Aerodynamics Computational Assignment #3: Flow over Thick Airfoils and Finite Wings

Assigned Date: March 7, 2023
Due Date: April 7, 2023

Collaboration Policy:

Collaboration is permitted on the computational labs. You may discuss the means and methods for formulating and solving problems and even compare answers, but you are not free to copy someone else's work. *Copying material from any resource (including solutions manuals) and submitting it as one's own is considered plagiarism and is an Honor Code violation.*

Matlab Code Policy:

Computational codes must be written individually and are expected to be written in MATLAB. If you have collaborated with others while writing your code be sure to acknowledge them in the header of your code, otherwise you may receive a zero for plagiarism. All code files required to successfully run the computational assignment driver script along with a pdf of your code and its execution (i.e. printed comments and figures) should be submitted via the course website by 11:59pm on the due date. Code files will not be accepted after the given due date.

Reflection Questions:

In this assignment, there are multiple reflection questions. These reflection questions are provided to help you review the functionality of your code, help you analyze and understand your results, and to test your understanding of the concepts being studied.

Learning Outcomes:

1. Understand the difference between the application and results of thin airfoil theory and the vortex panel method.

2. Understand how changes in wing section camber and thickness alter the sectional lift slope and zero-lift angle of attack.

3. Practice using Prandtl Lifting Line Theory to calculate lift and drag on a finite wing.

4. Understand how the number of terms in Prandtl Lifting Line Theory affects the resulting error in the solution.

Problem #1: Computation of the Lift Generated by a Thick Symmetric Airfoil

As the thickness of an airfoil is increased, it is no longer accurate to approximate the flow around it using thin airfoil theory, with a continuous vortex sheet located on the chord line. Instead a vortex sheet can be "wrapped" around the airfoil surface to better account for the thickness of the body and to directly enforce the boundary conditions on the surface, including no penetration of the surface streamline.

This process can be accomplished with a classical "Vortex Panel Method" as outlined by Anderson in Section 4.10 of the Textbook. The approach discussed by Anderson represents a first order method which approximates the vortex sheet strength as constant along each panel. It can be error prone and unstable. A better, and more complicated, second order method which allows the vortex sheet strength to linearly vary across each panel is defined in Section 5.10 of the textbook by Kuethe and Chow. A pdf copy of this reading is provided on the course website along with a MATLAB function `Vortex_Panel.m` which implements the FORTRAN code outlined in Kuethe and Chow, along with a few minor modifications. Note that this code computes the flow for any arbitrary two-dimensional body defined by a set of (x, y) coordinates that define its surface.

The provided function has the form:

```
function c_l = Vortex_Panel(x_b,y_b,V_inf,alpha)
```

where `c_l` is the sectional coefficient of lift (to be computed and returned), `x_b` is a vector containing the x-location of the panel boundary points (in feet), `y_b` is a vector containing the y-location of the panel boundary points (in feet), `V_inf` is the free-stream flow speed (in feet per second), and `alpha` is the angle of attack (in degrees). Note that the points in `x_b` and `y_b` should be ordered around the surface in a clockwise direction starting from and ending at the trailing edge to be consistent with the code from Kuethe and Chow.

Using the provided vortex panel code, compute (and print to the command window) the sectional coefficient of lift experienced by a NACA 0012 airfoil at an angle of attack of $\alpha = 10°$, and determine (and print to the command window) the number of total panels (e.g., panels on both the upper and lower surfaces) required to achieve a predicted sectional coefficient of lift with 1 percent relative error.

Problem #2: Study of the Effect of Airfoil Thickness on Lift

Using the provided vortex panel code and the number of total panels required to achieve a predicted sectional coefficient of lift with 1 percent relative error for a NACA 0018 airfoil at an angle of attack of $\alpha = 10°$, obtain plots of the sectional coefficient of lift versus angle of attack for the following airfoils:

- NACA 0006 (Relatively Thin Airfoil)
- NACA 0012 (Moderate Thickness Airfoil)
- NACA 0024 (Relatively Thick Airfoil)

Using these plots, estimate the sectional lift slope and zero-lift angle of attack for each of the airfoils (print these findings to the command window), and compare these results with thin airfoil theory (both in the plot and in the command window). It is recommended that you plot all of these together to provide a clearer comparison.

Reflection: How do changes in the wing section thickness alter the sectional lift slope and the zero-lift angle of attack? How accurate is the assumption of thin airfoil theory for each wing section?

Problem #3: Study of the Effect of Airfoil Camber on Lift

Using the provided vortex panel code and the number of total panels required to achieve a predicted sectional coefficient of lift with 1 percent relative error for a NACA 0012 airfoil at an angle of attack of $\alpha = 10°$, obtain plots of the sectional coefficient of lift versus angle of attack for the following airfoils:

- NACA 0012 (Symmetric Airfoil)
- NACA 2412 (Moderately Cambered Airfoil)
- NACA 4412 (Significantly Cambered Airfoil)

Using these plots, estimate the sectional lift slope and zero-lift angle of attack for each of the airfoils (print these findings to the command window), and compare these results with thin airfoil theory (both in the plot and in the command window). It is recommended that you plot all of these together to provide a clearer comparison.

Reflection: How do changes in the wing section camber alter the sectional lift slope and the zero-lift angle of attack? How accurate is the assumption of thin airfoil theory for each wing section?

