ASEN 3128 Lab 1 Assignment

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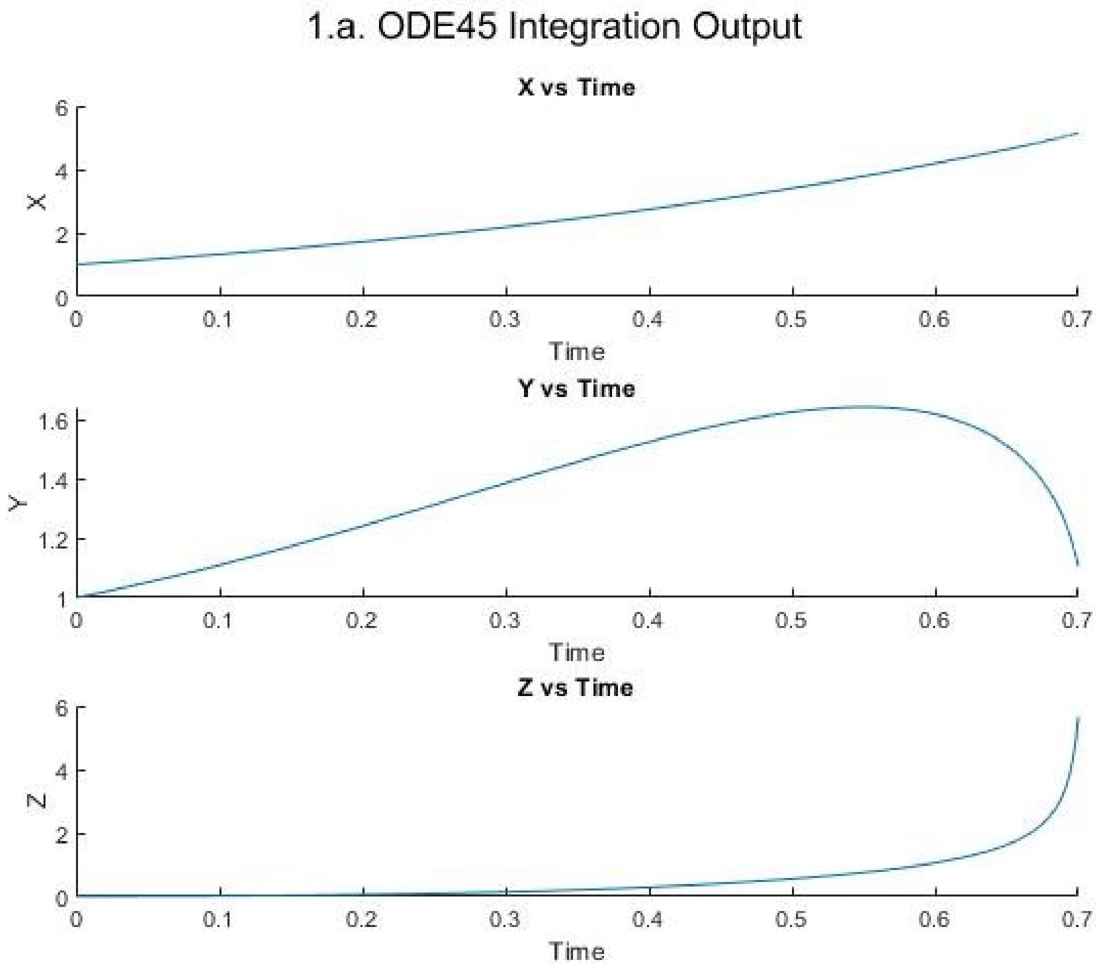
Problem 1. Nonlinear dynamic system

