

ASEN 6080 HW 3

- dan Faber, 108577813

- i. b. Use the filters with SNC to process the data from HW2 with J_3 in the dynamics, but not the filter model. Characterize the performance of the filters over the range $\sigma \in [10^{-15}, 10^{-2}] \text{ km/s}^2$.

- i. Plot the post-bit RMS of the measurements as a function of σ

See PDF

- ii. Plot the 3D position + velocity RMS vs. σ

See PDF

- iii. What is the optimal value of σ to use? Why?

I found that the optimal value of σ to use was

$$\sigma = 10^{-8} \text{ km/s}^2,$$

as it balances postbit RMS as close to 1 as possible while also minimizing 3D RMS. In fact, 10^{-8} km/s^2 results in postbit RMS's of 1.0006 for LKF and 1.0021 for EKf, with respective 3D RMS's of 0.155044 and 0.281466.

IV. Show the filter performance with the optimal σ by plotting the time history of the state errors with covariance bounds, as well as the postfit residuals.

See PDF.

C. Define Q in the RIC frame and discuss its performance relative to Q in the ECI frame.

See PDF for plots.

Defining Q in the RIC frame as diagonal with the same σ as in ECI didn't affect the performance at all. This makes sense, as defining Q as diagonal with all identical entries essentially defines the initial covariance ellipse as a sphere, which is symmetrical in every orientation. Thus, rotation doesn't have any effect.

d. How else could $T(t_i, t_{i-1})$ be evaluated other than analytically?

We could numerically integrate T instead. In fact, this is very easy to do based on how I've implemented my filter.

We know that

$$T'(t_i, t_{i-1}) = \int_{t_{i-1}}^{t_i} \Phi(t_i, \tau) B(\tau) d\tau,$$

where $B = [0_{3 \times 3}, I_{3 \times 3}]^T$. Thus,

$$T(t_i, t_{i-1}) = \int_{t_{i-1}}^{t_i} \Phi(t_i, \tau) d\tau \cdot B$$

Since I already integrate Φ in my filters from t_{i-1} to t_i , all I have to do is multiply the result by B and I get the numerically integrated T !

ASEN 6080 HW 3 Problem 1 Main Script

Table of Contents

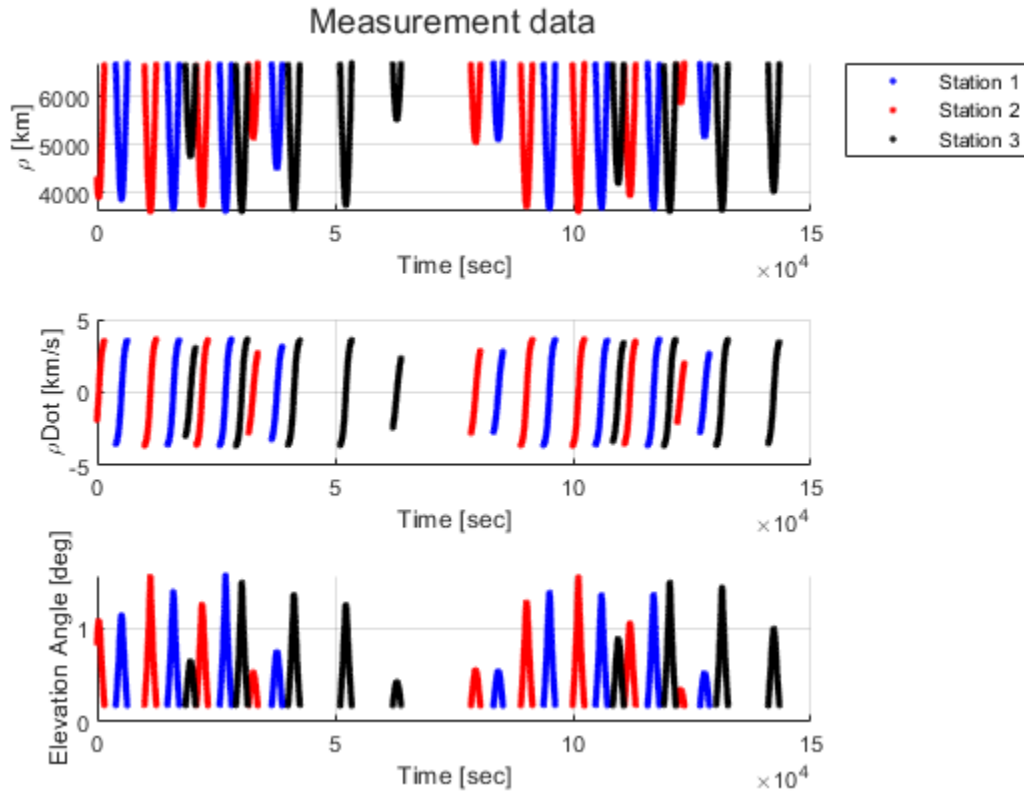
Housekeeping	2
Setup	2
Load Truth Data	2
Problem 1b: Filter setup	2
Problem 1b: Run filters once per sigma	2
Problem 1b: Plot results vs. sigma	2
Problem 1b: Choose optimal sigma and show state errors	4
Problem 1c: Define Q in the RIC frame and run again	12

By: Ian Faber

Housekeeping

Setup

Load Truth Data

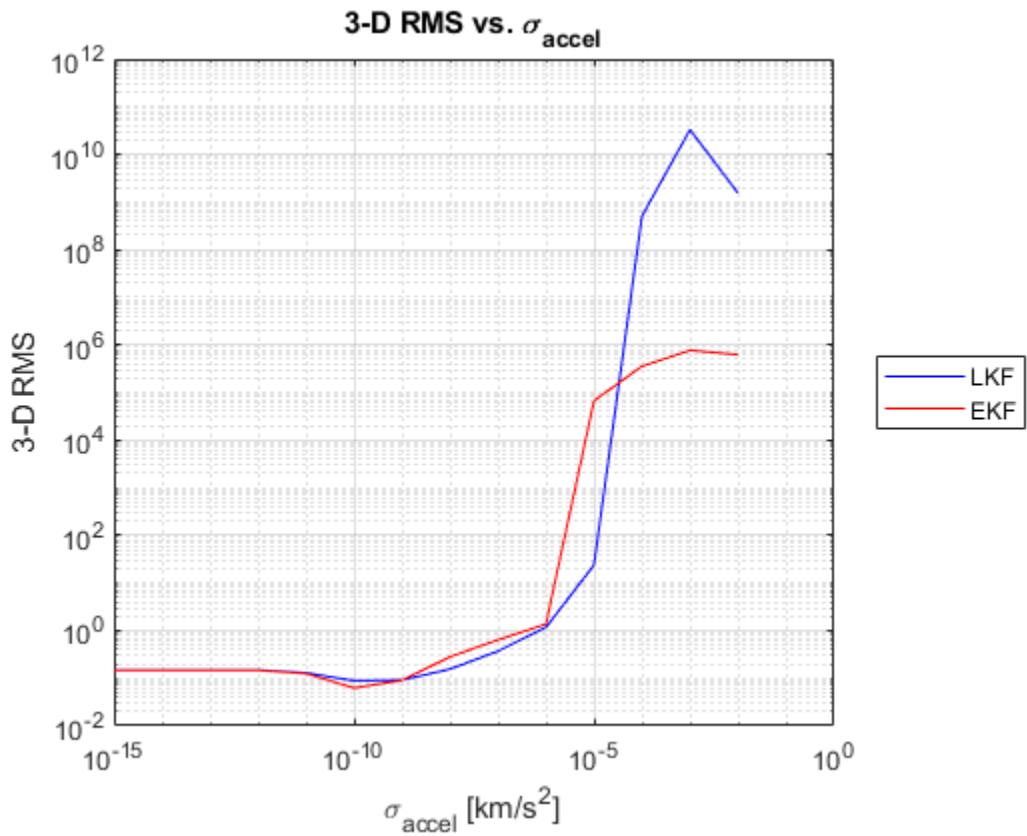
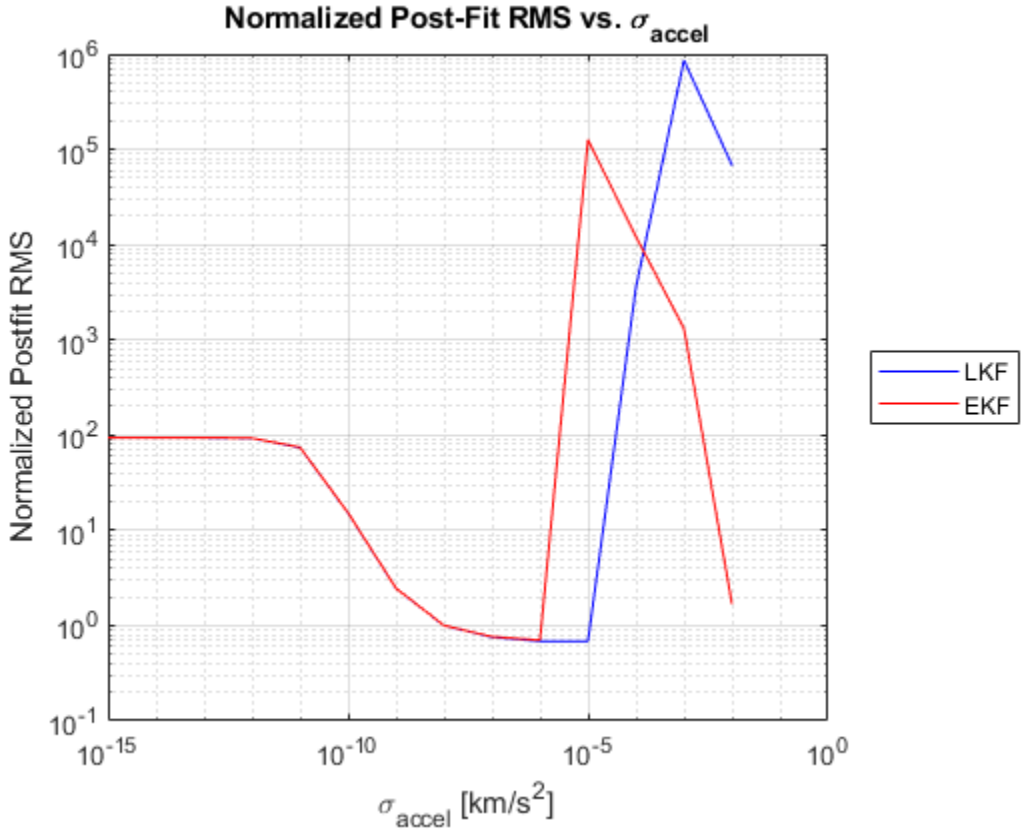


Problem 1b: Filter setup

Problem 1b: Run filters once per sigma

Problem 1b: Plot results vs. sigma

Plotting SNC filter results vs. sigma



Problem 1b: Choose optimal sigma and show state errors

Based on plots, $\sigma = 1\text{e-}8$ balances both postfit and 3D RMS

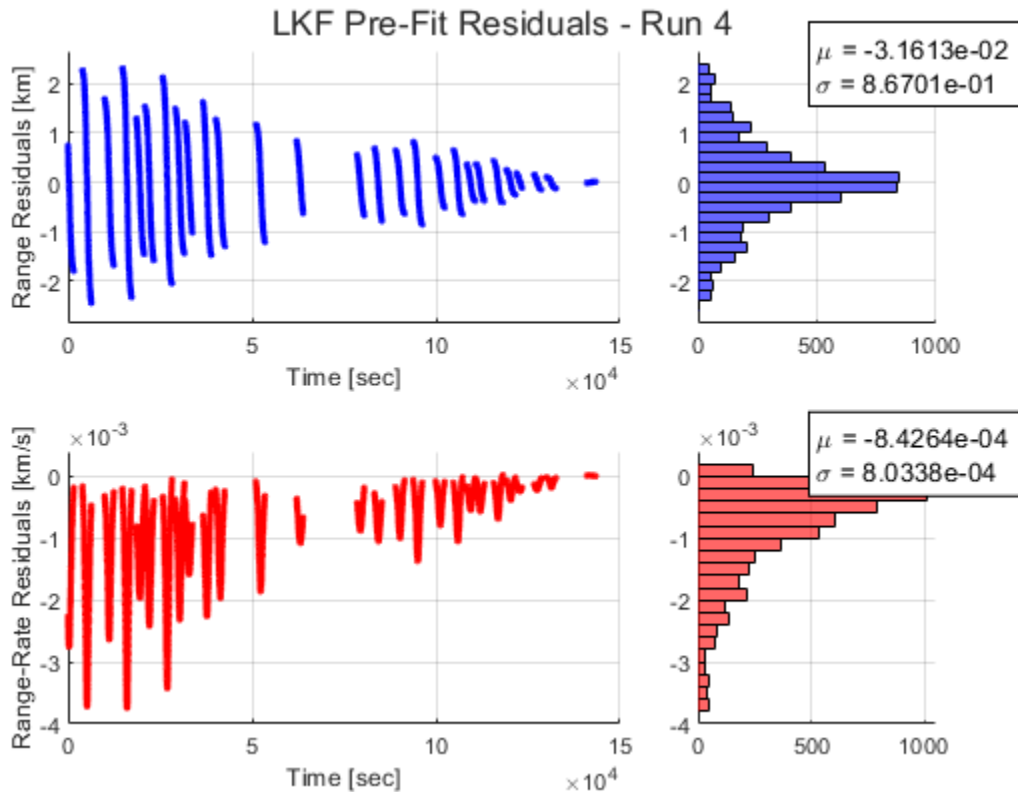
Plotting state errors vs. time for $\sigma = 1.000\text{e-}08 \text{ km/s}^2$

Running LKF:

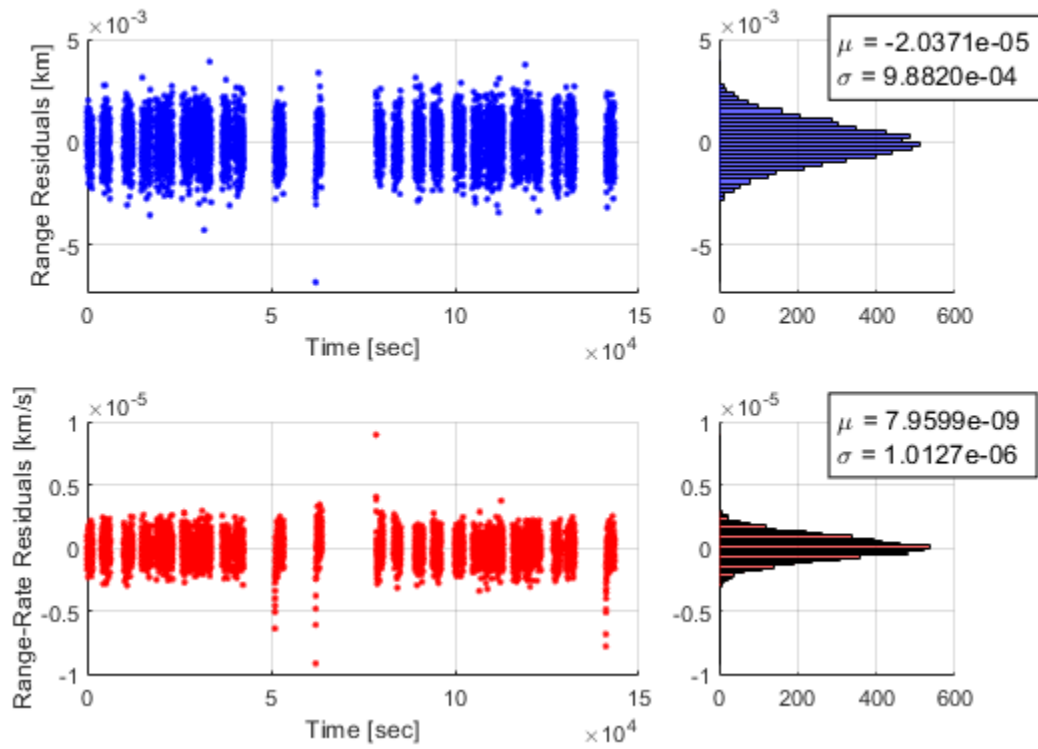
Prefit RMS: 9534.1497, Postfit RMS: 2.6087. Iterating LKF. Runs so far: 1
Prefit RMS: 8138.3783, Postfit RMS: 1.9022. Iterating LKF. Runs so far: 2
Prefit RMS: 3573.5092, Postfit RMS: 1.0638. Iterating LKF. Runs so far: 3
Prefit RMS: 1152.3100, Postfit RMS: 1.0010. Iterating LKF. Runs so far: 4
Final prefit RMS: 1026.6296. Converged after 4 runs
Final postfit RMS: 1.0006. Converged after 4 runs

Running EKF:

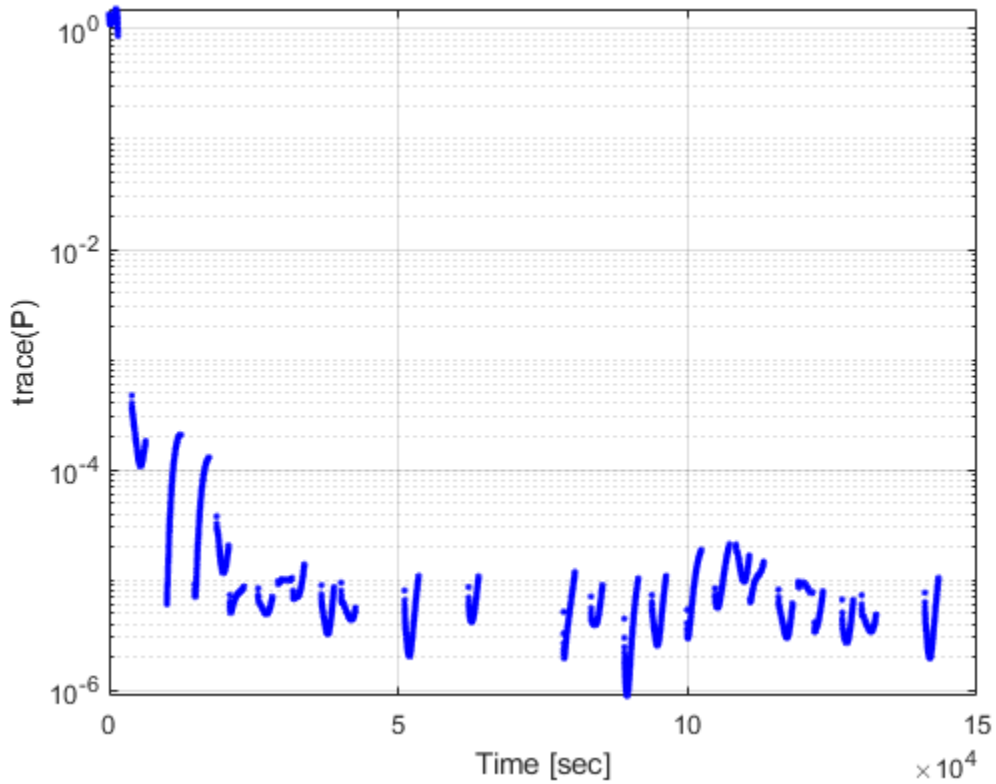
Prefit RMS: 1.0021
Postfit RMS: 1.0021

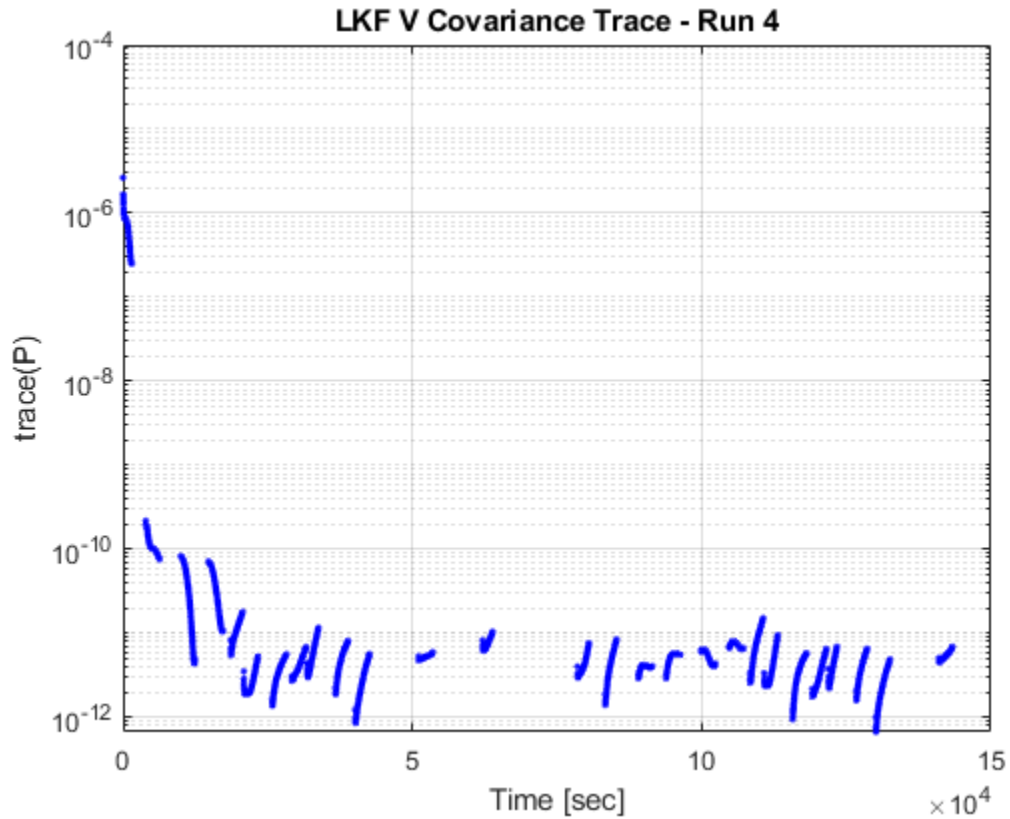


LKF Post-Fit Residuals - Run 4

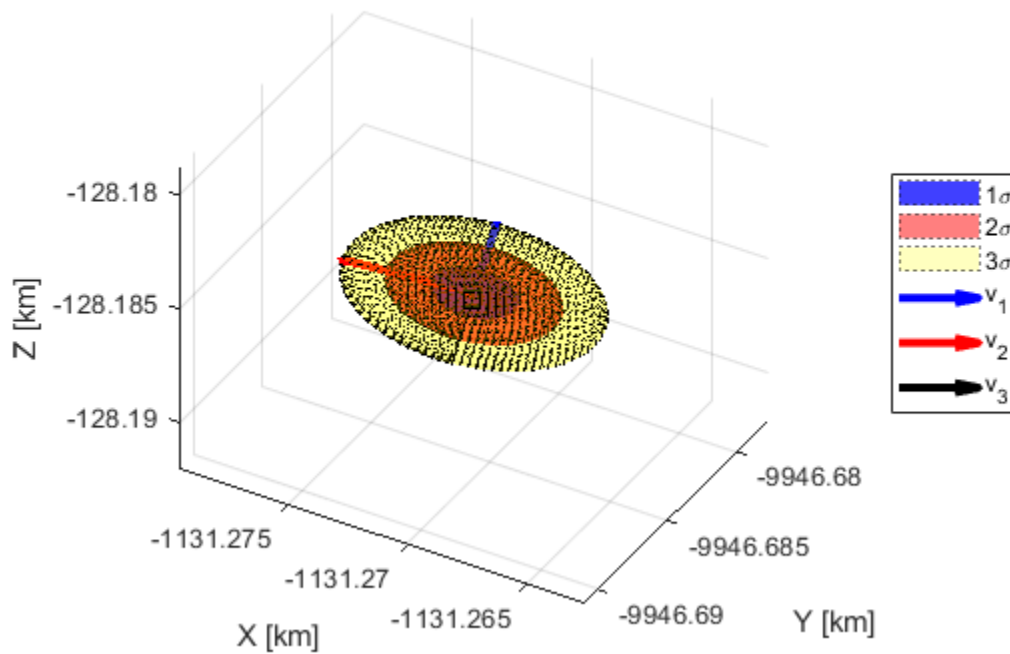


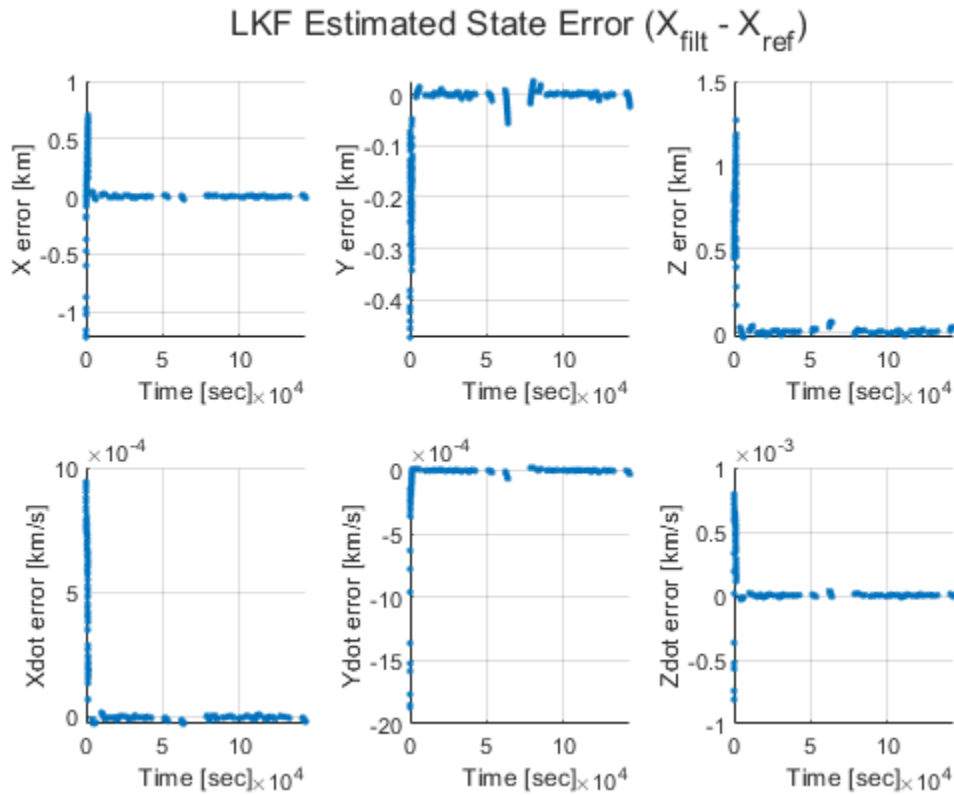
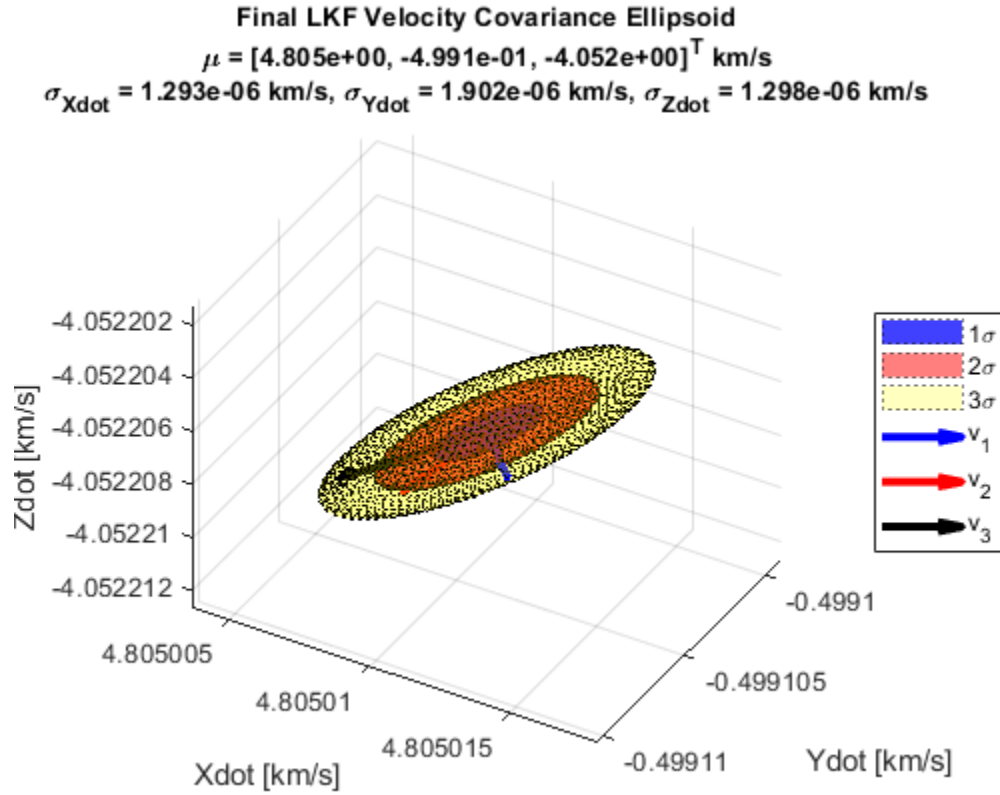
LKF R Covariance Trace - Run 4



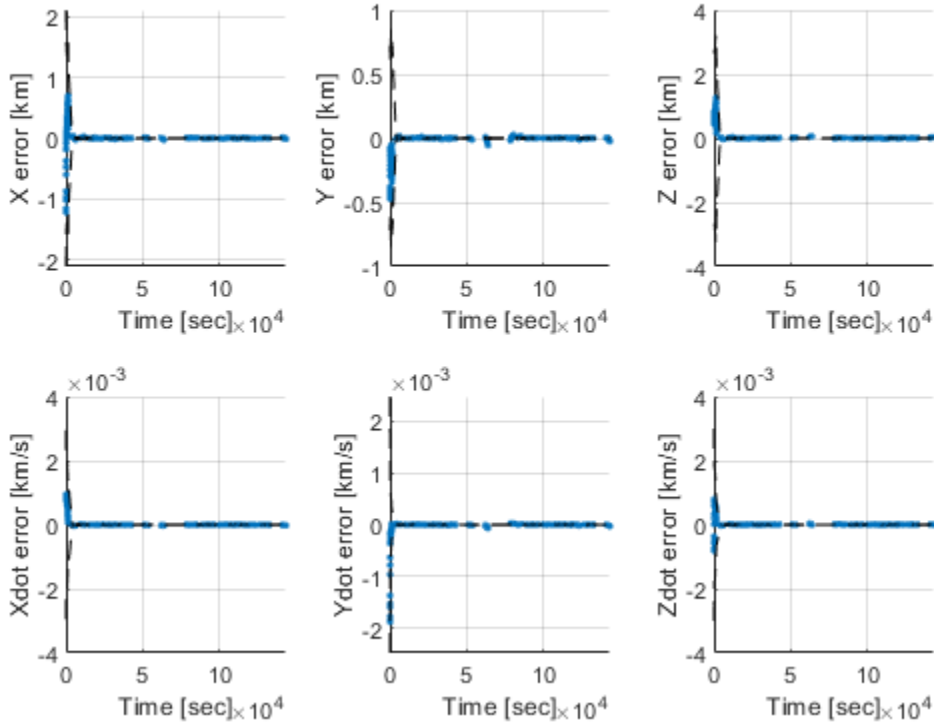


Final LKF Position Covariance Ellipsoid
 $\mu = [-1.131\text{e}+03, -9.947\text{e}+03, -1.282\text{e}+02]^T \text{ km}$
 $\sigma_X = 1.832\text{e}-03 \text{ km}, \sigma_Y = 1.453\text{e}-03 \text{ km}, \sigma_Z = 2.225\text{e}-03 \text{ km}$

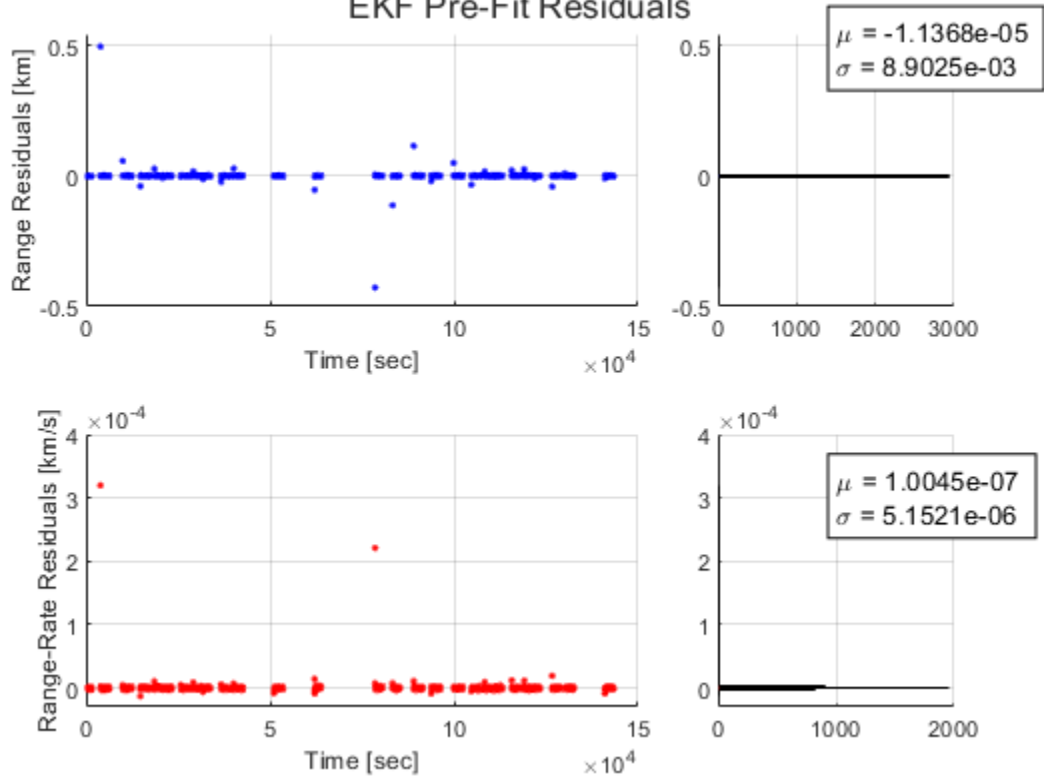




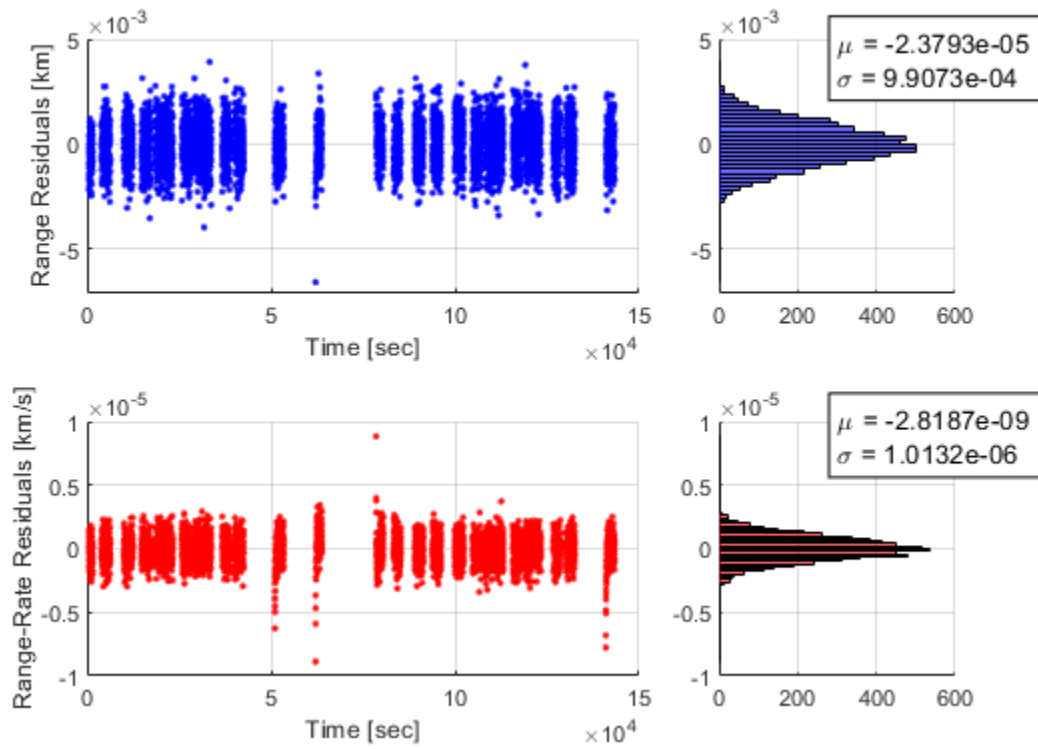
LKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



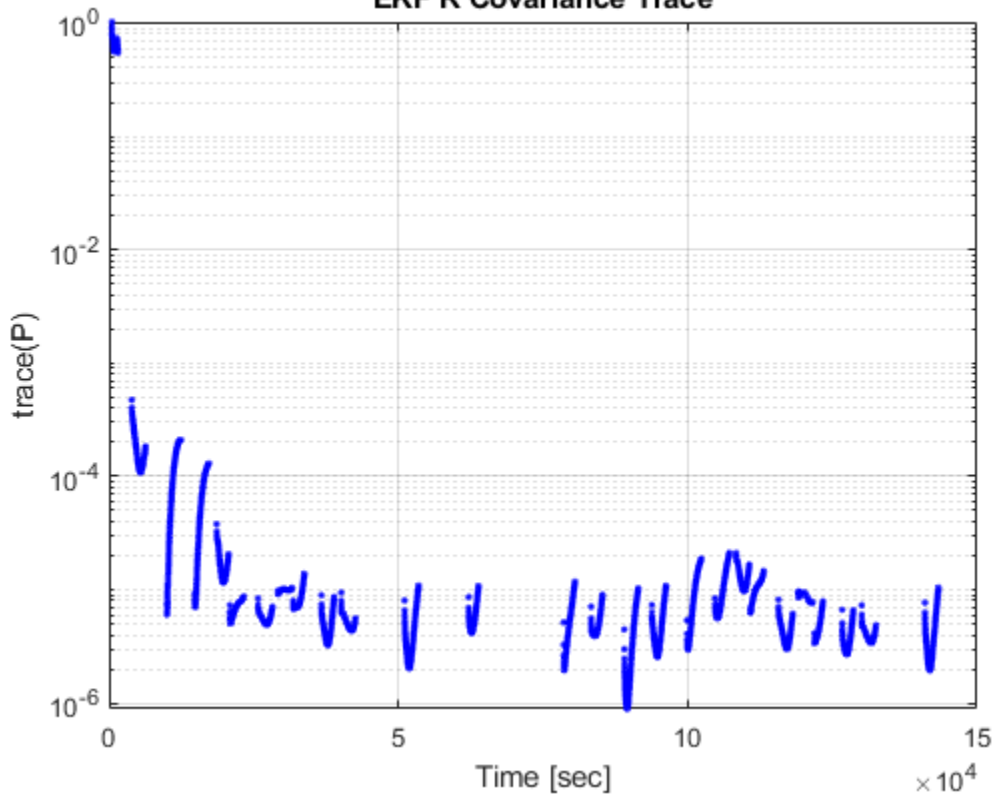
EKF Pre-Fit Residuals

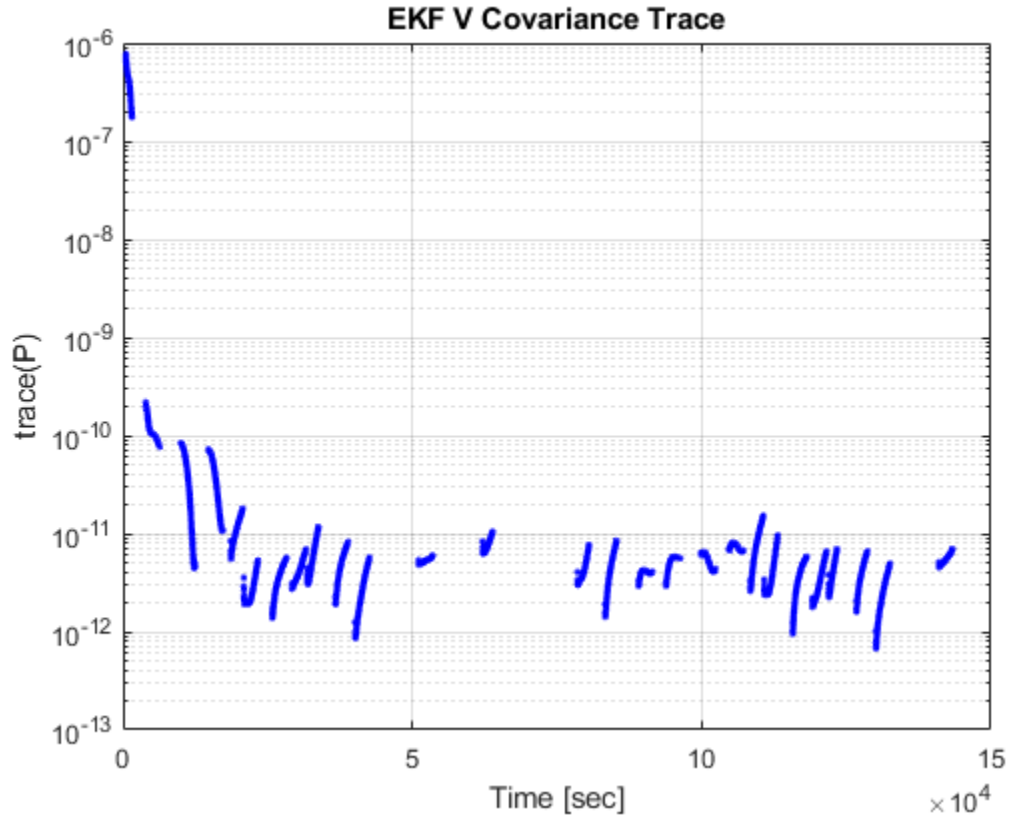


EKF Post-Fit Residuals

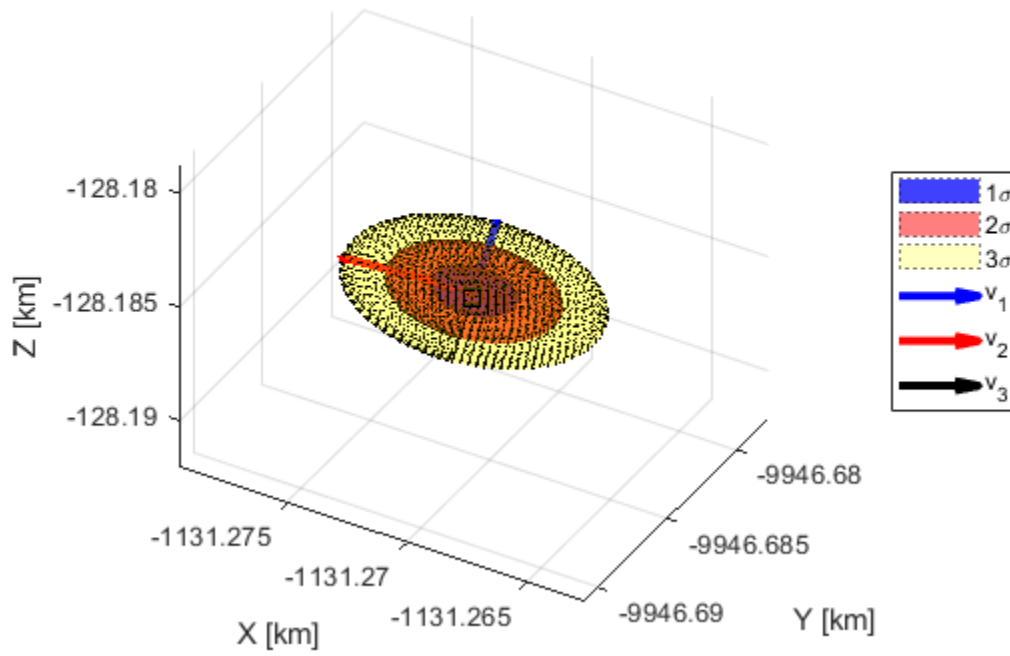


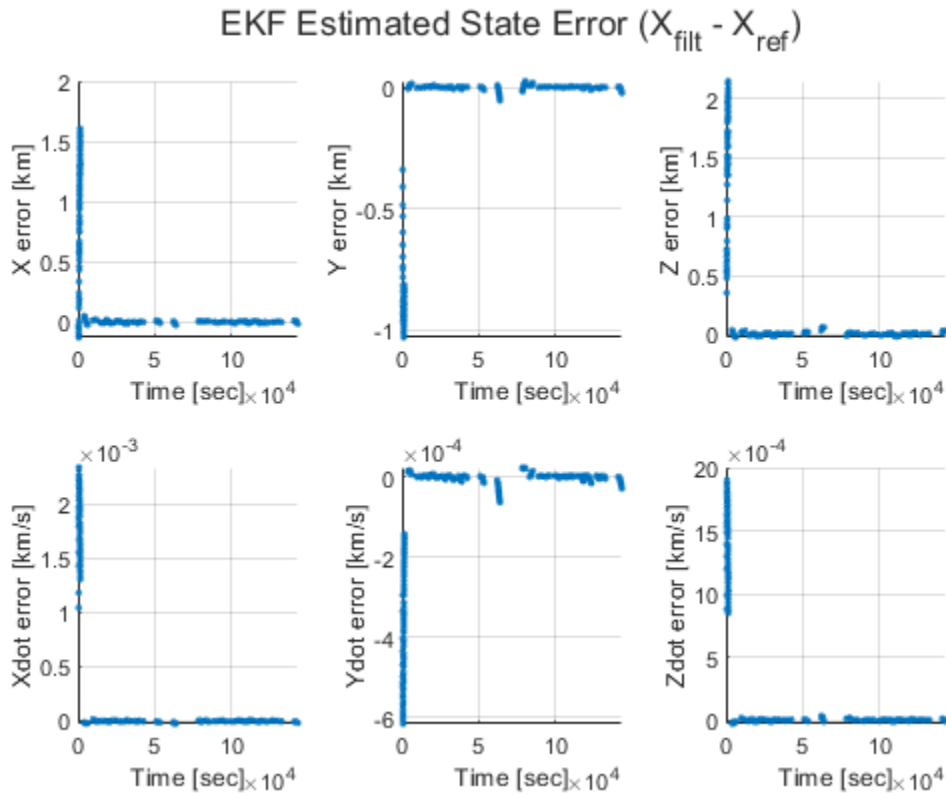
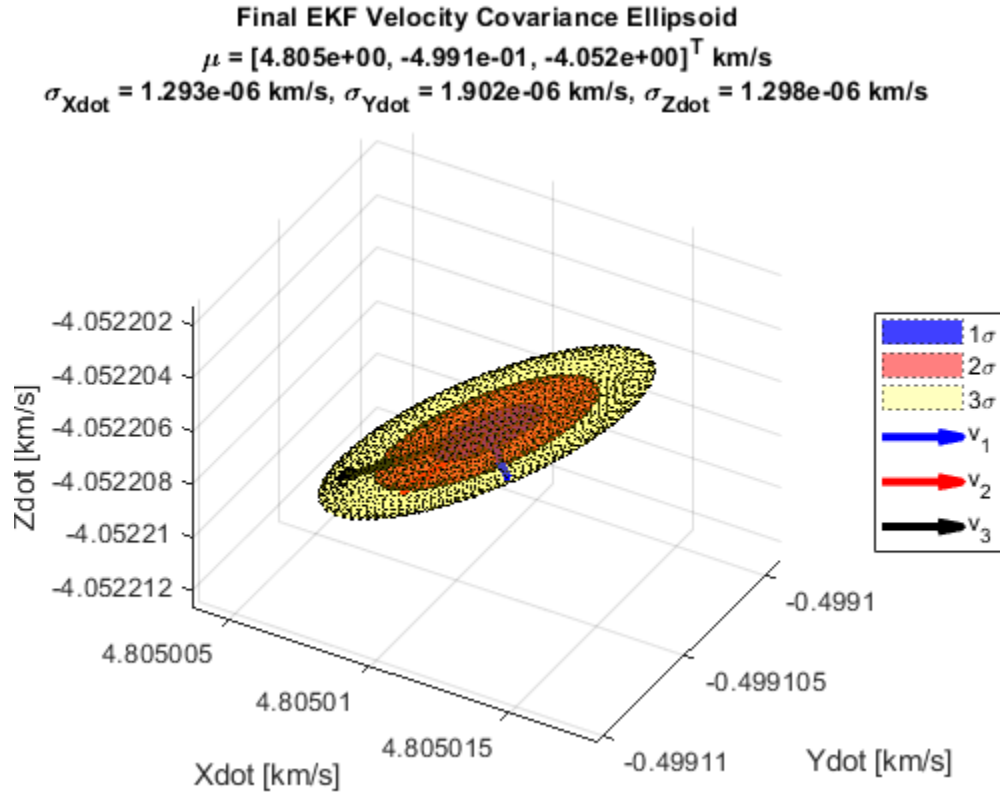
EKF R Covariance Trace

