

ASEN 6080 HW 2

- dan Fober, 108577813

1. Estimate the orbit from Homework 2 using a batch, CKF, and EKF. Take measurements with $\sigma_p = 1 \text{ m}$ & $\sigma_v = 1 \text{ mm/s}$, also create \bar{P}_0 using $\sigma_r = 1 \text{ km}$ and $\sigma_v = 1 \text{ m/s}$.

- a. choose a δx_0 within \bar{P}_0 to use for all 3 filters.

$$\bar{P}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1e-6 & 0 \\ 0 & 0 & 0 & 0 & 1e-6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Given \bar{P}_0 , I'm using

$$\delta x_0 = 0.5 \cdot [1 \ 1 \ 1 \ 1e-3 \ 1e-3]^T$$

- b. process the data using each filter and do the following:

- i. compute the error between the estimated + true states for the entire arc, and plot the $\pm 3\sigma$ bounds, what happens during the last set of measurements?

- ii. Plot the post-fit measurement residuals. Do these match the noise levels you added?

iii. Report the state RMS errors & post-fit residual RMS values, how do these values change if you ignore the first pass of data when computing RMS values?

iv. For the Batch filter, how many times did you iterate? Show how the RMS values change with each iteration

Batch Filter

i. See PDF for peat.

during the last set of measurements, I don't see anything meaningful happen to the state errors

ii. my post-fit residuals match the provided standard deviations of $\sigma_p = 1 \text{ m}$ ($1 \times 10^{-3} \text{ km}$) and $\sigma_i = 1 \text{ mm/s}$ ($1 \times 10^{-6} \text{ km/s}$) after the 3rd iteration of the batch filter!

iii. my state RMS errors after the 3rd iteration were

component: $[2.623 \times 10^{-5}, 1.2901 \times 10^{-5}, 2.198 \times 10^{-5}, 1.3236 \times 10^{-8}, 9.3911 \times 10^{-9}, 1.1176 \times 10^{-8}]^T$

full-state: 3.6572×10^{-5}

similarly, my post-fit residuals were as follows:

1st run: 25,7585

2nd run: 1,0052

3rd run: 1,0052

IV. my batch filter converged after the 3rd run, as the post-fit residual RMS error settled to $\approx 1,0052$.

CKF/LKF

i. see PDF for float

during the last set of measurements, I don't see anything meaningful happen other than a decrease in uncertainty.

ii. my post-fit residuals match the provided standard deviations of $\sigma_p = 1 \text{ m} (1e-3 \text{ km})$ and $\sigma_v = 1 \text{ mm/s} (1e-6 \text{ km/s})$ after the 3rd LKF iteration

iii. my RMS errors after the 3rd LKF iteration were

component: $[0.0124, 0.0077, 0.0153, 1.2085 \times 10^{-5}, 1.3783 \times 10^{-5}, 1.6003 \times 10^{-5}]^T$

full-state: 0.0212

if I ignore the first pass of data, my RMS errors reduce drastically! In fact, they become the same order as the batch filter.

EKF

i. See PDF for plot

again, during the last set of measurements I don't see anything meaningful happen other than a reduction in uncertainty.

ii. my post-fit residuals match the provided standard deviations of $\sigma_p = 1 \text{ m} (1e-3 \text{ km})$ and $\sigma_d = 1 \text{ mm/s}$ ($1e-6 \text{ km/s}$) after only 1 iteration when initialized with 157 LKF measurements.

iii. my state RMS errors were

component: $[7.1202 \times 10^{-4}, 1.1939 \times 10^{-4}, 5.3141 \times 10^{-4}, 6.4186 \times 10^{-7}, 2.6282 \times 10^{-7}, 7.85 \times 10^{-7}]^T$

full-state: 8.9848×10^{-4}

if I ignore the first pass of data, these values reduce by about an order of 10!

c. discuss the comparison between the performance of the different filters.

In terms of full-state RMS errors, the batch filter has the smallest, followed by the EKF and finally the LKF. In terms of past-fit residual RMS, this order is reversed! LKF has the smallest, followed by EKF and then batch. This makes sense, as the batch filter processes all measurements at once - resulting in less overall error - while the EKF and LKF have uncertainty that propagates from timestep to timestep - getting more certain as more measurements are taken.

In terms of implementation, the EKF converged immediately, while the LKF and batch both needed 3 runs to converge. However, the EKF required a number of LKF runs first, which means that it too needs "3 runs" to converge.

Finally, it seems that the batch algorithm ran the fastest, at ~3 seconds, followed by the EKF and LKF at ~3.5 and ~5.5 seconds respectively.

d. change $\sigma_r = 1000 \text{ km}$ and $\sigma_v = 1 \text{ km/s}$. How does this change each filter's performance?

changing σ_r and σ_v didn't change the batch filter's performance much at all.

changing σ_r and σ_v increased the number of runs needed for the LKF to converge and increased its final RMS errors, as many of the state errors landed outside of the predicted range. ct broke!

changing σ_r and σ_v introduced outliers to the EKF's position residuals and increased its RMS errors, but it still ran in the same amount of time and resulted in plausible state errors.

e. change $\delta x_0 = 100\delta x_{0,\text{old}}$. How does this change each filter's performance? did any stop working?

changing δx_0 didn't affect the Batch filter performance at all!

changing δx_0 further increased my LKF RMS errors and resulted in state errors outside the predicted bounds. ct broke!

Since my EKF is initialised with an LKF (which broke), it also broke. However, initialising it with my batch filter made it work again.

In general, it seems like the batch filter is the most resistant to initialisation while the LKF is very sensitive. Likewise, the EKF will behave as long as the filter that initialised it also behaves.

f. What happens to the solution at the original $P_0 + \delta x_0$, but only half the measurements are processed?

If only half the measurements are processed, the batch filter's final state errors vary more, but the LKF and EKF seem relatively unaffected! This makes sense, as the more measurements the batch filter processes the better it is, while the LKF and EKF are "online" estimators, meaning they only need one measurement at a time and their state estimates tend to get better with each measurement.

ASEN 6080 HW 2 Problem 1 Main Script

Table of Contents

Housekeeping	1
Setup	1
Make Truth Data	1
Problem 1a: Filter setup	1
Problem 1b: Batch Filter	1
Problem 1b: CKF/LKF	4
Problem 1b: EKF	7
Problem 1d: Increase x_0	9
Problem 1e: Increase P_0	16
Problem 1f: Process half the measurements	23

By: Ian Faber

Housekeeping

Setup

Make Truth Data

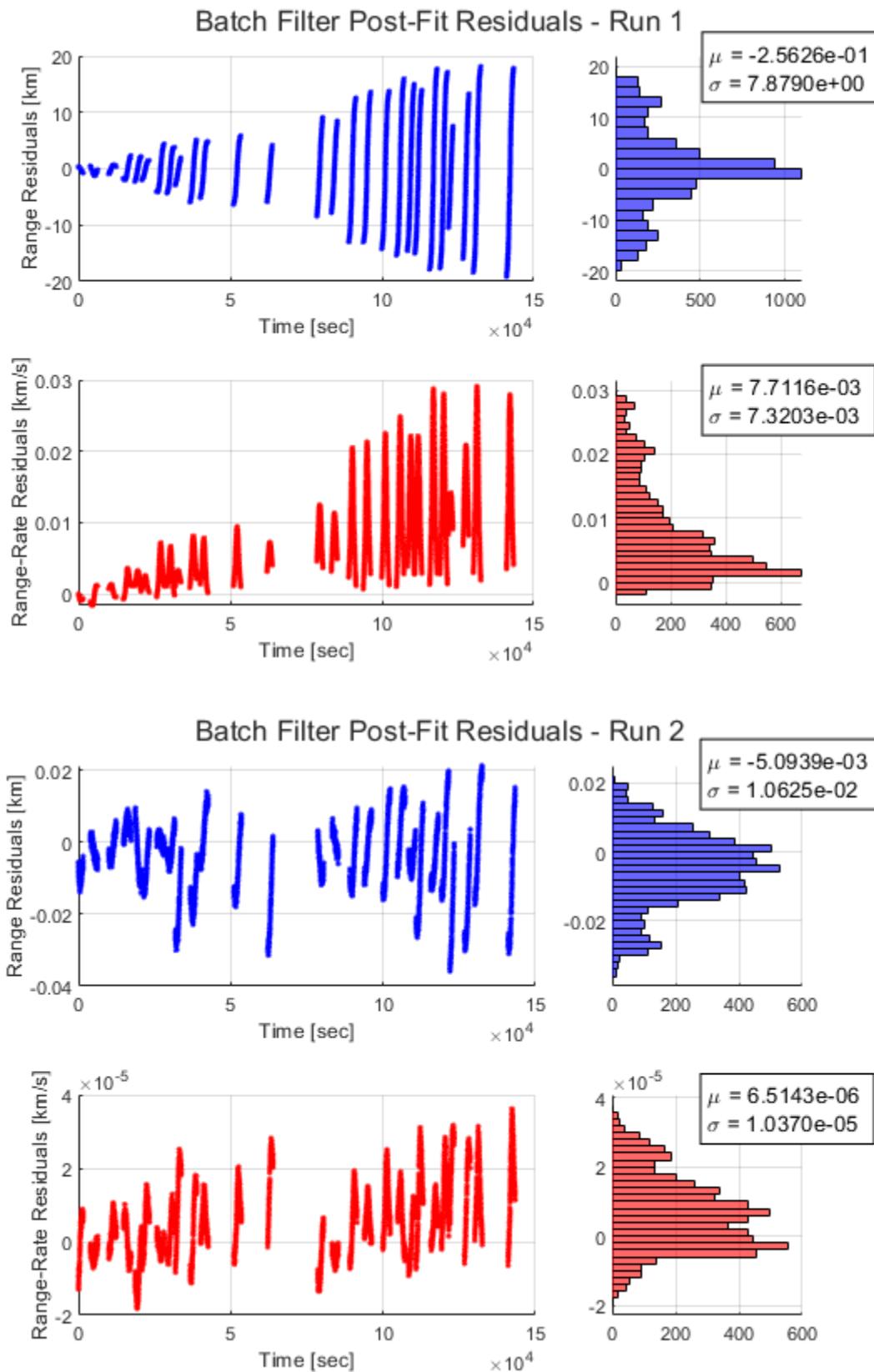
Problem 1a: Filter setup

Problem 1b: Batch Filter

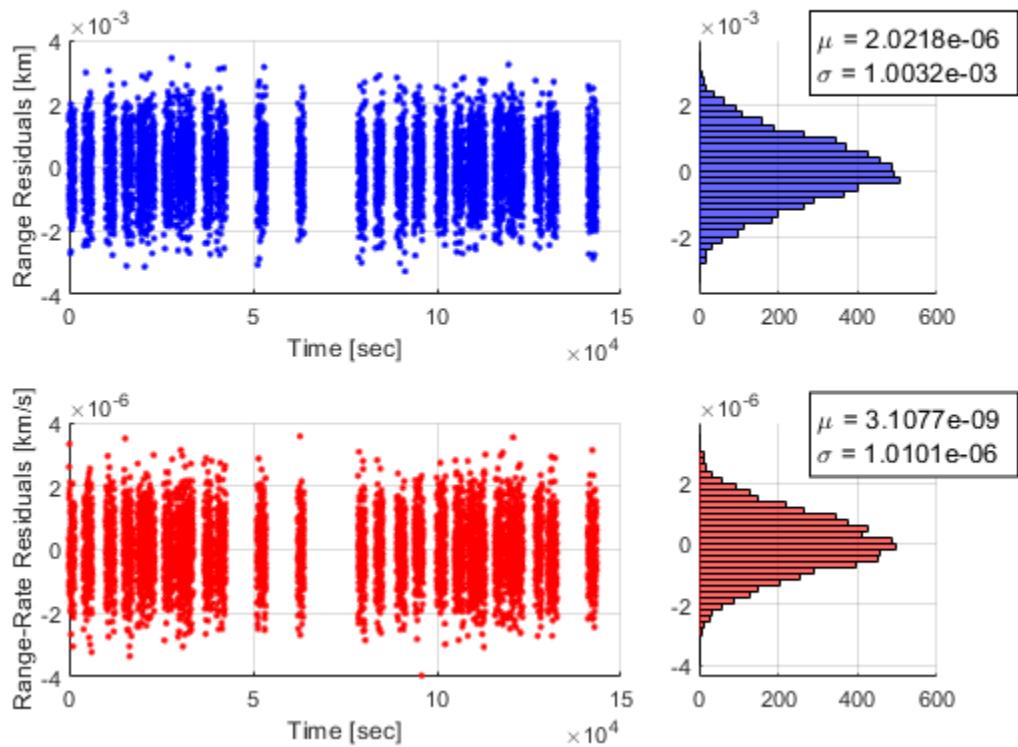
1b. Batch Filter

Running Batch Filter:

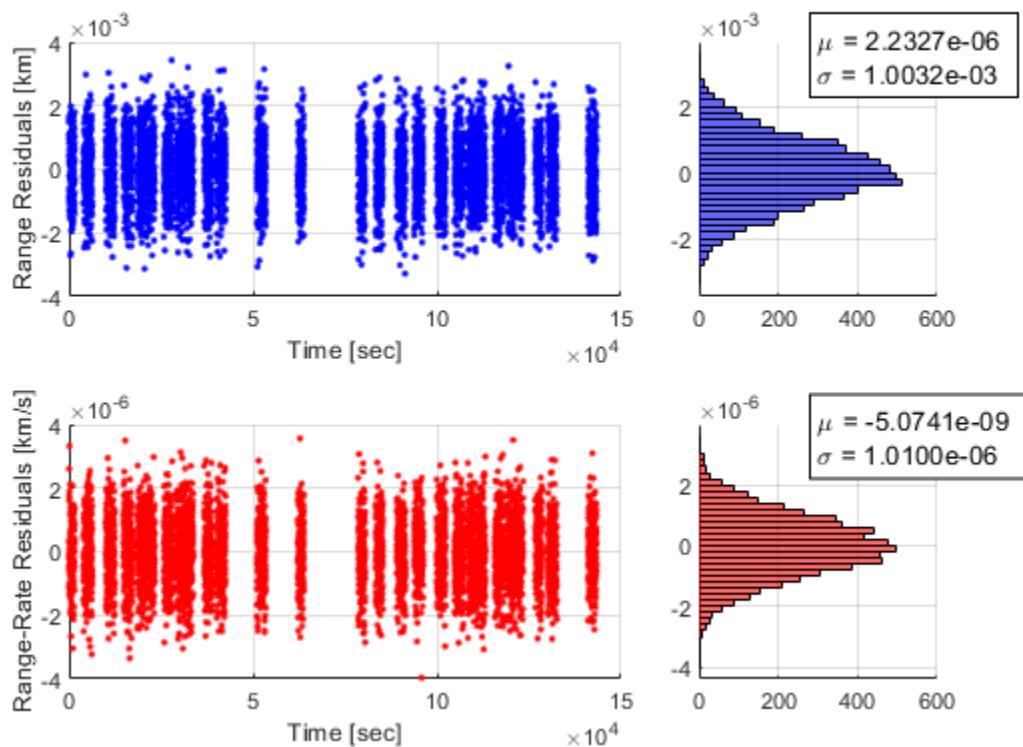
```
Postfit RMS: 25.7449. Iterating Batch with x0_batch = [1.067e-02;
-4.072e-03; 1.021e-02; 1.778e-05; -4.425e-06; 2.383e-05]. Runs so far: 1
Postfit RMS: 1.0065. Iterating Batch with x0_batch = [-2.875e-05; 2.892e-06;
-2.544e-05; -1.682e-08; -1.178e-09; -2.762e-08]. Runs so far: 2
Postfit RMS: 1.0065. Iterating Batch with x0_batch = [-2.879e-05; 2.845e-06;
-2.535e-05; -1.693e-08; -1.208e-09; -2.768e-08]. Runs so far: 3
Final postfit RMS: 1.0065. Converged after 4 runs
```