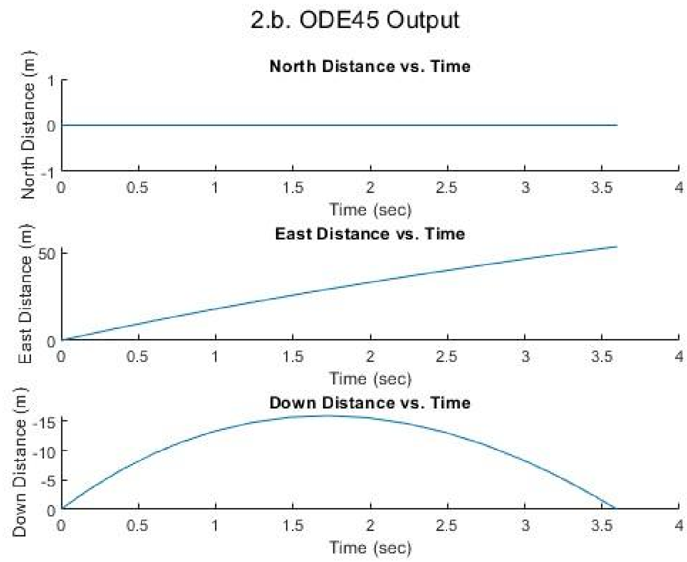


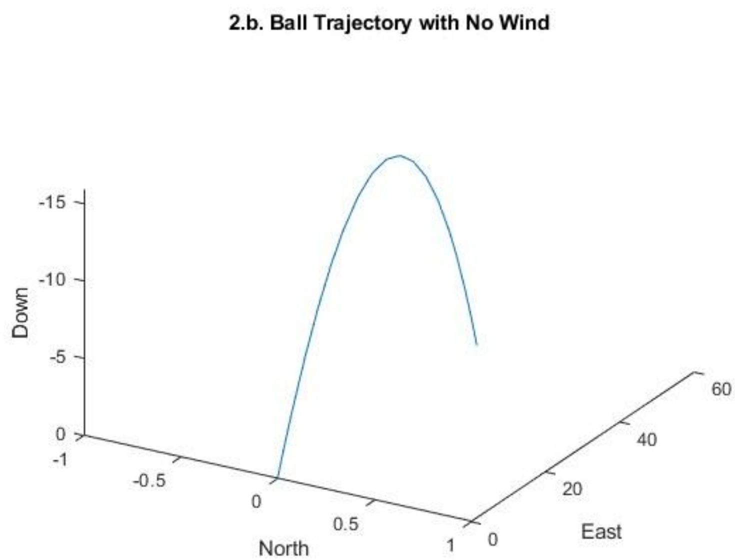
Problem 2. Golf ball trajectory simulation with drag and wind

2.a. ObjectEOM function

```
function xdot = ObjectEOM(t,x,rho,Cd,A,m,g,wind)
%
% Inputs:   t = Time vector (sec)
%           x = State vector (m, m/s)
%             = [x; y; z; vx; vy; vz]
%           rho = Air density (kg/m^3)
%           Cd = Coefficient of drag
%           A = Cross-sectional area of golf ball relative to motion (m^2)
%           m = Mass of golf ball (kg)
%           g = Freefall acceleration due to gravity (m/s^2)
%           wind = Wind vector (m/s)
%                = [windx; windy; windz]
%
% Outputs: xdot = rate of change of state vector (m/s, m/s^2)
%               = [vx; vy; vz; ax; ay; az]
%
% Methodology: Function used by ode45 to calculate golf ball trajectory,
%               problem 2 part a
% Extract inertial velocity and calculate wind-relative velocity
Ve = x(4:6);
V = Ve - wind;
% Position rate of change is inertial velocity
pdot = Ve;
% Calculate speed and the wind-relative velocity unit vector
mag = norm(V);
unitV = V/mag;
% Calculate drag and weight forces
Fdrag = (-0.5*rho*(mag^2)*Cd*A) * unitV;
Fgrav = m*g;
% Combine forces into total force vector
F = Fdrag + Fgrav;
% Velocity rate of change is a = F/m (F = ma)
vdot = F/m;
% Format output vector
xdot = [pdot; vdot];
end
```

2.b. Base trajectory, no wind





This trajectory makes sense for multiple reasons. The first is that since there is no North wind in this simulation, there should be no displacement in the North axis, which is clearly shown to be true in the North Distance vs. Time plot. The second reason is that we can see a slight curvature in the East displacement on the East Distance vs. Time plot, which indicates the presence of drag. Without drag, the displacement curve on the East axis would be a straight line. Finally, given the common aircraft body frame of North-East-Down, we should expect the "down" displacement to get more negative ("gain more height") up to a maximum, then decrease until reaching 0 (the ball hitting the ground) like any other ballistic trajectory. This is exactly what is shown in the Down Distance vs. Time plot. Furthermore, the plot also shows that it takes slightly longer for the ball to fall than rise, meaning drag has slowed the ball down.