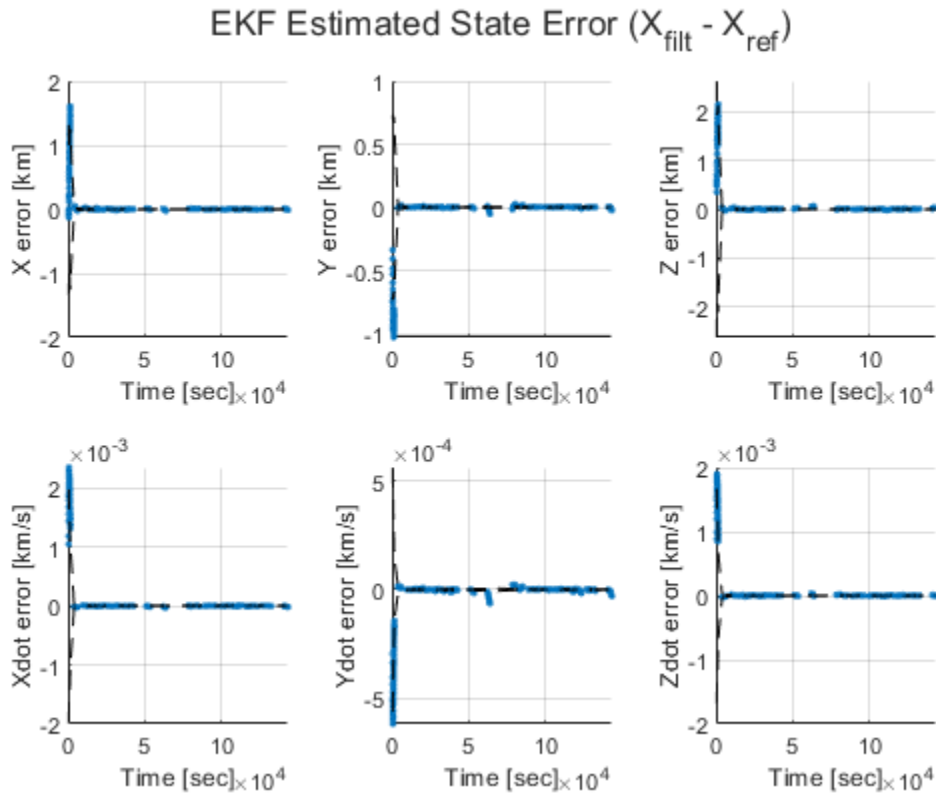




Final EKF Position Covariance Ellipsoid
 $\mu = [-1.131\text{e}+03, -9.947\text{e}+03, -1.282\text{e}+02]^T \text{ km}$
 $\sigma_X = 1.832\text{e}-03 \text{ km}, \sigma_Y = 1.453\text{e}-03 \text{ km}, \sigma_Z = 2.225\text{e}-03 \text{ km}$







Problem 1c: Define Q in the RIC frame and run again

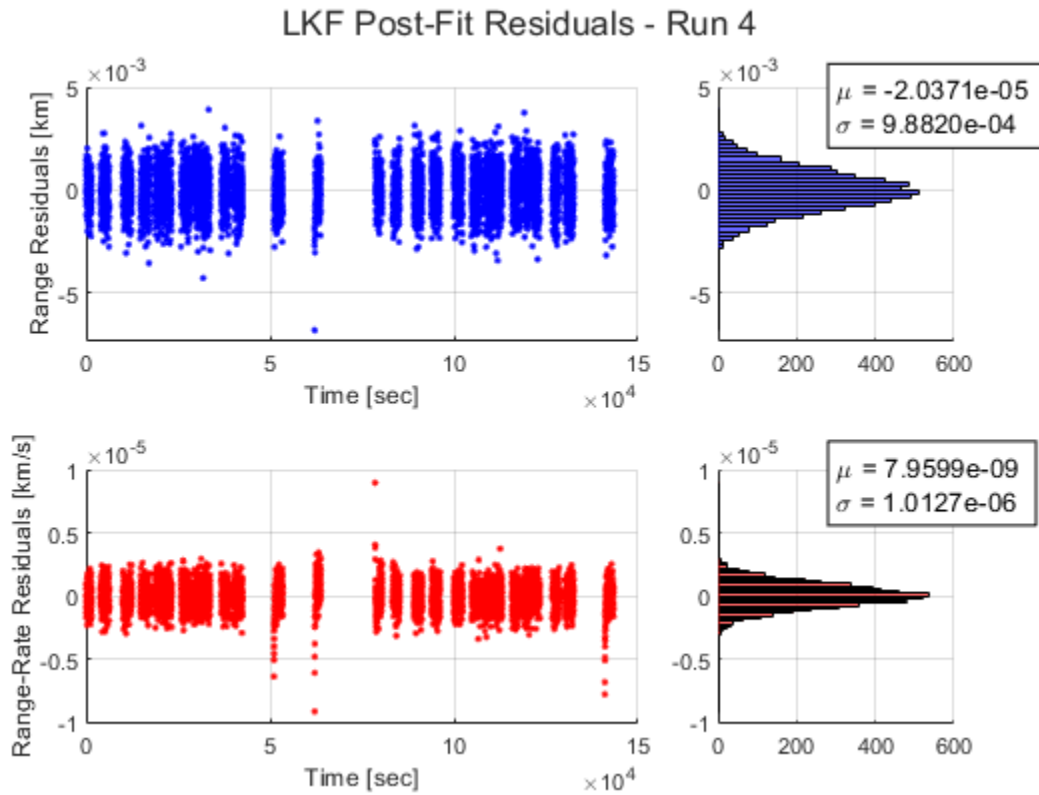
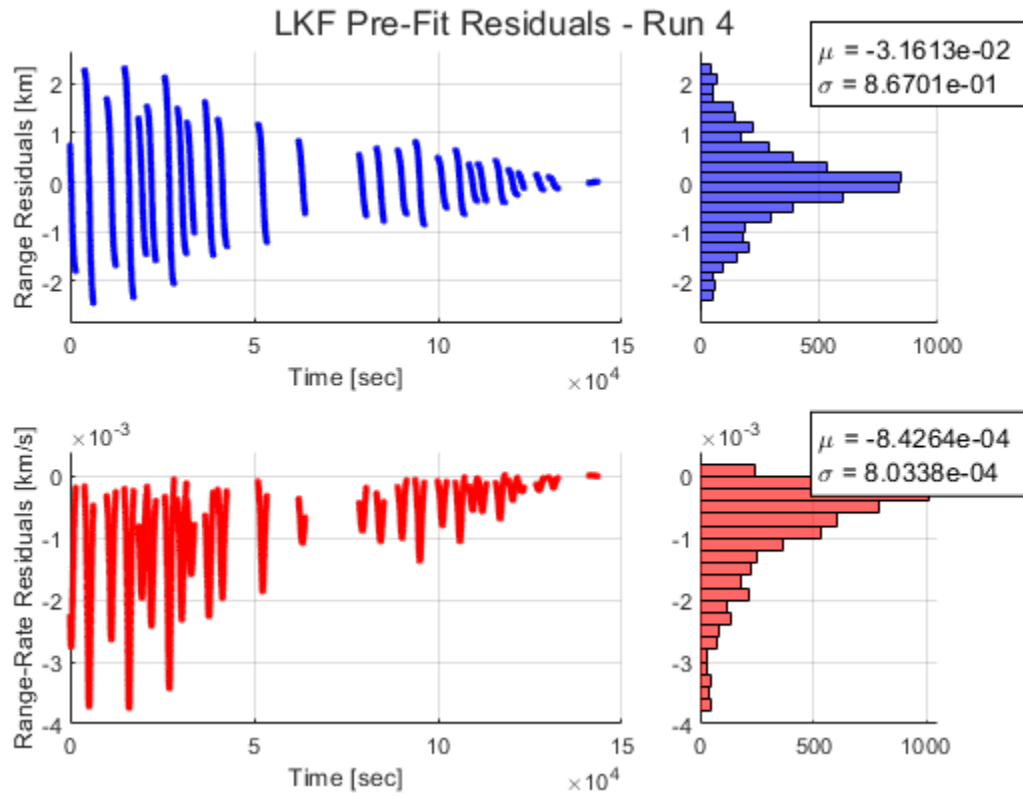
Running filters again with Q defined in RIC frame

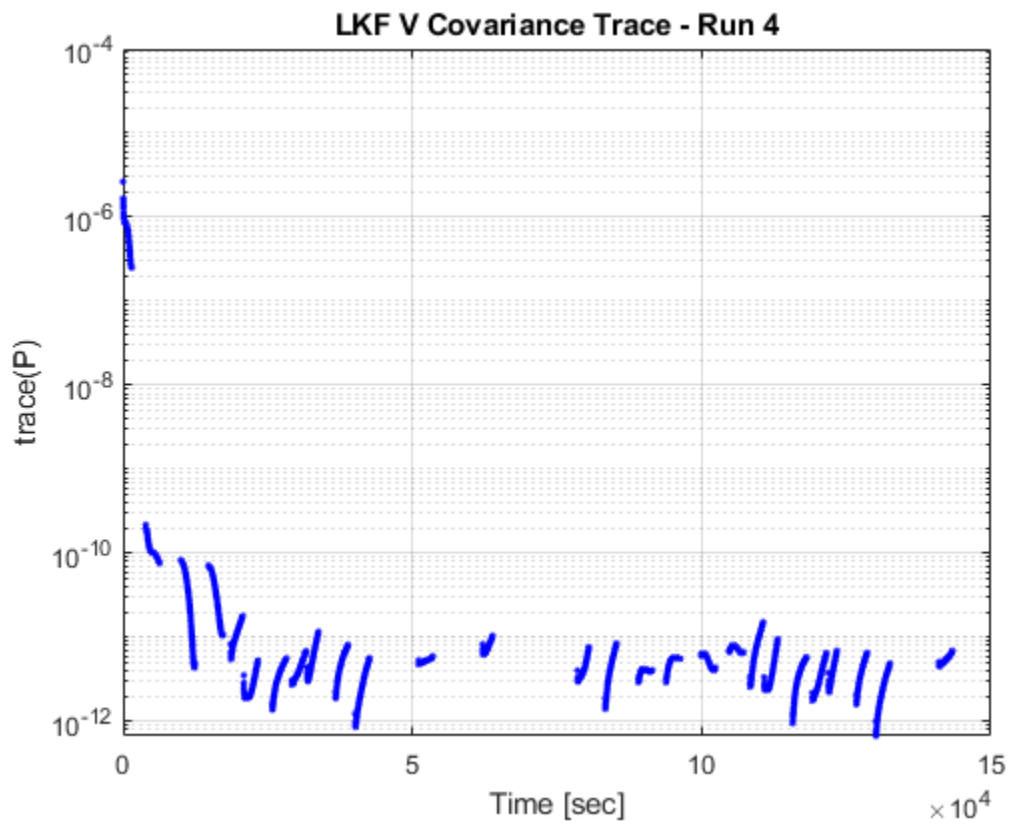
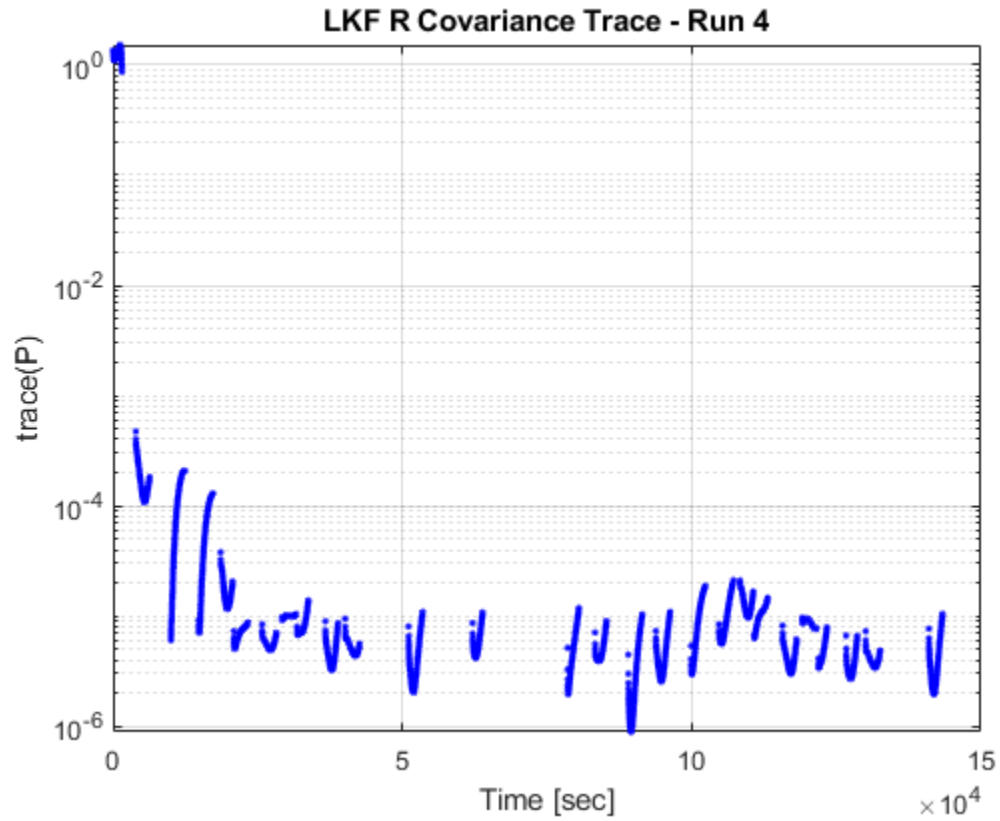
Running LKF:

*Prefit RMS: 9534.1497, Postfit RMS: 2.6087. Iterating LKF. Runs so far: 1
Prefit RMS: 8138.3783, Postfit RMS: 1.9022. Iterating LKF. Runs so far: 2
Prefit RMS: 3573.5092, Postfit RMS: 1.0638. Iterating LKF. Runs so far: 3
Prefit RMS: 1152.3100, Postfit RMS: 1.0010. Iterating LKF. Runs so far: 4
Final prefit RMS: 1026.6296. Converged after 4 runs
Final postfit RMS: 1.0006. Converged after 4 runs*

Running EKF:

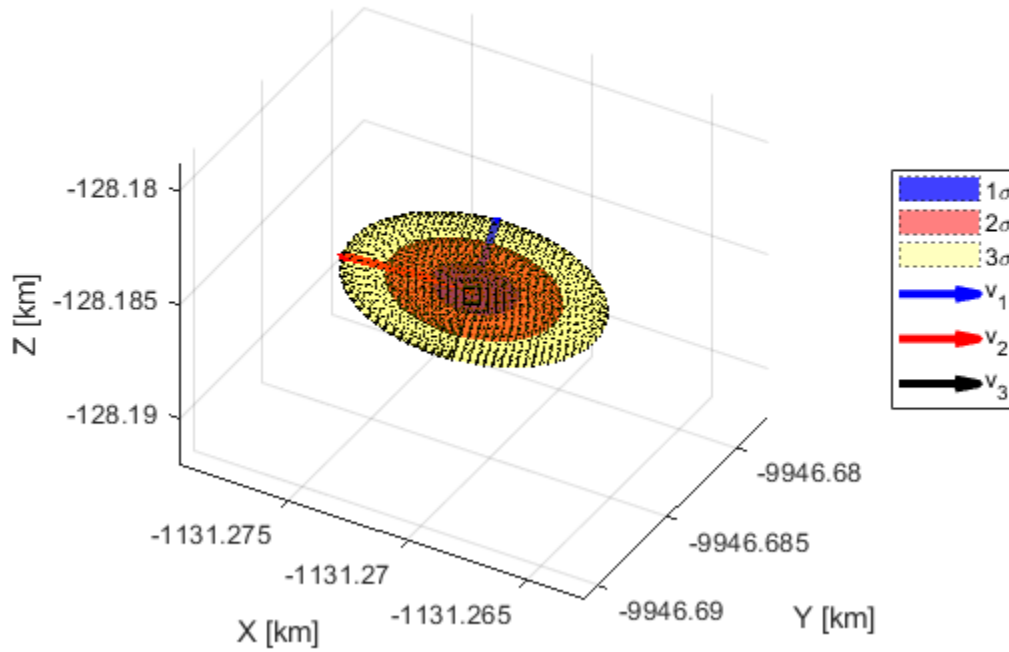
*Prefit RMS: 1.0021
Postfit RMS: 1.0021*





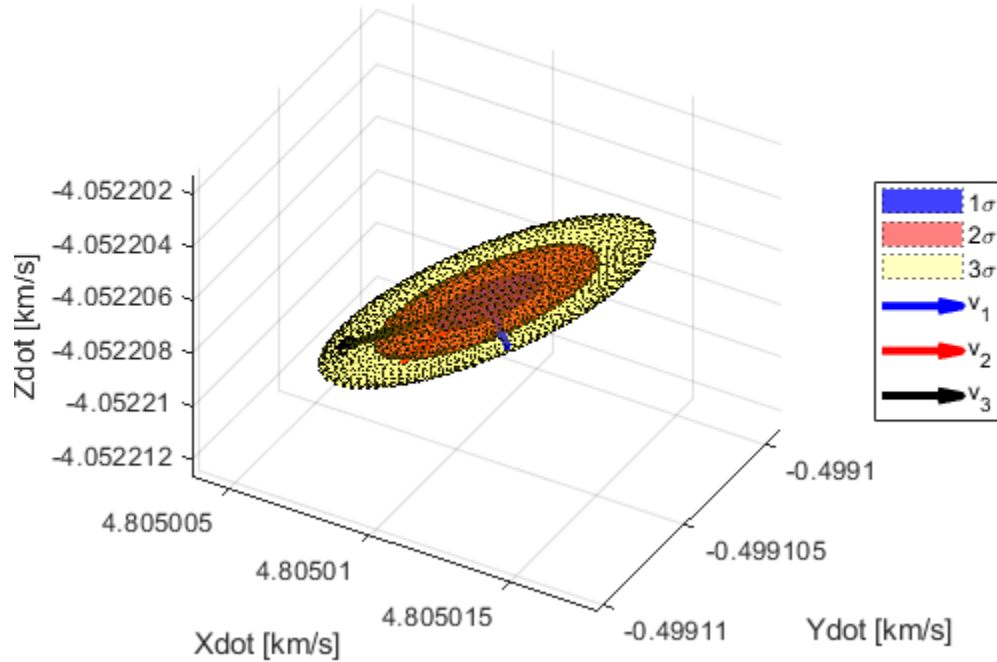
Final LKF Position Covariance Ellipsoid

$$\mu = [-1.131e+03, -9.947e+03, -1.282e+02]^T \text{ km}$$
$$\sigma_X = 1.832e-03 \text{ km}, \sigma_Y = 1.453e-03 \text{ km}, \sigma_Z = 2.225e-03 \text{ km}$$

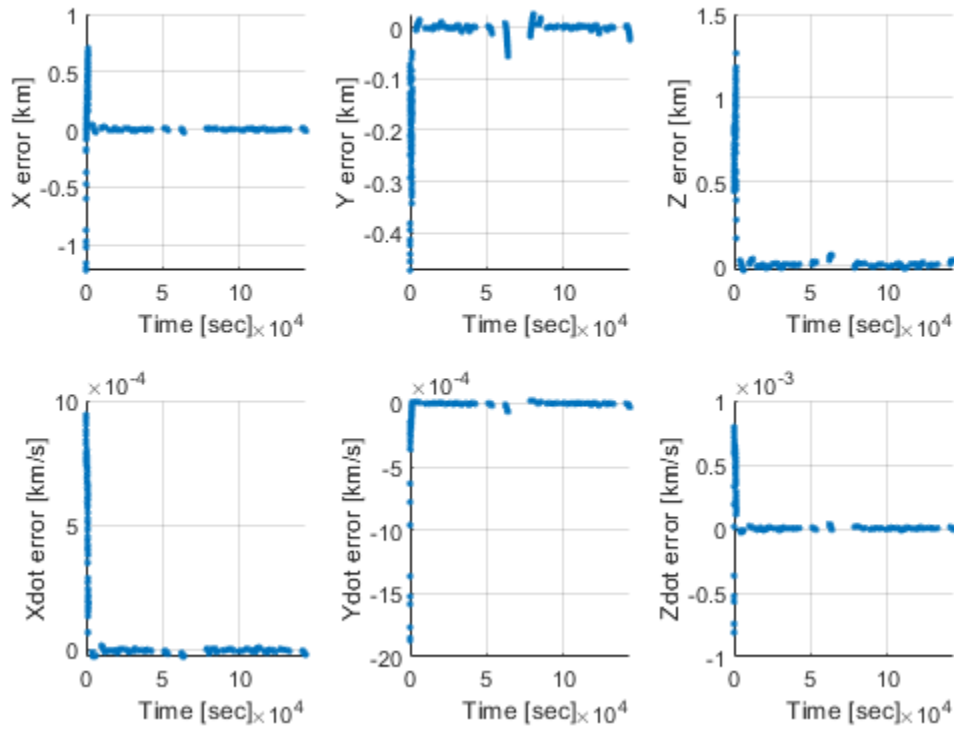


Final LKF Velocity Covariance Ellipsoid

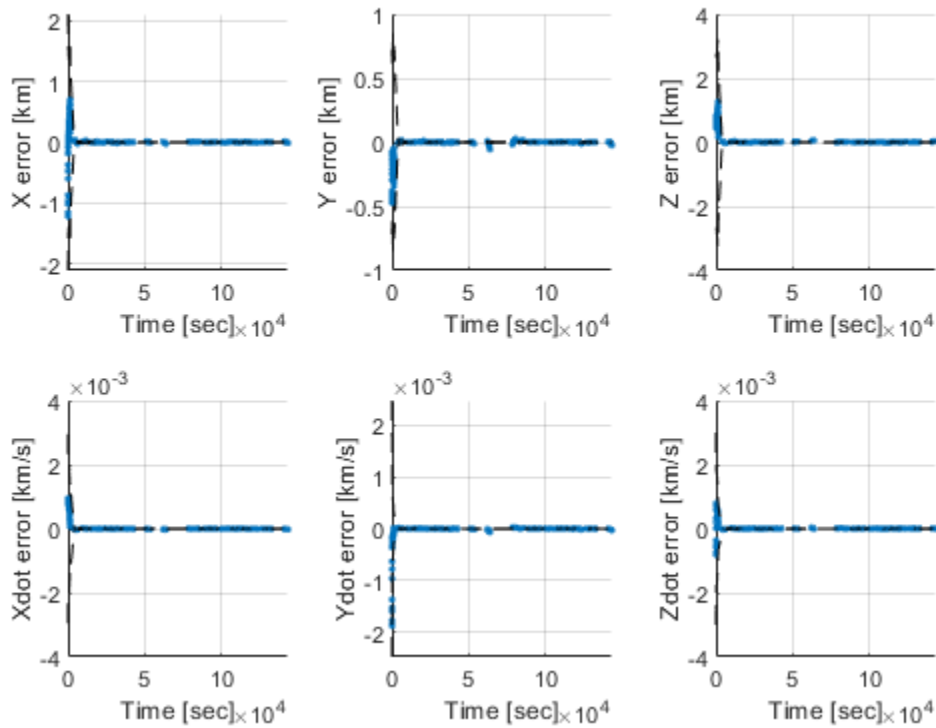
$$\mu = [4.805e+00, -4.991e-01, -4.052e+00]^T \text{ km/s}$$
$$\sigma_{Xdot} = 1.293e-06 \text{ km/s}, \sigma_{Ydot} = 1.902e-06 \text{ km/s}, \sigma_{Zdot} = 1.298e-06 \text{ km/s}$$

