ASEN 6080 HW 8

-dan Faber, 108577813

1. a. Create an orbit simulation with M, J_2, + drag in the dynamics, with the orbit in LEO.

chosen parameters:

$a = 6800$ km, $e = 0.001$, $i = 40°$,
$\Omega = 80°$, $\omega = 40°$, $\nu = 0°$

b. Propagate 1000 monte carlo runs for 24 hours, with cases drawn from $\sigma_r = 1$ km and $\sigma_v = 1$ m/s

see PDF for plots

c. Every 6 hours, make corner plots and list the σ, mean, and nominal value for each state component.

see PDF for plots and summaries at

$t = 0, 6, 12, 18, + 24$ hr

2. a/b. Use CKF time update to propagate the initial covariance + estimate + produce the same I.C. results

See PDF for plots + results summaries

c. What percentage of monte carlo cases are inside the 2σ bounds?

a decreasing percentage of the cases are inside the 2σ bounds, until at 24 hrs next to none of the cases are within the bounds.

3. a/b. Use the UKF time update to propagate the initial covariance and estimate + produce the same I.C. results

See PDF for plots + results summaries

c. What percentage of the monte carlo cases are inside the 2σ bounds?

almost all of the cases are inside the 2σ bounds, but at 24 hours the covariance is grossly inflated.

4. a & b, use a Gaussian sums method to propagate the initial covariance and estimate + reproduce IC.

want $\mu_{6mm} = x_0$ and $P_{6mm} = P_0$

$\alpha_i = [0.25, 0.5, 0.25]$ ← $L = 3$

$M_i = [a x_0, b x_0, c x_0]$, $\sum_{i=1}^{3} \alpha_i M_i = x_0$

$(0.25a + 0.5b + 0.25c) x_0 = x_0$

$\therefore a = 0.75$, $c = 1.25$, $b = 0.8125$

$M_i = [0.75 x_0, 0.8125 x_0, 1.25 x_0]$

$$P_{mm} = \sum_{i=1}^{3} \alpha_i (P_i + \mu_i \mu_i^T) - \mu_{6mm} \mu_{6mm}^T$$

$$= P_0$$

$$= 0.25(P_1 + 0.75^2 x_0 x_0^T) + x_0 x_0^T +$$

$$0.5(P_2 + 0.8125^2 x_0 x_0^T) - x_0 x_0^T +$$

$$0.25(P_3 + 1.25^2 x_0 x_0^T) - x_0 x_0^T - x_0 x_0^T$$

$$P_0 = 0.25 P_1 + 0.5 P_2 + 0.25 P_3 - 0.13867\ldots x_0 x_0^T$$

$\therefore P_1 = P_3 = 0.25 P_0$, $P_2 = 2(0.875 P_0 + 0.1386\ldots x_0 x_0^T)$

so,

Gaussian 1: $N(0.75x_0, 0.75P_0)$, $\alpha_1 = 0.25$

Gaussian 2: $N(0.8125x_0, P_2)$, $\alpha_2 = 0.5$

Gaussian 3: $N(1.25x_0, 0.75P_0)$, $\alpha_3 = 0.25$

where $P_2 = 2(0.875P_0 + 0.1386718875x_0x_0^T)$.

see PDF for plots + results summaries

c. what percentage of the cases are inside the 2σ bounds?

I don't think I coded this correctly and ran out of time. all things were marking, I imagine GMM would do a good job.

5. compare + contrast.

ideally, GMM would do the best, followed by UKF and then LKF. nonlinear is the most accurate, but results in a mean orbit that isn't valid.