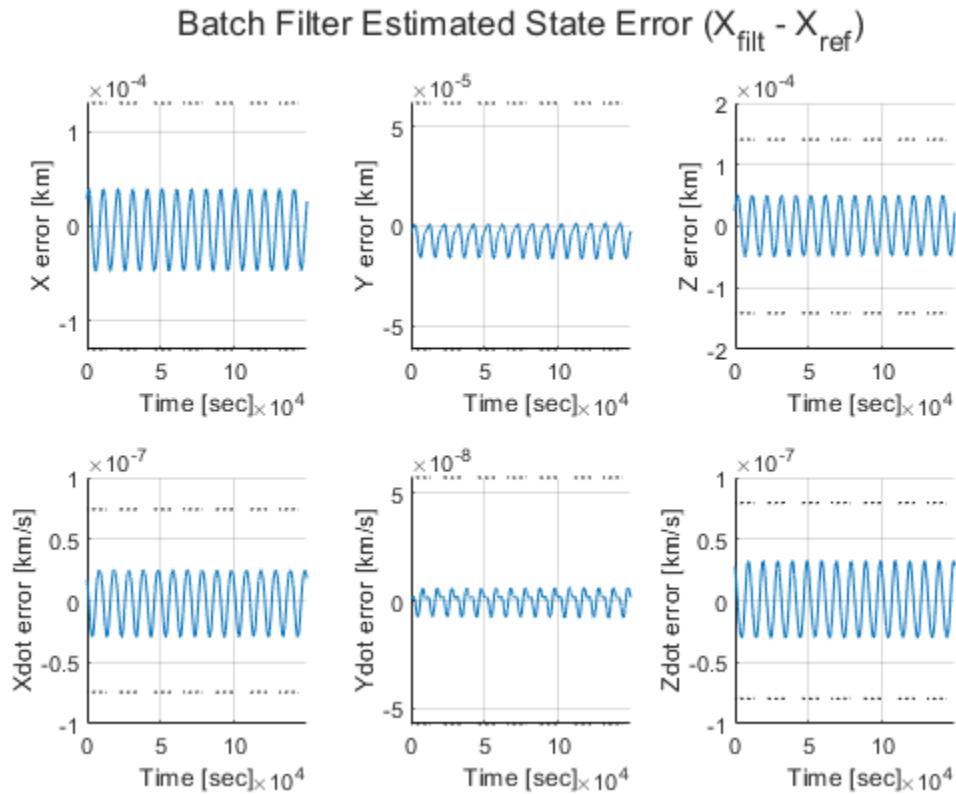


Batch Filter Post-Fit Residuals - Run 3



Batch Filter Post-Fit Residuals - Run 4



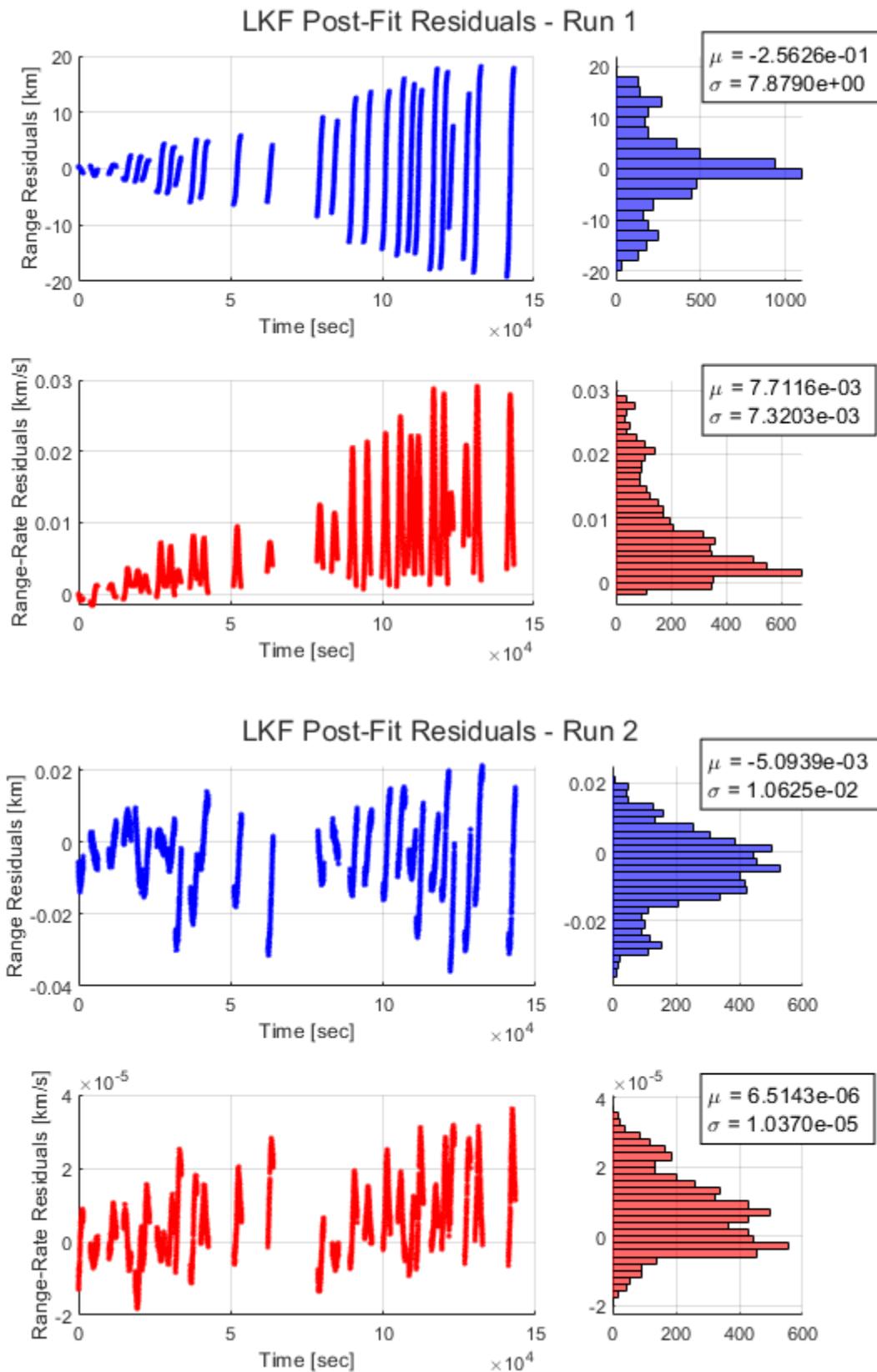


Problem 1b: CKF/LKF

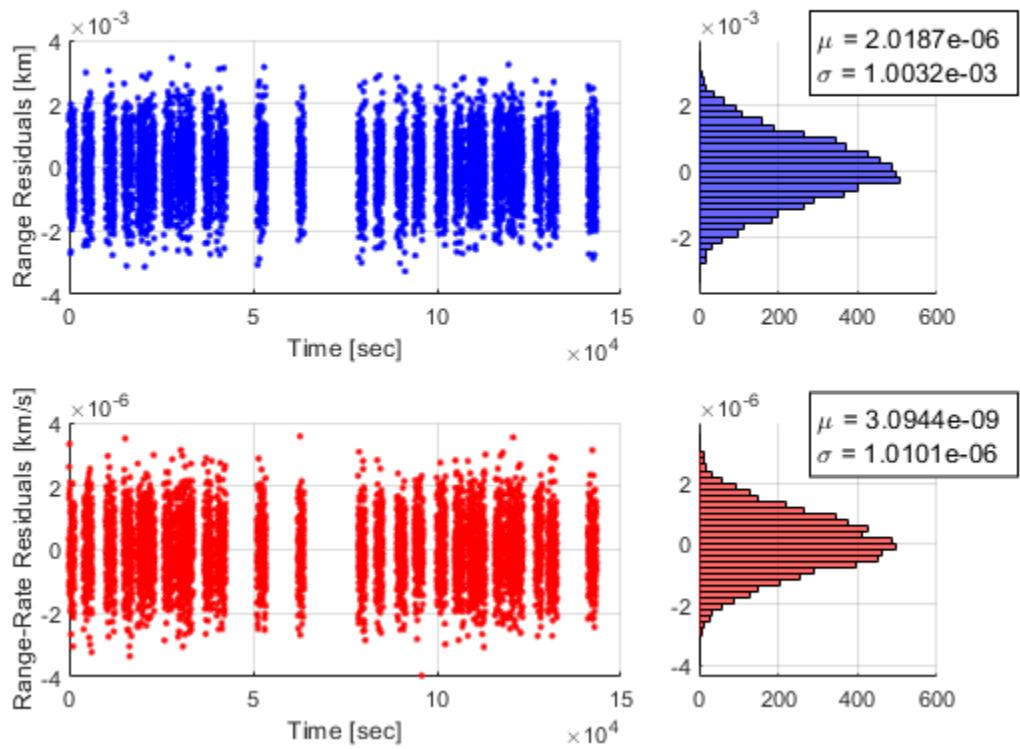
1b. LKF

Running LKF:

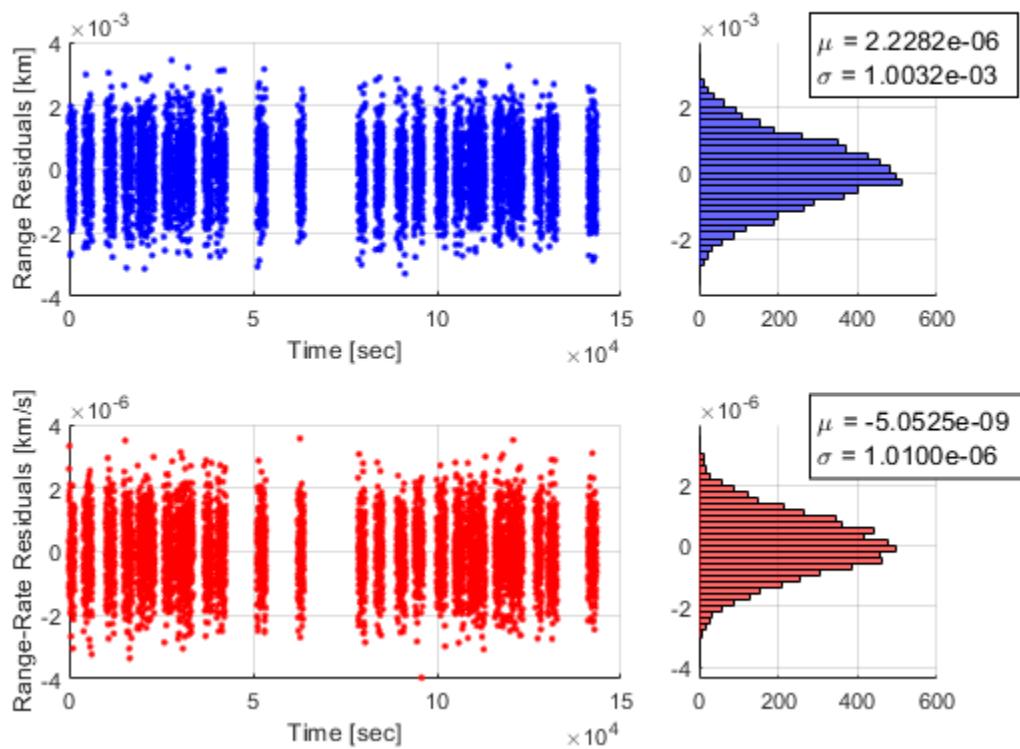
```
Postfit RMS: 25.7373. Iterating LKF with x0_LKF = [4.893e-01; 5.041e-01;
4.898e-01; 4.822e-04; 5.044e-04; 4.762e-04]. Runs so far: 1
Postfit RMS: 1.0029. Iterating LKF with x0_LKF = [1.070e-02; -4.075e-03;
1.023e-02; 1.780e-05; -4.424e-06; 2.385e-05]. Runs so far: 2
Postfit RMS: 1.0029. Iterating LKF with x0_LKF = [3.439e-08; 4.107e-08;
-9.244e-08; 1.191e-10; 2.333e-11; 5.341e-11]. Runs so far: 3
Final postfit RMS: 1.0029. Converged after 4 runs
```

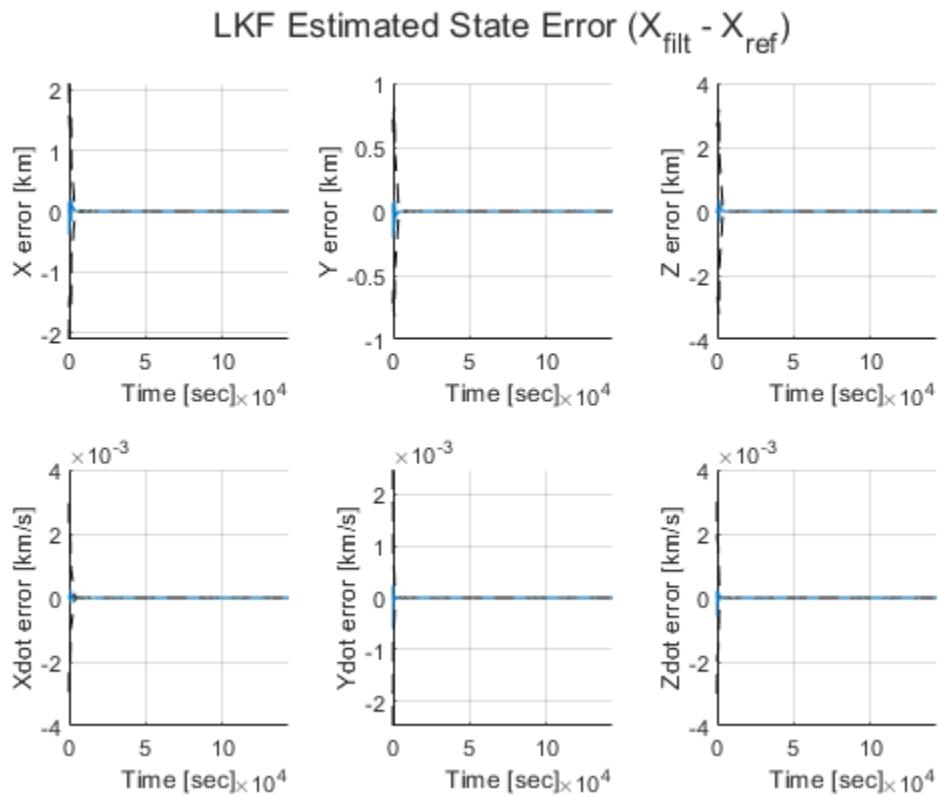


LKF Post-Fit Residuals - Run 3



LKF Post-Fit Residuals - Run 4



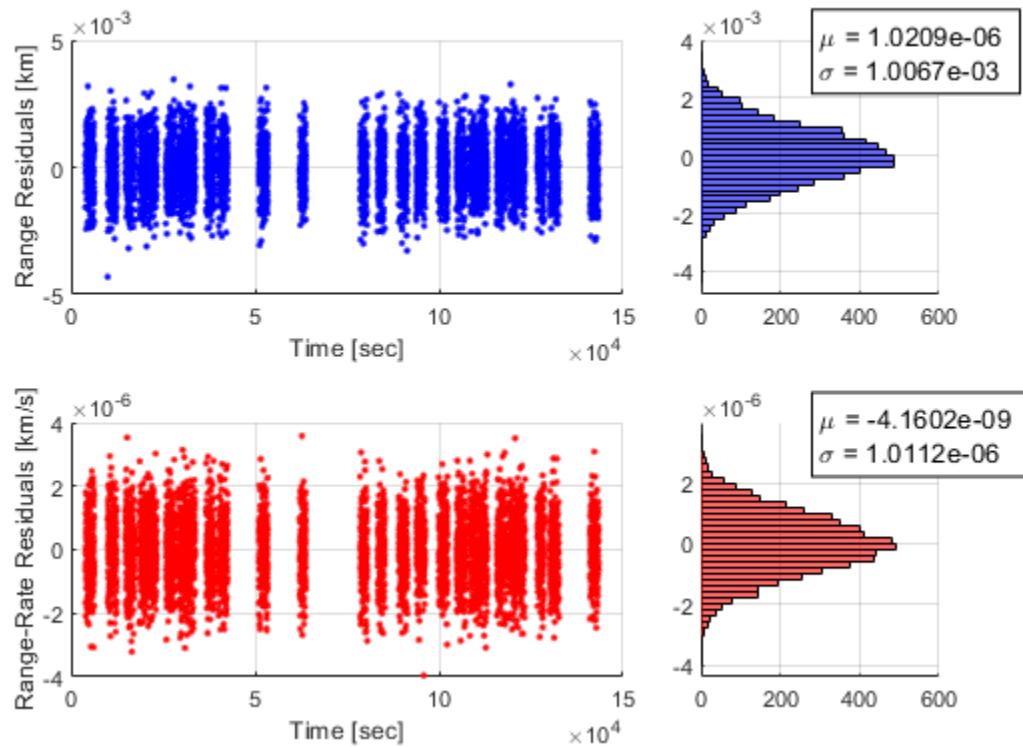


Problem 1b: EKF

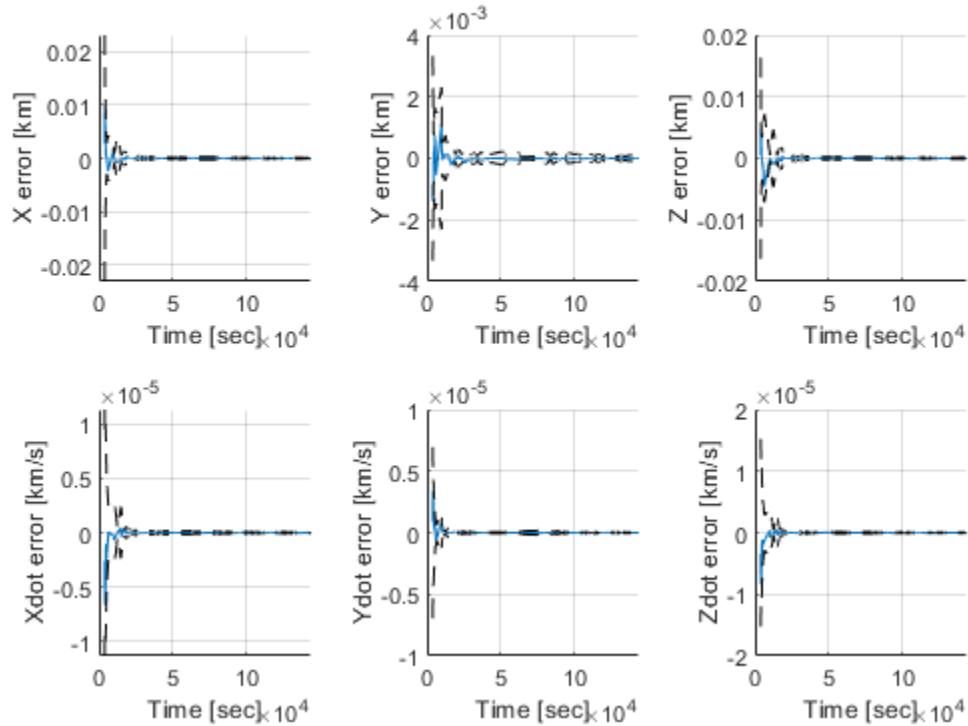
1b. EKF

*Running EKF:
Postfit RMS: 1.0043*

EKF Post-Fit Residuals



EKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



Problem 1d: Increase x0

1d. Increased x0

Running Batch Filter:

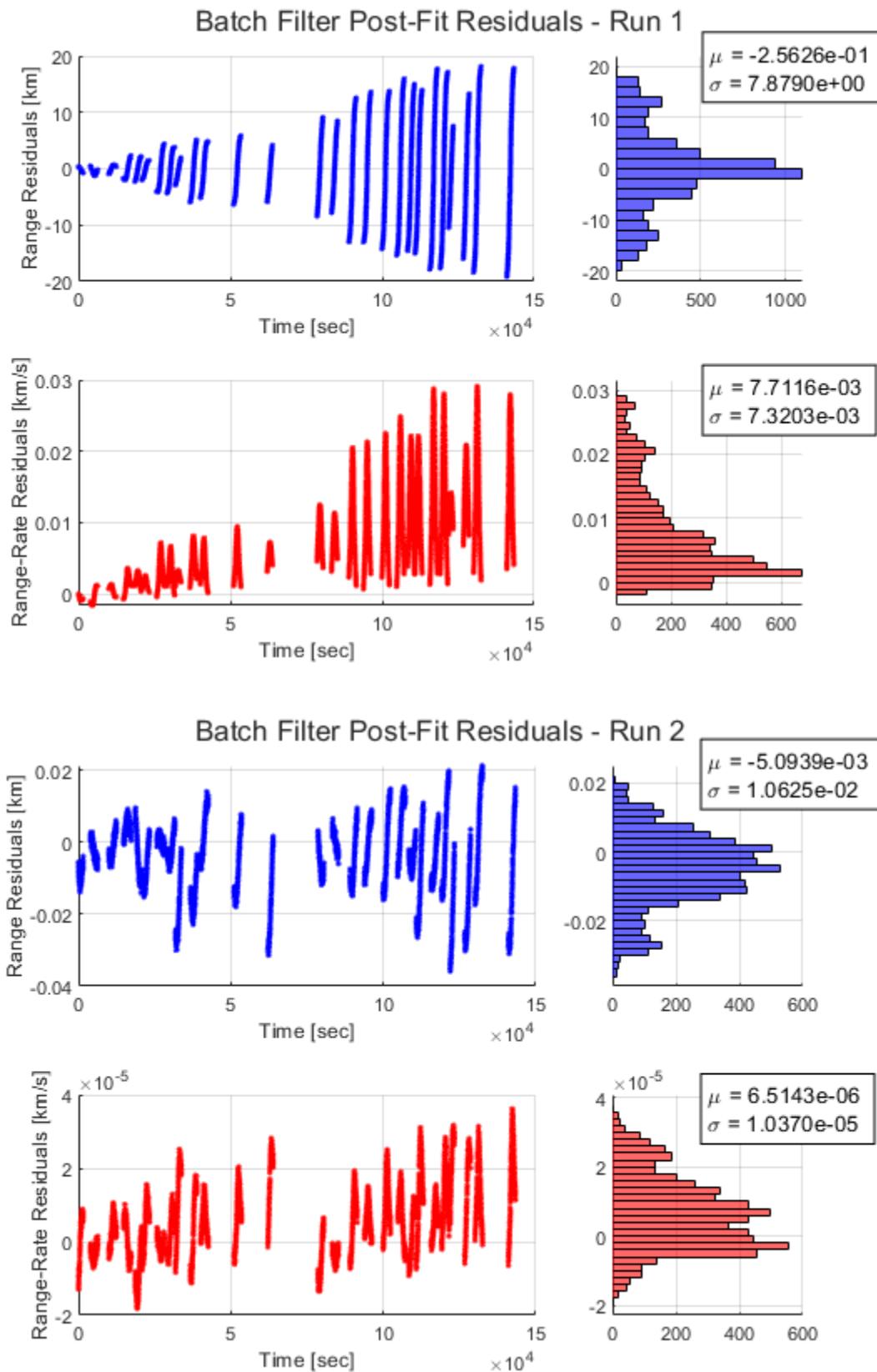
```
Postfit RMS: 25.7449. Iterating Batch with x0_batch = [4.951e+01; 4.950e+01;  
4.951e+01; 4.952e-02; 4.950e-02; 4.952e-02]. Runs so far: 1  
Postfit RMS: 1.0065. Iterating Batch with x0_batch = [4.950e+01; 4.950e+01;  
4.950e+01; 4.950e-02; 4.950e-02; 4.950e-02]. Runs so far: 2  
Postfit RMS: 1.0065. Iterating Batch with x0_batch = [4.950e+01; 4.950e+01;  
4.950e+01; 4.950e-02; 4.950e-02; 4.950e-02]. Runs so far: 3  
Final postfit RMS: 1.0065. Converged after 4 runs
```

Running LKF:

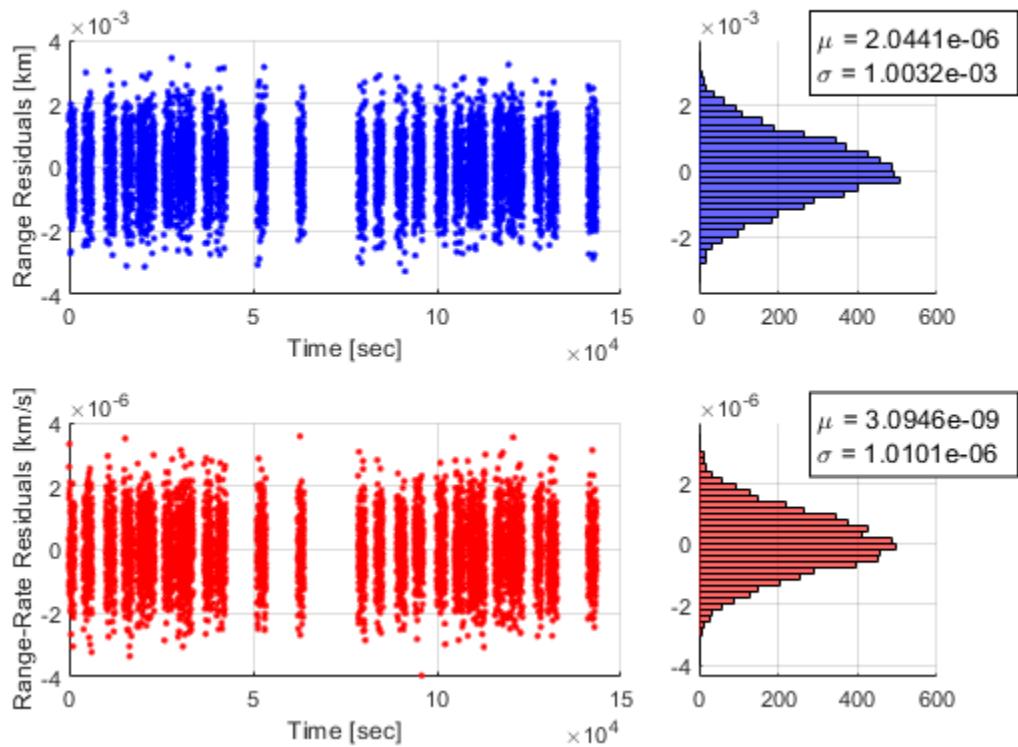
```
Postfit RMS: 25.7483. Iterating LKF with x0_LKF = [4.893e-01; 5.041e-01;  
4.898e-01; 4.822e-04; 5.044e-04; 4.762e-04]. Runs so far: 1  
Postfit RMS: 1.0029. Iterating LKF with x0_LKF = [1.070e-02; -4.075e-03;  
1.023e-02; 1.780e-05; -4.424e-06; 2.385e-05]. Runs so far: 2  
Postfit RMS: 1.0029. Iterating LKF with x0_LKF = [3.592e-08; 4.190e-08;  
-9.354e-08; 1.185e-10; 2.419e-11; 5.383e-11]. Runs so far: 3  
Final postfit RMS: 1.0029. Converged after 4 runs
```

Running EKF:

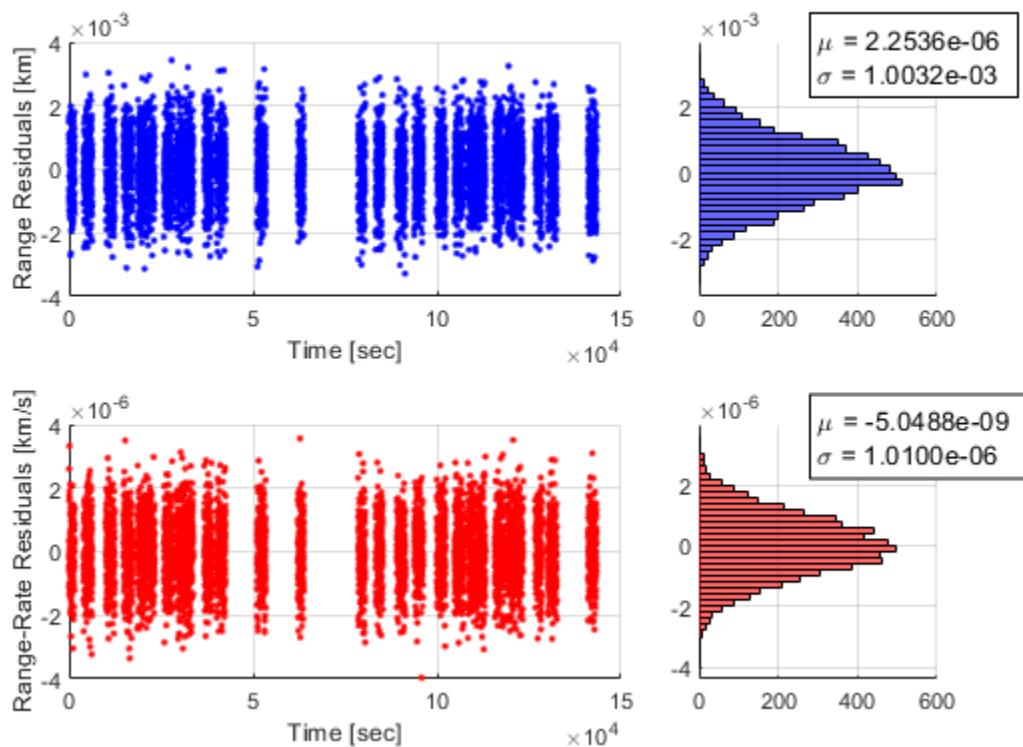
```
Postfit RMS: 1.0043
```



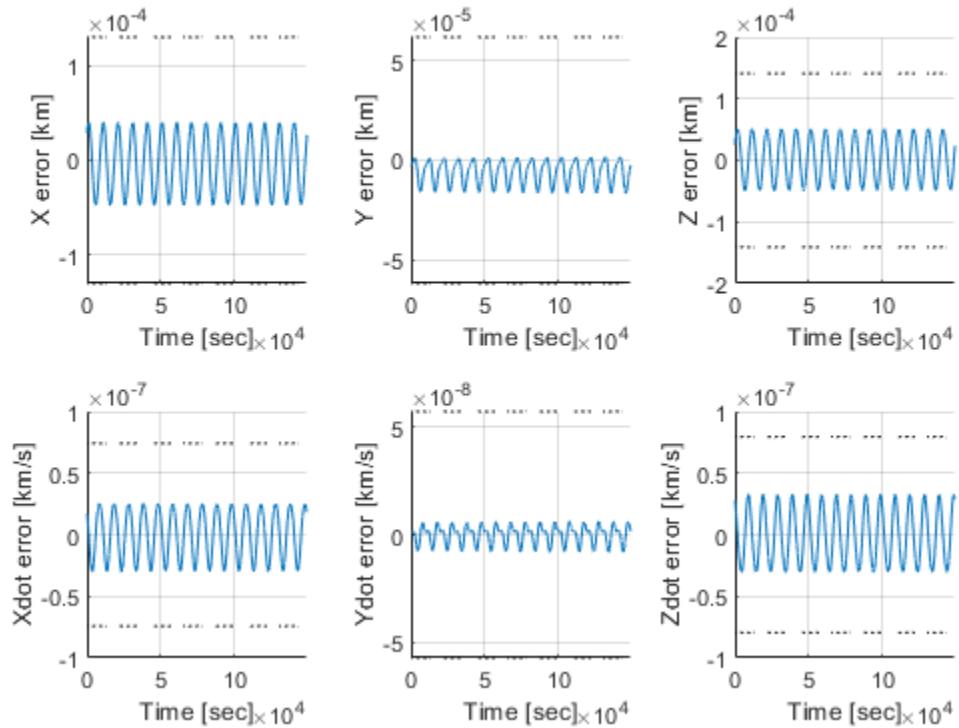
Batch Filter Post-Fit Residuals - Run 3



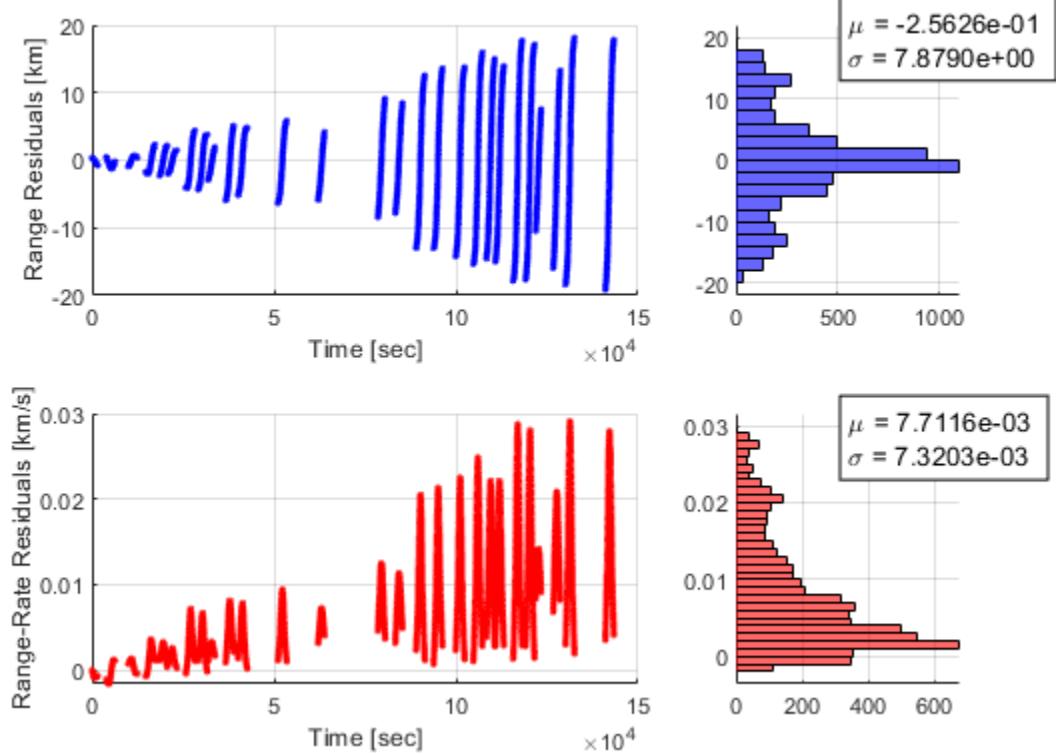
Batch Filter Post-Fit Residuals - Run 4

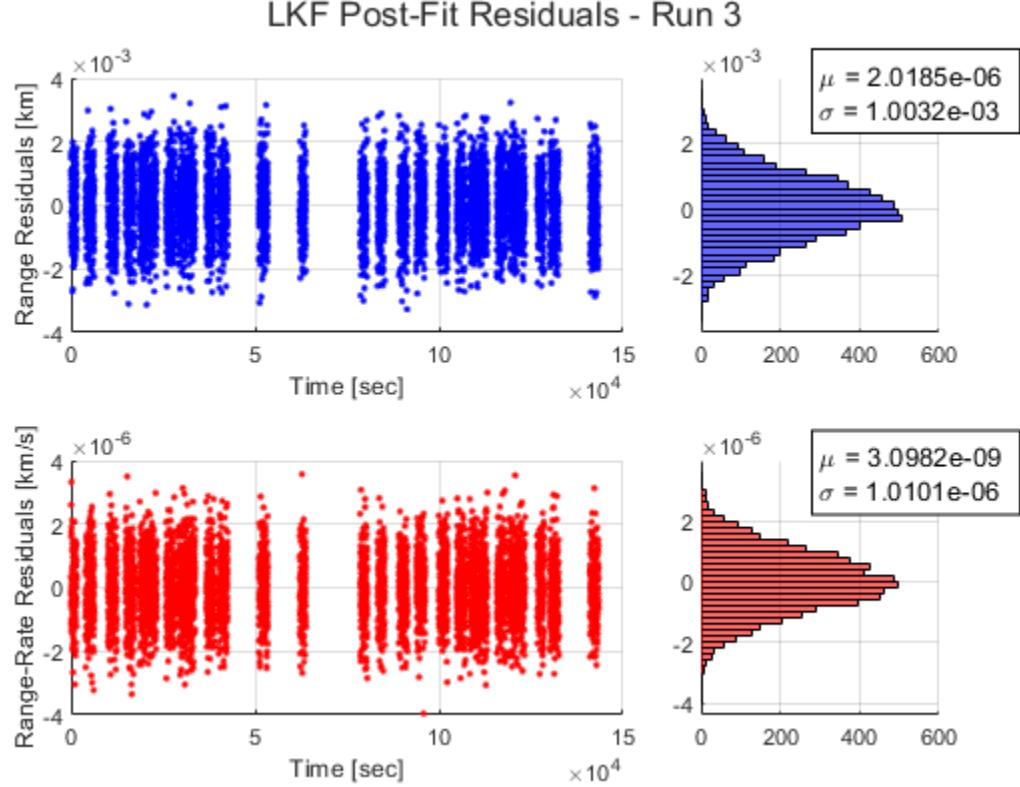
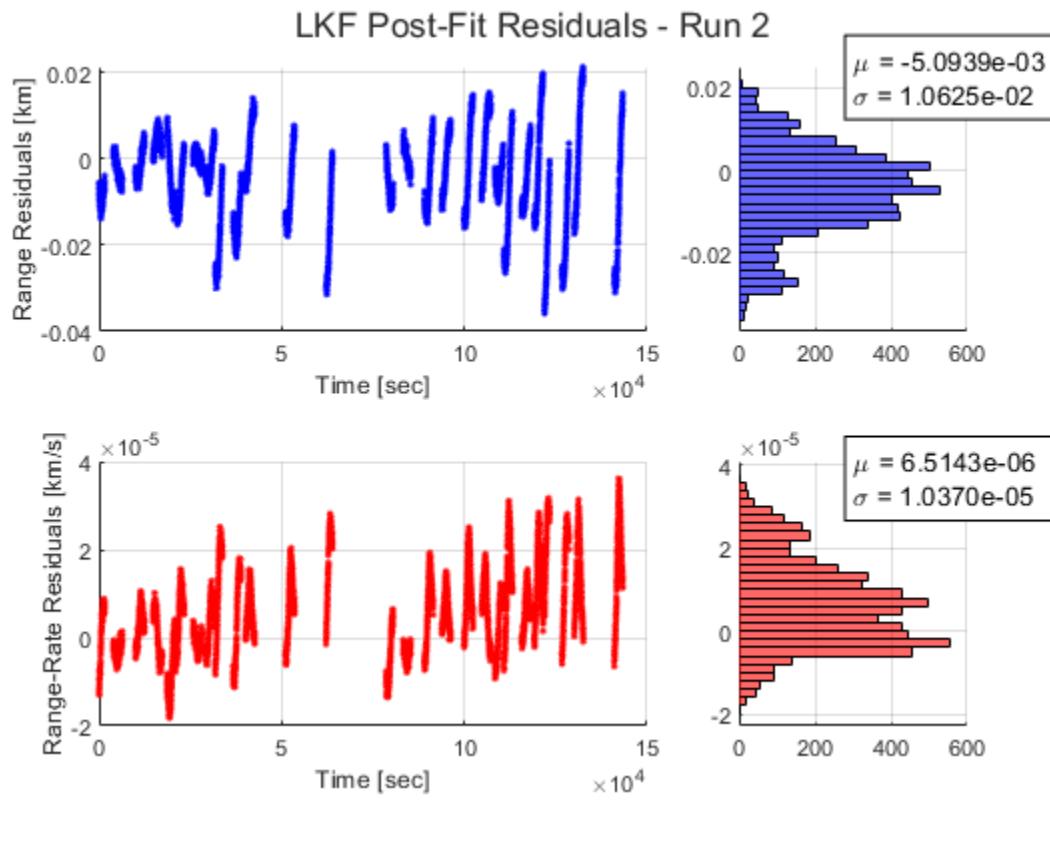


Batch Filter Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)

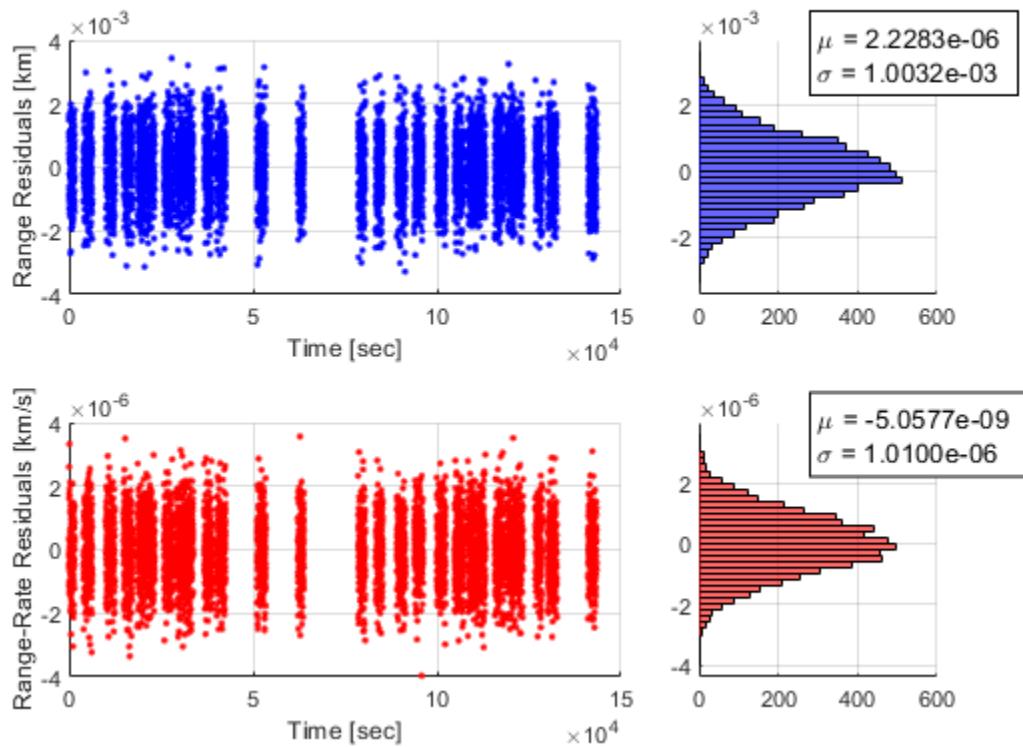


LKF Post-Fit Residuals - Run 1

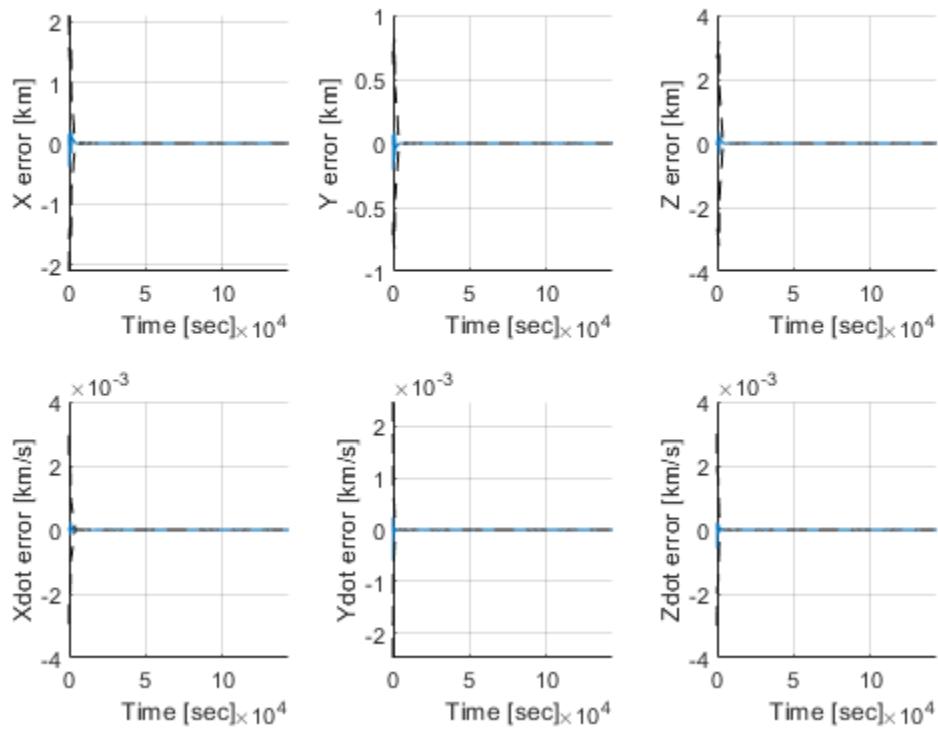




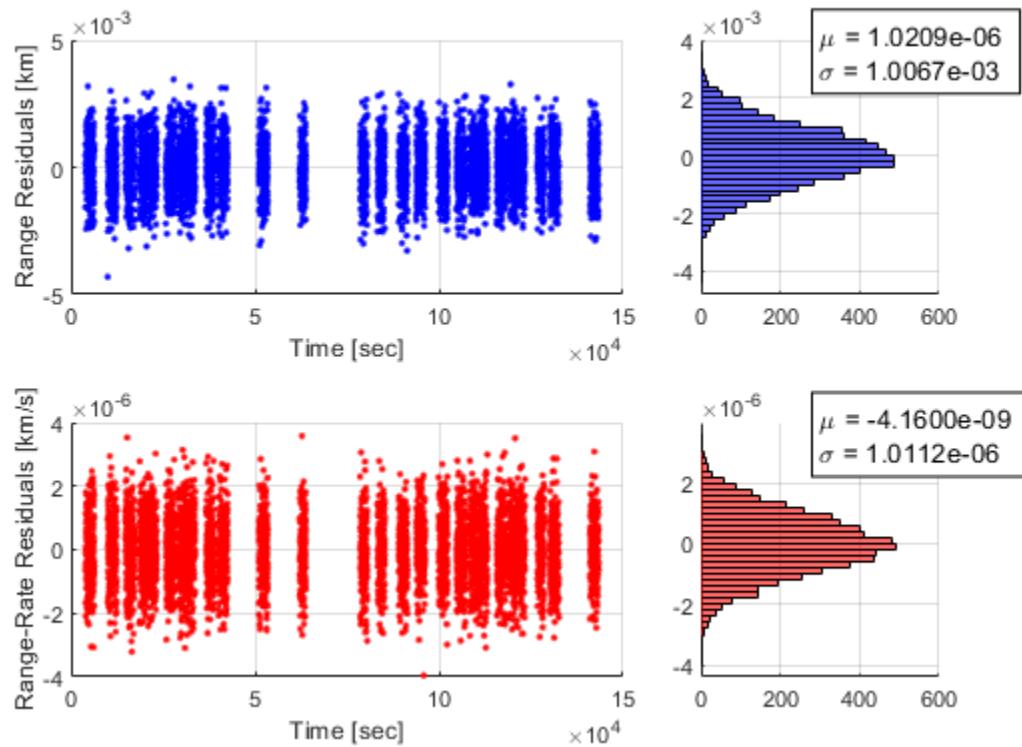
LKF Post-Fit Residuals - Run 4



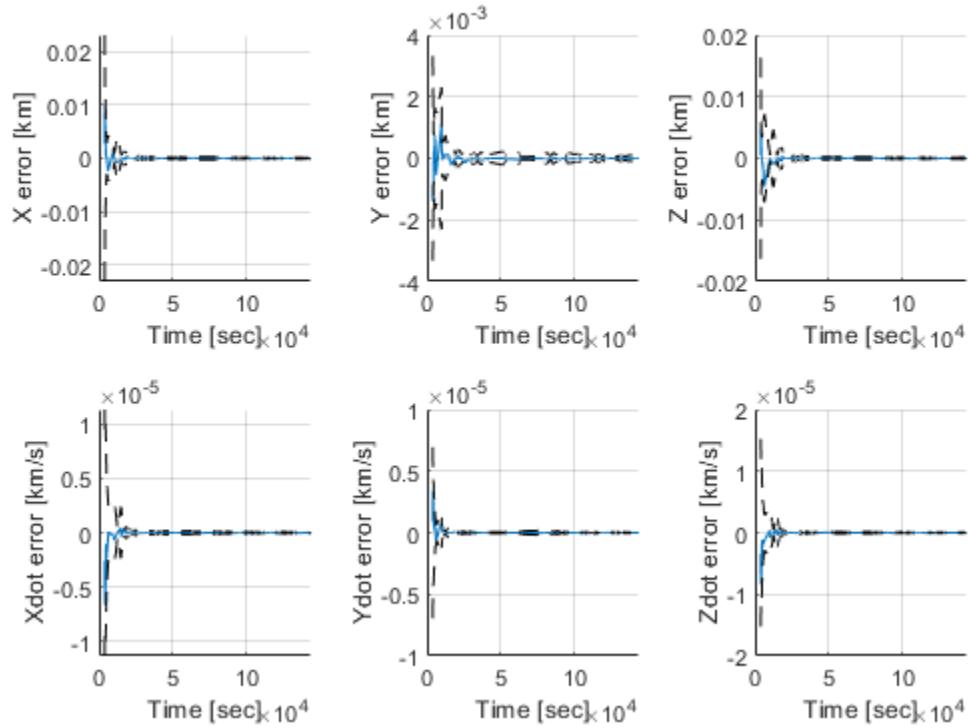
LKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



EKF Post-Fit Residuals



EKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



Problem 1e: Increase P0

1e. Increased P_0

Running Batch Filter:

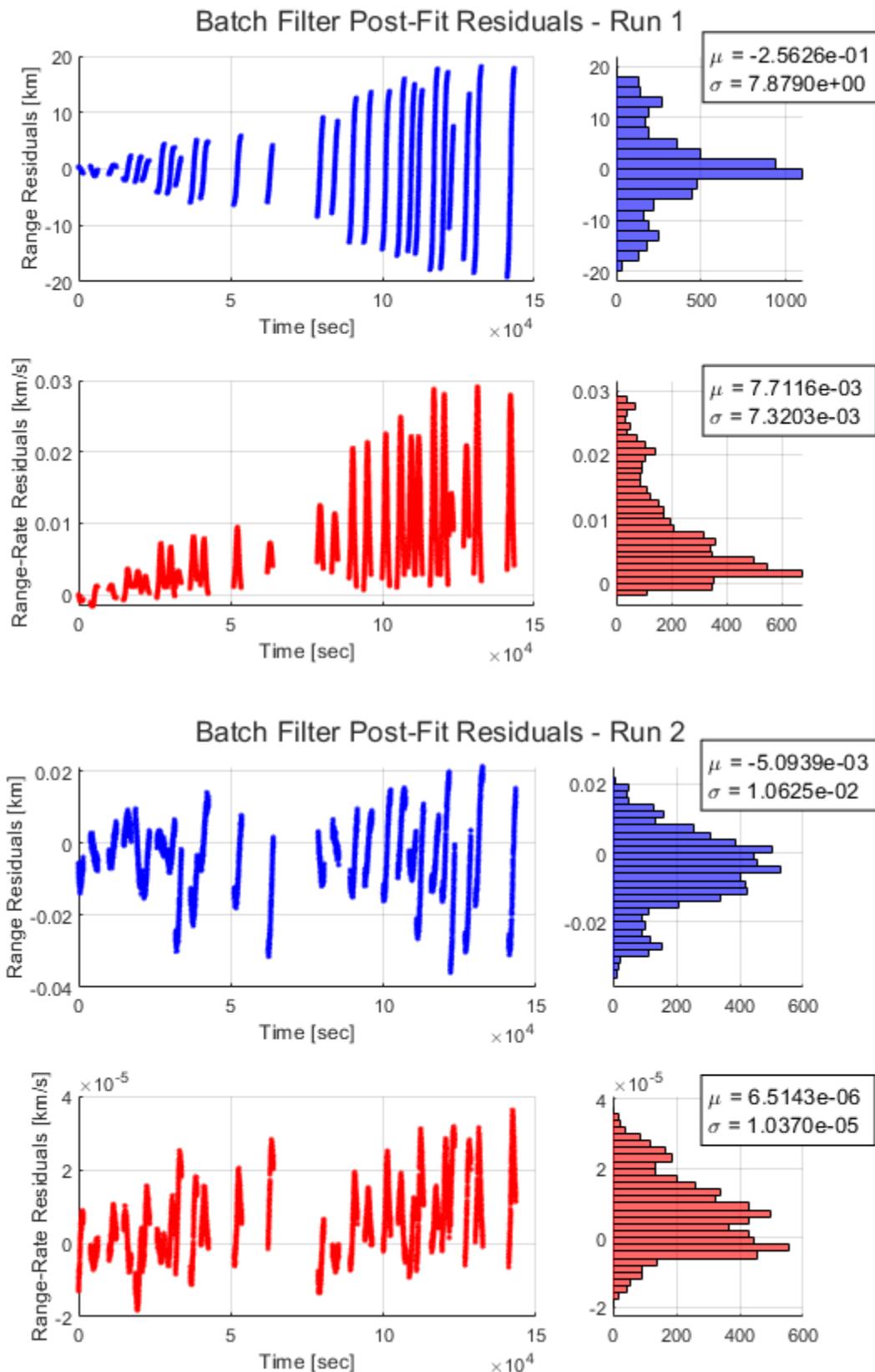
```
Postfit RMS: 25.7449. Iterating Batch with x0_batch = [4.951e+01; 4.950e+01;
4.951e+01; 4.952e-02; 4.950e-02; 4.952e-02]. Runs so far: 1
Postfit RMS: 1.0065. Iterating Batch with x0_batch = [4.950e+01; 4.950e+01;
4.950e+01; 4.950e-02; 4.950e-02; 4.950e-02]. Runs so far: 2
Postfit RMS: 1.0065. Iterating Batch with x0_batch = [4.950e+01; 4.950e+01;
4.950e+01; 4.950e-02; 4.950e-02; 4.950e-02]. Runs so far: 3
Final postfit RMS: 1.0065. Converged after 4 runs
```

Running LKF:

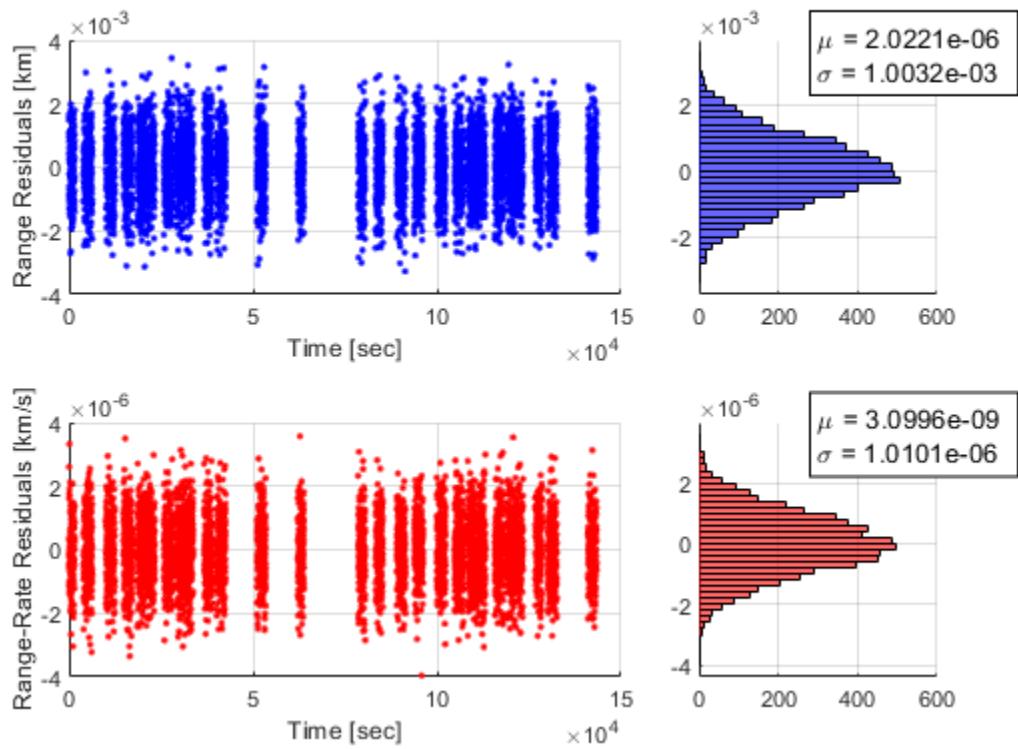
```
Postfit RMS: 25.7500. Iterating LKF with x0_LKF = [4.884e-01; 5.039e-01;
4.895e-01; 4.822e-04; 5.042e-04; 4.759e-04]. Runs so far: 1
Postfit RMS: 8.8852. Iterating LKF with x0_LKF = [1.143e-02; -3.944e-03;
1.047e-02; 1.781e-05; -4.248e-06; 2.412e-05]. Runs so far: 2
Postfit RMS: 24.2410. Iterating LKF with x0_LKF = [1.288e-04; 7.696e-05;
-3.431e-05; -1.585e-08; 7.017e-08; 4.587e-08]. Runs so far: 3
Postfit RMS: 5.9785. Iterating LKF with x0_LKF = [-7.009e-05; -9.834e-05;
1.114e-04; 3.027e-08; -7.961e-08; -2.710e-08]. Runs so far: 4
Final postfit RMS: 392.4142. Hit maximum number of 5 runs
Warning: Imaginary parts of complex X and/or Y arguments ignored.
Warning: Imaginary parts of complex X and/or Y arguments ignored.
Warning: Imaginary parts of complex X and/or Y arguments ignored.
Warning: Imaginary parts of complex X and/or Y arguments ignored.
Warning: Imaginary parts of complex X and/or Y arguments ignored.
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Warning: Imaginary parts of complex X and/or Y arguments ignored.
Warning: Imaginary parts of complex X and/or Y arguments ignored.
Warning: Imaginary parts of complex X and/or Y arguments ignored.
Warning: Imaginary parts of complex X and/or Y arguments ignored.
```

Running EKF:

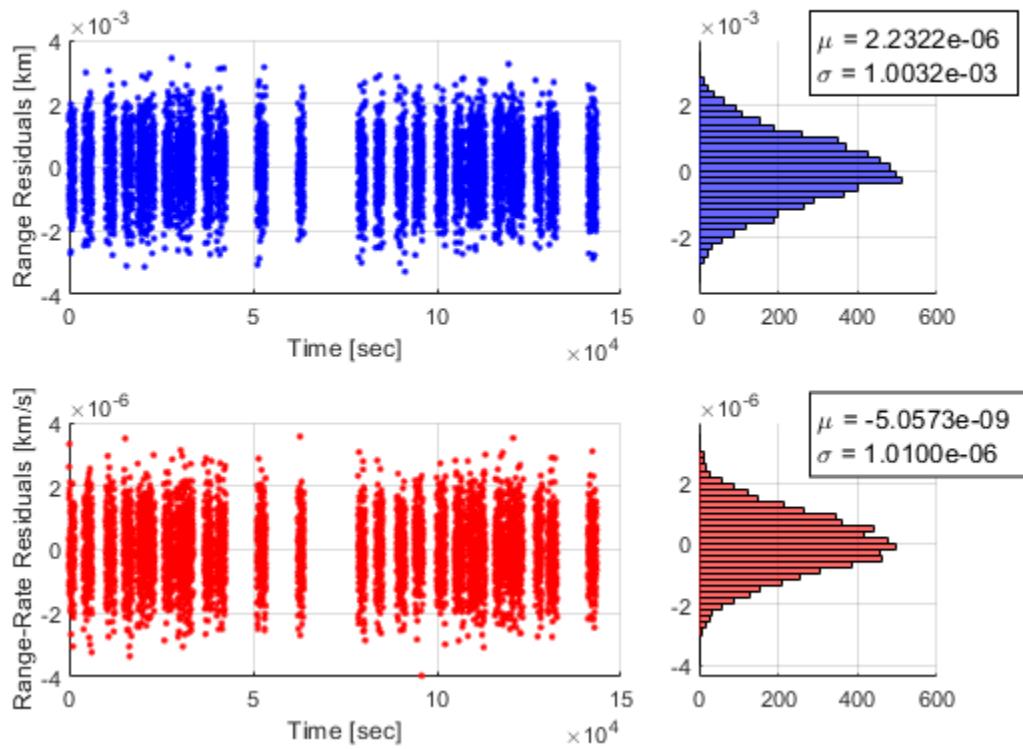
```
Postfit RMS: 1.0282
```



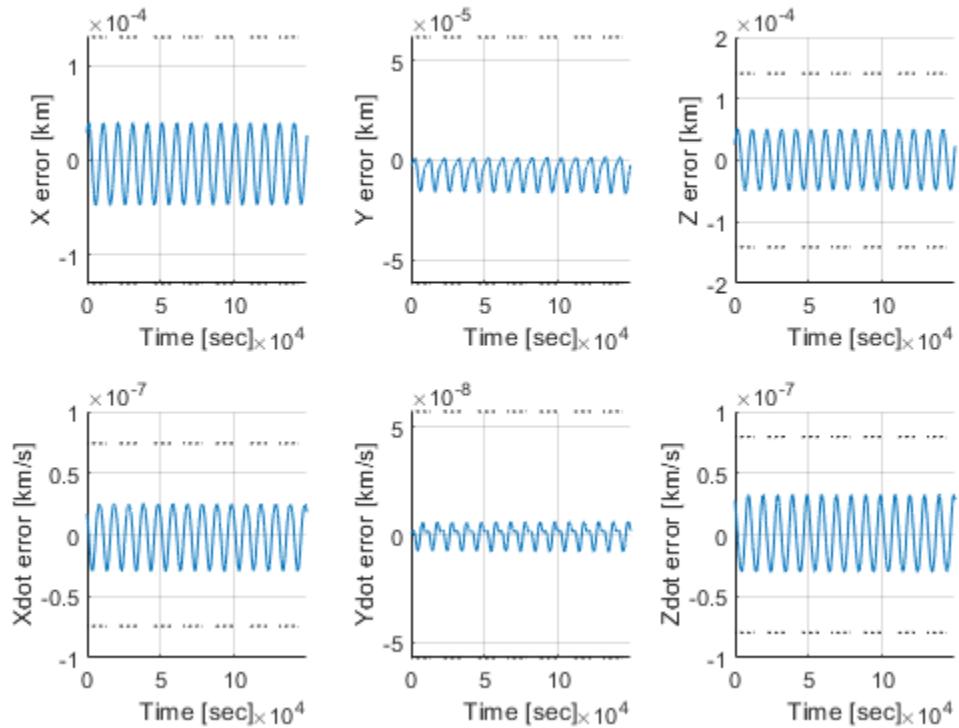
Batch Filter Post-Fit Residuals - Run 3



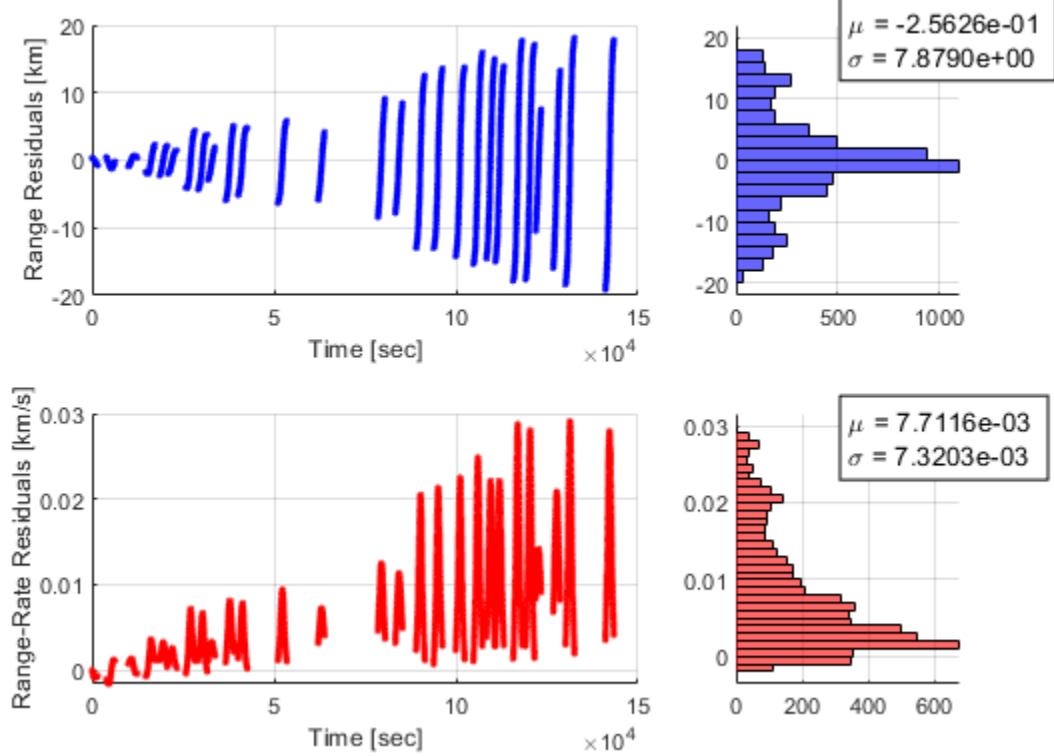
Batch Filter Post-Fit Residuals - Run 4

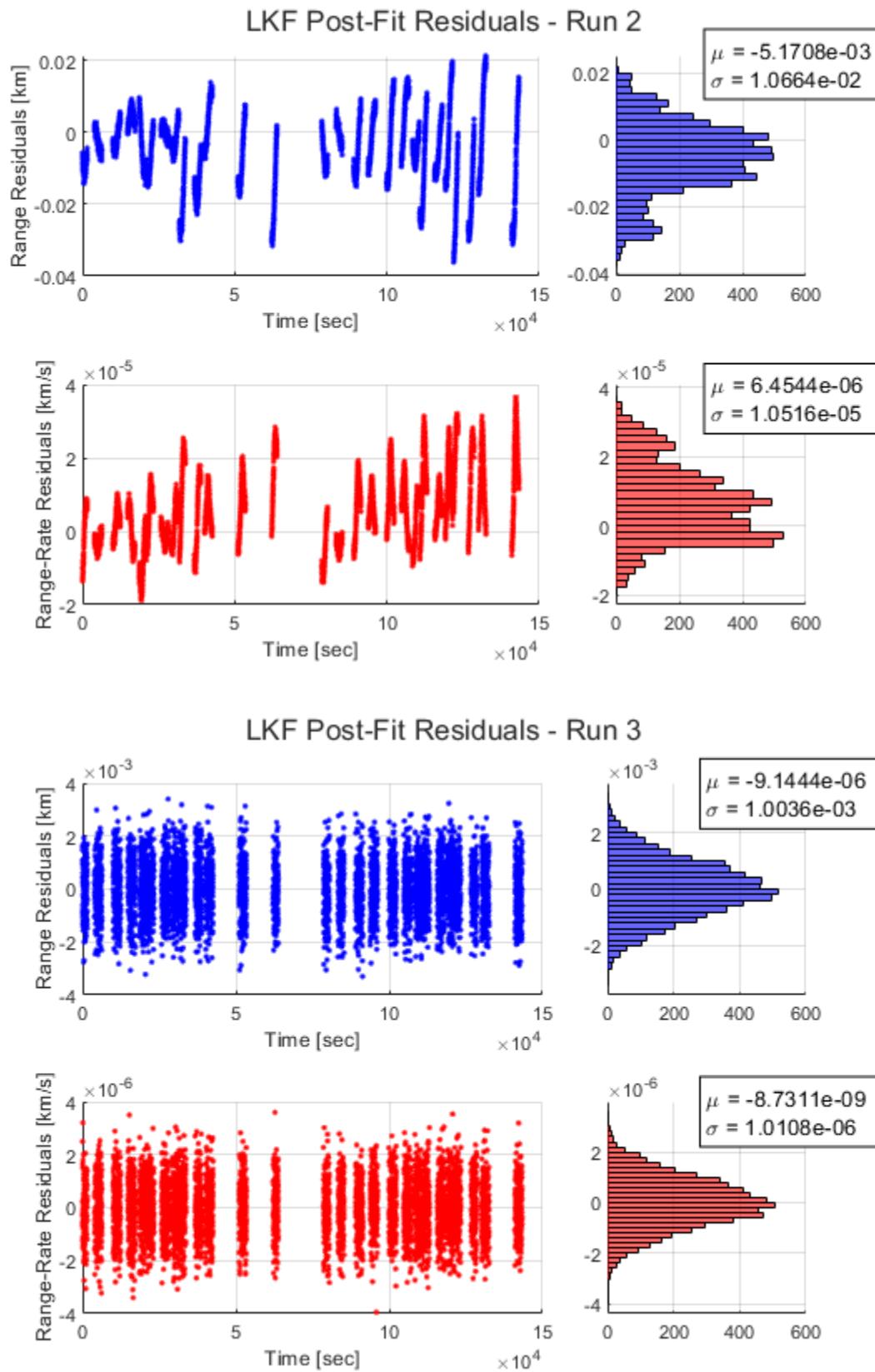


Batch Filter Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)