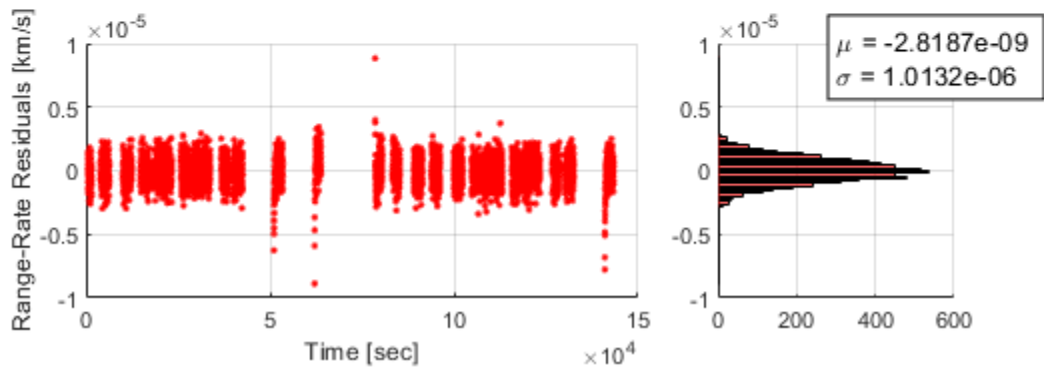
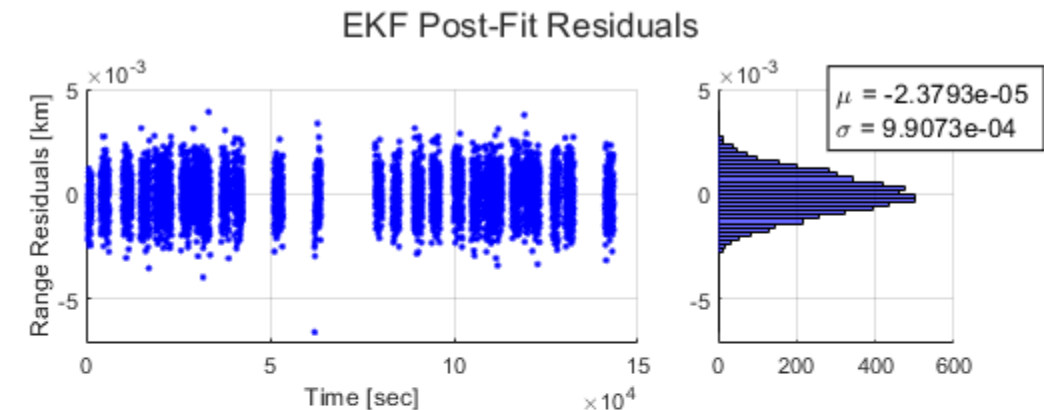
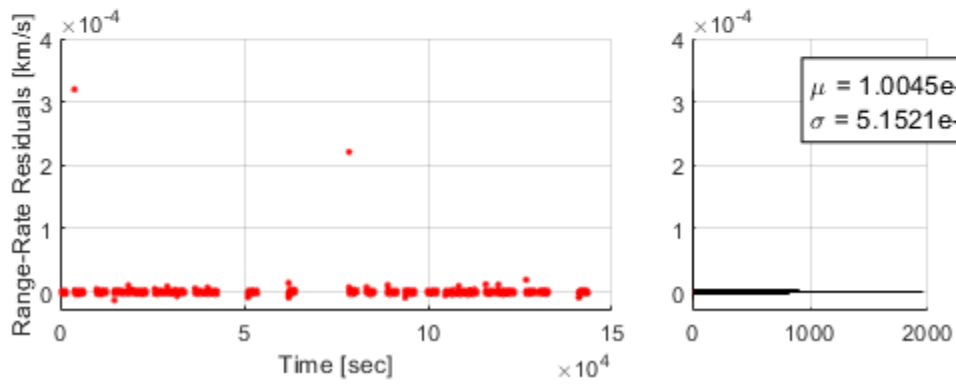
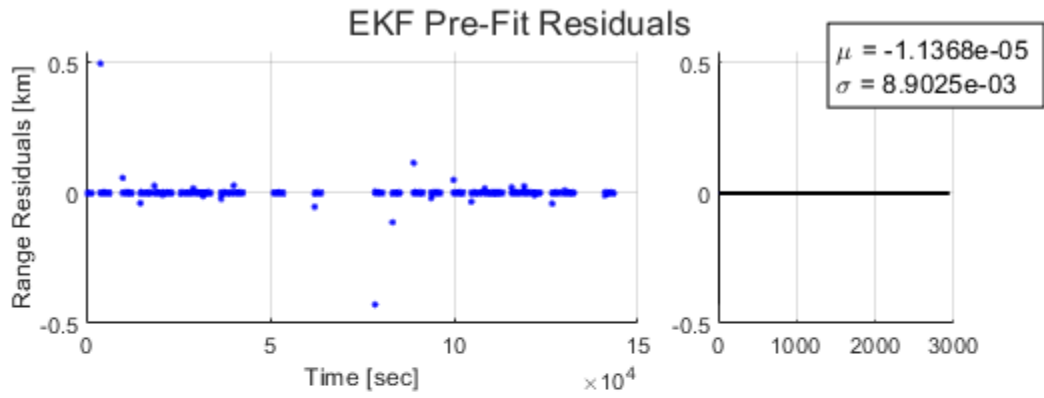


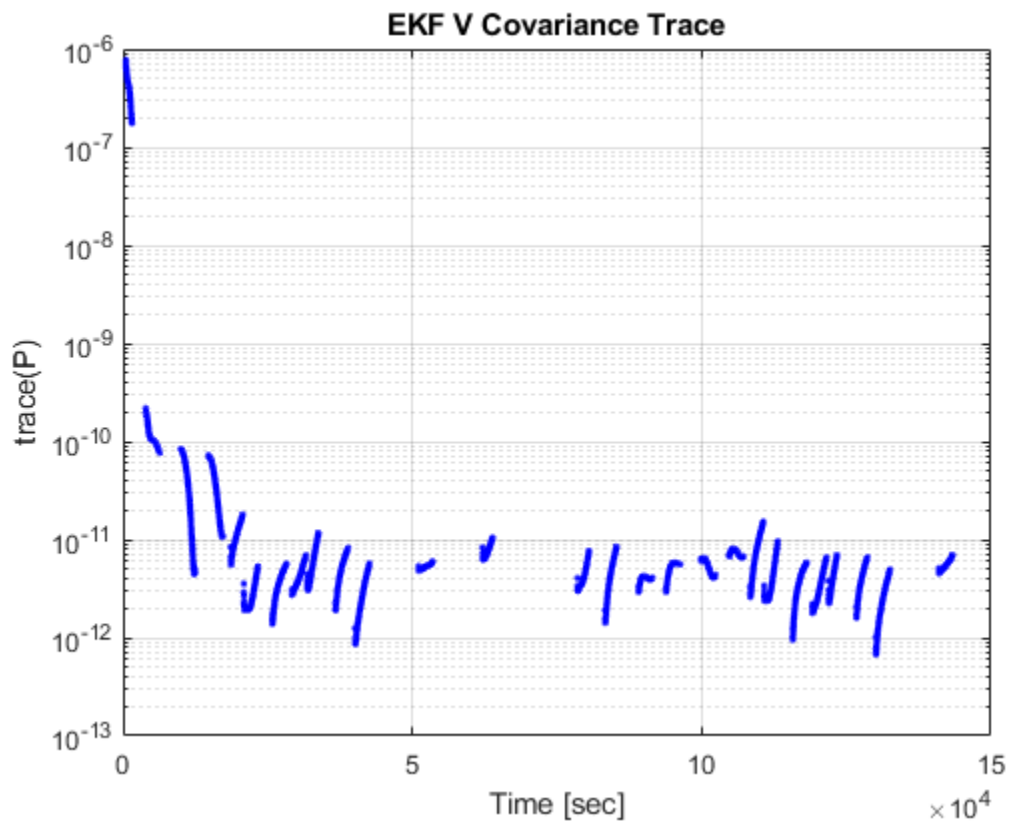
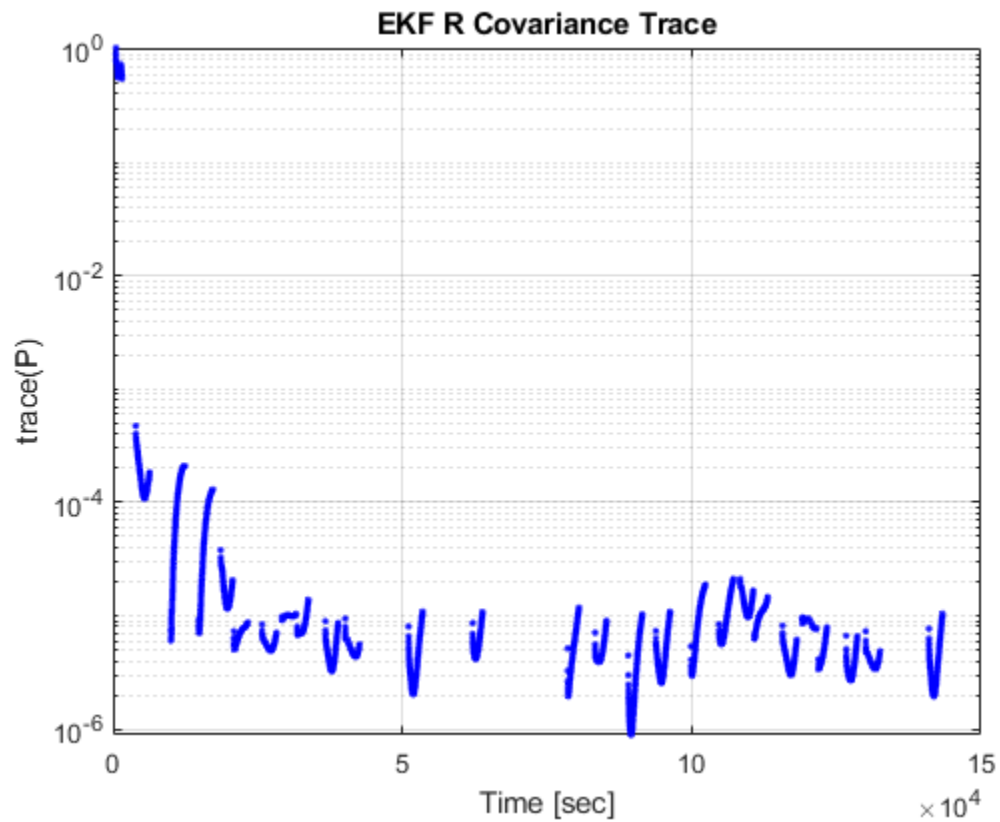
LKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



LKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)

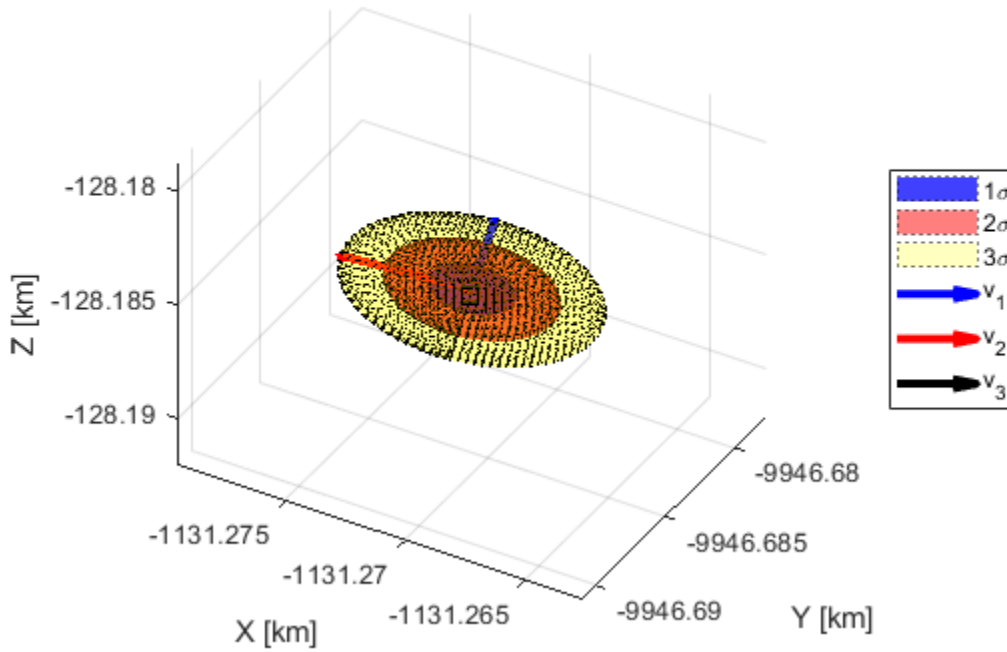






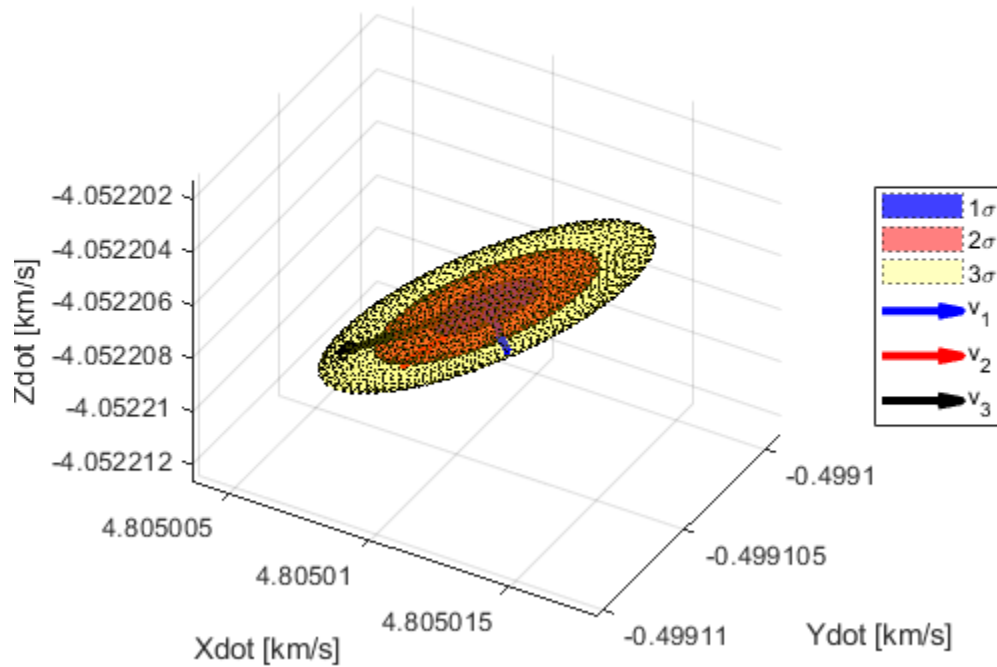
Final EKF Position Covariance Ellipsoid

$$\mu = [-1.131e+03, -9.947e+03, -1.282e+02]^T \text{ km}$$
$$\sigma_X = 1.832e-03 \text{ km}, \sigma_Y = 1.453e-03 \text{ km}, \sigma_Z = 2.225e-03 \text{ km}$$

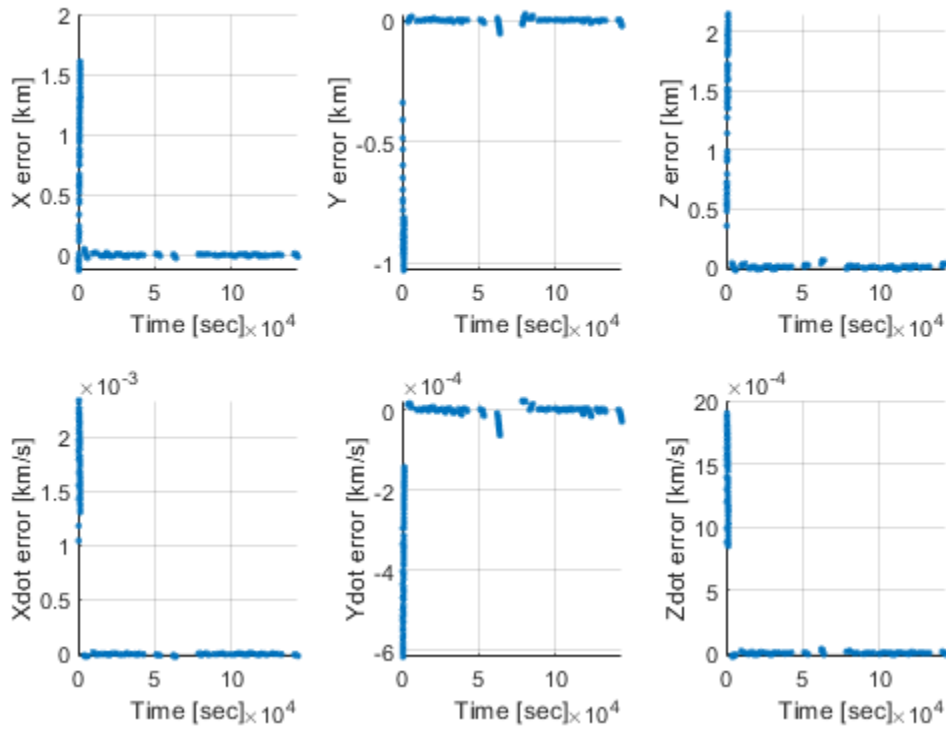


Final EKF Velocity Covariance Ellipsoid

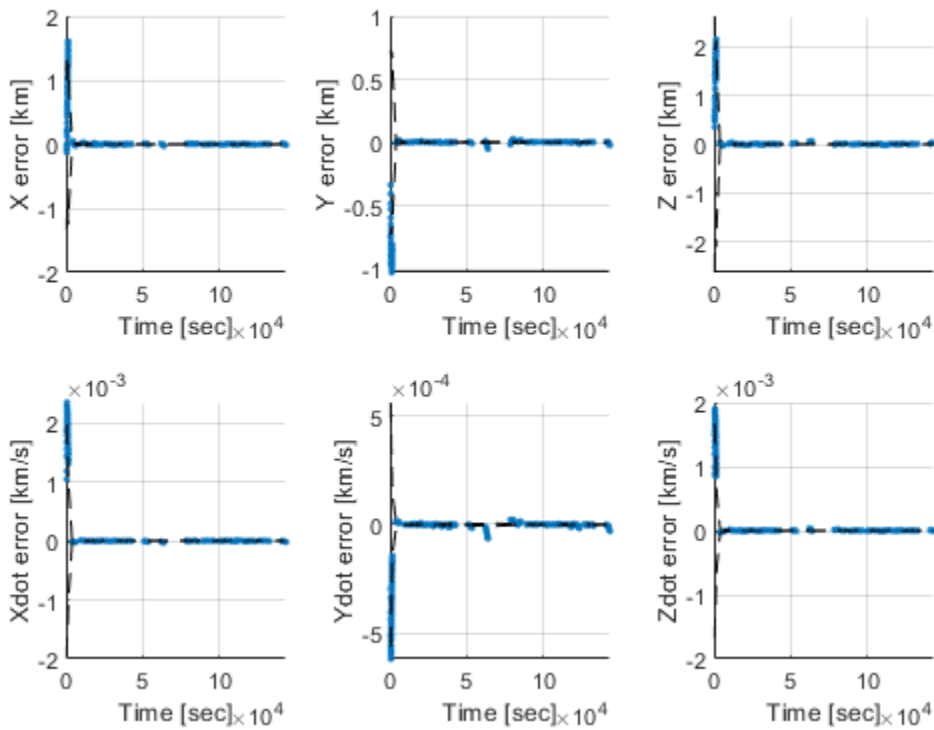
$$\mu = [4.805e+00, -4.991e-01, -4.052e+00]^T \text{ km/s}$$
$$\sigma_{Xdot} = 1.293e-06 \text{ km/s}, \sigma_{Ydot} = 1.902e-06 \text{ km/s}, \sigma_{Zdot} = 1.298e-06 \text{ km/s}$$



EKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



EKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



Published with MATLAB® R2023b

2. Repeat problem 1 using VMC with $\bar{n} = \vec{0}$ and $\sigma = T/30$, where T is the orbit period in seconds

i. What is the optimal value for σ ? why?

I found that $\sigma = 1 \times 10^{-10}$ balanced the normalized postbit RMS near 1 while minimizing 3D RMS

ii. For the optimal σ , plot the T_z accelerations and filter estimates vs. time, as well as their errors with covariance bounds. How well does the filter estimate the acceleration?

The LKF does a decent job at estimating the acceleration, while the EKF exhibits some serious deviations from the true T_z acceleration. The deviations occur at the start of each measurement window, which makes sense as the exponential model we use is maximized at t_0 , i.e. the start of each measurement window. This could be alleviated by reducing σ , but then postbit RMS suffers.

iii. choose at least 1 different value for τ . How does this change the filter performance?

changing τ_z from $T/30$ to $T/600$ doesn't affect the final estimates for R and V , but it causes \hat{W}_z to no longer approximate the z component of T_z , instead approaching noise vs. a deterministic acceleration. Likewise, increasing τ results in a \hat{W} that responds too slowly to the true T_z acceleration.

C. Discuss how DMC could be used to account for an unknown maneuver.

An unknown maneuver would occur relatively quickly on the orbital timescale and at a specific time. I thus, to use DMC to account for a maneuver, I would use a small τ and only apply the \hat{Q} from DMC at the probable maneuver times.

