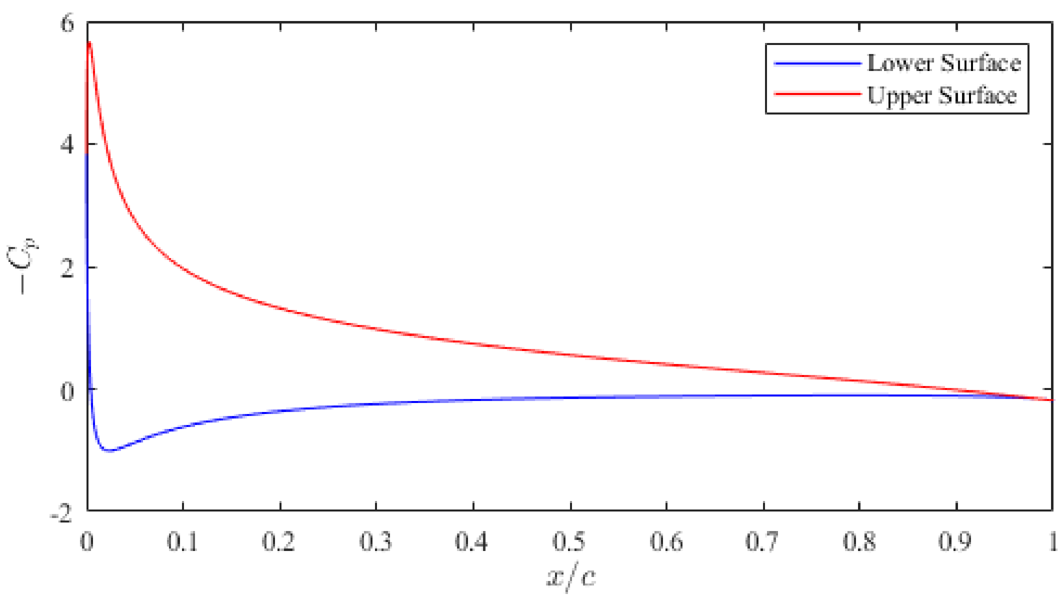
**Reflection:** How do the two methods compare? Would they still perform the same way if you were given experimental data in the form of a discrete array of pressure coefficient values instead of being given an exact solution/equation?

**Problem #2:**

Consider turbulent flow over a NACA 0012 airfoil at an angle of attack of $\alpha = 10°$ and a Reynolds number of $Re = V_\infty c/\nu = 10$ million where $V_\infty$ is the freestream airspeed, $c$ is the chord length, and $\nu$ is the kinematic viscosity. This flow is depicted graphically below:




For this flow problem, the coefficient of pressure cannot be determined analytically. However, one can instead turn to computational fluid dynamics simulations using a turbulence model to predict the coefficient of pressure around the airfoil. One such turbulence model is the widely used Spalart-Allmaras model, introduced by Philippe Spalart and Steven Allmaras in 1992. Application of the Spalart-Allmaras model to the problem at hand yields the following coefficient of pressure approximations along the upper and lower surfaces of the NACA 0012 airfoil respectively:




The Spalart-Allmaras model results have been further interpolated using splines and the results are stored within a MATLAB .mat file `Cp.mat` located in the Lab directory on the course website. To open the MATLAB file, type `load Cp` into the Command Window. This will load two spline variables, `Cp_upper` and `Cp_lower`, into the Workspace. Then, to evaluate the coefficient of pressure along some location $x/c$ along the upper surface, simply type `fnval(Cp_upper, x/c)`. Similarly, to evaluate the coefficient of pressure then along some location $x/c$ along the lower surface, type `fnval(Cp_lower, x/c)`. Note that you need to replace the variable $x/c$ with a numeric position to get the $C_p$ value at that position.

Using the MATLAB spline variables `Cp_upper` and `Cp_lower` and the composite trapezoidal rule, determine and print to the command window the lift and drag (per unit span) on a stationary NACA 0012 airfoil with chord length $c = 3$ m at 10° angle of attack in a turbulent flow with freestream airspeed $V_\infty = 50$ m/s, air density $\rho_\infty = 1.225$ kg/m$^3$, and pressure $p_\infty = 101.3 \times 10^3$ Pa. Ignore skin friction contributions in your calculations.

In addition to the above, complete the following tasks:

- Determine and print to the command window the number of equispaced (with respect to chord line distance, $x$) integration points, n, required to obtain a lift solution to within one percent relative error.
- Determine and print to the command window the number of equispaced (with respect to chord line distance, $x$) integration points, n, required to obtain a drag solution to within one percent relative error.

Above n refers to the total number of integration points along the airfoil surface, not just the points on either the lower or upper surfaces.

**Reflection:** Given the number of required equispaced integration points required to obtain accurate lift and drag solutions, how should one go about measuring pressure in the wind tunnel to experimentally determine the lift and drag coefficients? If the number of experimental pressure ports is limited how can you best locate these ports along to surface to improve the accuracy in the lift and drag estimates?

**Note:** The formula for the shape of a NACA 00xx airfoil, with "xx" being replaced by the percentage of thickness to chord, is:

$$y_t = \frac{t}{0.2}c\left[0.2969\sqrt{\frac{x}{c}} - 0.1260\left(\frac{x}{c}\right) - 0.3516\left(\frac{x}{c}\right)^2 + 0.2843\left(\frac{x}{c}\right)^3 - 0.1036\left(\frac{x}{c}\right)^4\right]$$

where $c$ is the chord length, $x$ is the position along the chord from 0 to $c$, $y_t$ is the half thickness at a given value of $x$ (centerline to surface), and $t$ is the maximum thickness as a fraction of the chord (i.e., $t = \text{xx}/100$).