ASEN 1320 Fall 2020
Homework Assignment - Free Fall Motion
Due: 11:59pm on October 30 (Friday)

Problem: Astronauts who are orbiting the Earth experience sensations of weightlessness. Although free fall and weightlessness describe the same condition, free fall is the more accurate term. Free fall is the special type of motion that happens when the only force acting upon an object is gravity. There are no other forces such as friction, air drag, tension, or anything else pushing or pulling on the object. Let's consider the free fall motion of an object that is thrown into the air from a position (x_0, y_0) at time t_0 with some initial velocity (vx_0, vy_0) in the horizontal `x` and vertical `y` coordinate system. Newton's second law of motion tells us that in free fall, all objects will fall with the same gravitational acceleration, regardless of their mass. The equations that govern the free falling object's trajectory are thus given as follows.

$$\begin{aligned} vx(t) &= vx_0 \\ vy(t) &= vy_0 - g\,(t - t_0) \\ x(t) &= x_0 + vx_0\ (t - t_0) \\ y(t) &= y_0 + vy_0\ (t - t_0) - \frac{1}{2} g\,(t - t_0)^2 \end{aligned} \tag{1}$$

where g is gravitational acceleration ($9.81\ m/s^2$). The time t_g when the object hits the ground can be found by solving the following quadratic equation:

$$0 = y(t_g) = y_0 + vy_0\ (t_g - t_0) - \frac{1}{2} g\,(t_g - t_0)^2 \tag{2}$$

Note that this equation needs to be rearranged into the standard form (e.g., $at_g^2 + bt_g + c = 0$) to use the quadratic formula. There are two roots (answers) to a quadratic equation, and the negative root will provide a valid value for the ground impact t_g.
Tasks: Write a MATLAB program that performs the following tasks.

1. Compute the time t_g of the object's ground impact.
2. Calculate the trajectory $(x(t), y(t))$ and velocity $(vx(t), vy(t))$ of the object from the initial time t_0 to the time of impact t_g.
3. Make a plot of the object's free fall motion trajectory in the `x` and `y` coordinates, a plot of the object's velocity and position from t_0 to t_g in the `y` direction, and another plot in the `x` direction.

Each task should be performed by separate MATLAB functions, named **calcImpact**, **calcTrajectory**, and **makePlot**, respectively. You need to write a MATLAB script that sets initial values for $t0, vx_0, vy_0, x_0, y_0$ as well as an time step size dt, and calls these three functions for testing, but only **calcImpact**, **calcTrajectory**, and **makePlot** functions are required for MATLAB Grader submission. Use $9.81\ m/s^2$ for gravitational acceleration g.

- Main Script
  - Set an initial time value $t0$, initial conditions for the equations of motion vx_0, vy_0, x_0, y_0, and a time step size dt
  - Set up a column vector of length 4 that contains vx_0, vy_0, x_0, y_0 in this order to form an initial state vector $\mathbf{s}_0$
  - Call **calcImpact**, **calcTrajectory**, and **makePlot** functions
- **calcImpact** Function
  - Inputs: t_0, vy_0, y_0 (in this order)
  - Tasks: Solve the quadratic equation (Equation (2)) for t_g. Use the negative root for the ground impact time t_g
  - Outputs: t_g
- **calcTrajectory** Function
  - Inputs: t_0, dt, t_g, $\mathbf{s}_0$ (in this order)
  - Tasks: Generate a column vector of evenly spaced time points between t_0 and t_g with an time step size of dt to form a time vector $\mathbf{t}$ and evaluate equations of motion (Equation (1)) for $vx(t)$, $vy(t)$, $x(t)$, $y(t)$ at all time points in $\mathbf{t}$. Place the result in a state matrix $\mathbf{S}_{0g}$. The first column of $\mathbf{S}_{0g}$ should contain $vx(t_0), vx(t_0 + dt), \cdots, vx(t_g)$, the second column should contain $vy(t_0), vy(t_0 + dt), \cdots, vy(t_g)$, the third column should contain $x(t_0), x(t_0+dt), \cdots, x(t_g)$, and the forth column should contain $y(t_0), y(t_0+dt), \cdots, y(t_g)$. The total number of rows for $\mathbf{S}_{0g}$ should match the total length of $\mathbf{t}$.
  - Outputs: $\mathbf{t}$, $\mathbf{S}_{0g}$ (in this order)
- **makePlot** Function
  - Inputs: $\mathbf{t}$, $\mathbf{S}_{0g}$ (in this order)
  - Tasks: Make the following 2D line plots
    1. Plot $x(t_0), x(t_0 + dt), \cdots, x(t_g)$ in the horizontal axis and $y(t_0), y(t_0 + dt), \cdots, y(t_g)$ in the vertical axis. Axes should be labeled appropriately.
    2. Make plots of $y(t_0), y(t_0 + dt), \cdots, y(t_g)$ and $vy(t_0), vy(t_0 + dt), \cdots, vy(t_g)$ in the same figure. Use time as the horizontal axis and vertical position as the vertical axis. Axes should be labeled appropriately. Include a legend for both line plots.
    3. Make plots of $x(t_0), x(t_0 + dt), \cdots, x(t_g)$ and $vx(t_0), vx(t_0 + dt), \cdots, vx(t_g)$ in the same figure. Use time as the horizontal axis and horizontal position as the vertical axis. Axes should be labeled appropriately. Include a legend for both line plots.
  - Outputs: NONE