ASEN 6080 HW 3 Problem 2 Main Script

Table of Contents

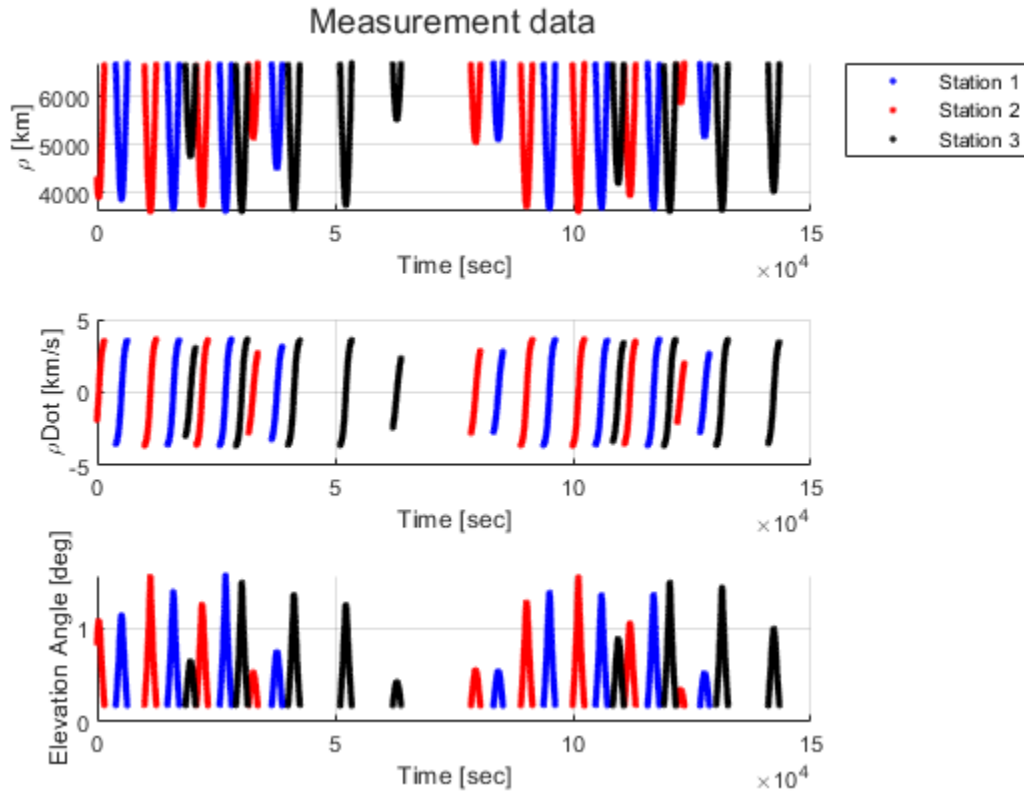
Housekeeping	2
Setup	2
Load Truth Data	2
Problem 2b: Filter setup	2
Problem 2b: Test values of sigma	2
Problem 2b: Plot results vs. sigma	2
Problem 2b: Choose optimal sigma	4
Problem 2b: Plot J3 accels and estimated filter estimates vs. time	12
Problem 2b: Different value of tau	14

By: Ian Faber

Housekeeping

Setup

Load Truth Data

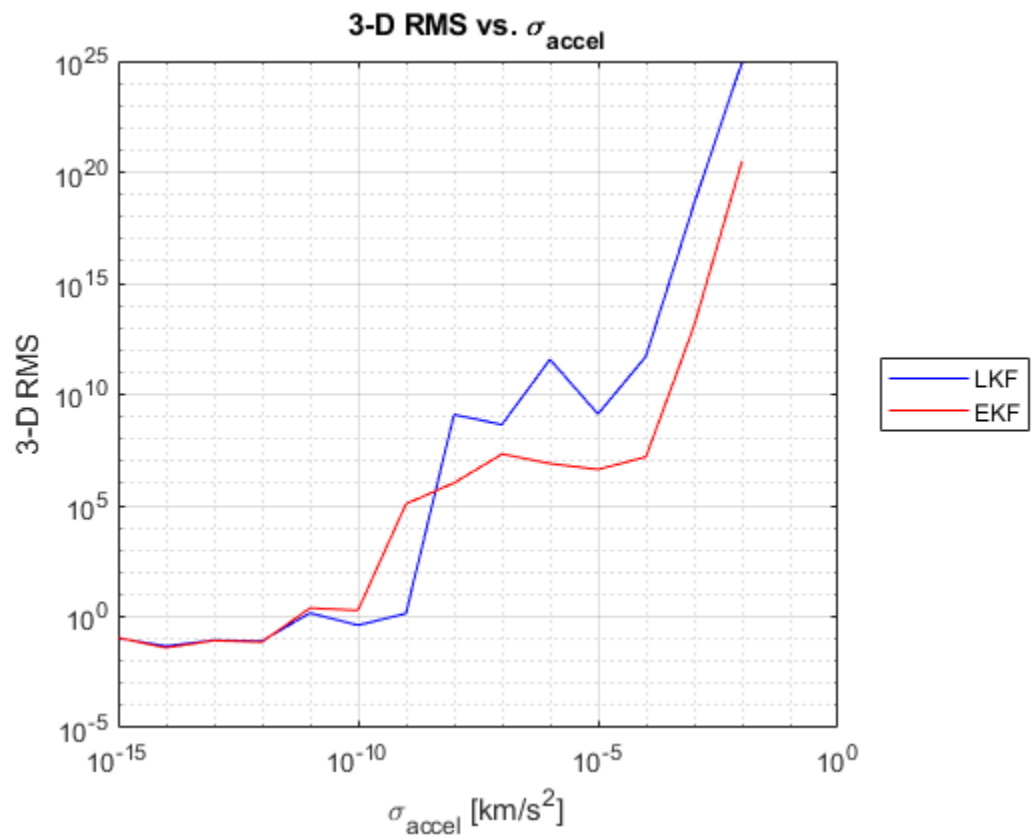
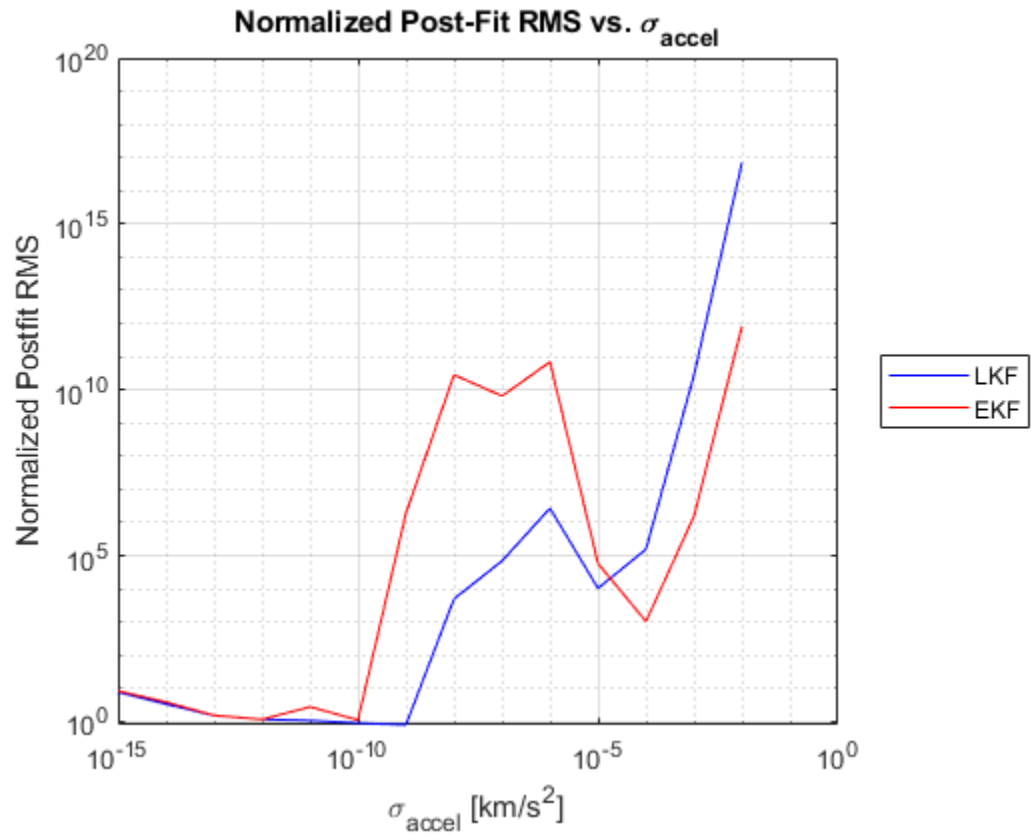


Problem 2b: Filter setup

Problem 2b: Test values of sigma

Problem 2b: Plot results vs. sigma

Plotting DMC filter results vs. sigma



Problem 2b: Choose optimal sigma

Based on plots, $\sigma = 1\text{e-}10$ balances both postfit and 3D RMS

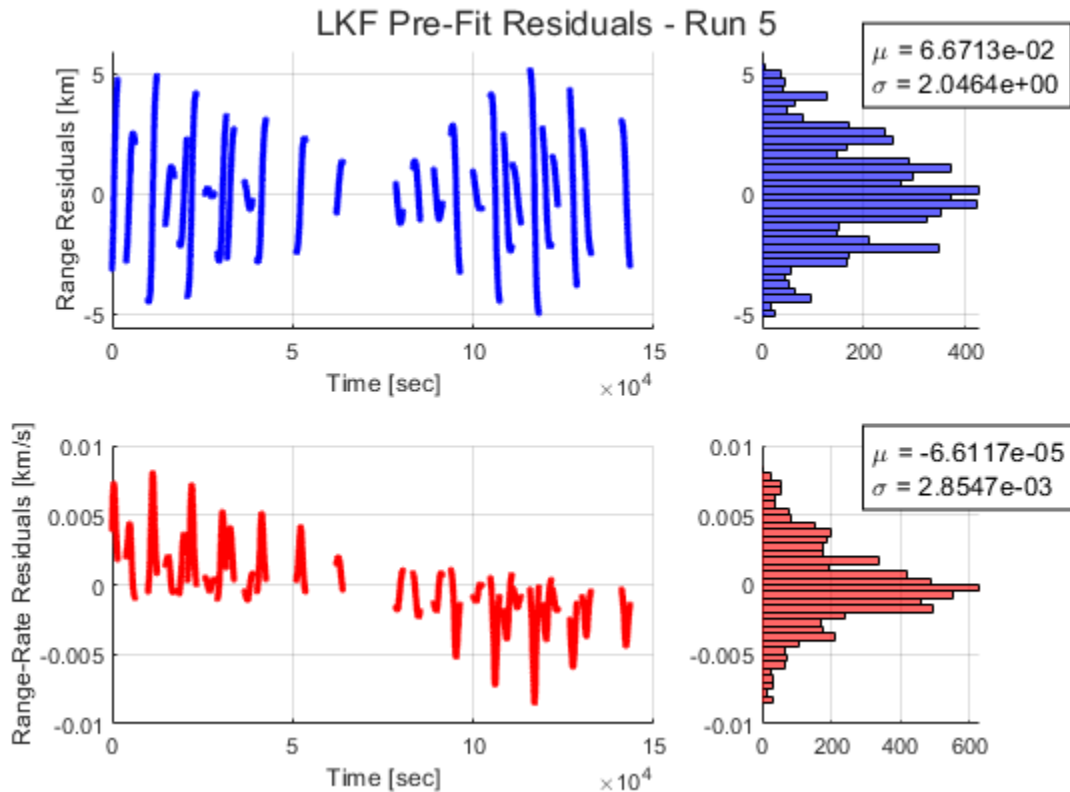
Plotting state errors vs. time for $\sigma = 1.000\text{e-}10 \text{ km/s}^2$

Running LKF:

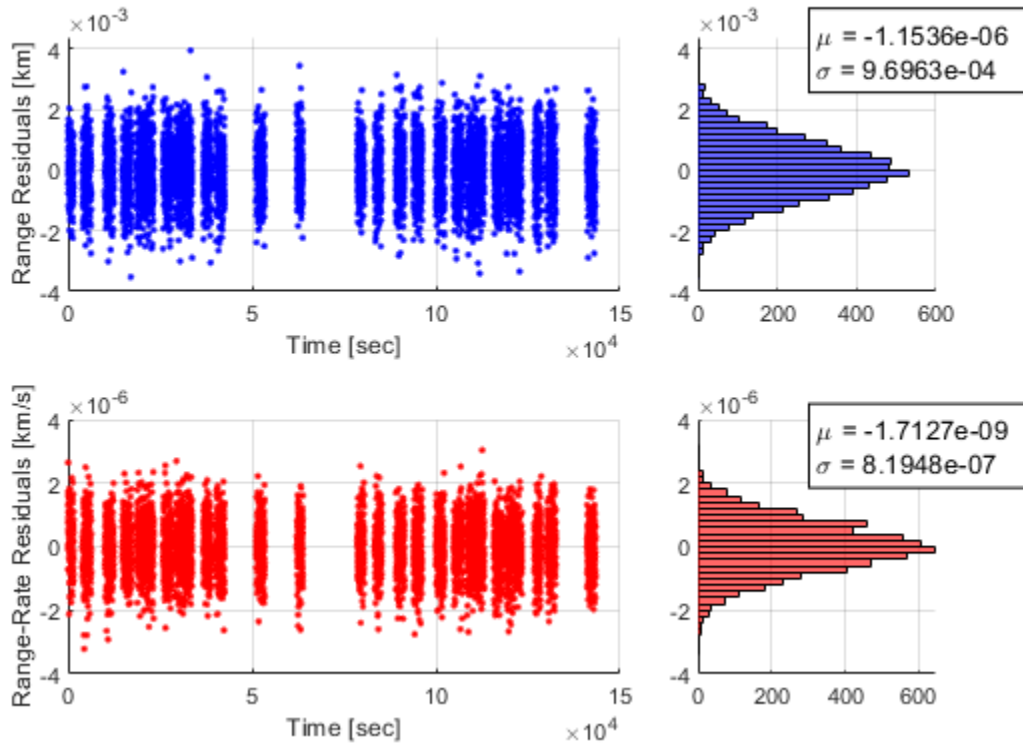
```
Prefit RMS: 9534.1497, Postfit RMS: 0.9366. Iterating LKF. Runs so far: 1
Prefit RMS: 5479.2712, Postfit RMS: 0.9244. Iterating LKF. Runs so far: 2
Prefit RMS: 3992.5455, Postfit RMS: 0.9218. Iterating LKF. Runs so far: 3
Prefit RMS: 2550.0405, Postfit RMS: 0.8967. Iterating LKF. Runs so far: 4
Prefit RMS: 2484.3232, Postfit RMS: 0.8976. Hit max LKF iterations. Runs so far: 5
Final prefit RMS: 2484.3232. Hit maximum number of 5 runs
Final postfit RMS: 0.8976. Hit maximum number of 5 runs
```

Running EKF:

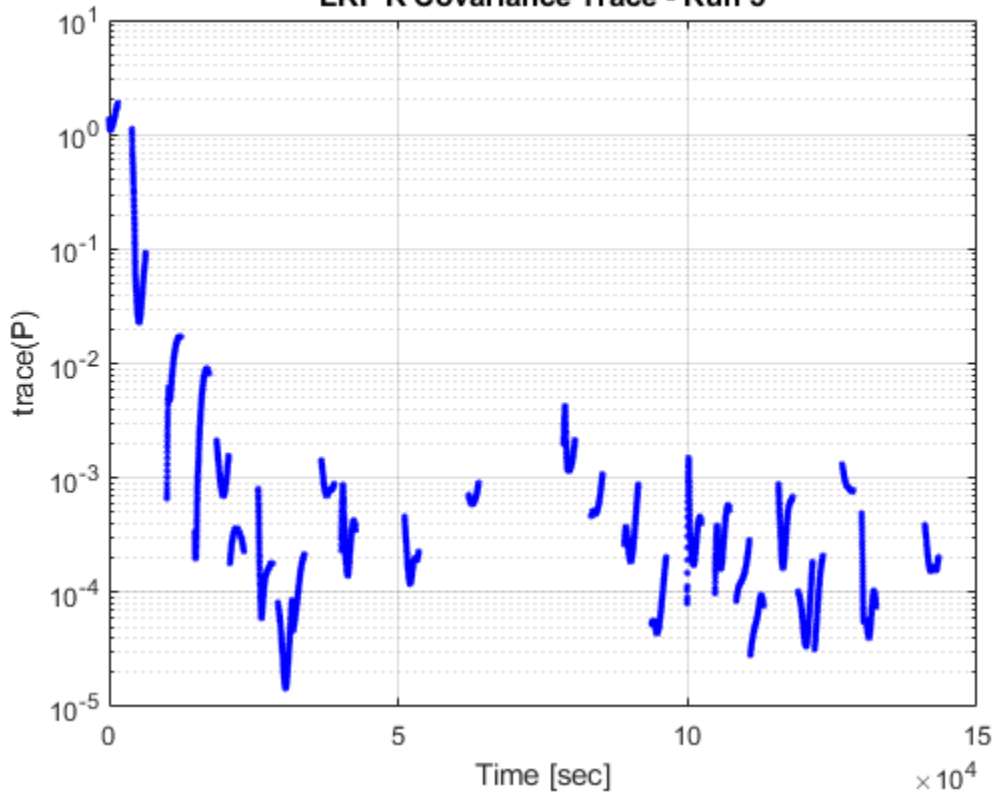
```
Prefit RMS: 0.8933
Postfit RMS: 0.8933
```

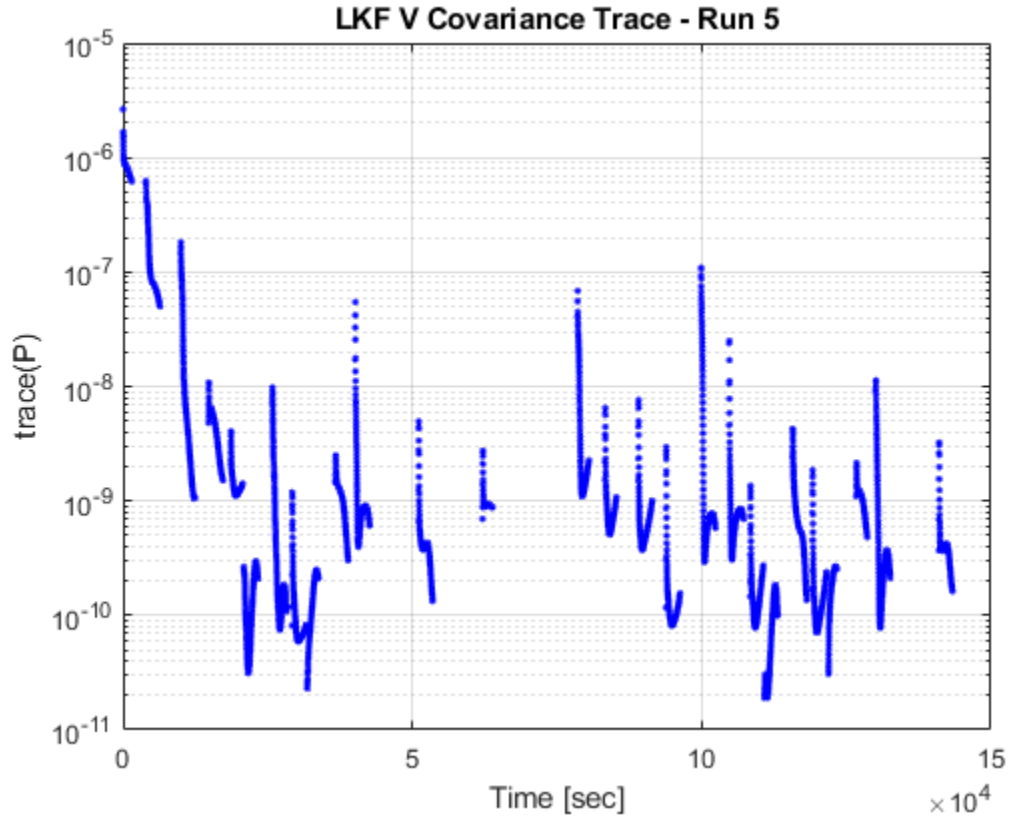


LKF Post-Fit Residuals - Run 5

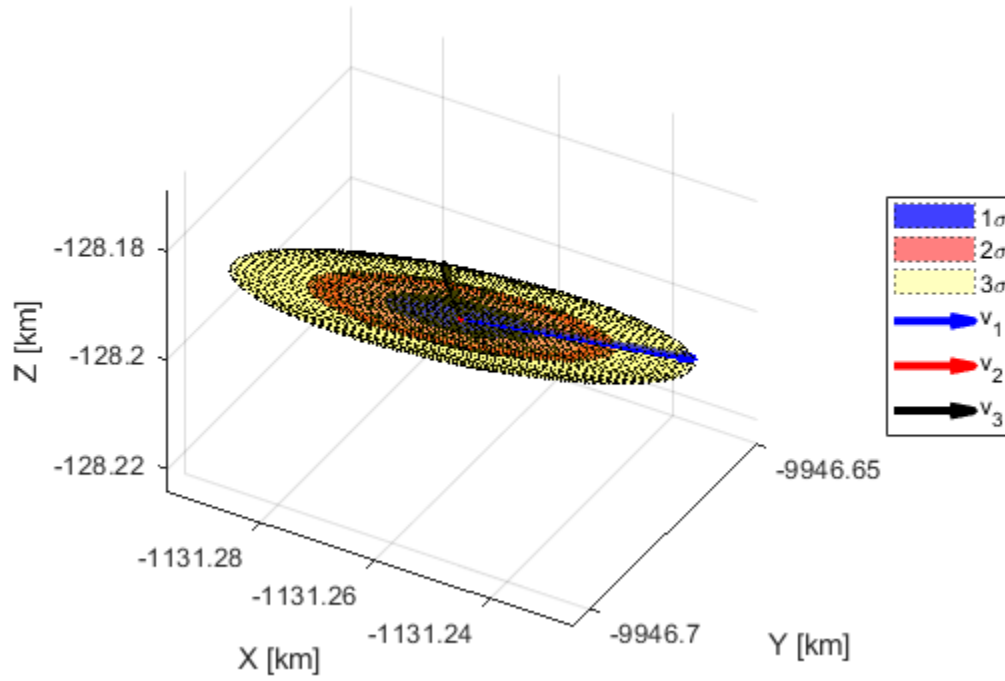


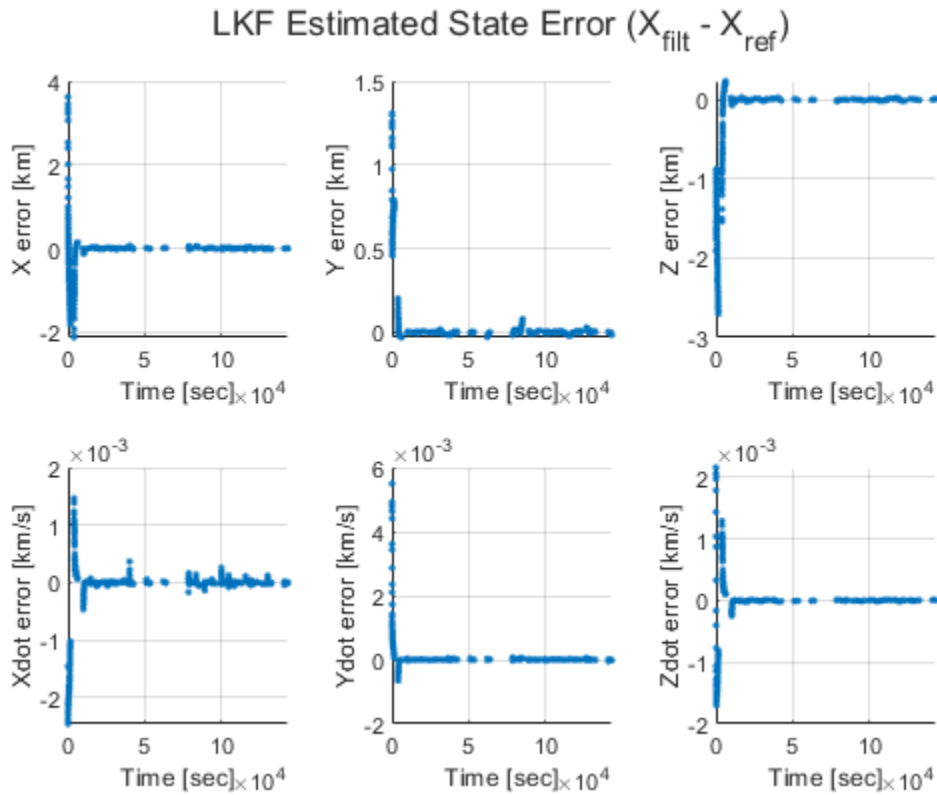
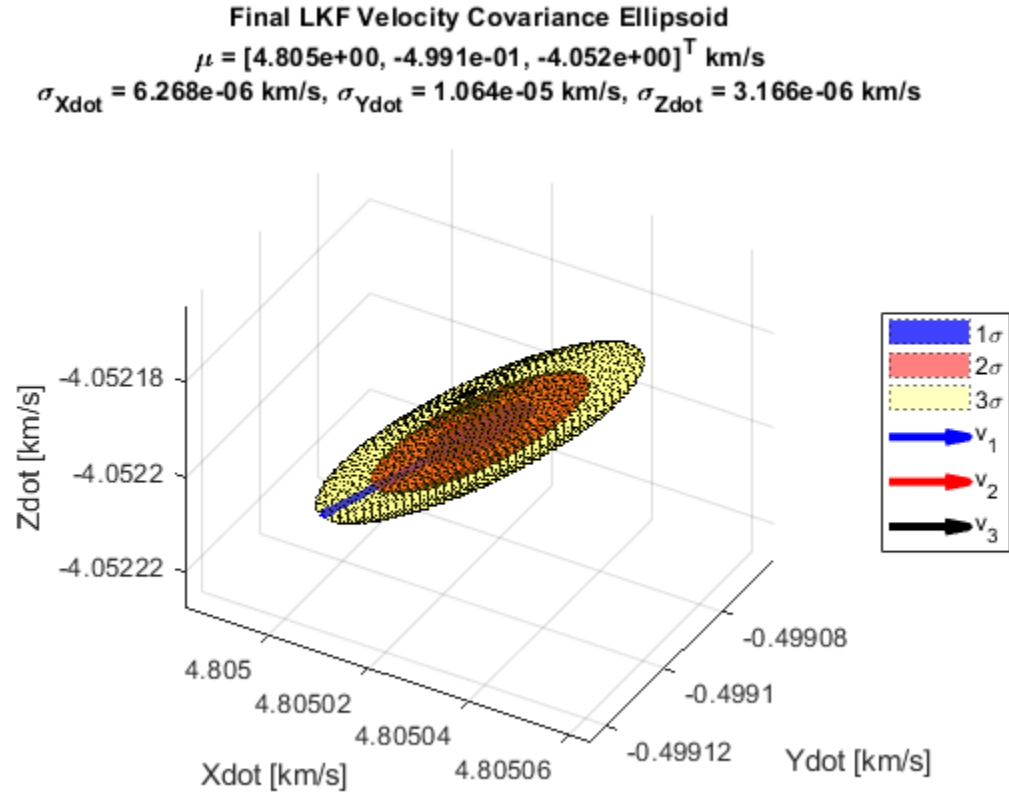
LKF R Covariance Trace - Run 5