ASEN 6080 HW 8 Main Script

Table of Contents

By: Ian Faber

Housekeeping

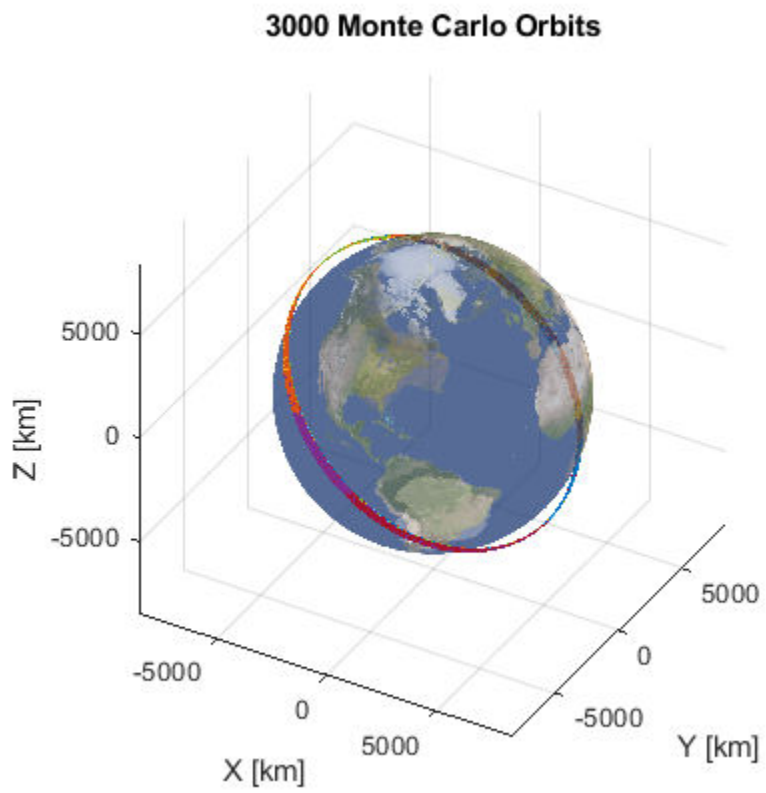
Setup

Path logistics

Part 1a/b. Generate Monte Carlo orbit data (nonlinear propagation)

rng(69420); % Set rng seed for consistency

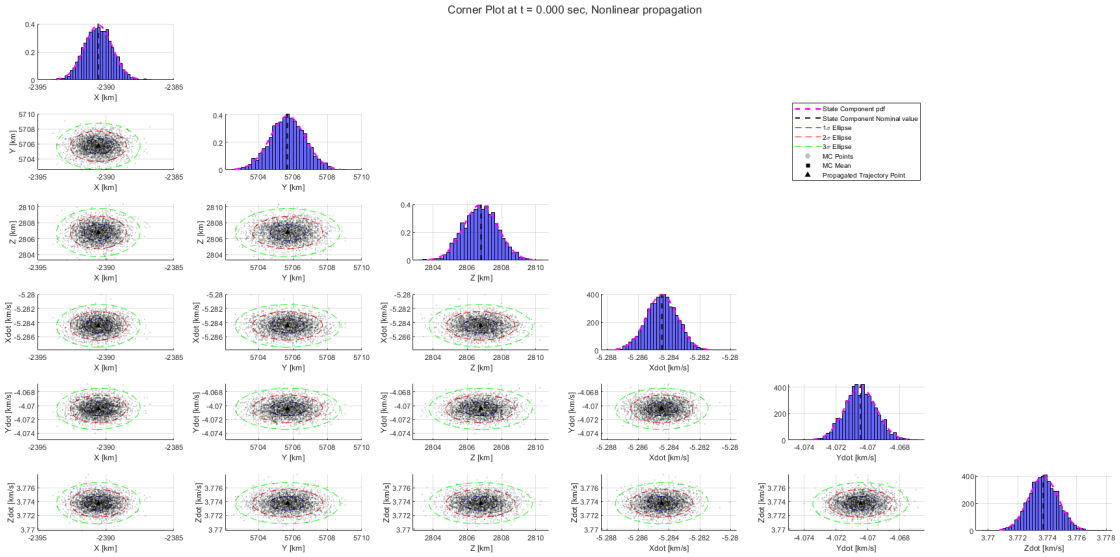
```
Loading Monte Carlo Data...
Loaded Data!
```