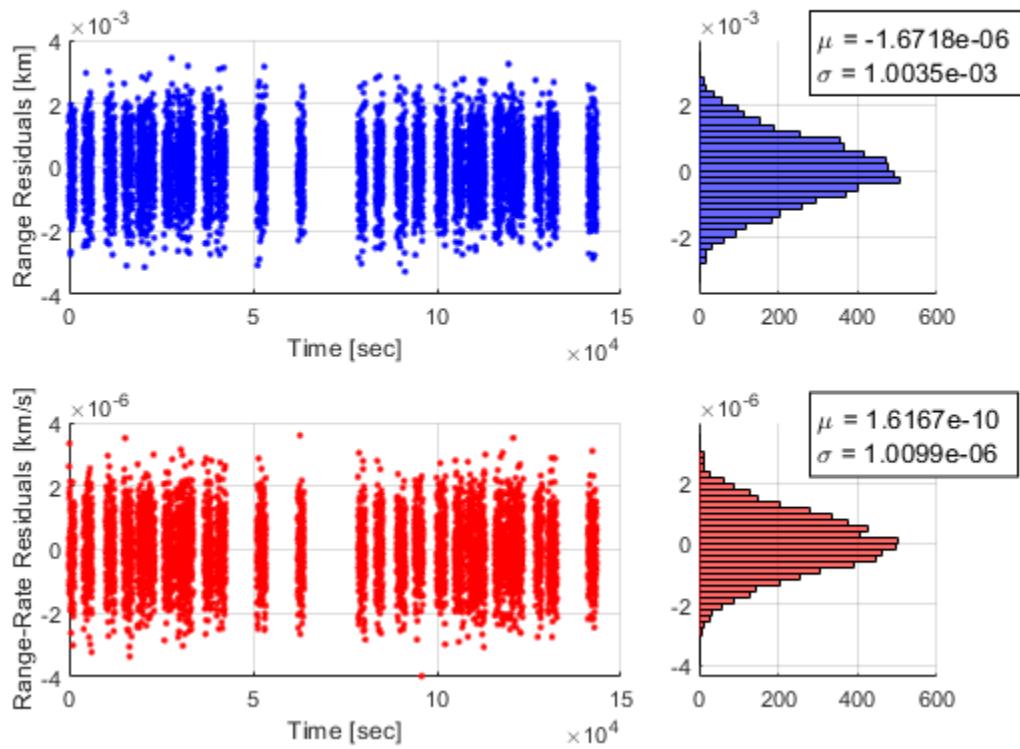


LKF Post-Fit Residuals - Run 1

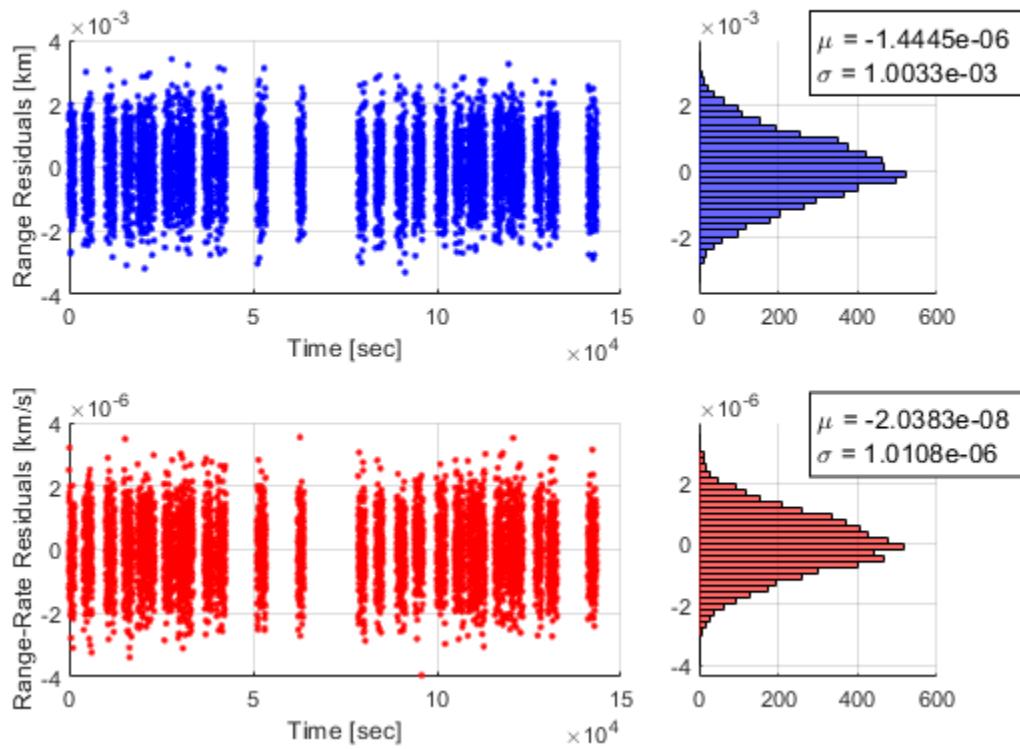




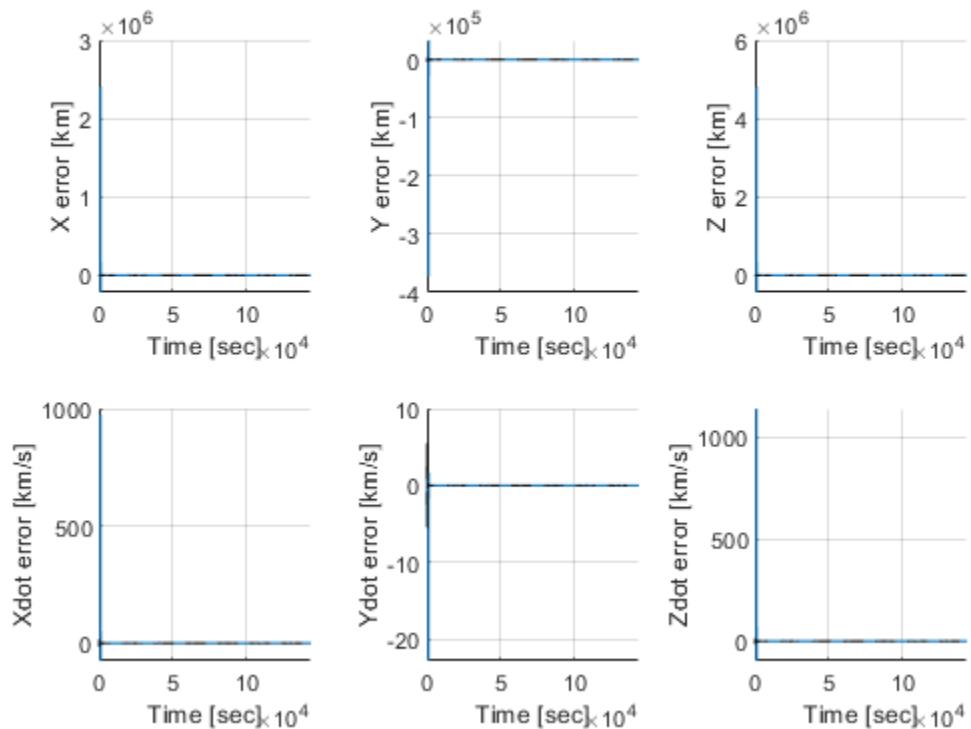
LKF Post-Fit Residuals - Run 4



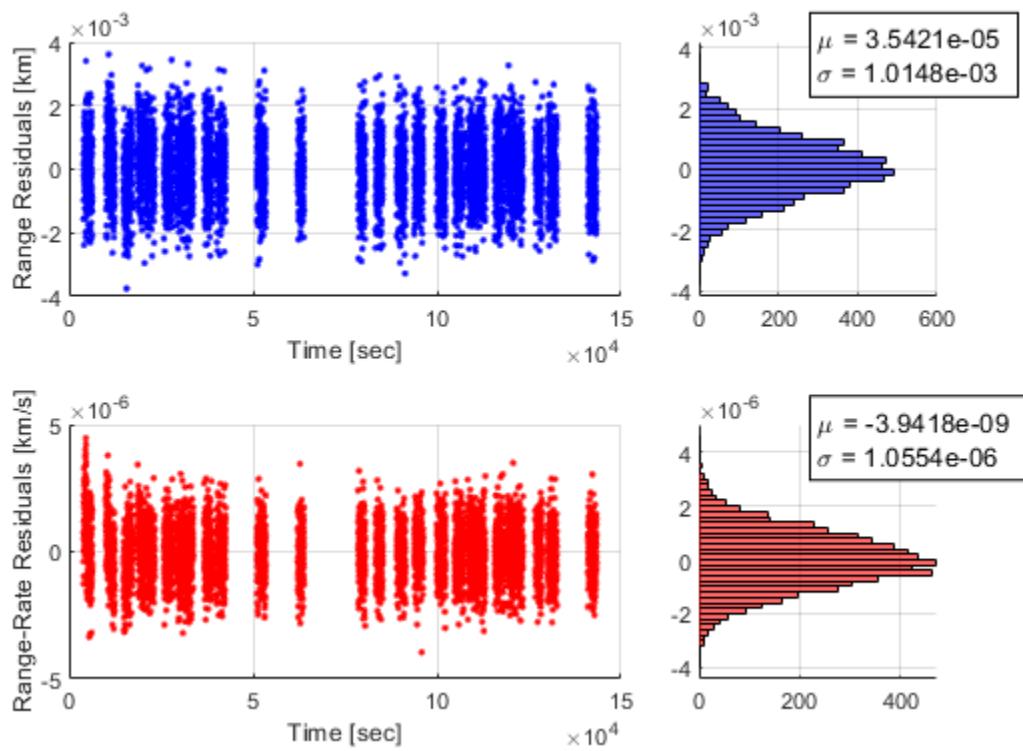
LKF Post-Fit Residuals - Run 5

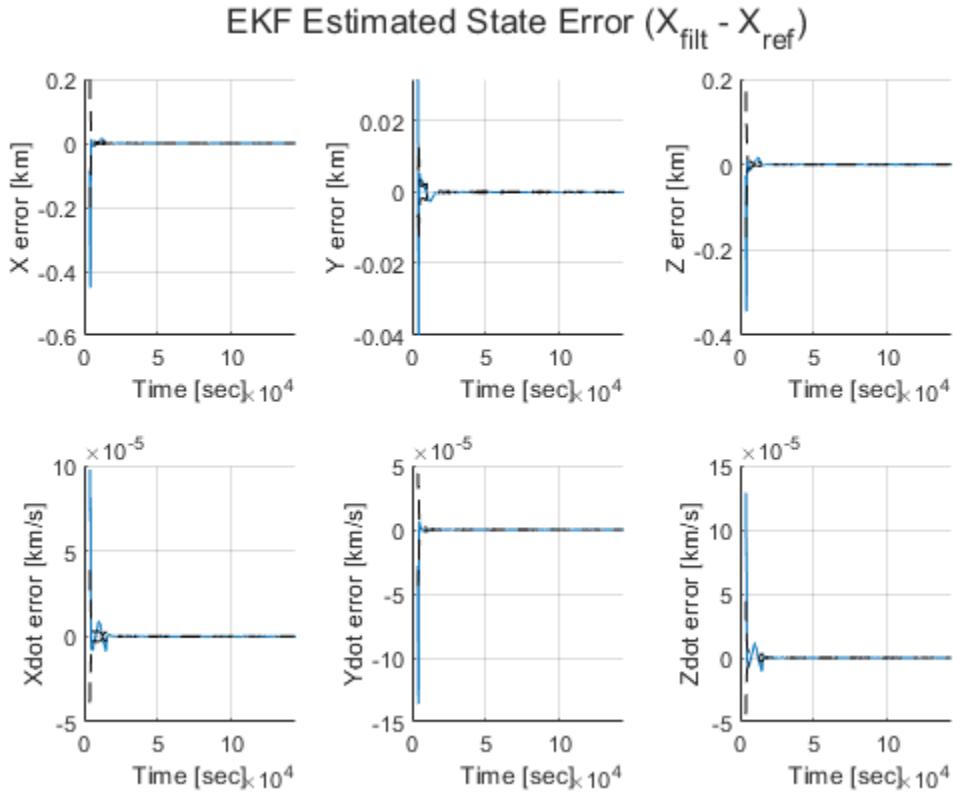


LKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



EKF Post-Fit Residuals





Problem 1f: Process half the measurements

Warning: Integer operands are required for colon operator when used as index.

Warning: Integer operands are required for colon operator when used as index.

Warning: Integer operands are required for colon operator when used as index.

Warning: Integer operands are required for colon operator when used as index.

Warning: Integer operands are required for colon operator when used as index.