2.c. Landing location sensitivity to north wind

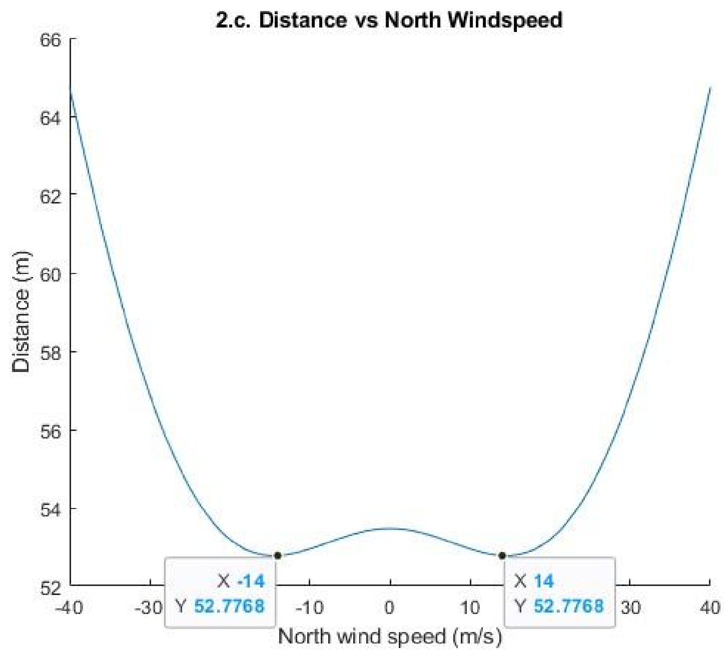


Figure 2.c.1. Plot of the relationship between North wind speed and the distance from the origin to the landing location of the golf ball

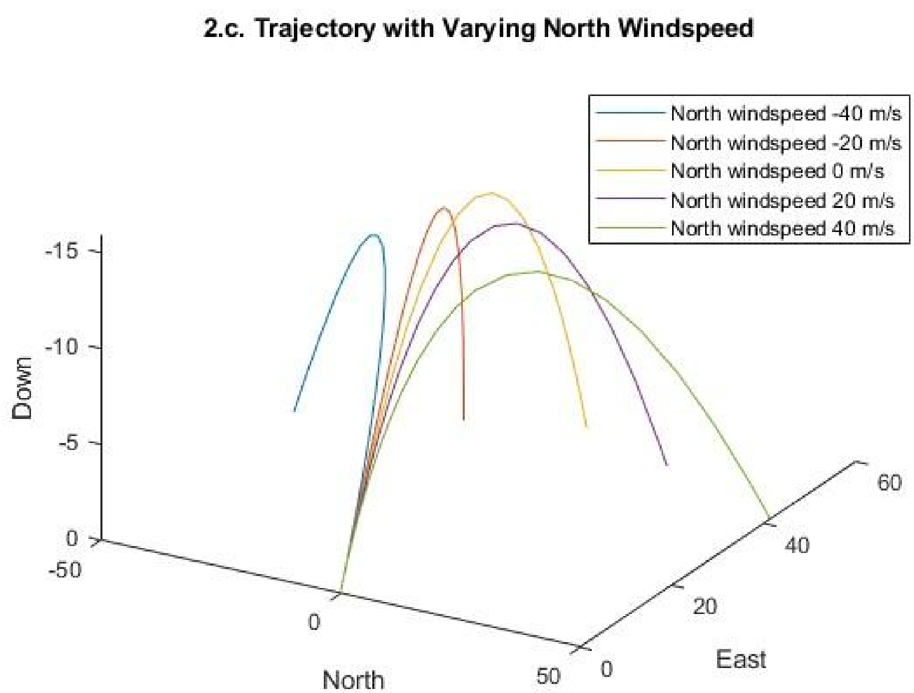


Figure 2.c.2. 3-D plot of the trajectory of the golf ball with a subset (every 20 m/s) of the tested North windspeeds