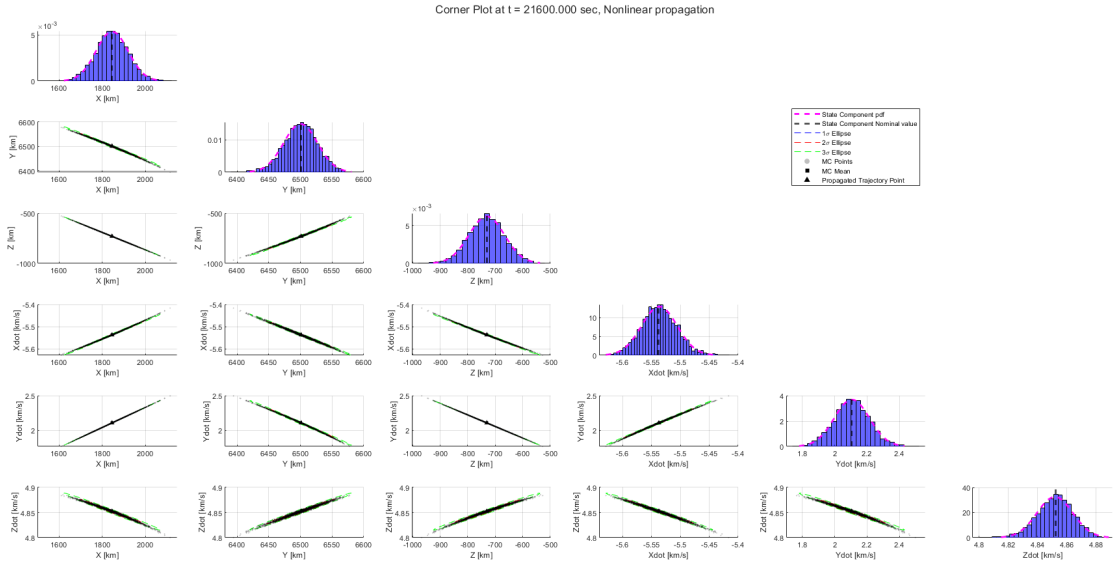


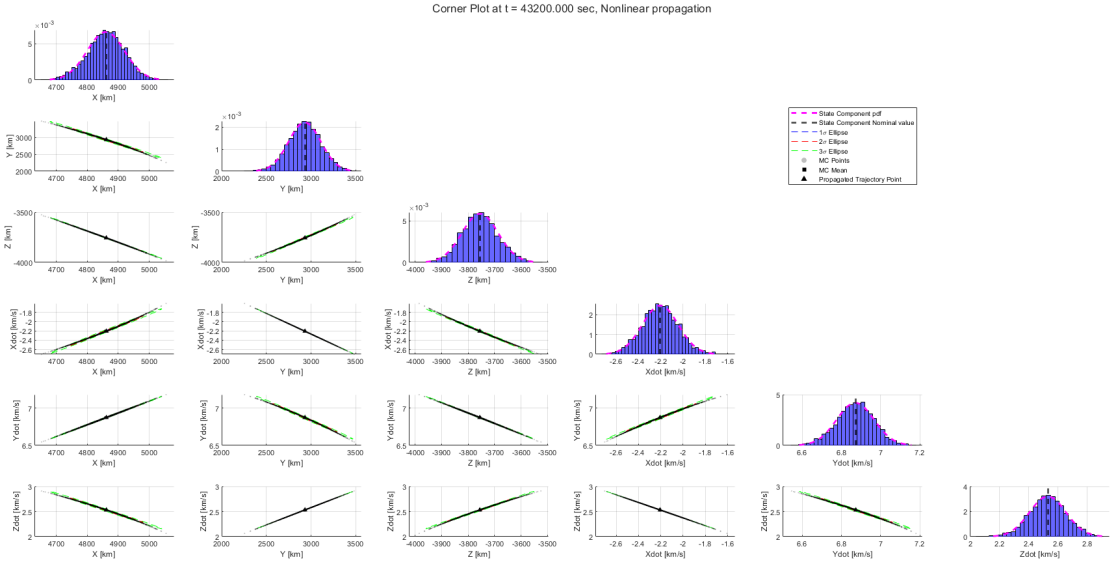
Part 1c. Analyze runs at 6 hour intervals