1f. Half measurements

Running Batch Filter:

Postfit RMS: 4.1014. Iterating Batch with $x0_batch = [5.332e-03; -2.111e-03;$

$7.824e-03; 3.520e-06; -1.241e-06; 4.037e-06]$. Runs so far: 1

Postfit RMS: 1.0172. Iterating Batch with $x0_batch = [-6.308e-05;$

$-2.934e-06; -2.034e-05; -3.293e-09; -1.624e-08; -3.591e-08]$. Runs so far: 2

Final postfit RMS: 1.0172. Converged after 3 runs

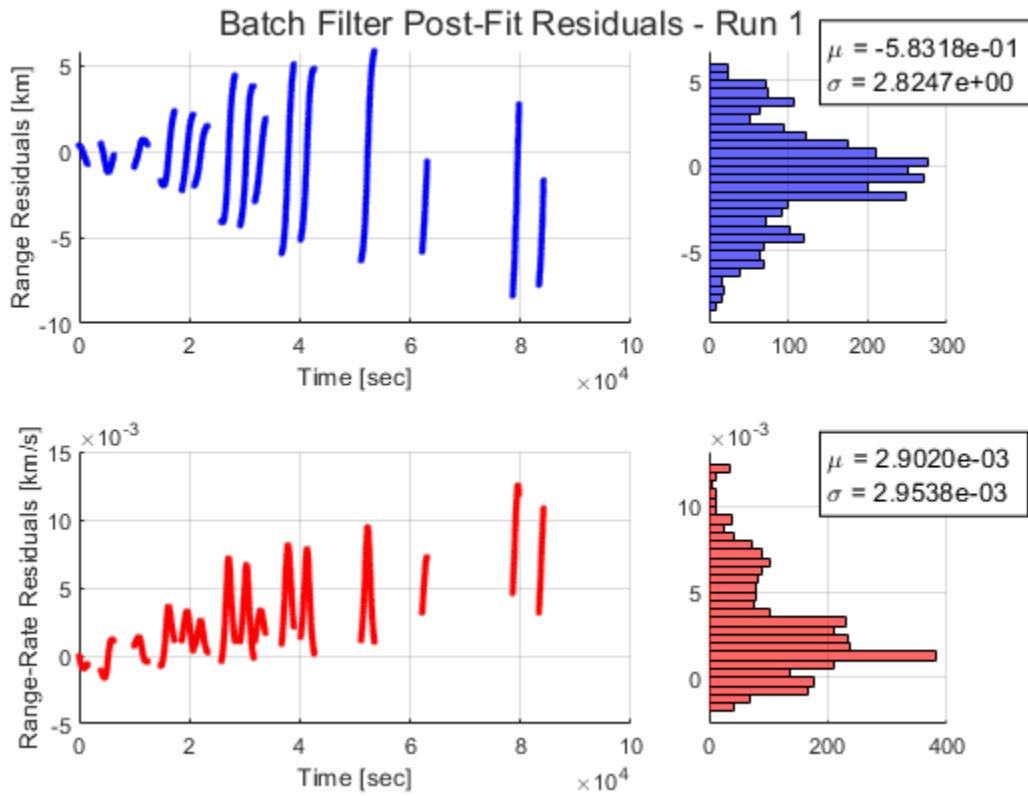
Running LKF:

Postfit RMS: 4.0959. Iterating LKF with $x0_LKF = [4.947e-01; 5.021e-01;$

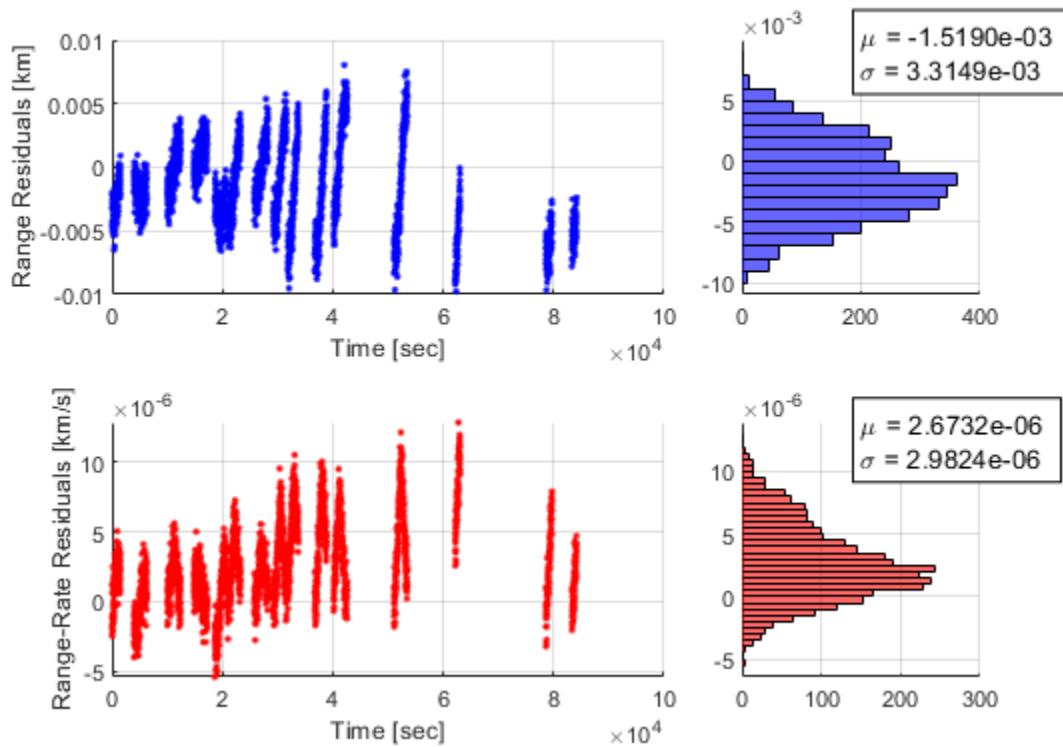
ASEN 6080 HW 2 Problem 1 Main
Script

```
4.922e-01; 4.965e-04; 5.012e-04; 4.960e-04]. Runs so far: 1
Postfit RMS: 1.0105. Iterating LKF with x0_LKF = [5.395e-03; -2.108e-03;
7.845e-03; 3.523e-06; -1.225e-06; 4.073e-06]. Runs so far: 2
Postfit RMS: 1.0105. Iterating LKF with x0_LKF = [5.323e-09; -1.762e-09;
-6.386e-09; 7.267e-12; -2.229e-12; 4.500e-12]. Runs so far: 3
Final postfit RMS: 1.0105. Converged after 4 runs
```

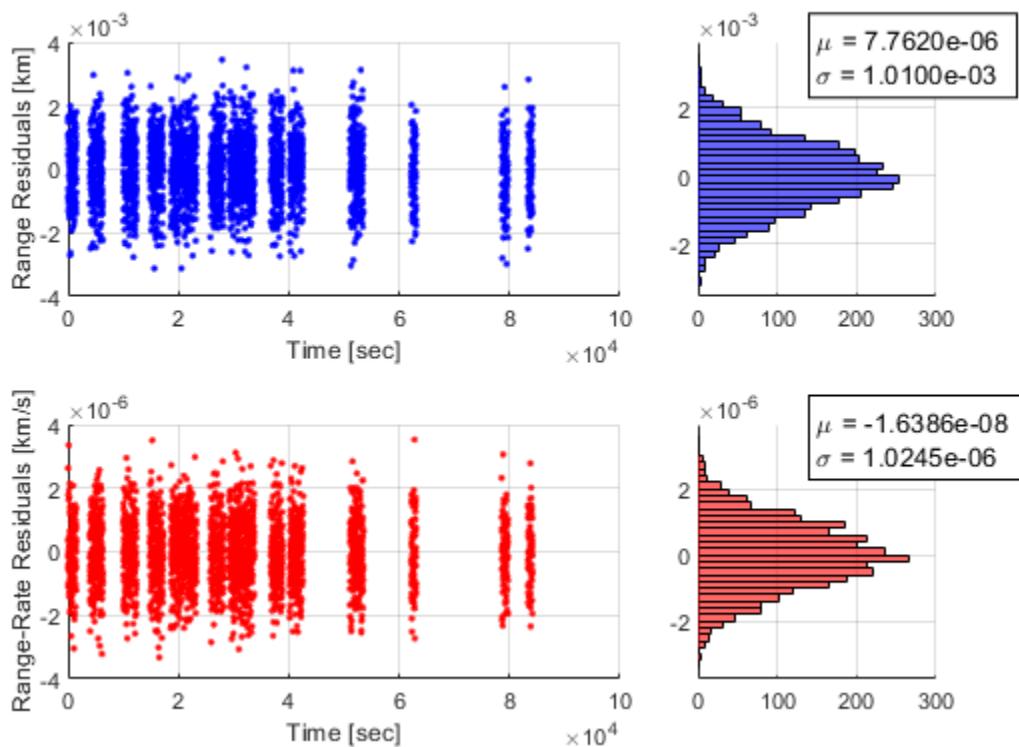
Running EKF:
Postfit RMS: 1.0137



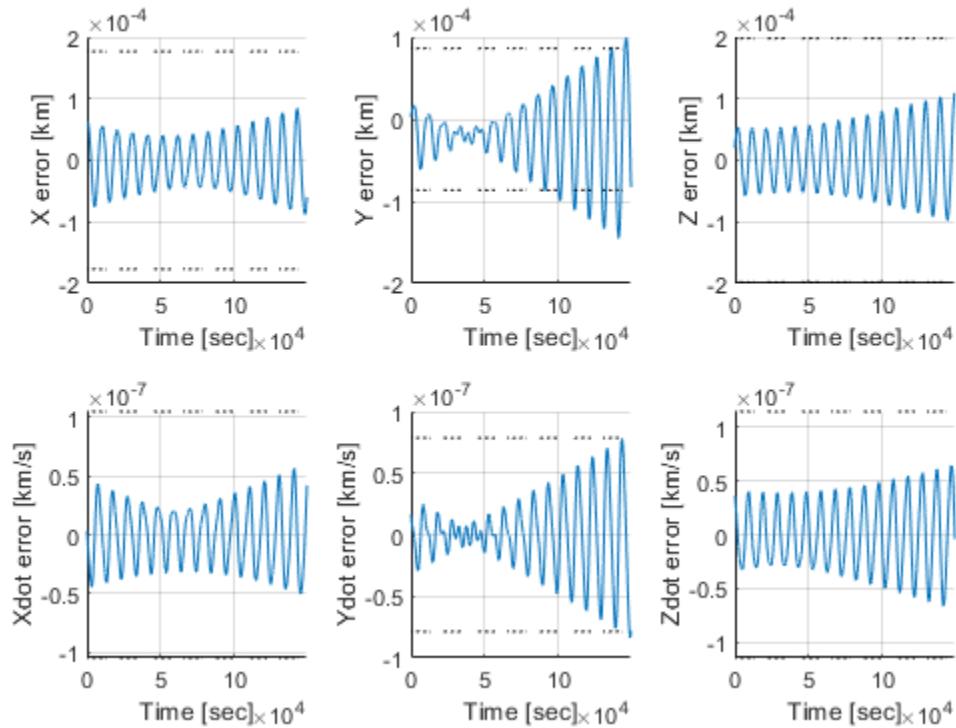
Batch Filter Post-Fit Residuals - Run 2



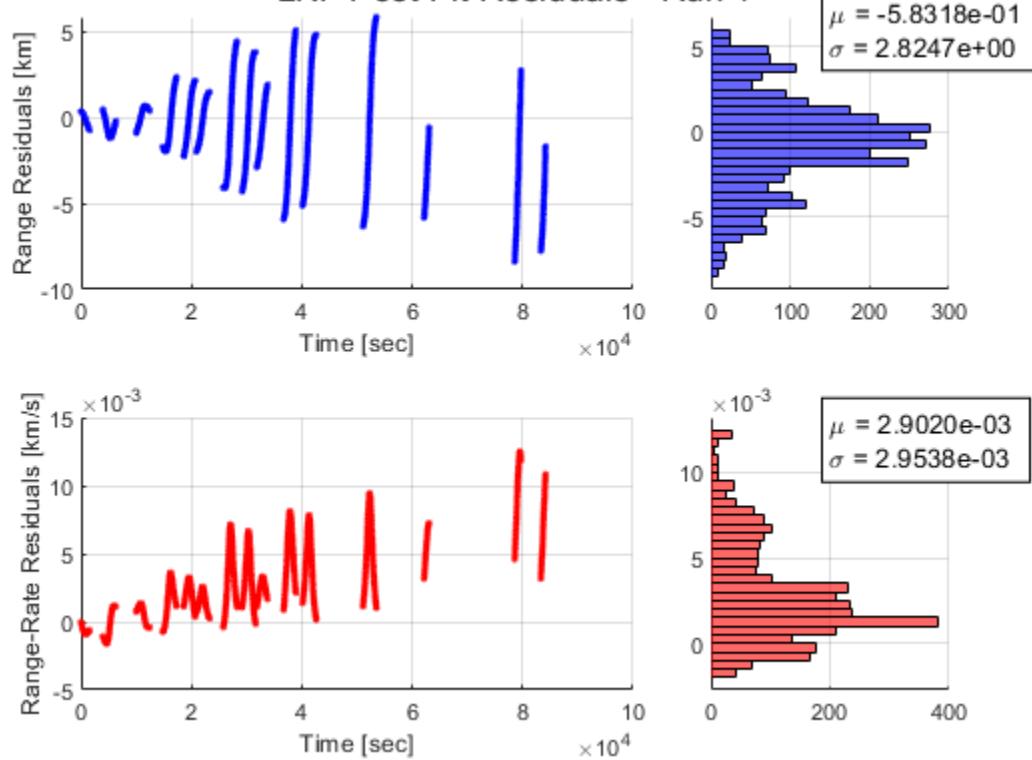
Batch Filter Post-Fit Residuals - Run 3



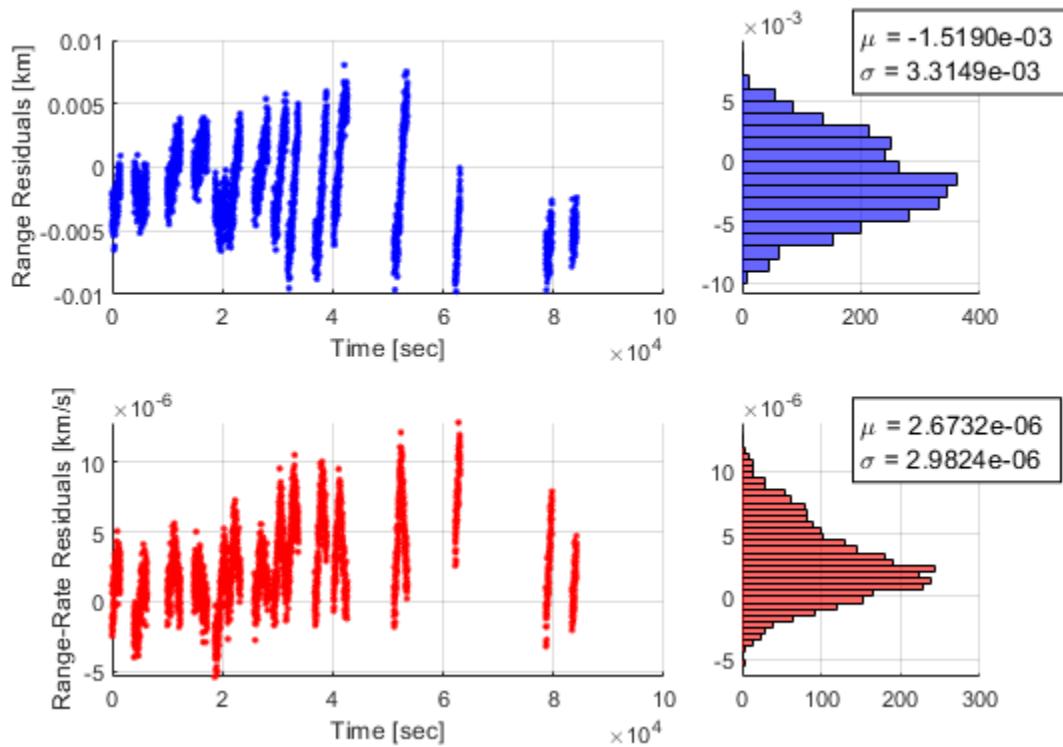
Batch Filter Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