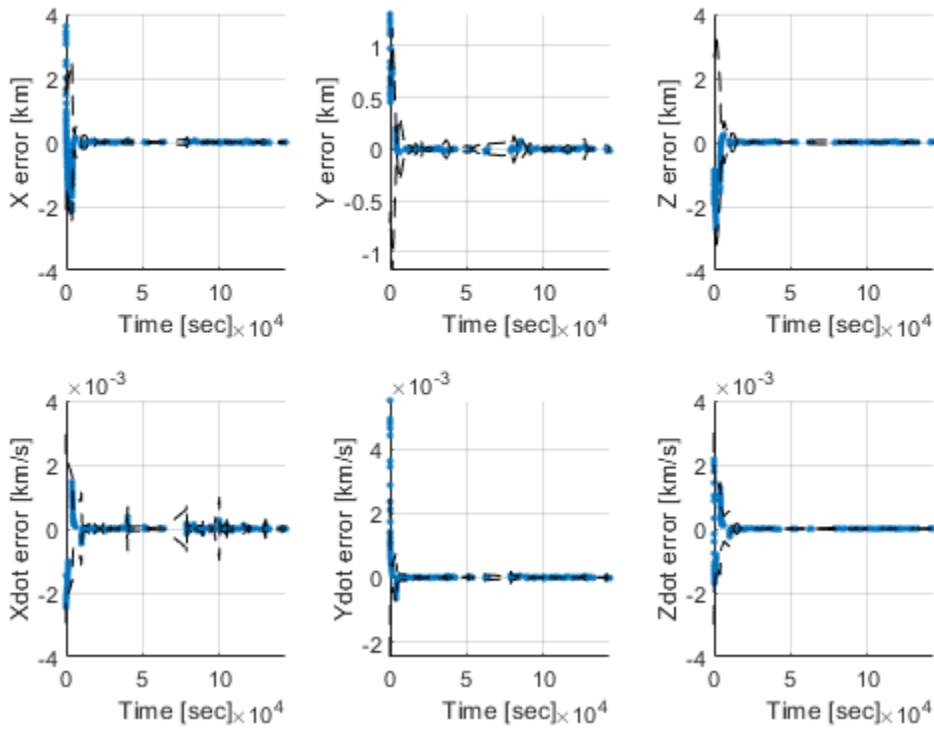


Final LKF Position Covariance Ellipsoid
 $\mu = [-1.131\text{e}+03, -9.947\text{e}+03, -1.282\text{e}+02]^T \text{ km}$
 $\sigma_X = 1.175\text{e}-02 \text{ km}, \sigma_Y = 5.133\text{e}-03 \text{ km}, \sigma_Z = 6.208\text{e}-03 \text{ km}$

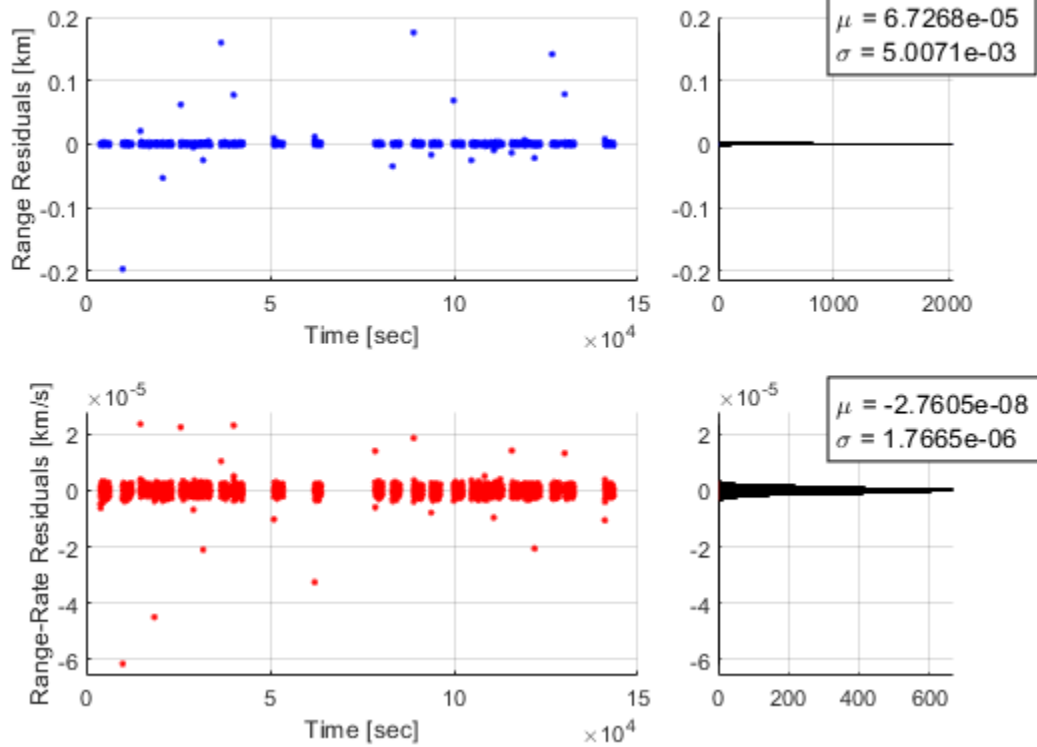




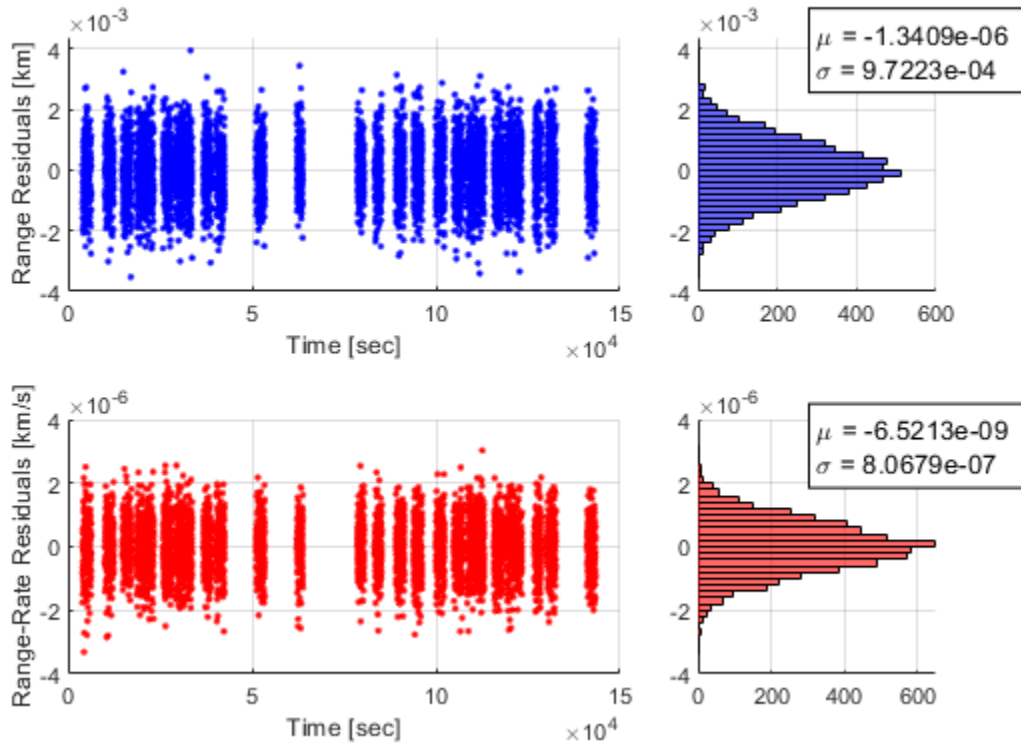
LKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



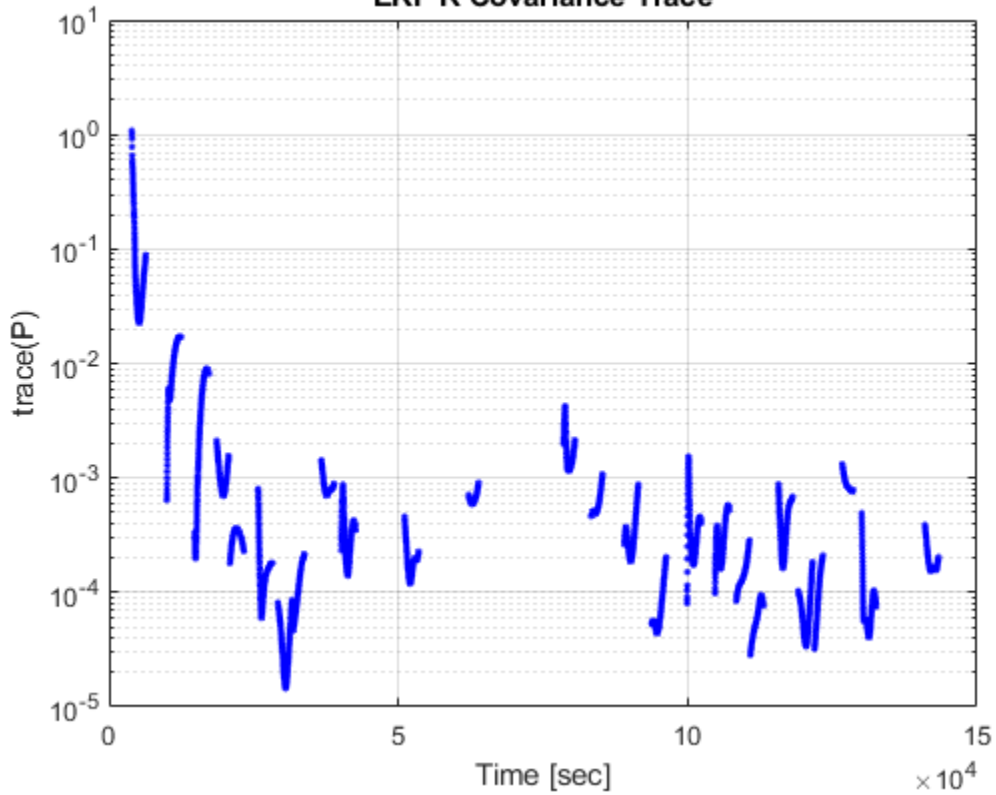
EKF Pre-Fit Residuals

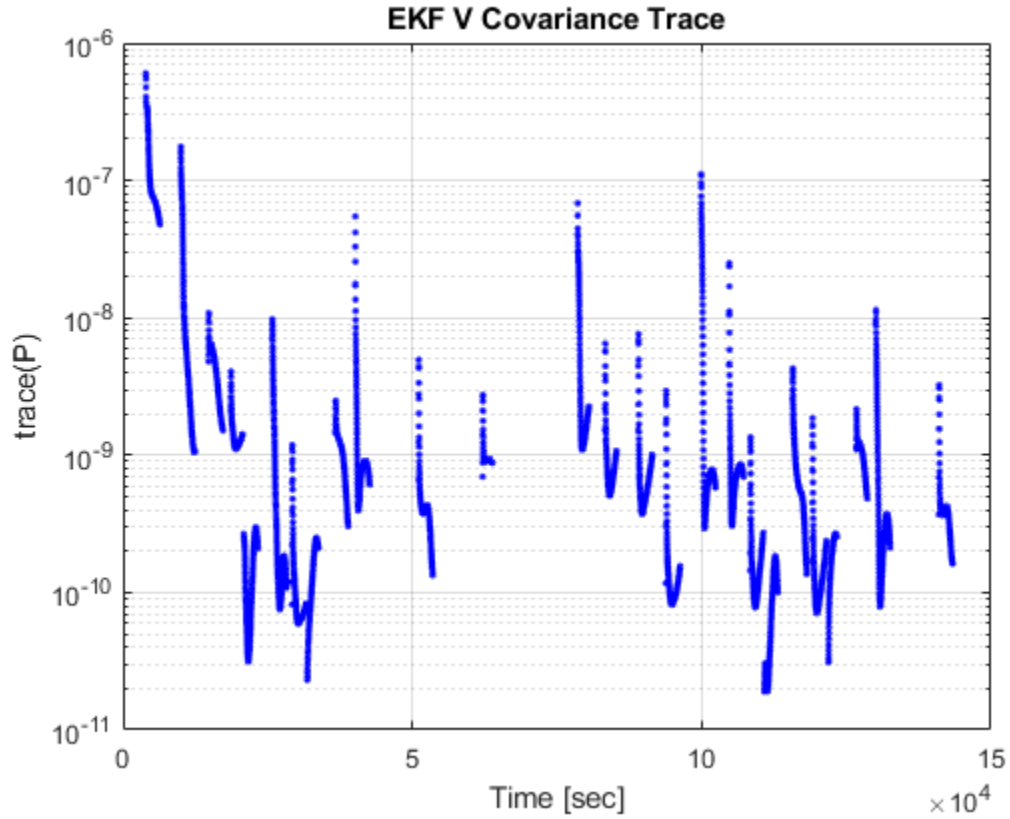


EKF Post-Fit Residuals

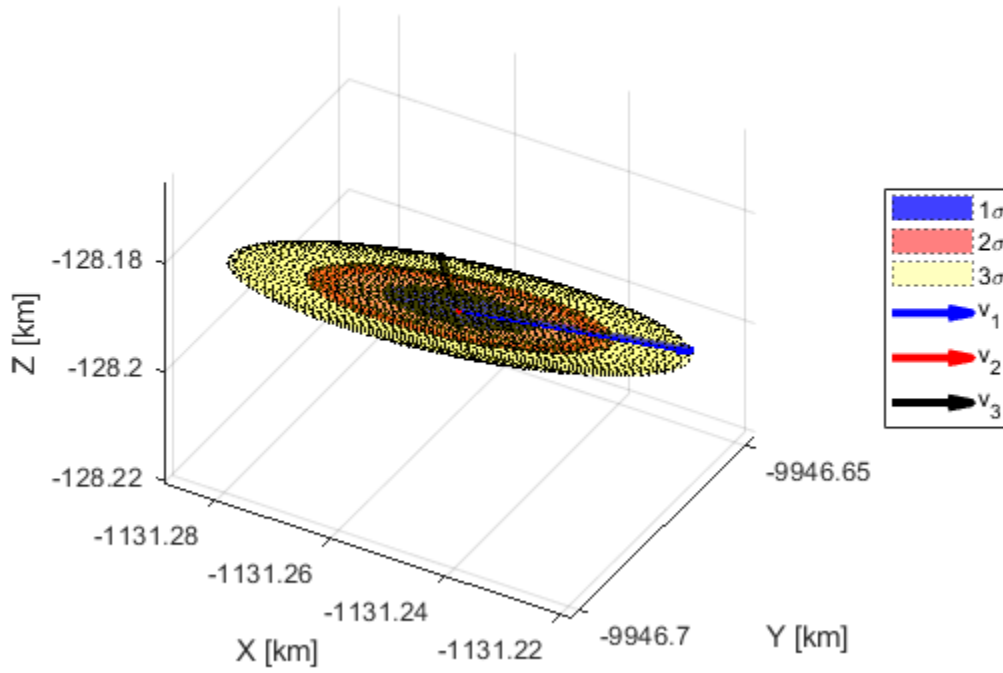


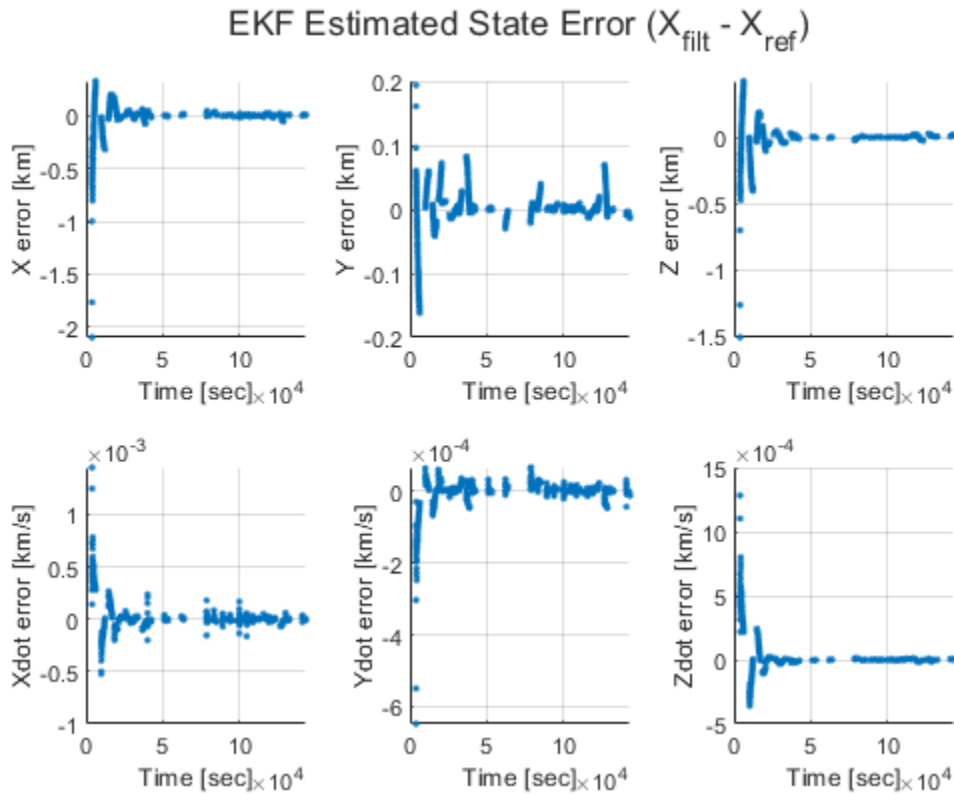
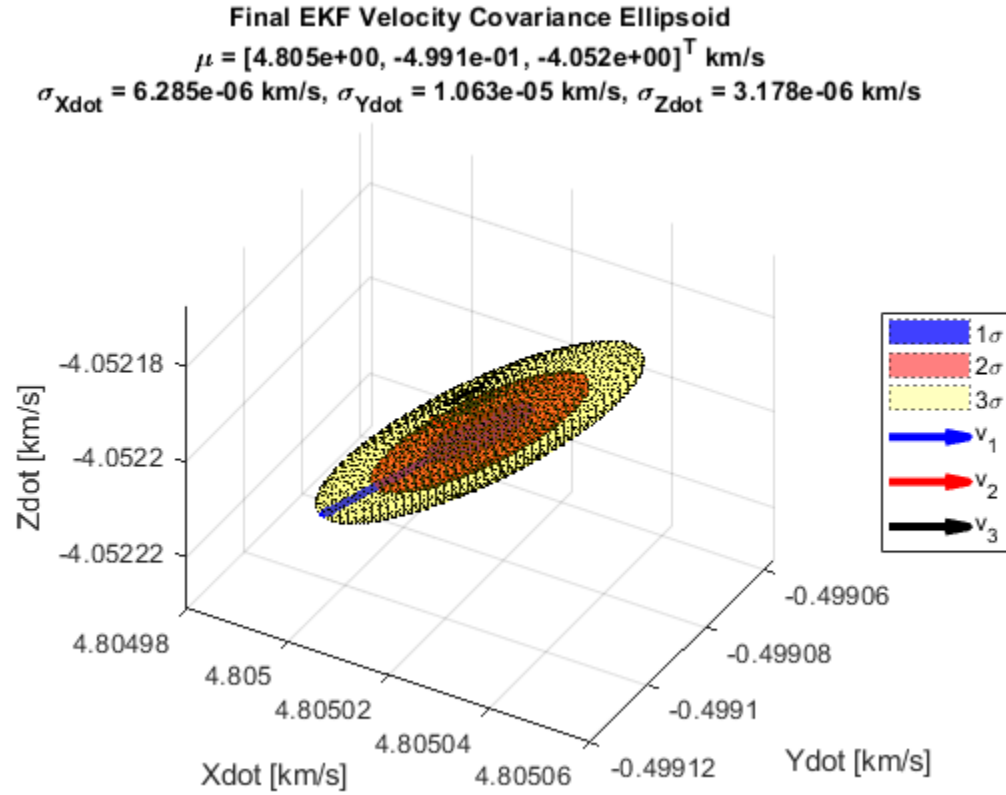
EKF R Covariance Trace

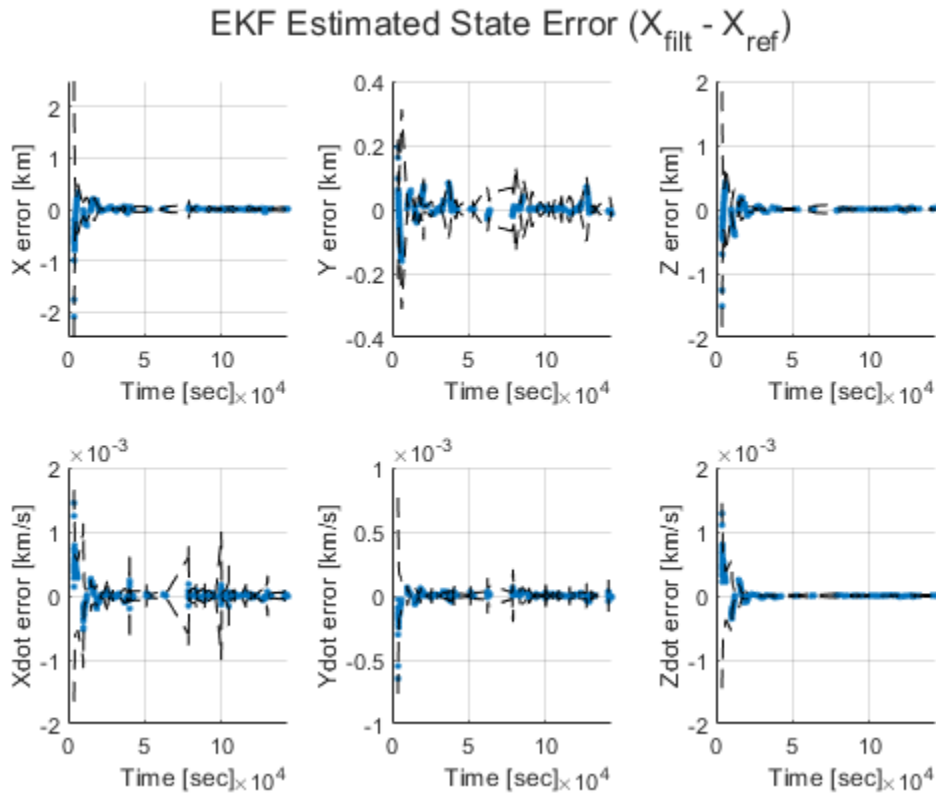




Final EKF Position Covariance Ellipsoid
 $\mu = [-1.131\text{e}+03, -9.947\text{e}+03, -1.282\text{e}+02]^T \text{ km}$
 $\sigma_X = 1.176\text{e}-02 \text{ km}, \sigma_Y = 5.118\text{e}-03 \text{ km}, \sigma_Z = 6.216\text{e}-03 \text{ m}$



