ASEN 5014 Linear Control Design

Midtern Exam 2 2024

Instructions: Do not communicate with any other person concerning the exam, except the instructor. Sign the Honor Code Pledge at the end of this exam. Any violation will receive a 0 for the exam. Be sure to verify any numerical answer, and explain any code used. Answers without explanations will earn no credit. Explanations using the terminology and results from class are expected. Illegible solutions will not be graded. The exam is due at 11:59 pm on November 13, 2024 on Canvas (in a single PDF file).

1. (60 pts) For each of the following statements, either show why it is true, or show why it is false, and fix it to make it true and provide a corresponding explanation why.
    a. A minimum error solution exists for any linear equation.
    b. In a direct sum, the components in the sum must be orthogonal complements.
    c. For a set of linear equations, multiple solutions exist when the number of unknowns exceeds the number of equations.
    d. For a set of linear equations, no solutions exist when the number of equations exceeds the number of unknowns.
    e. For a set of linear equations with equal numbers of equations and unknowns, a unique solution exists.
    f. Dim(LN(M)) > 0 implies M is not invertible.
    g. A full rank matrix M is invertible.
    h. For a linear mapping M, dim(RN(M)) $\leq$ dim(RS(M)).
    i. For a linear mapping M, dim(CS(M)) = dim(RN(M)).
    j. A minimum length solution to a linear equation always exists.

2. (40 pts) Given the matrix M of a linear mapping below
    a. Find basis sets for CS(M), LN(M), RN(M), and RS(M).
    b. Determine the components of $\underline{y}$ (given below) in the CS(M) and LN(M) subspaces. What is the error in a minimm error solution?
    c. Describe the set of all solutions to $\underline{z} = M\underline{x}$ for $\underline{z}$ given below. What is the minimm length solution?
    d. Is the set of solutons from part c.) a subspace of the domain of M?

$$M = \begin{bmatrix} 1 & -3 & 5 & 1 & 6 \\ 0 & -1 & 1 & 0 & 3 \\ 3 & -4 & 10 & 3 & 3 \\ 1 & -1 & 3 & 1 & 0 \end{bmatrix}$$

$$\underline{y}^T = \begin{bmatrix} 1 & 1 & 2 & -2 \end{bmatrix}$$

$$\underline{z}^T = \begin{bmatrix} -1 & -2 & 7 & 3 \end{bmatrix}$$

Honor Code Pledge

On my honor, as a University of Colorado Boulder student, the solutions submitted are entirely my own, and I have not communicated with any other person (except the instructor) concerning the exam. Also, I have not utilized any materials except my own notes, the posted materials for this class on Canvas, and the textbook for the class.

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