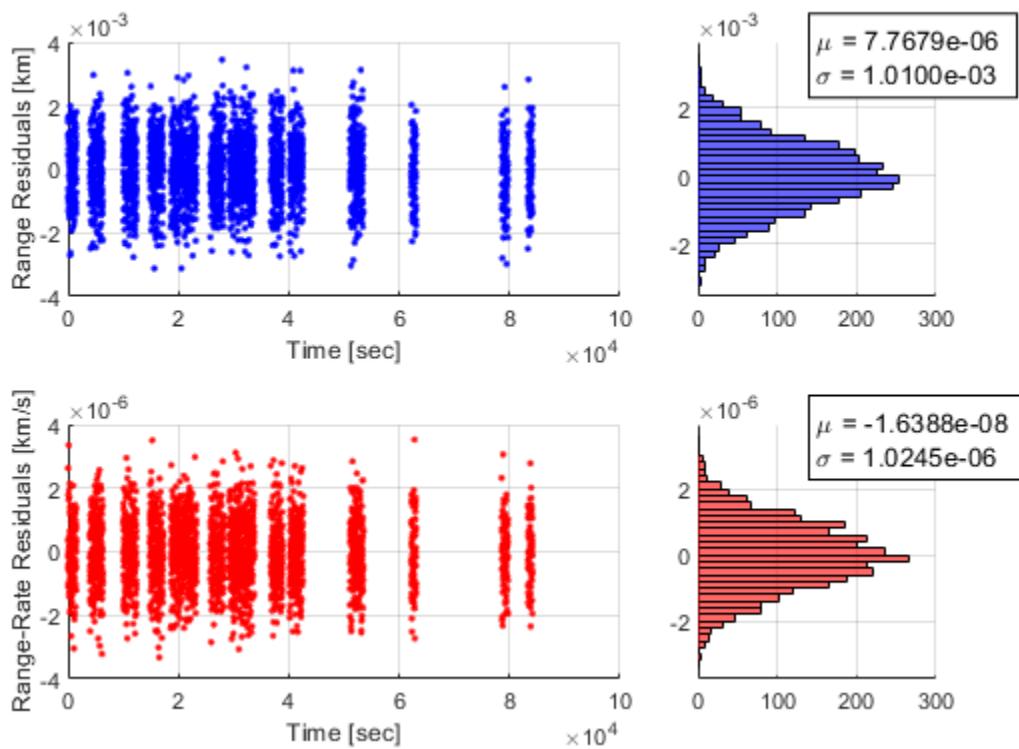
LKF Post-Fit Residuals - Run 1



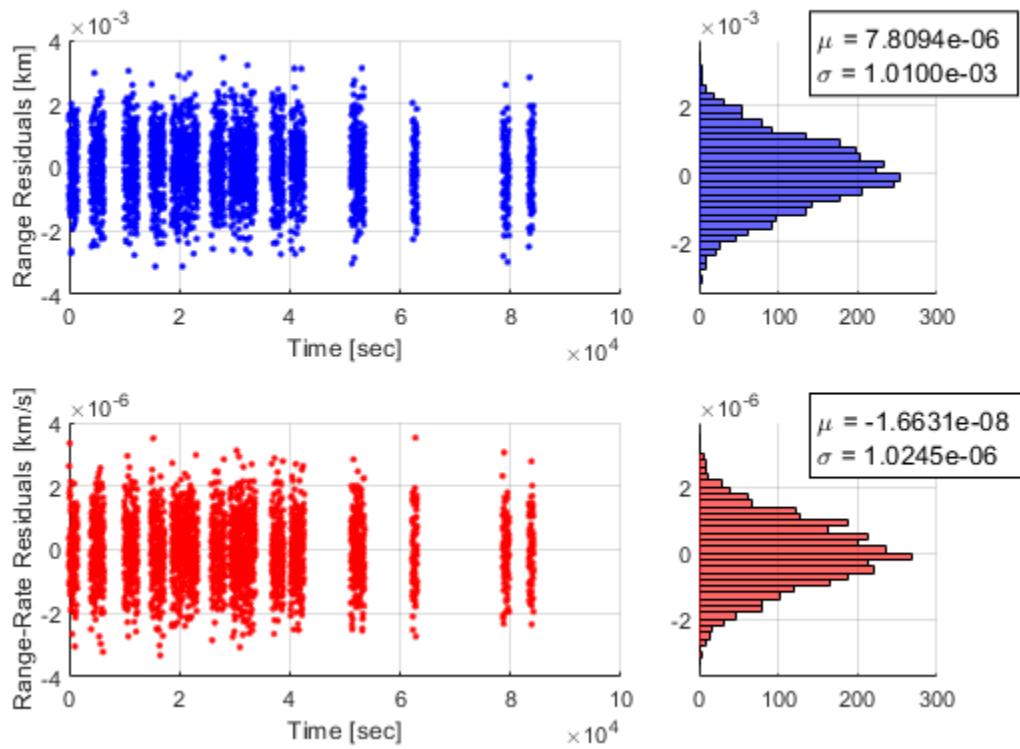
LKF Post-Fit Residuals - Run 2



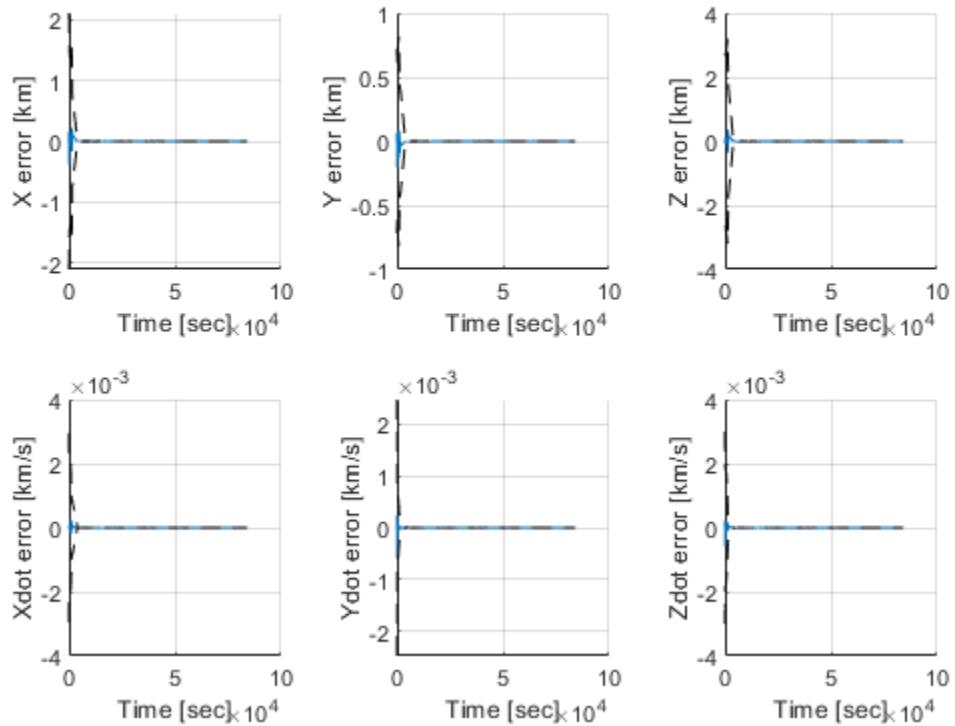
LKF Post-Fit Residuals - Run 3



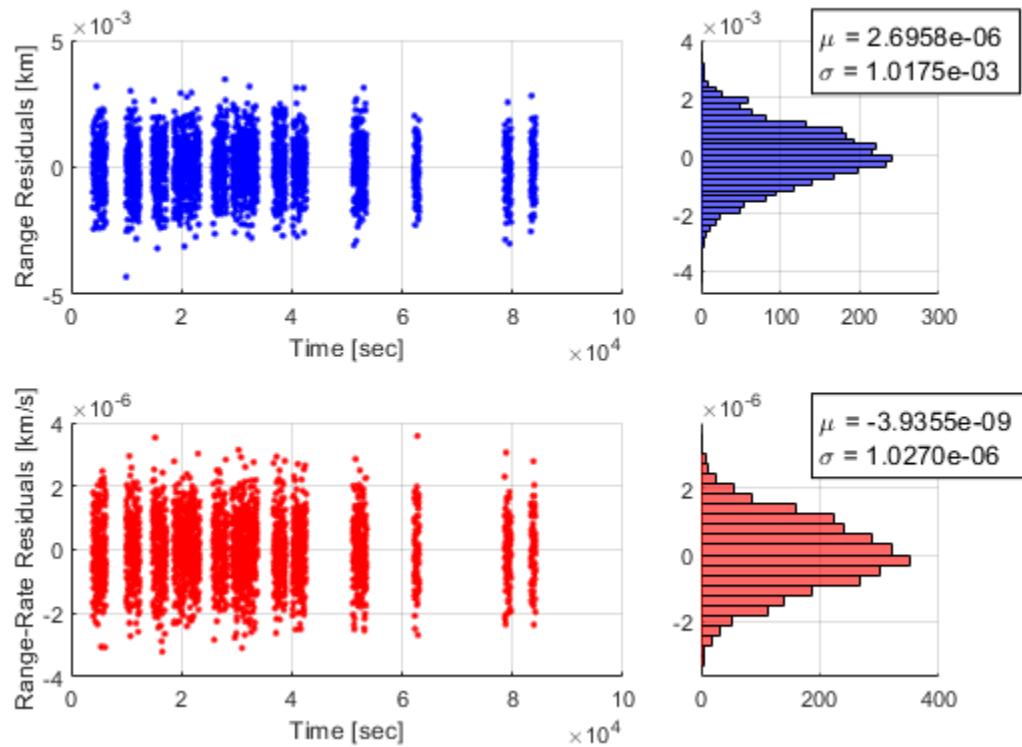
LKF Post-Fit Residuals - Run 4



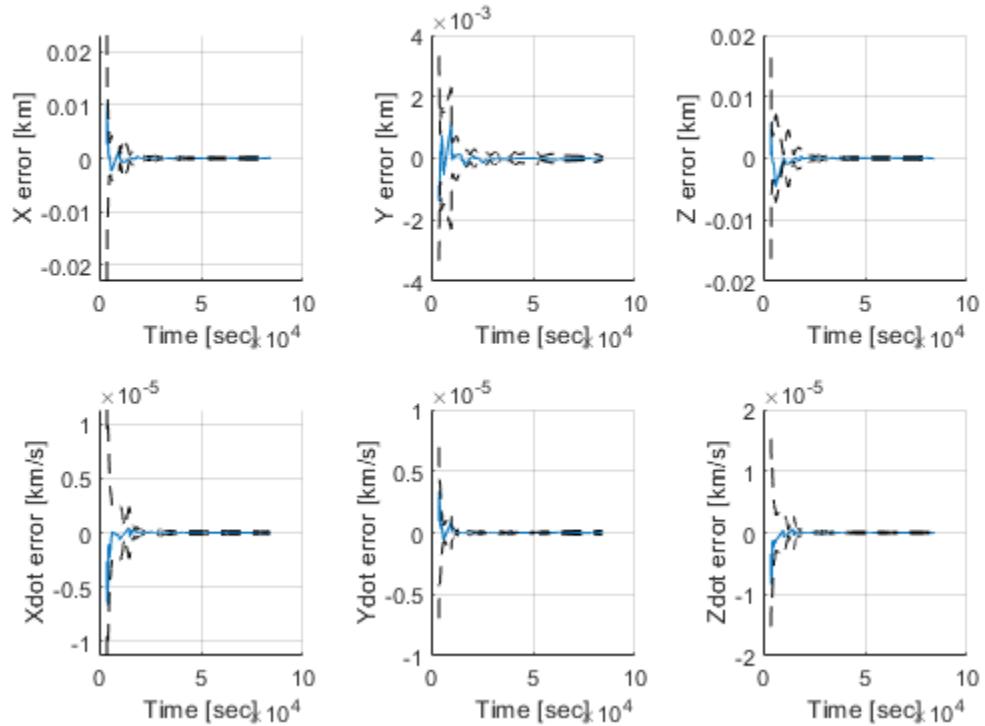
LKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



EKF Post-Fit Residuals



EKF Estimated State Error ($X_{\text{filt}} - X_{\text{ref}}$)



2. Simulate a new truth orbit + measurements w/ J_3
 - a. make a new set of truth data including J_3 and plot the differences of this new data and that from HW1.
See PDF for plot
 - b. process the data above using just J_2 dynamics in the filter. How does this change filter performance?

When J_3 isn't included in the filter dynamics, every filter breaks. Their RMS errors are large, and their state errors fall outside of their predicted bounds. In particular,

The batch filter errors exhibit strange trends

The LKF error oscillates wildly, as do the EKF's

Finally, none of the post-fit residuals are zero-mean Gaussian, implying unmodeled dynamics (like J_2 !)

ASEN 6080 HW 2 Problem 2 Main Script

Table of Contents

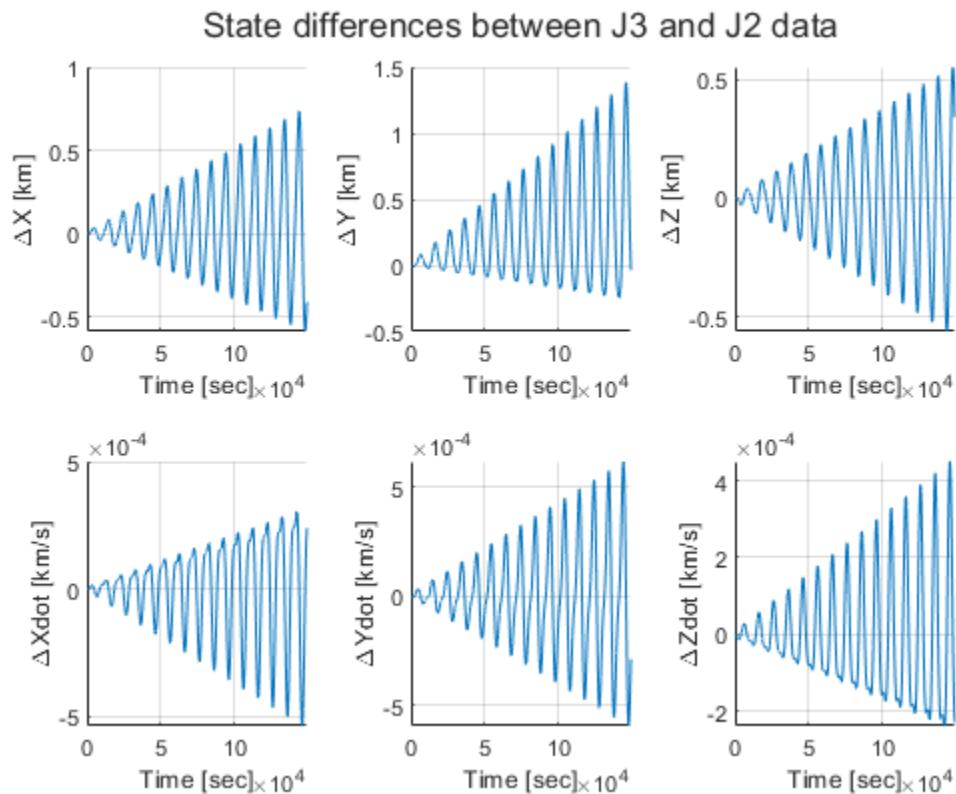
Housekeeping	1
Setup	1
Problem 2a: Make Truth Data with J3	1
Problem 2b: Filter setup	2
Problem 2b: Run filters on new data	2

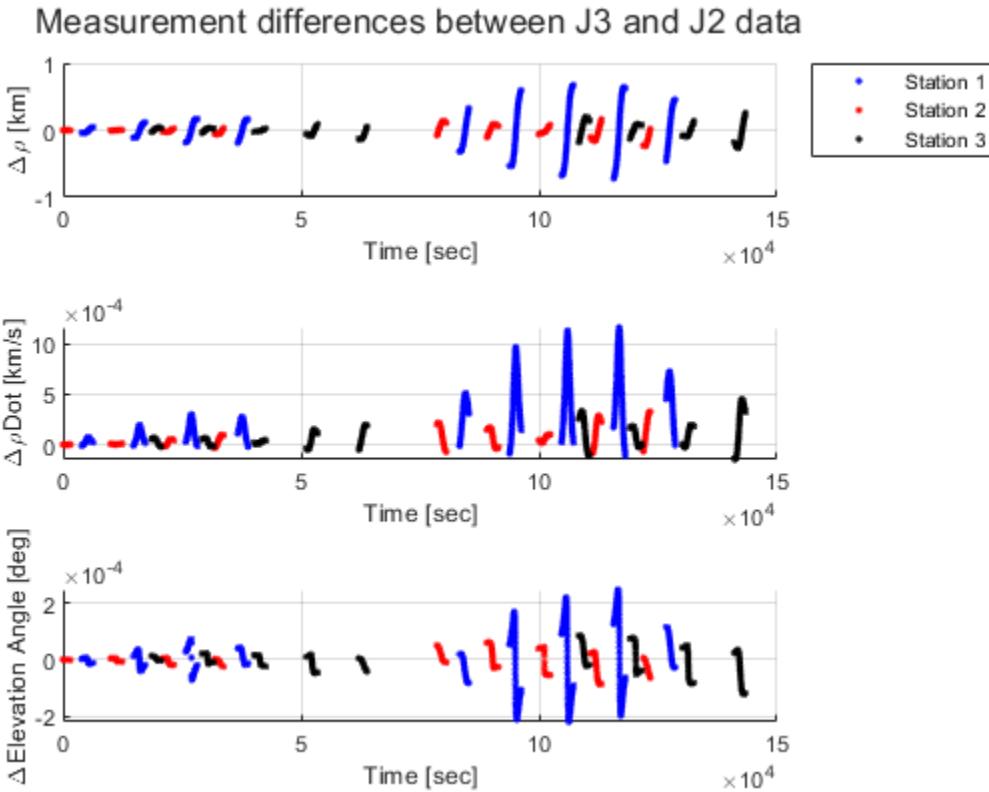
By: Ian Faber

Housekeeping

Setup

Problem 2a: Make Truth Data with J3





Problem 2b: Filter setup

Problem 2b: Run filters on new data

Running Batch Filter:

```
Postfit RMS: 97.8506. Iterating Batch with x0_batch = [-1.860e-02;
-1.256e-01; -2.042e-02; -3.525e-05; -3.391e-05; 5.462e-05]. Runs so far: 1
Postfit RMS: 93.5787. Iterating Batch with x0_batch = [-2.901e-02;
-1.212e-01; -3.210e-02; -5.429e-05; -2.915e-05; 2.986e-05]. Runs so far: 2
Final postfit RMS: 93.5787. Converged after 3 runs
```

Running LKF:

```
Postfit RMS: 97.7726. Iterating LKF with x0_LKF = [5.186e-01; 6.256e-01;
5.204e-01; 5.352e-04; 5.339e-04; 4.454e-04]. Runs so far: 1
Postfit RMS: 93.5017. Iterating LKF with x0_LKF = [1.041e-02; -4.457e-03;
1.168e-02; 1.905e-05; -4.761e-06; 2.476e-05]. Runs so far: 2
Final postfit RMS: 93.5017. Converged after 3 runs
```

Running EKF:

```
Postfit RMS: 93.5599
```