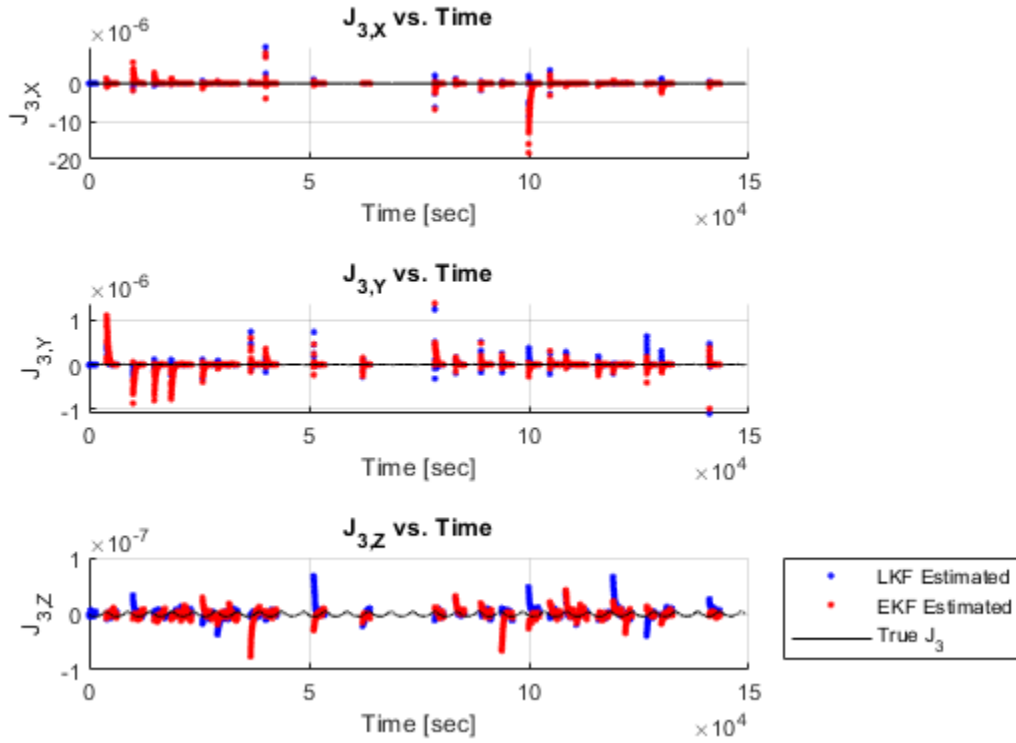




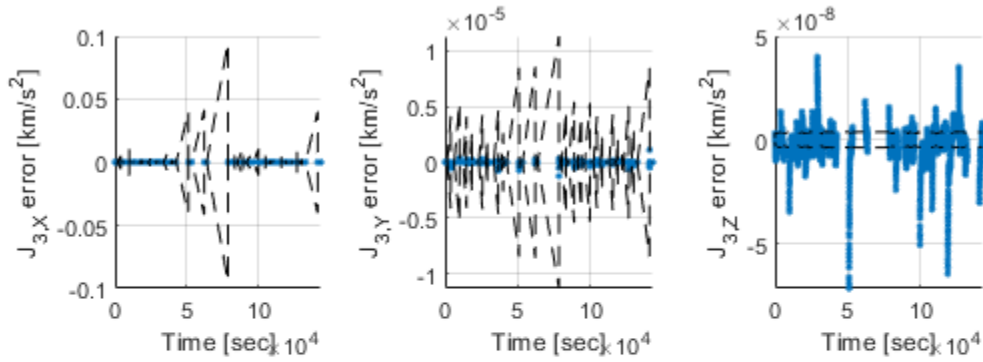
Problem 2b: Plot J3 accels and estimated filter estimates vs. time

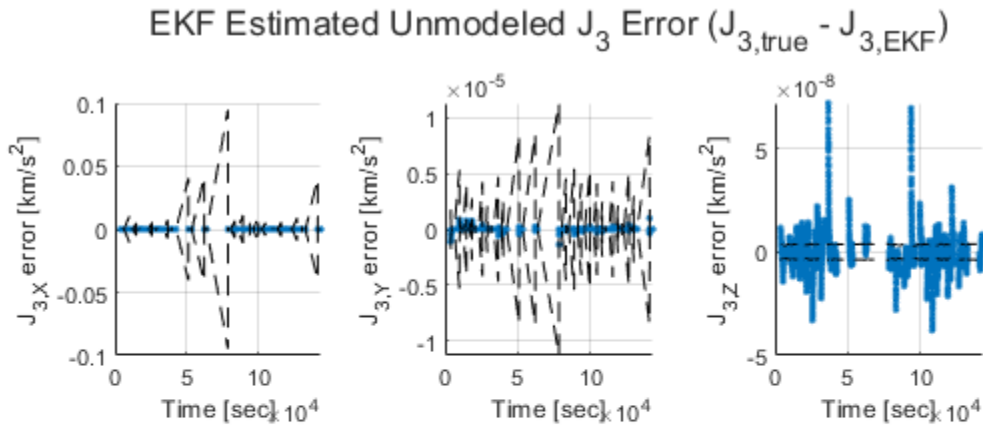
Plotting true J3 accels vs. filter estimates

J3 Accelerations vs. time



LKF Estimated Unmodeled J_3 Error ($J_{3,true} - J_{3,LKF}$)





Problem 2b: Different value of tau

Running filters with $\tau_x = 3.173e+02$, $\tau_y = 3.173e+02$, $\tau_z = 1.587e+01$

Running LKF:

Prefit RMS: 9534.1497, Postfit RMS: 0.9887. Iterating LKF. Runs so far: 1

Prefit RMS: 5324.0627, Postfit RMS: 0.9317. Iterating LKF. Runs so far: 2

Prefit RMS: 3521.5149, Postfit RMS: 0.9247. Iterating LKF. Runs so far: 3

Prefit RMS: 2330.8700, Postfit RMS: 0.9041. Iterating LKF. Runs so far: 4

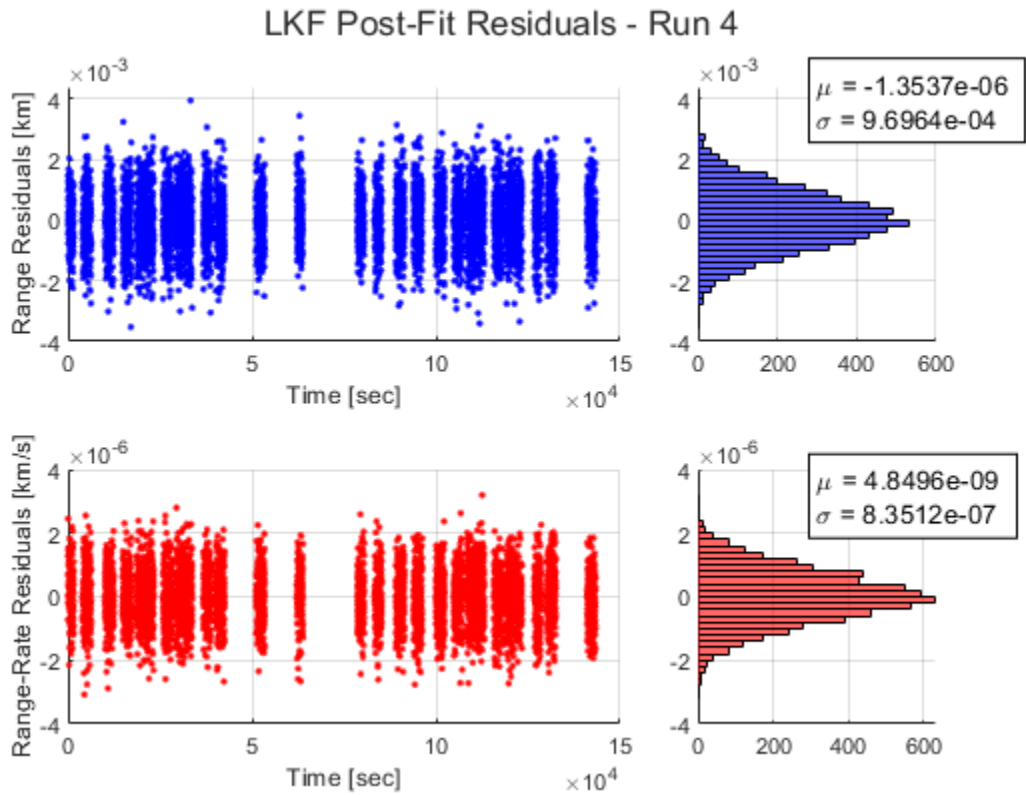
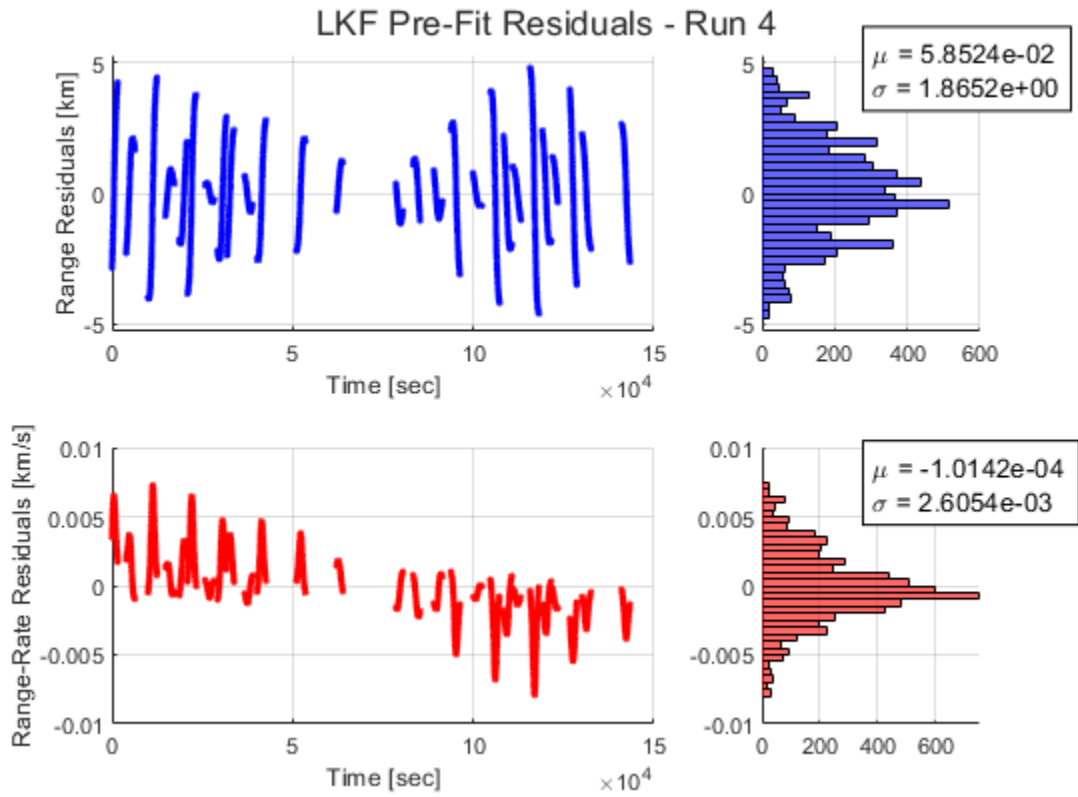
Final prefit RMS: 2267.0795. Converged after 4 runs

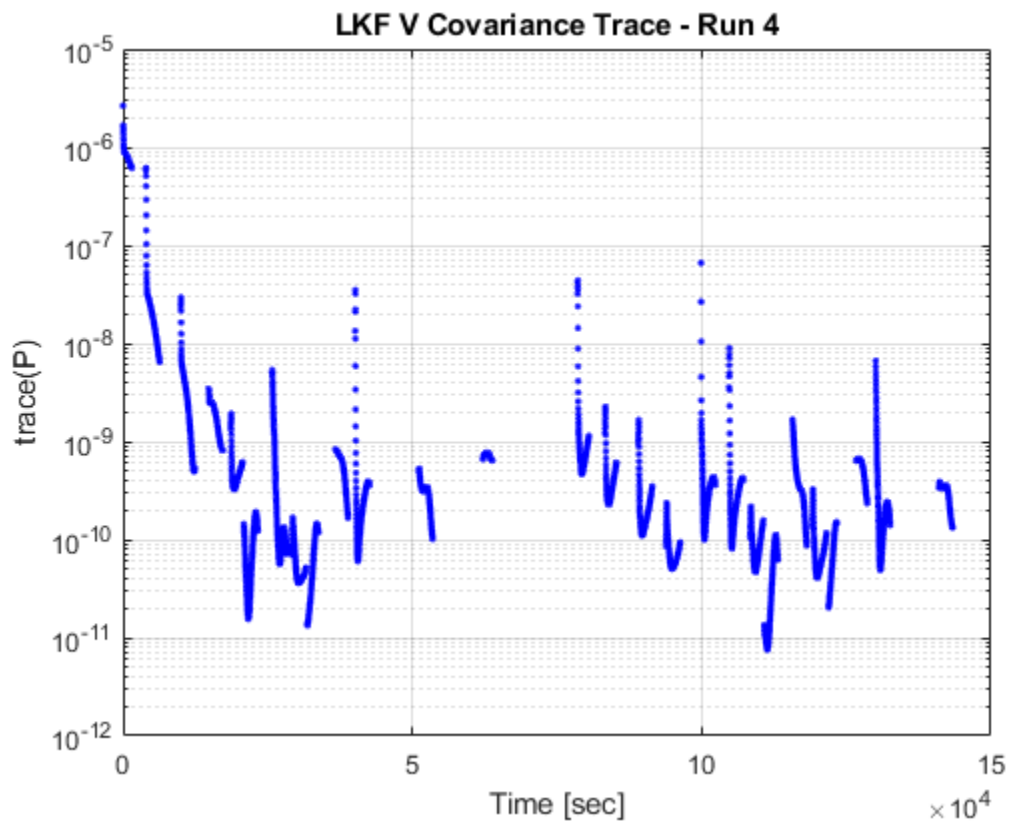
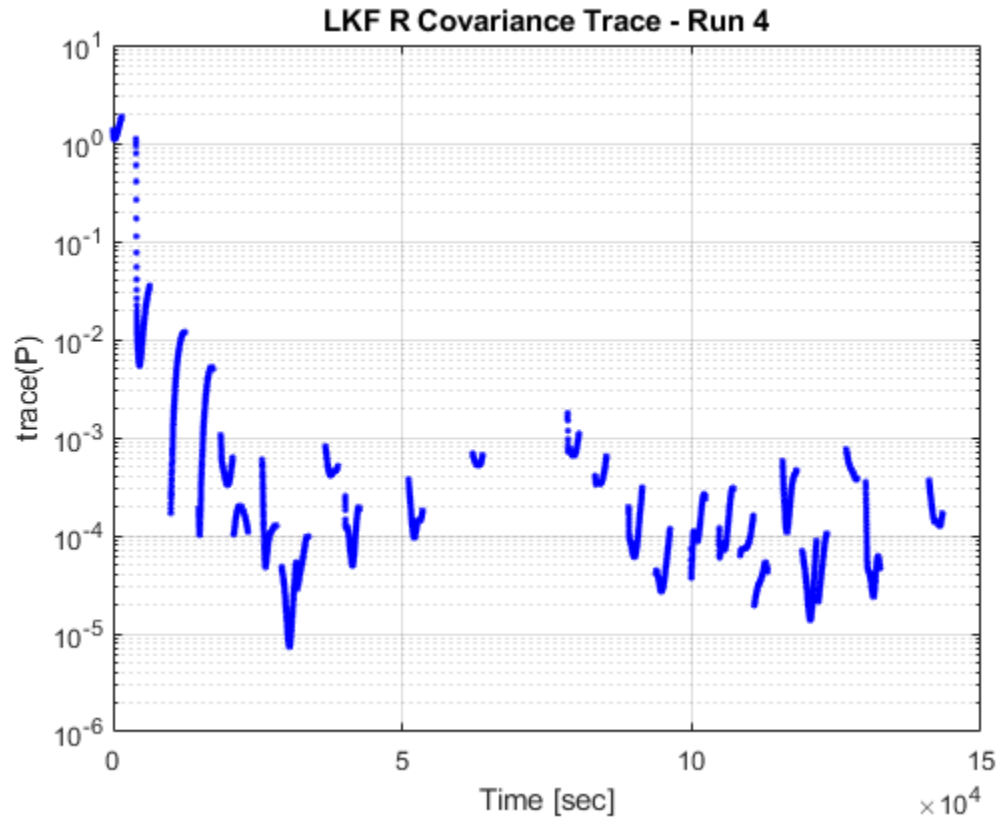
Final postfit RMS: 0.9048. Converged after 4 runs

Running EKF:

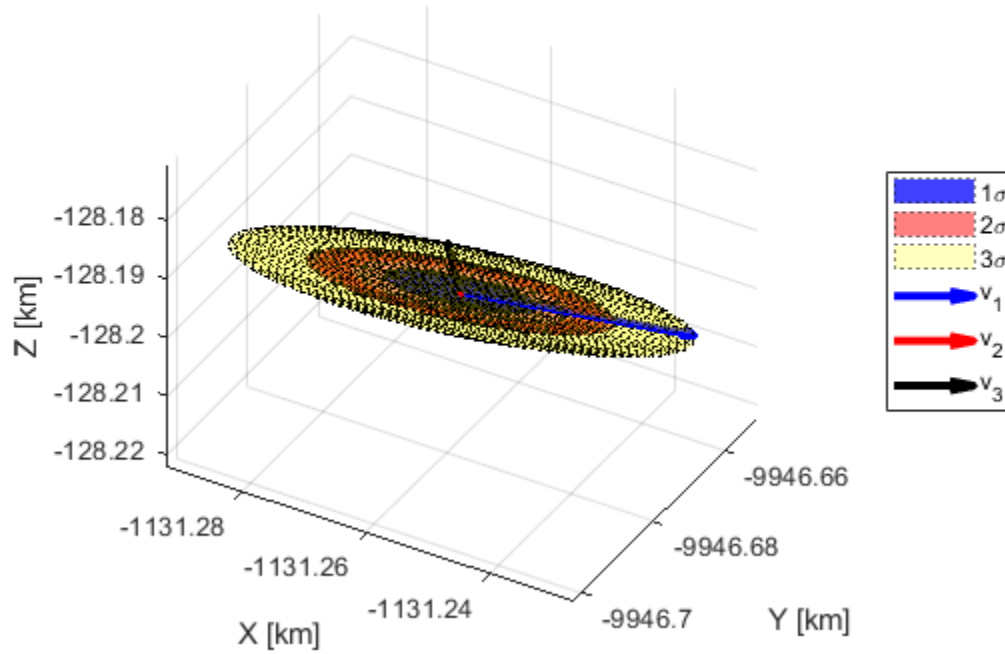
Prefit RMS: 0.9017

Postfit RMS: 0.9017

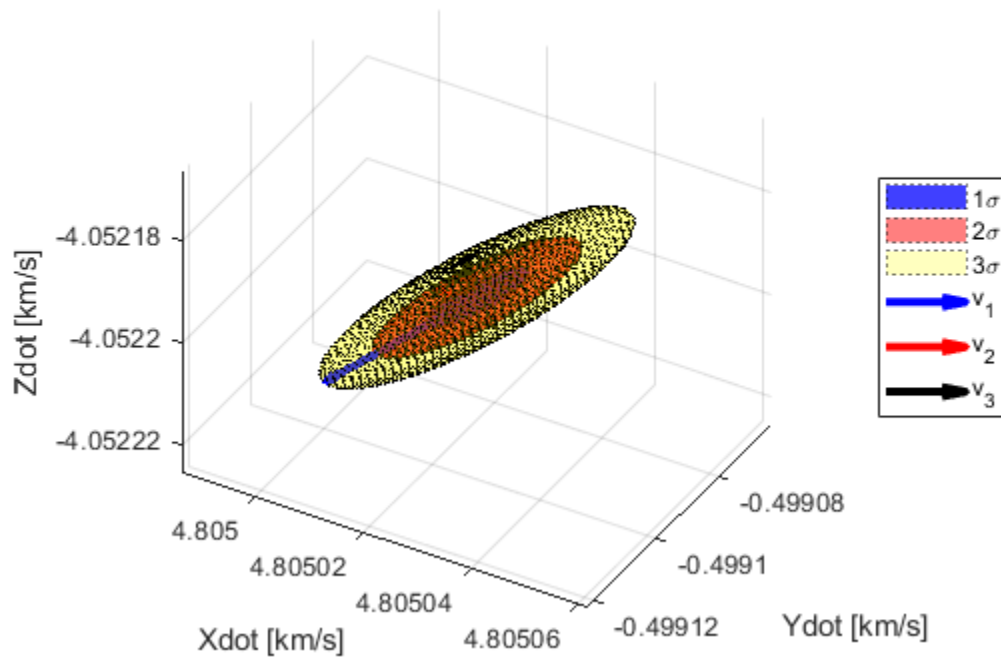




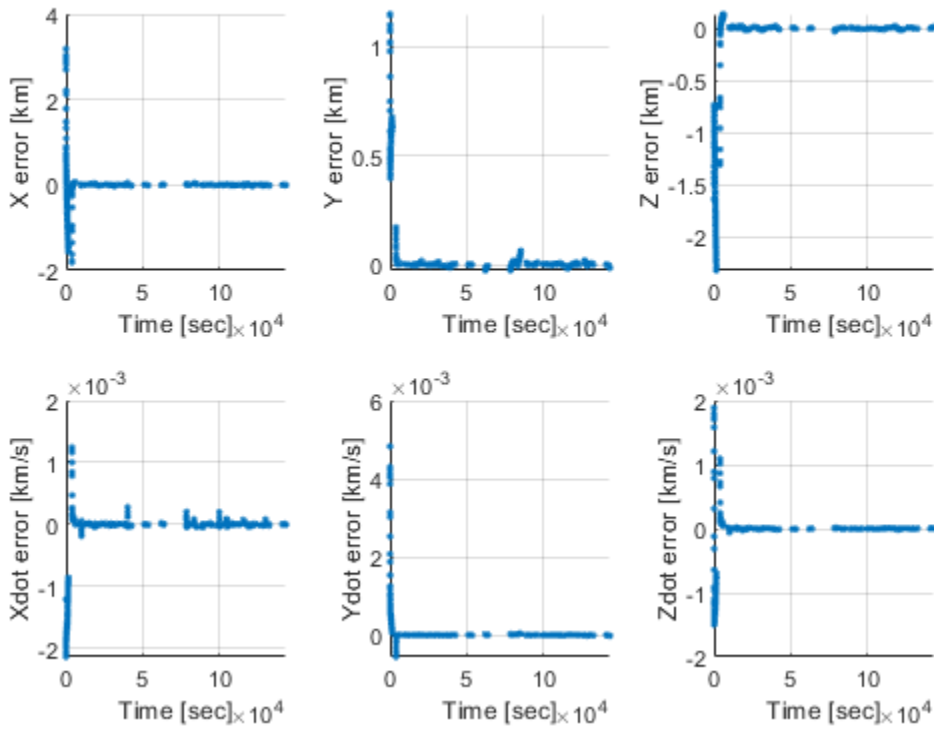
Final LKF Position Covariance Ellipsoid
 $\mu = [-1.131e+03, -9.947e+03, -1.282e+02]^T$ km
 $\sigma_X = 1.092e-02$ km, $\sigma_Y = 4.615e-03$ km, $\sigma_Z = 5.373e-03$ km



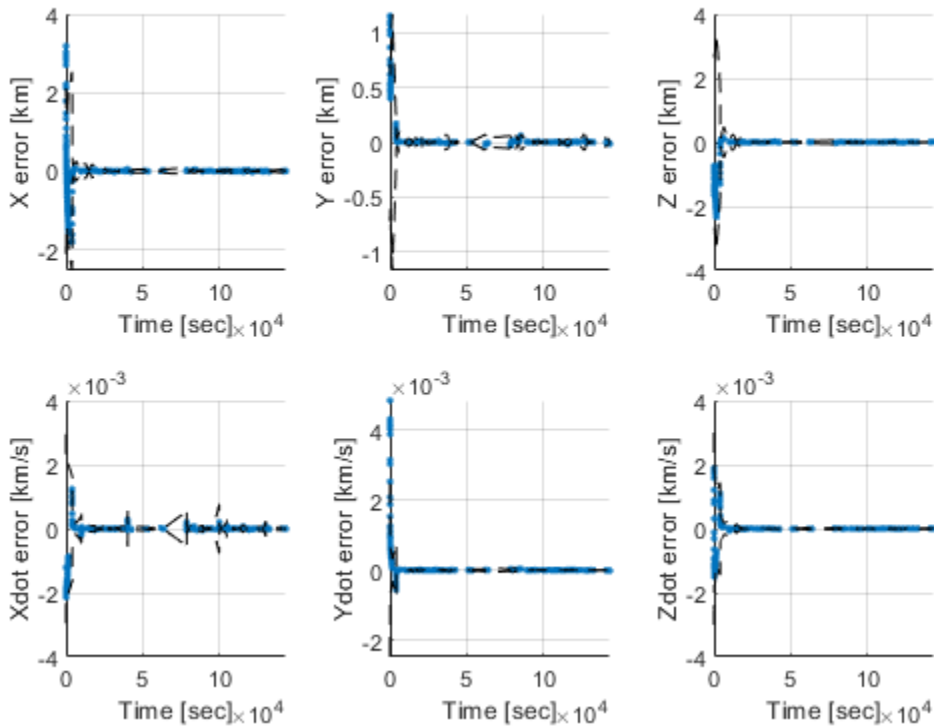
Final LKF Velocity Covariance Ellipsoid
 $\mu = [4.805e+00, -4.991e-01, -4.052e+00]^T$ km/s
 $\sigma_{Xdot} = 5.235e-06$ km/s, $\sigma_{Ydot} = 9.886e-06$ km/s, $\sigma_{Zdot} = 2.768e-06$ km/s