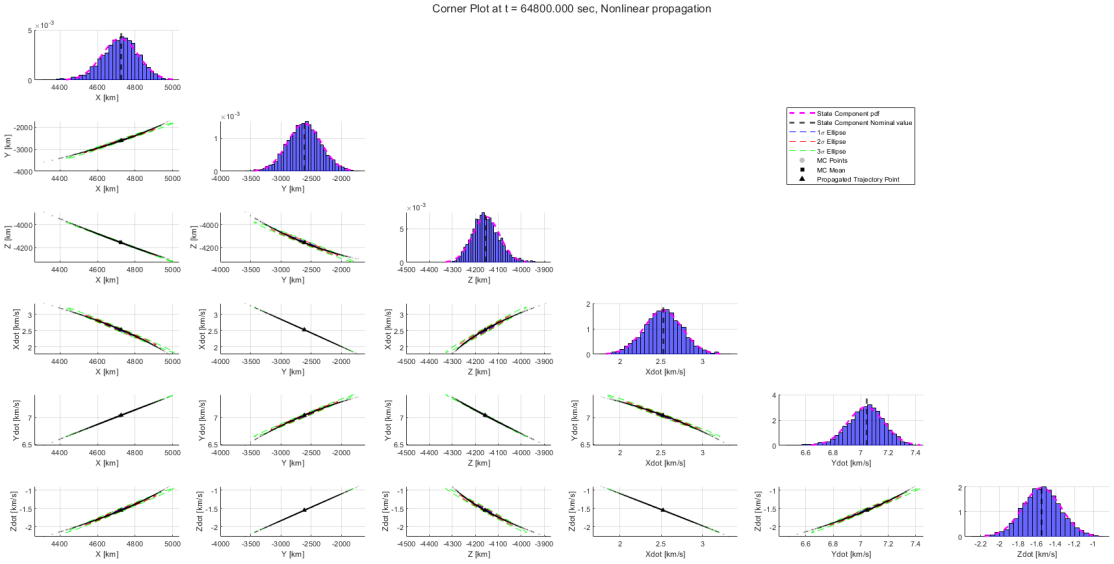
```
Analyzing Monte Carlo Data via Nonlinear Propagation...

    Summary at t = 0.000, nl prop:
        State component standard deviations:
            X: 1.021,    Y: 1.017,    Z: 1.004,    Xdot: 0.001,    Ydot:
0.001,    Zdot: 0.001
        State component means:
            X: -2390.538,    Y: 5705.687,    Z: 2806.776,    Xdot:
-5.284,    Ydot: -4.070,    Zdot: 3.774
        State component propagated nonlinear values:
            X: -2390.533,    Y: 5705.687,    Z: 2806.787,    Xdot:
-5.284,    Ydot: -4.070,    Zdot: 3.774

    Summary at t = 21600.000, nl prop:
        State component standard deviations:
            X: 74.208,    Y: 26.812,    Z: 64.723,    Xdot: 0.031,    Ydot:
0.108,    Zdot: 0.012
        State component means:
            X: 1845.015,    Y: 6500.322,    Z: -730.712,    Xdot: -5.536,
Ydot: 2.106,    Zdot: 4.852
        State component propagated nonlinear values:
            X: 1844.579,    Y: 6501.266,    Z: -730.214,    Xdot: -5.537,
```

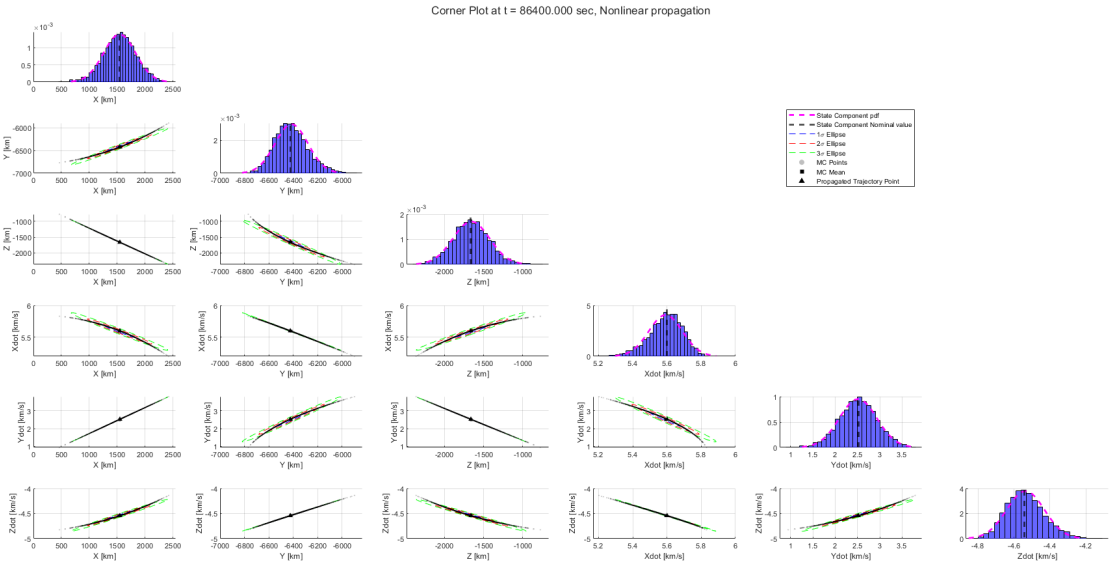
```
Ydot: 2.105,    Zdot: 4.853

    Summary at t = 43200.000, nl prop:
        State component standard deviations:
            X: 60.329,    Y: 180.343,    Z: 68.271,    Xdot: 0.161,    Ydot:
0.095,    Zdot: 0.124
        State component means:
            X: 4859.834,    Y: 2928.602,    Z: -3754.975,    Xdot:
-2.205,    Ydot: 6.871,    Zdot: 2.530
        State component propagated nonlinear values:
            X: 4861.431,    Y: 2931.385,    Z: -3756.005,    Xdot:
-2.207,    Ydot: 6.873,    Zdot: 2.532

    Summary at t = 64800.000, nl prop:
        State component standard deviations:
            X: 96.418,    Y: 277.240,    Z: 58.623,    Xdot: 0.231,    Ydot:
0.132,    Zdot: 0.203
        State component means:
            X: 4719.974,    Y: -2612.107,    Z: -4152.838,    Xdot:
2.522,    Ydot: 7.037,    Zdot: -1.550
        State component propagated nonlinear values:
            X: 4725.285,    Y: -2612.352,    Z: -4157.274,    Xdot:
2.523,    Ydot: 7.045,    Zdot: -1.550

    Summary at t = 86400.000, nl prop:
        State component standard deviations:
            X: 288.745,    Y: 134.649,    Z: 233.577,    Xdot: 0.098,
Ydot: 0.420,    Zdot: 0.106
        State component means:
            X: 1543.133,    Y: -6414.883,    Z: -1657.647,    Xdot:
5.592,    Ydot: 2.513,    Zdot: -4.536
        State component propagated nonlinear values:
            X: 1548.055,    Y: -6424.628,    Z: -1662.317,    Xdot:
5.600,    Ydot: 2.520,    Zdot: -4.543
```