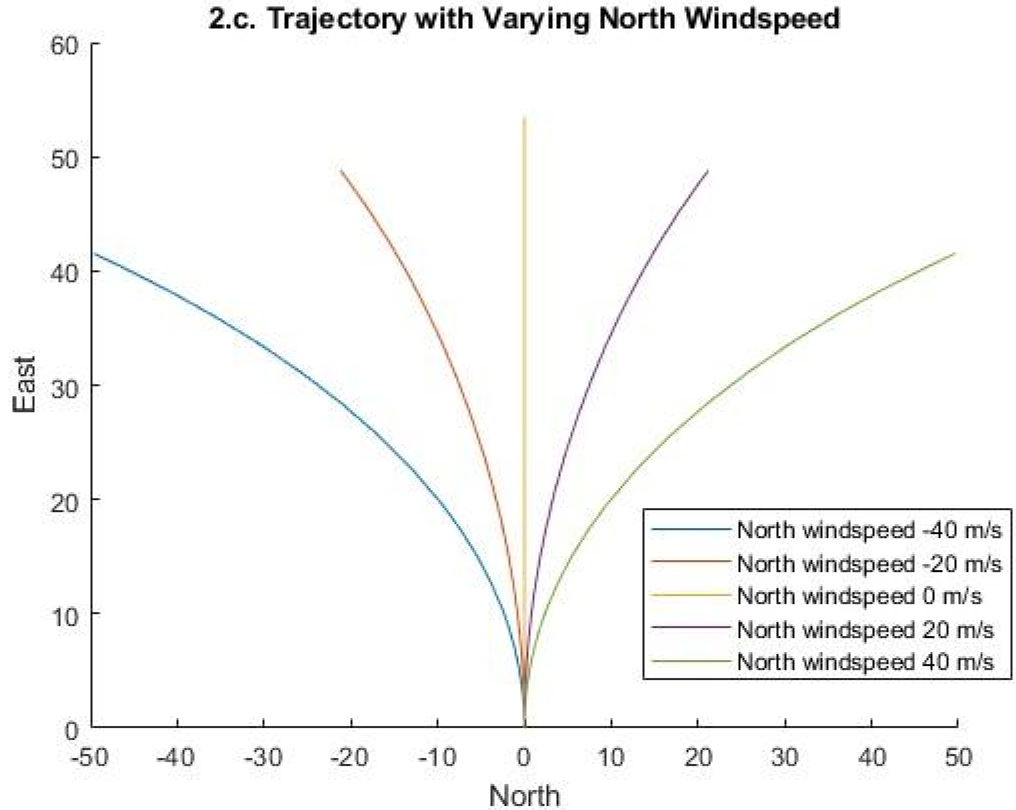


Figure 2.c.3. Top view of the trajectory of the golf ball under a subset (every 20 m/s) of the tested North windspeeds

Based on Figure 2.c.1, we concluded that the landing location of the golf ball is very sensitive to windspeeds higher than ~3 m/s. After that, the distance from the origin decreases to a minimum at a windspeed of ~14 m/s, then increases rapidly with any windspeed greater than 14 m/s. In terms of coordinates, Figure 2.c.3 shows that any amount of wind will decrease the magnitude of the East coordinate of the landing location and increase the magnitude of the North coordinate of the landing location. The sign of the North coordinate matches the sign of the windspeed vector's North component, and the sign of the East coordinate is always positive. Wind will also cause the maximum height of the golf ball to decrease, as seen in Figure 2.c.2, but that has little to no effect on the ball's final landing location.