Problem #4: Prandtl Lifting Line Code

Write a MATLAB function which solves the fundamental equation of Prandtl Lifting Line Theory for finite wings with thick airfoils:

$$\alpha(\theta) = \frac{4b}{a_0(\theta)c(\theta)} \sum_{n=1}^{\infty} A_n \sin(n\theta) + \alpha_{L=0}(\theta) + \sum_{n=1}^{\infty} nA_n \frac{\sin(n\theta)}{\sin(\theta)}$$

by satisfying the equation at the N prescribed locations:

$$\theta_i = \frac{i\pi}{2N}, \quad i = 1, \ldots, N$$

and truncating the series expansion for circulation using N odd terms:

$$\Gamma(\theta) = 2bV_\infty \sum_{j=1}^{N} A_{(2j-1)} \sin((2j-1)\theta)$$

Your function should be general enough to work for an arbitrary number of terms in the series expansion for circulation and should allow for a linear spanwise variation of the cross-sectional lift slope, the local chord length, the aerodynamic twist, and the geometric twist. Your function should return as output the span efficiency factor as well as the coefficient of lift and coefficient of induced drag. Consequently, your function should take the form:

```
function [e,c_L,c_Di] = PLLT(b,a0_t,a0_r,c_t,c_r,aero_t,aero_r,geo_t,geo_r,N)
```

where `e` is the span efficiency factor (to be computed and returned), `c_L` is the coefficient of lift (to be computed and returned), `c_Di` is the induced coefficient of drag (to be computed and returned), `b` is the span (in feet), `a0_t` is the cross-sectional lift slope at the tips (per radian), `a0_r` is the cross-sectional lift slope at the root (per radian), `c_t` is the chord at the tips (in feet), `c_r` is the chord at the root (in feet), `aero_t` is the zero-lift angle of attack at the tips (in degrees), `aero_r` is the zero-lift angle of attack at the root (in degrees), `geo_t` is the geometric angle of attack at the tips (in degrees), `geo_r` is the geometric angle of attack at the root (in degrees), and N is the number of odd terms to include in the series expansion for circulation.

To validate that your function is working correctly, reproduce Figure 5.20 from Anderson's *Fundamental's of Aerodynamics* of the induced drag factor, δ, as a function of taper ratio, c_t/c_r.

Reflection: In this lab only the odd terms are utilized in the PLLT series expansion. Why is this the case? When would both the odd and even terms be required?

Problem #5: Analysis of Approximate Cessna 150 Wing Performance

Apply your Prandtl Lifting Line Function to a wing with a span of 33 ft 4 in and a straight taper from 5 ft 4 in root chord to 3 ft 8.5 in tip chord. The root airfoil is chosen to be a NACA 2412 while the tip airfoil is chosen to be a NACA 0012. This results in a linear spanwise variation of cross-sectional lift slope and zero-lift angle of attack. The wing is also twisted such that the geometric angle of attack varies linearly from 1° at the root to 0° at the tips.

If the aircraft is flying at a cruise angle of attack of 3° then determine (and print to the command window) the lift and induced drag for the wing at a cruise speed of 60 knots and $15,000$ ft altitude (standard atmosphere). Moreover, complete the following tasks:

- Determine the number of odd terms required in the series expansion for circulation to obtain lift and induced drag solutions with ten percent relative error. Print this value to the command window.
- Determine the number of odd terms required in the series expansion for circulation to obtain lift and induced drag solutions with one percent relative error. Print this value to the command window.
- Determine the number of odd terms required in the series expansion for circulation to obtain lift and induced drag solutions with 1/10 percent relative error. Print this value to the command window.

Note: To compute the cross-sectional lift slope and zero-lift angle of attack at the root and tips, use the provided vortex panel code.

Suggested Approach:

You will need to build 4-digit NACA airfoils repeatedly throughout this lab. As such, it is suggested that you build a MATLAB function to construct panels for a given NACA airfoil. For instance, your MATLAB function may take the form:

```
[x,y] = function NACA_Airfoils(m,p,t,c,N)
```

where `x` is a vector containing the x-location of the boundary points, `y` is a vector containing the y-location of the boundary points, `m` is the maximum camber, `p` is the location of maximum camber, `t` is the thickness, `c` is the chord length, and `N` is the number of employed panels.

Note: The formula for the shape of a NACA 4-digit series airfoil with camber is a bit involved. The first ingredient is the thickness distribution of the airfoil from the mean camber line, which is given by:

$$y_t = \frac{t}{0.2}c\left[0.2969\sqrt{\frac{x}{c}} - 0.1260\left(\frac{x}{c}\right) - 0.3516\left(\frac{x}{c}\right)^2 + 0.2843\left(\frac{x}{c}\right)^3 - 0.1036\left(\frac{x}{c}\right)^4\right]$$

where c is the chord length, x is the position along the chord from 0 to c, y_t is the half thickness at a given value of x (mean camber line to surface), and t is the maximum thickness aas a fraction of the chord. As with the case of a symmetric NACA airfoil, the last two digits in the NACA XXXX description gives $100t$. The second ingredient is the formula for the mean camber line, which is:

$$y_c = \begin{cases} m\frac{x}{p^2}\left(2p - \frac{x}{c}\right), & 0 \le x \le pc \\ m\frac{c-x}{(1-p)^2}\left(1 + \frac{x}{c} - 2p\right), & pc \le x \le c \end{cases}$$

where m is the maximum camber and p is the location of maximum camber. The first digit in the NACA XXXX description gives $100m$ while the second digit gives $10p$. Then, the coordinates (x_U, y_U) and (x_L, y_L) of the upper and lower airfoil surface, respectively, become:

$$x_U = x - y_t \sin\xi \qquad y_U = y_c + y_t \cos\xi$$
$$x_L = x + y_t \sin\xi \qquad y_L = y_c - y_t \cos\xi$$

where

$$\xi = \arctan\left(\frac{dy_c}{dx}\right).$$

Note that for the NACA 4415 airfoil, $m = 4/100$, $p = 4/10$, and $t = 15/100$.