Corner Plot at t = 0.000 sec, Nonlinear propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]


Corner Plot at t = 21600.000 sec, Nonlinear propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]


Corner Plot at t = 43200.000 sec, Nonlinear propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]


Corner Plot at t = 64800.000 sec, Nonlinear propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]



Part 2. Propagate Uncertainty Using LKF

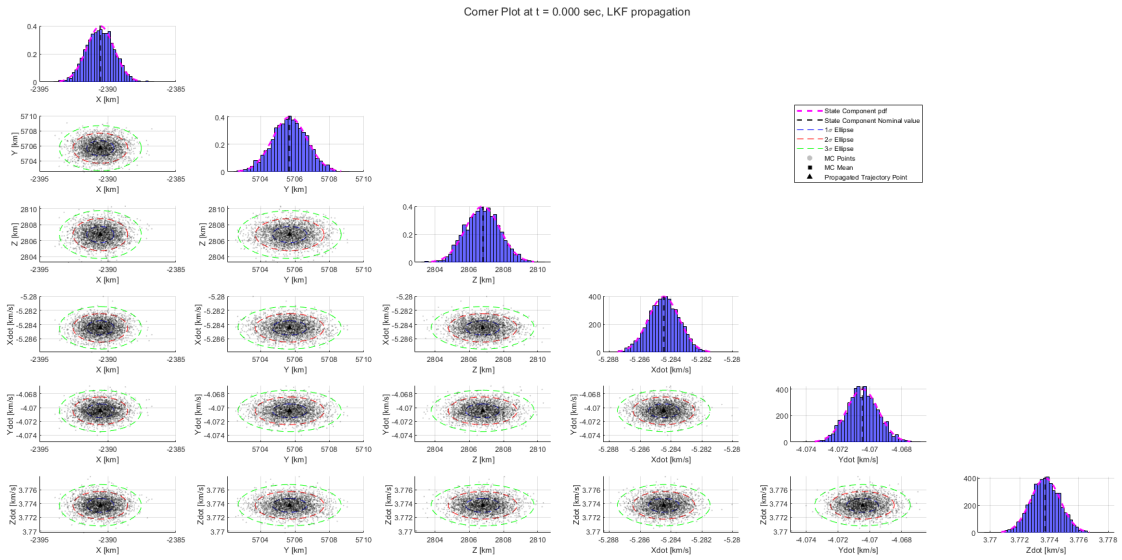
Analyzing Monte Carlo Data via LKF Propagation...