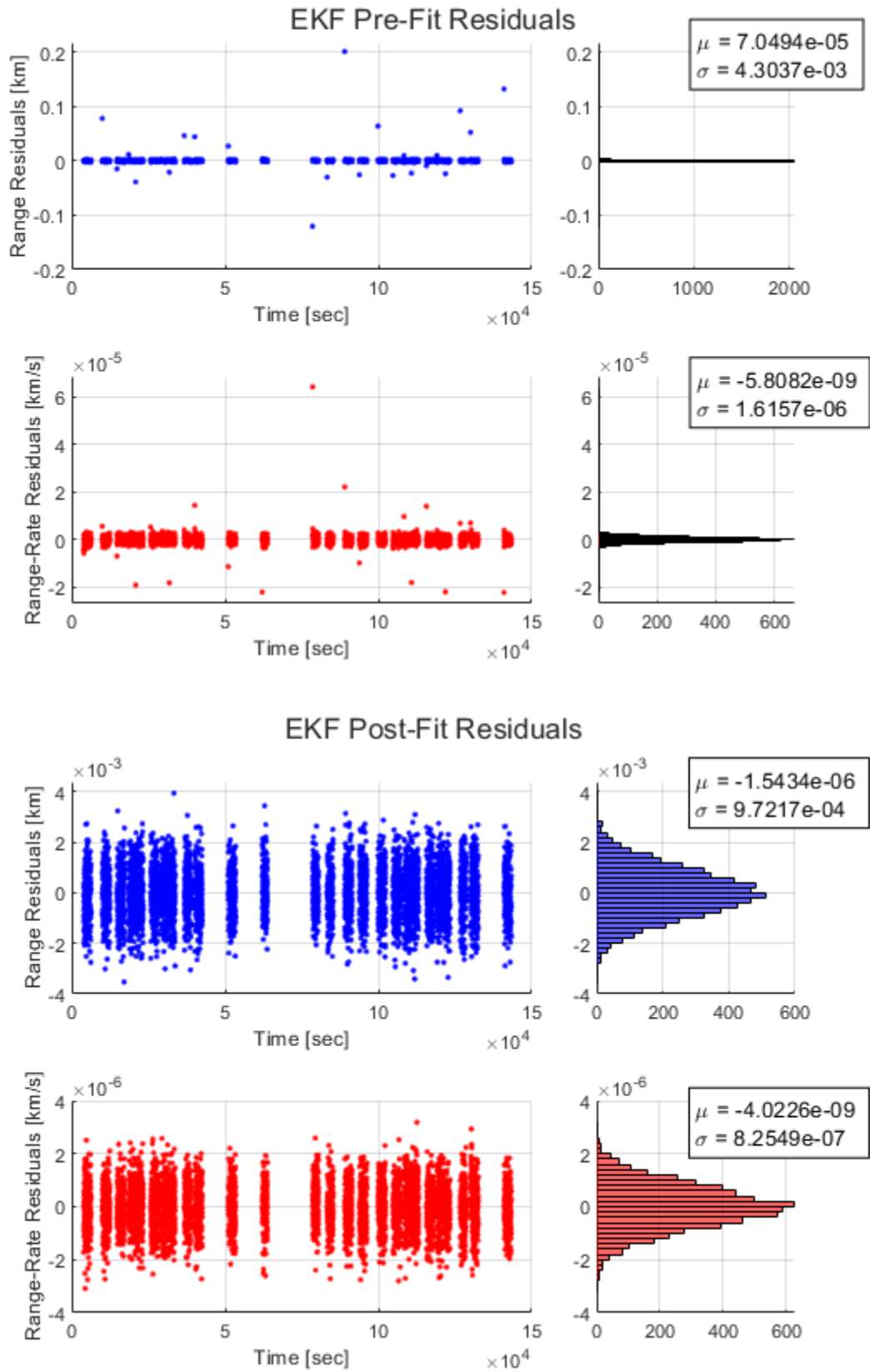


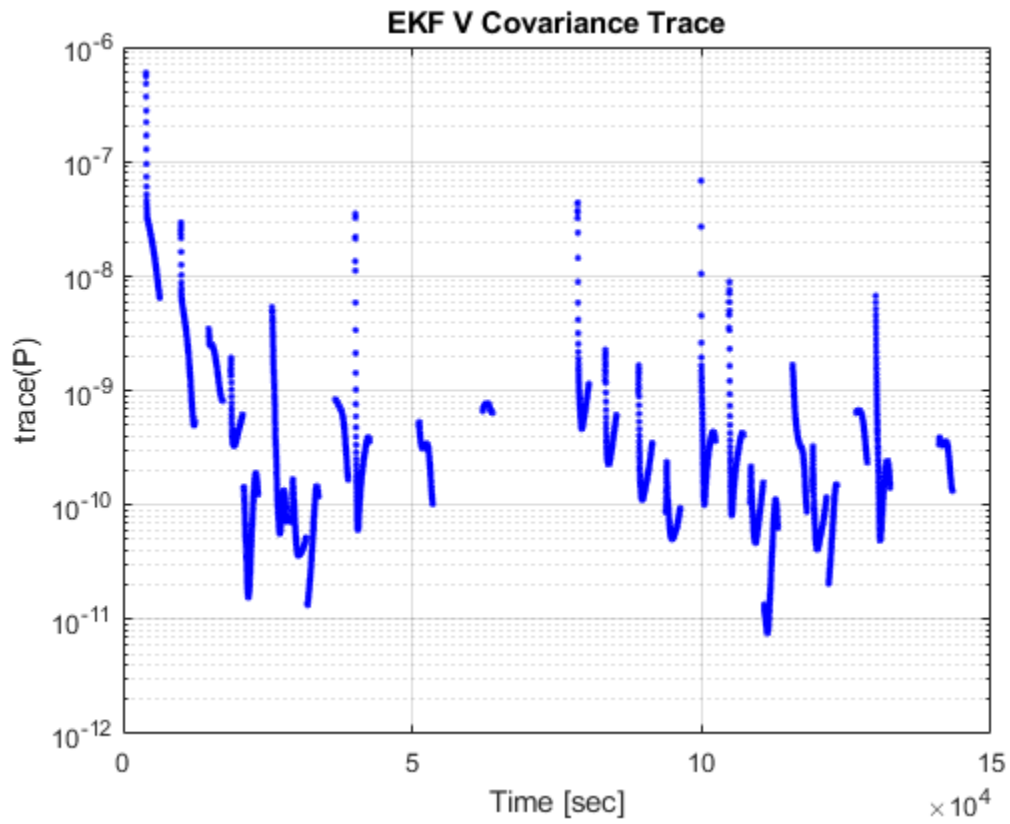
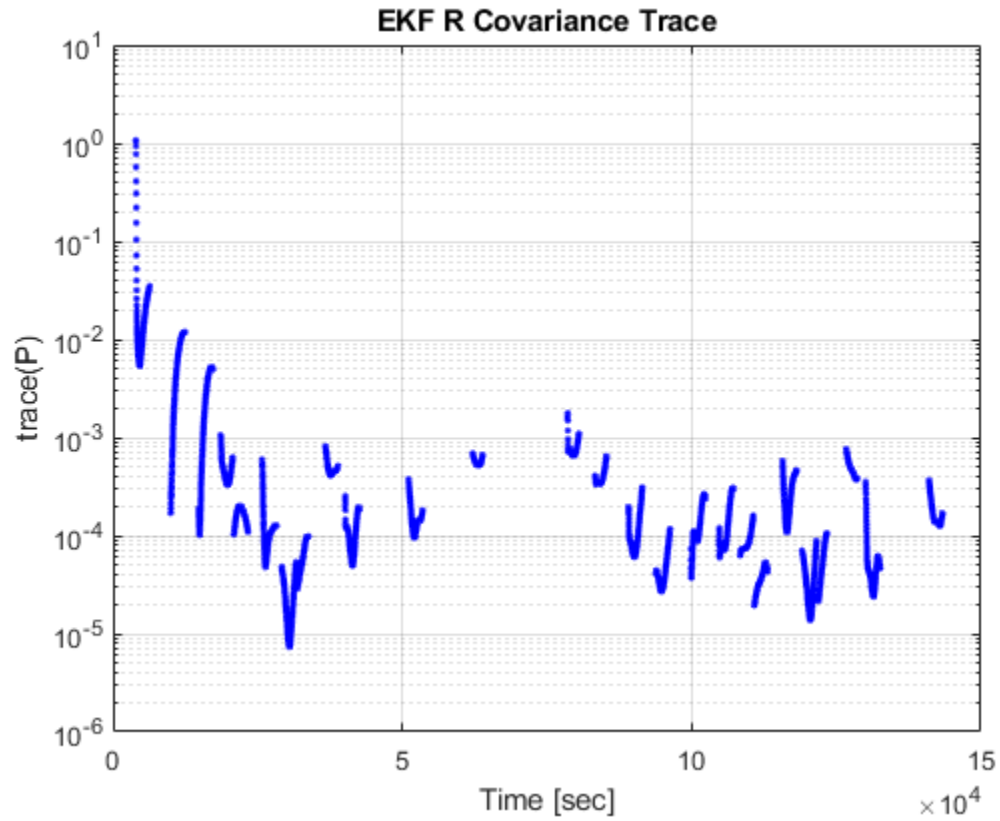
LKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)

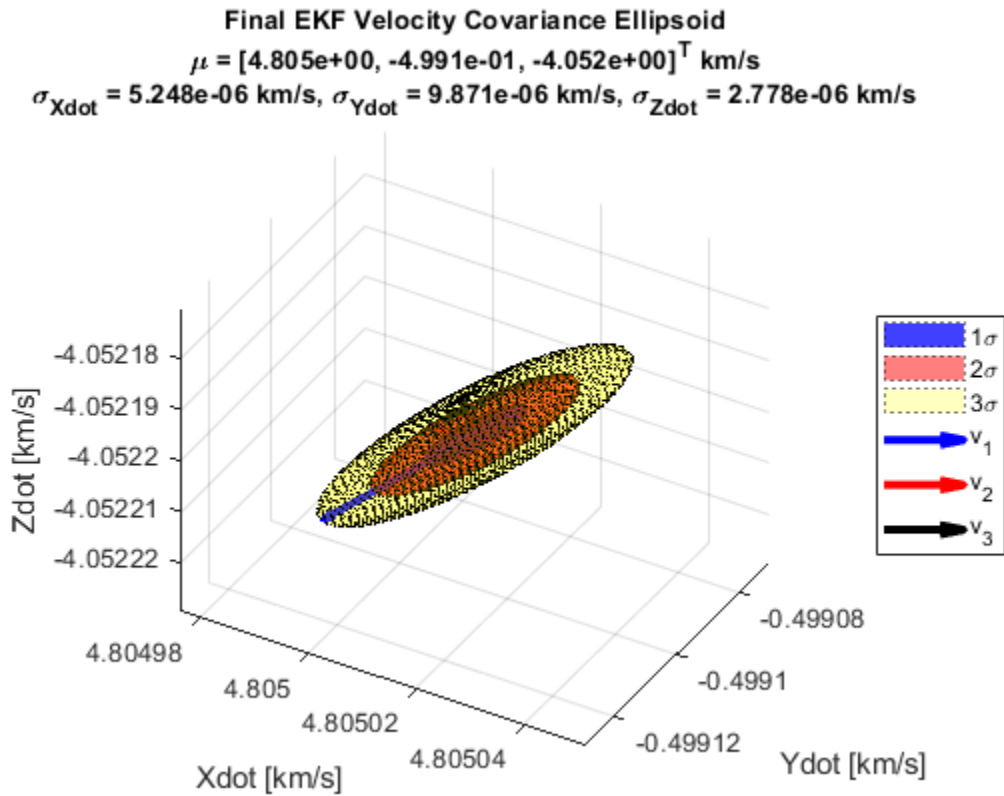
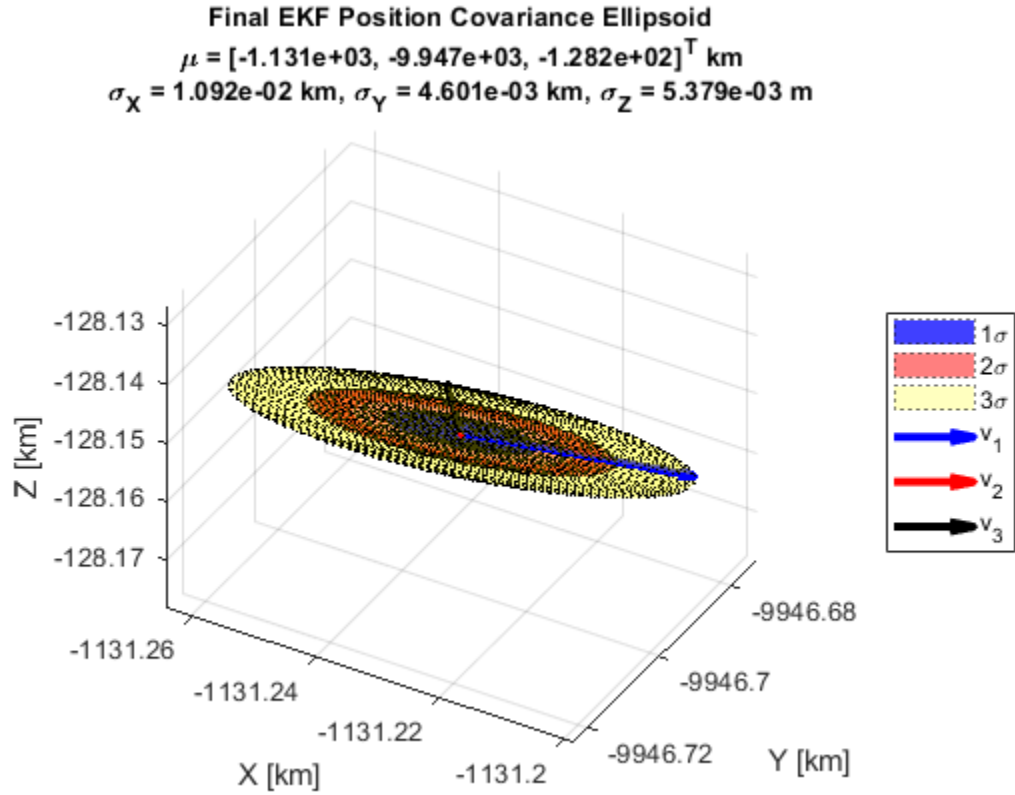


LKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)

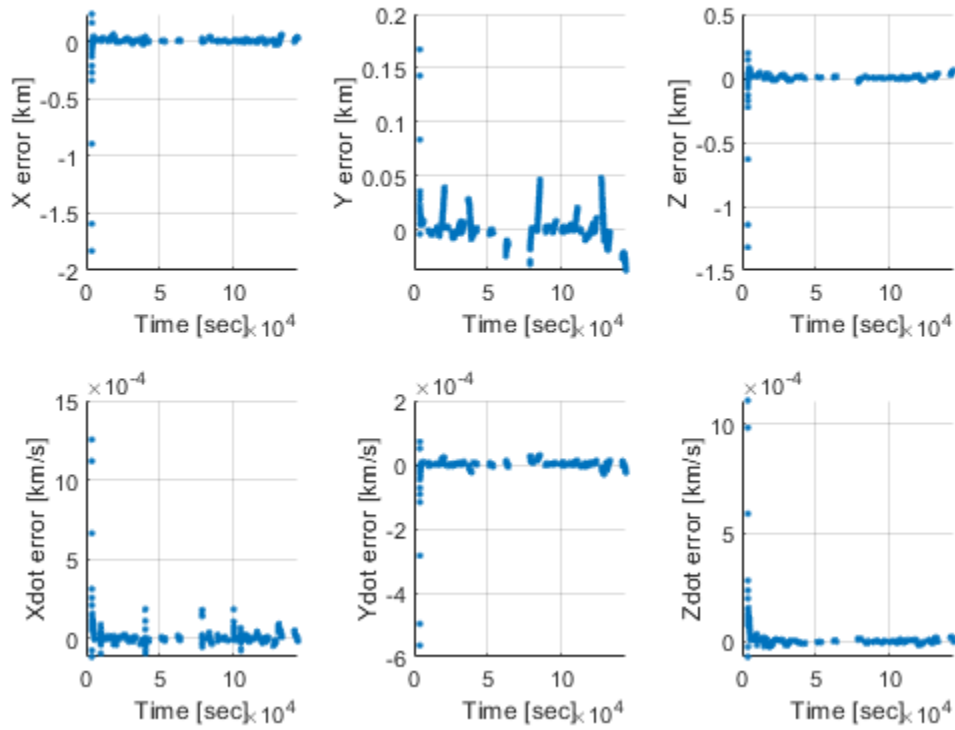




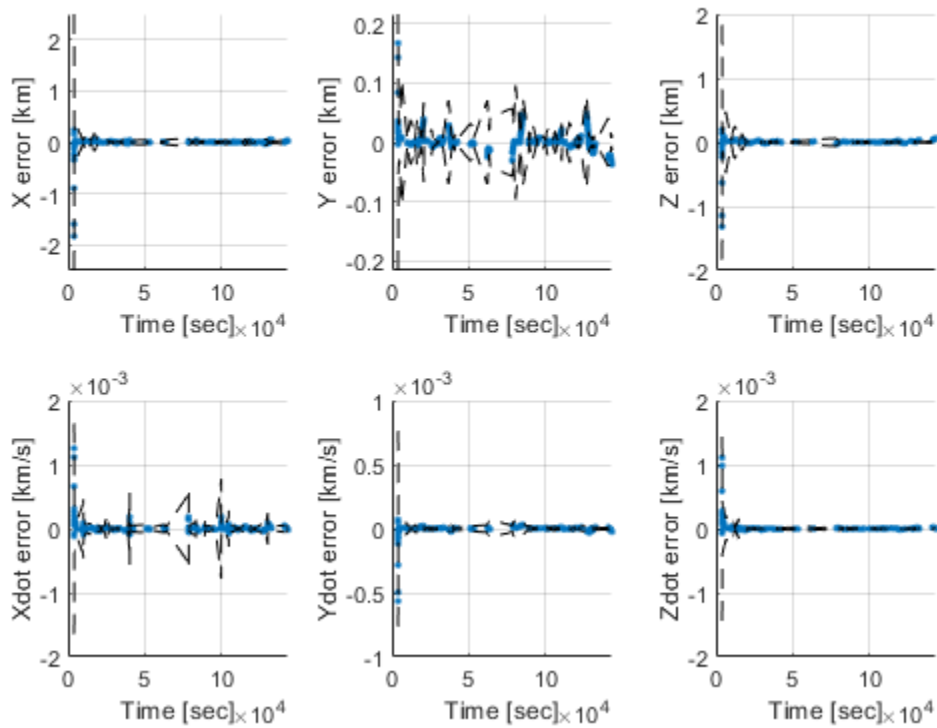


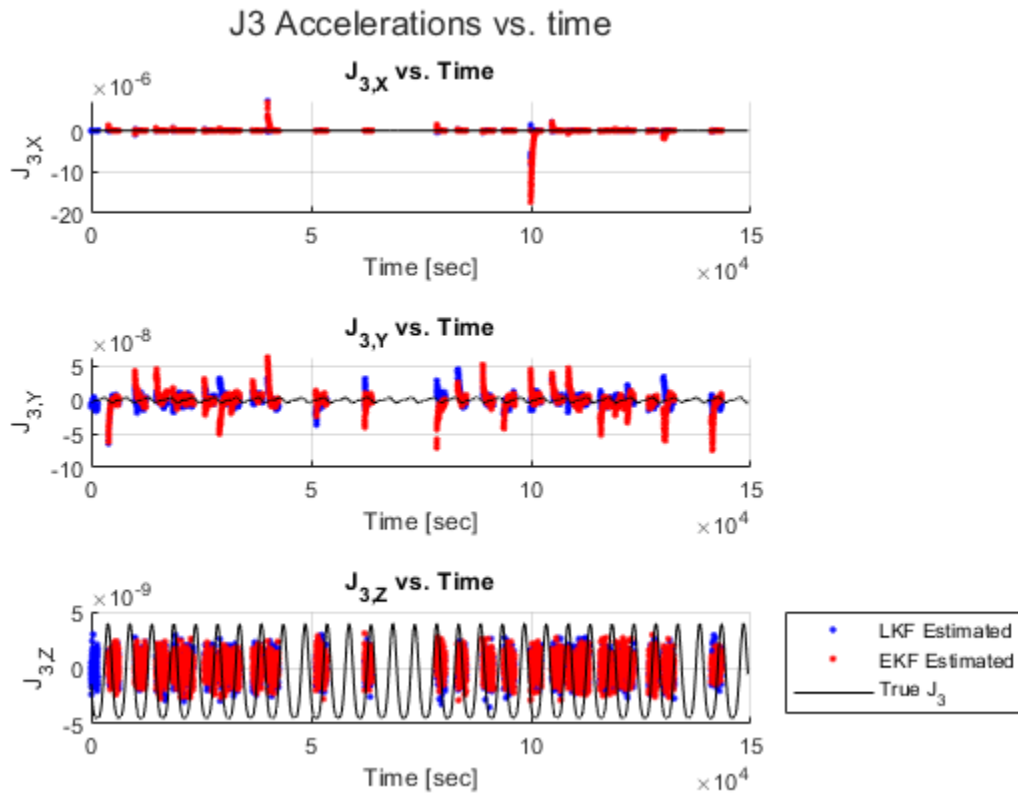


EKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)

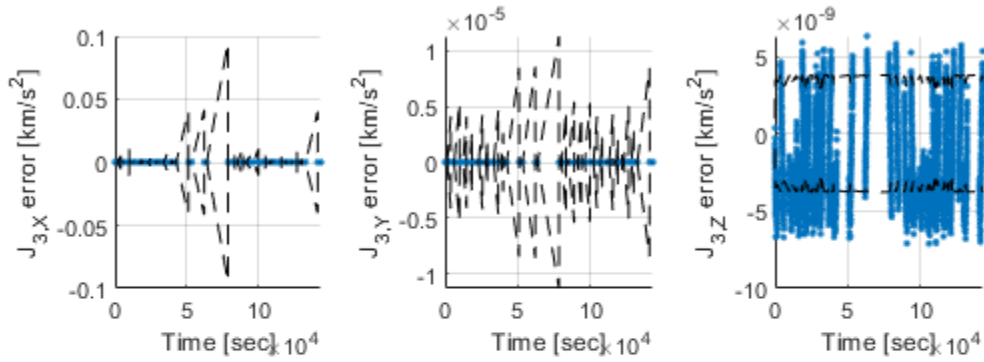


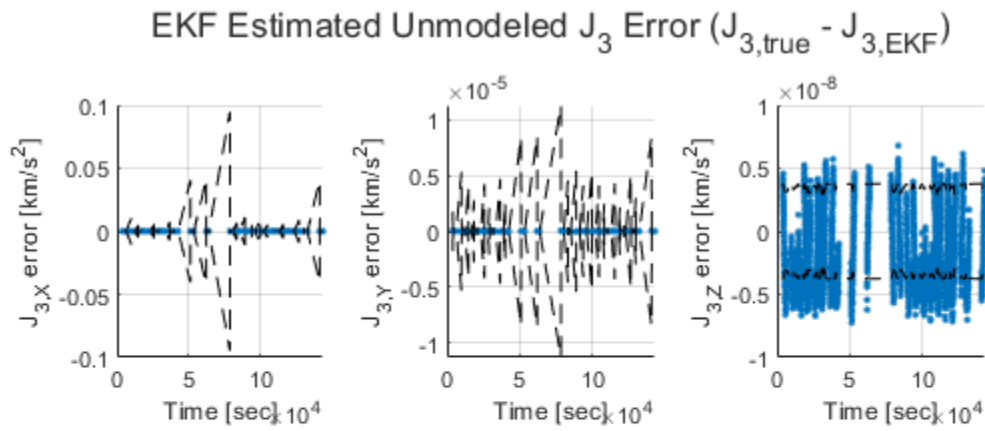
EKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)





LKF Estimated Unmodeled J₃ Error ($J_{3,true} - J_{3,LKF}$)





Published with MATLAB® R2023b