2.d. Limited kinetic energy, ball mass variation

To examine the effect different golf ball masses had on the landing location, we first needed to calculate the amount of kinetic energy available to the golf player. To do this, we found the speed of the golf ball in part 2.a, then substituted that value into the equation for kinetic energy.

$$\begin{aligned} V_E &= \begin{bmatrix} 0 \\ 20 \\ -20 \end{bmatrix}_E \\ \|V\| &= \sqrt{0^2 + 20^2 + (-20)^2} = 20\sqrt{2}\, m/s \\ KE &= \frac{1}{2} m_{ball} \|V\|^2 = \frac{1}{2}(0.02)\left(20\sqrt{2}\right)^2 = 12\, J \end{aligned}$$

The next thing we needed to do was derive an equation for speed in relation to the mass of the ball, given the constant kinetic energy of 12 J.

$$\begin{aligned} KE &= \frac{1}{2} m_{ball} \|V\|^2 \\ \frac{2 \cdot KE}{m_{ball}} &= \|V\|^2 \\ \|V\| &= \sqrt{\frac{2 \cdot KE}{m_{ball}}} \\ &= \sqrt{\frac{24}{m_{ball}}} \end{aligned}$$

With this equation relating the speed of the ball to any given ball mass, we were able to simulate the trajectory of the golf ball with a set of different ball masses and obtain a relationship between the distance from the origin to the landing location and the mass of the golf ball, given a limited kinetic energy.

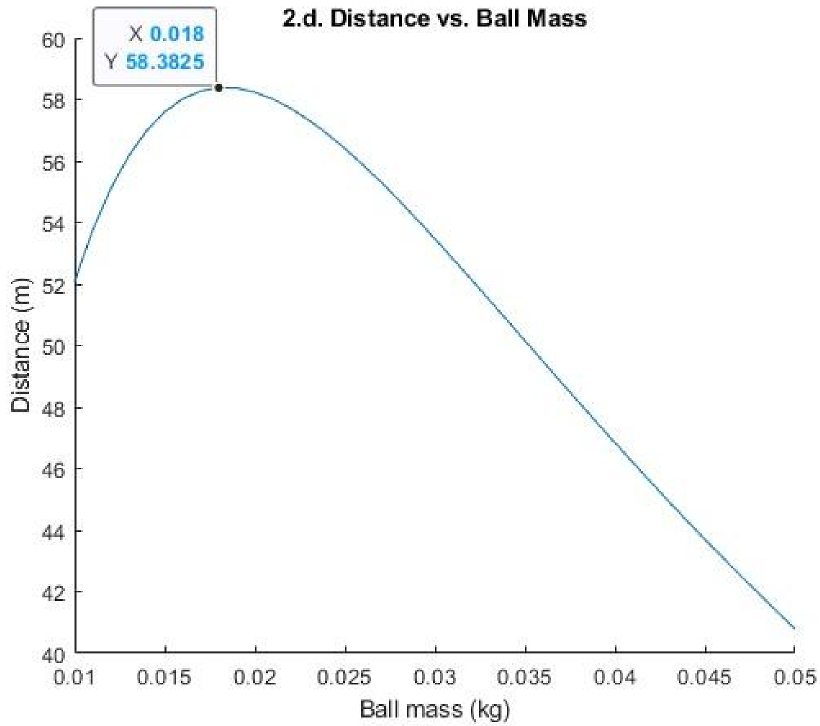


Figure 2.d.1. Plot of the relationship between golf ball mass and distance from the origin to the landing location of the golf ball