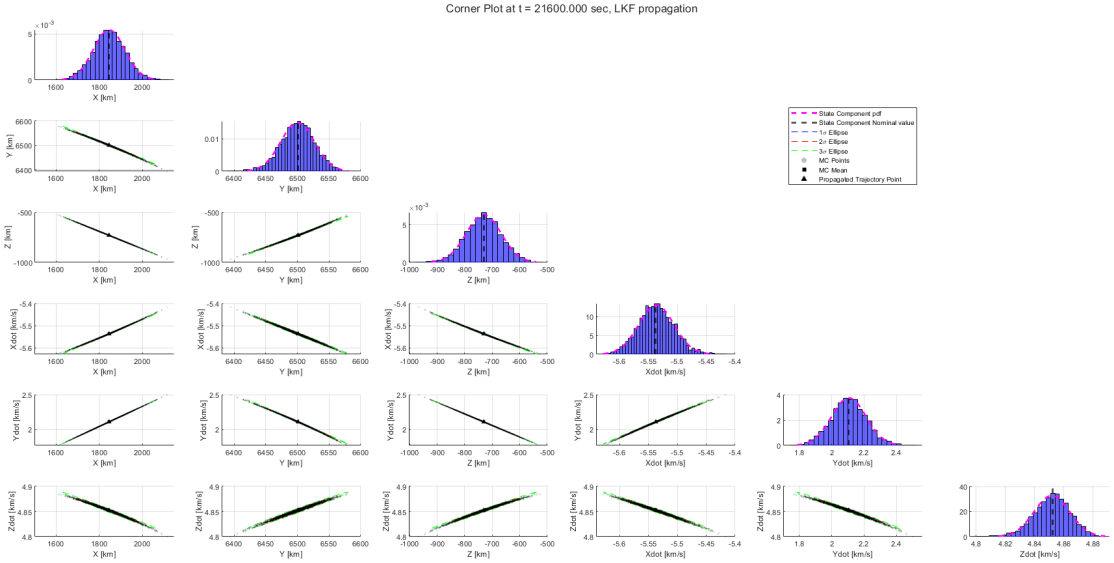
Summary at t = 0.000, LKF prop:
State component standard deviations:
X: 1.000, Y: 1.000, Z: 1.000, Xdot: 0.001, Ydot: 0.001, Zdot: 0.001
State component means:
X: -2390.538, Y: 5705.687, Z: 2806.776, Xdot: -5.284, Ydot: -4.070, Zdot: 3.774
State component propagated LKF values:
X: -2390.533, Y: 5705.687, Z: 2806.787, Xdot: -5.284, Ydot: -4.070, Zdot: 3.774

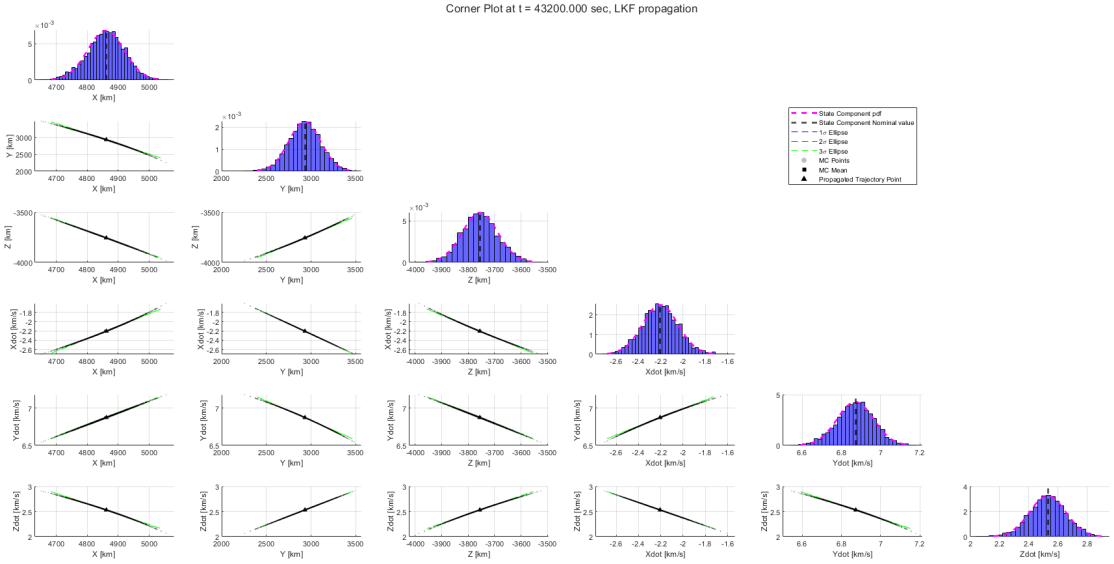
Summary at t = 21600.000, LKF prop:
State component standard deviations:
X: 73.099, Y: 26.580, Z: 63.812, Xdot: 0.030, Ydot: 0.107, Zdot: 0.012
State component means:
X: 1845.015, Y: 6500.322, Z: -730.712, Xdot: -5.536, Ydot: 2.106, Zdot: 4.852
State component propagated LKF values:
X: 1844.579, Y: 6501.266, Z: -730.214, Xdot: -5.537, Ydot: 2.105, Zdot: 4.853