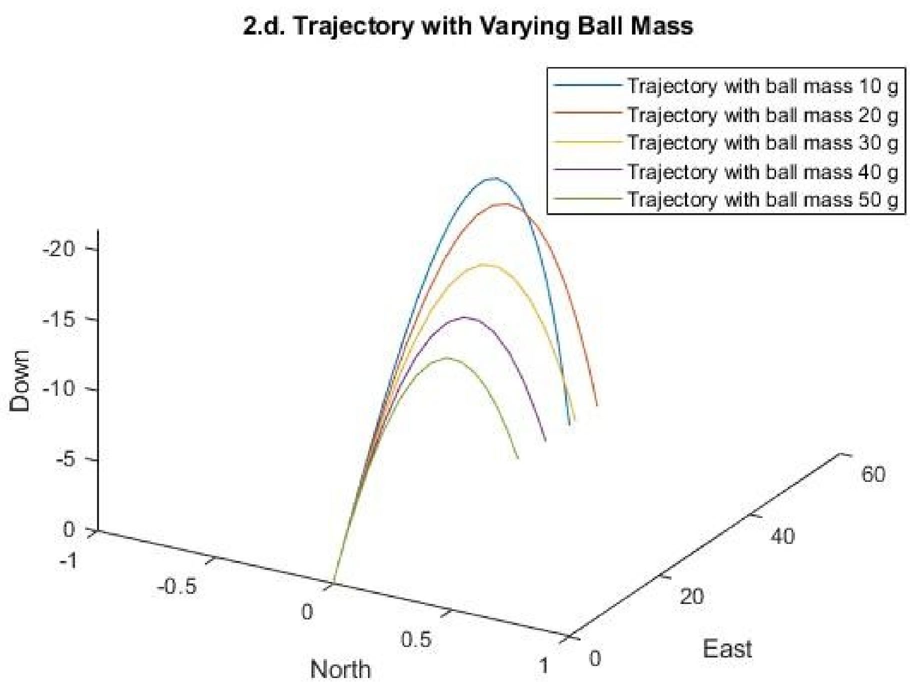


Figure 2.d.2. 3-D plot of the trajectory of the golf ball with a subset (every 10 g) of the tested golf ball masses

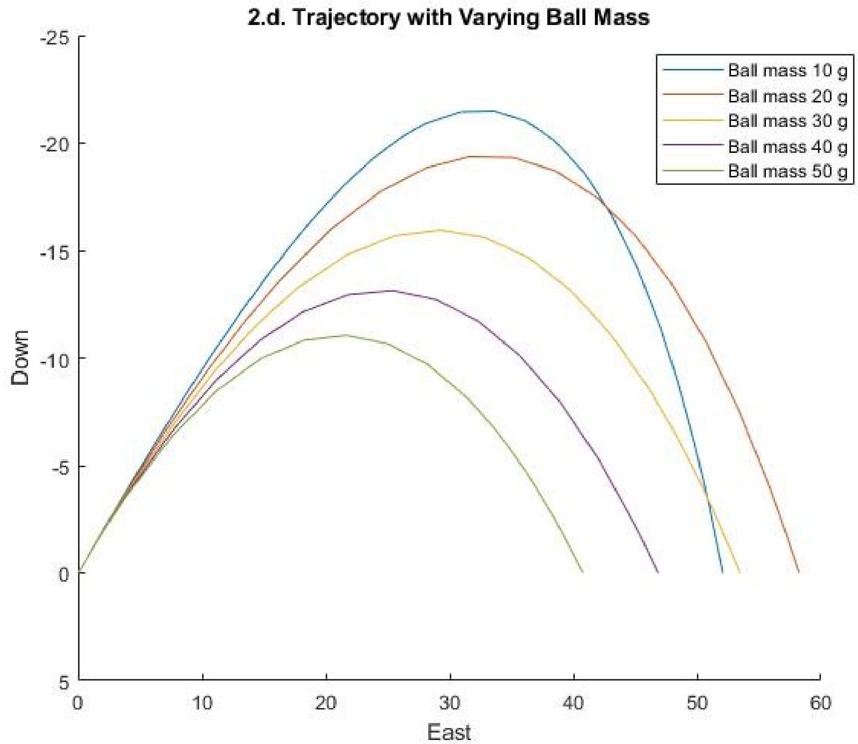


Figure 2.d.3. Side view of the trajectory of the golf ball with a subset (every 10 g) of the tested golf ball masses

Based on Figure 2.d.1, we concluded that there was an ideal golf ball mass at ~18 g, before and after which the distance from the origin to the landing location decreased. Thus, we concluded that a longer distance would be achieved by using a golf ball that was lighter than the original mass of 30 g, down to ~11 g. Figure 2.d.3 shows that a ball mass of 10 g does slightly worse than the original mass of 30 g, and a ball mass of 20 g outperforms all other masses plotted. This confirms our finding of an “ideal” ball mass for a given limited kinetic energy.

Appendix

See attached PDFs for all code