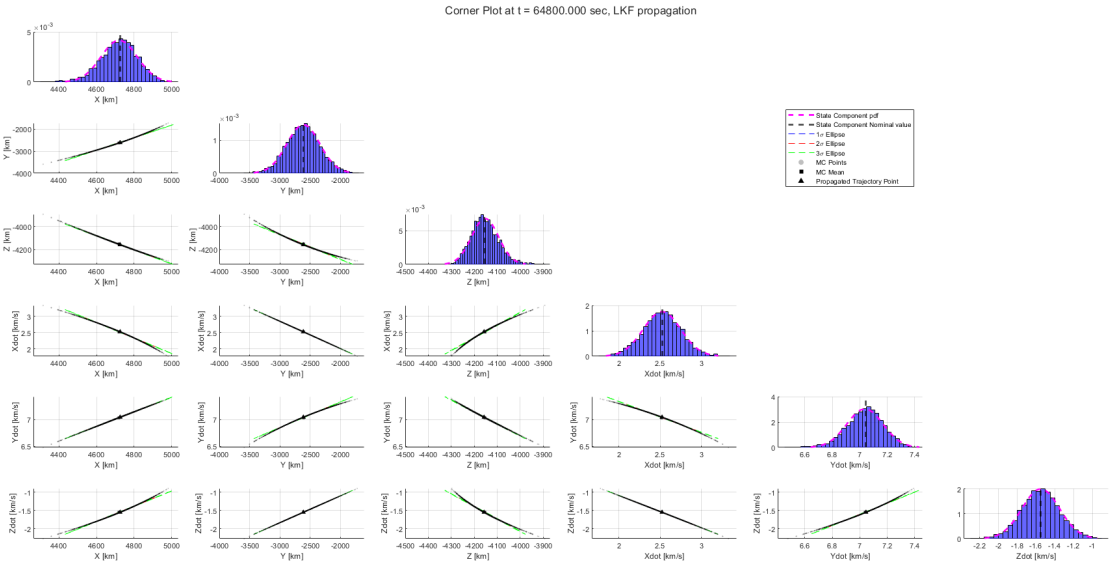
Summary at t = 43200.000, LKF prop:
State component standard deviations:
X: 58.936, Y: 178.110, Z: 66.938, Xdot: 0.159, Ydot: 0.093, Zdot: 0.123
State component means:

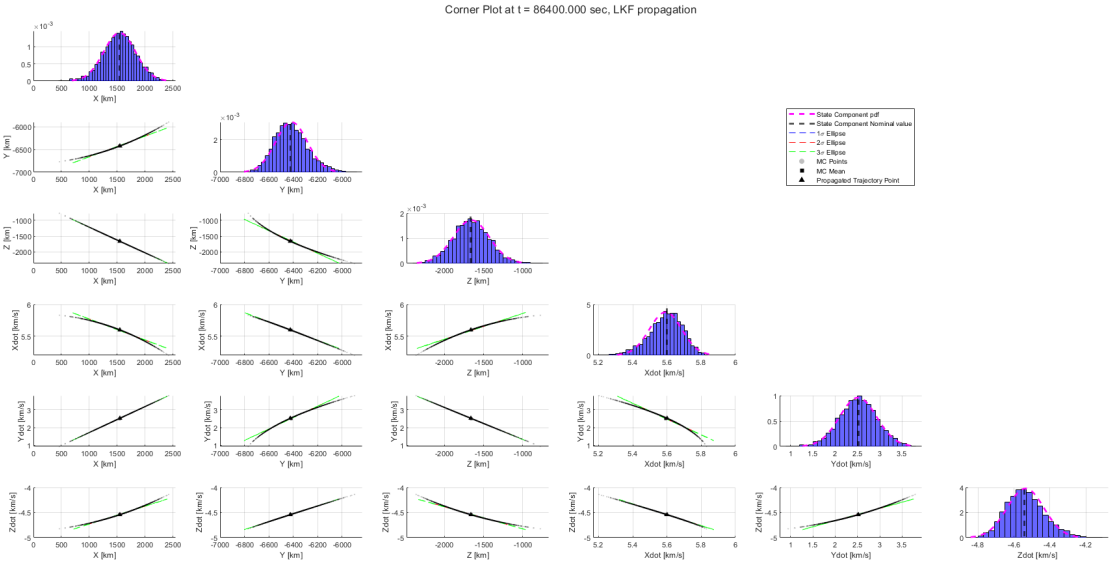
```
            X: 4859.834,    Y: 2928.602,    Z: -3754.975,    Xdot:
-2.205,    Ydot: 6.871,    Zdot: 2.530
        State component propagated LKF values:
            X: 4861.431,    Y: 2931.385,    Z: -3756.005,    Xdot:
-2.207,    Ydot: 6.873,    Zdot: 2.532

    Summary at t = 64800.000, LKF prop:
        State component standard deviations:
            X: 95.883,    Y: 272.913,    Z: 58.471,    Xdot: 0.228,    Ydot:
0.132,    Zdot: 0.200
        State component means:
            X: 4719.974,    Y: -2612.107,    Z: -4152.838,    Xdot:
2.522,    Ydot: 7.037,    Zdot: -1.550
        State component propagated LKF values:
            X: 4725.285,    Y: -2612.352,    Z: -4157.274,    Xdot:
2.523,    Ydot: 7.045,    Zdot: -1.550

    Summary at t = 86400.000, LKF prop:
        State component standard deviations:
            X: 285.511,    Y: 129.620,    Z: 231.205,    Xdot: 0.094,
Ydot: 0.415,    Zdot: 0.102
        State component means:
            X: 1543.133,    Y: -6414.883,    Z: -1657.647,    Xdot:
5.592,    Ydot: 2.513,    Zdot: -4.536
        State component propagated LKF values:
            X: 1548.055,    Y: -6424.628,    Z: -1662.317,    Xdot:
5.600,    Ydot: 2.520,    Zdot: -4.543
```




Corner Plot at t = 21600.000 sec, LKF propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]


Corner Plot at t = 43200.000 sec, LKF propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]

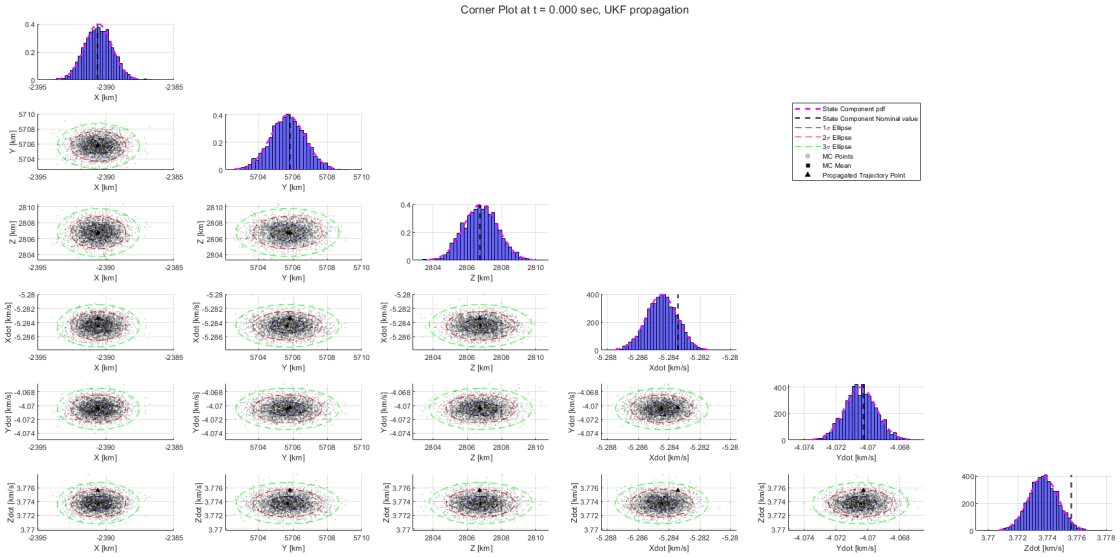


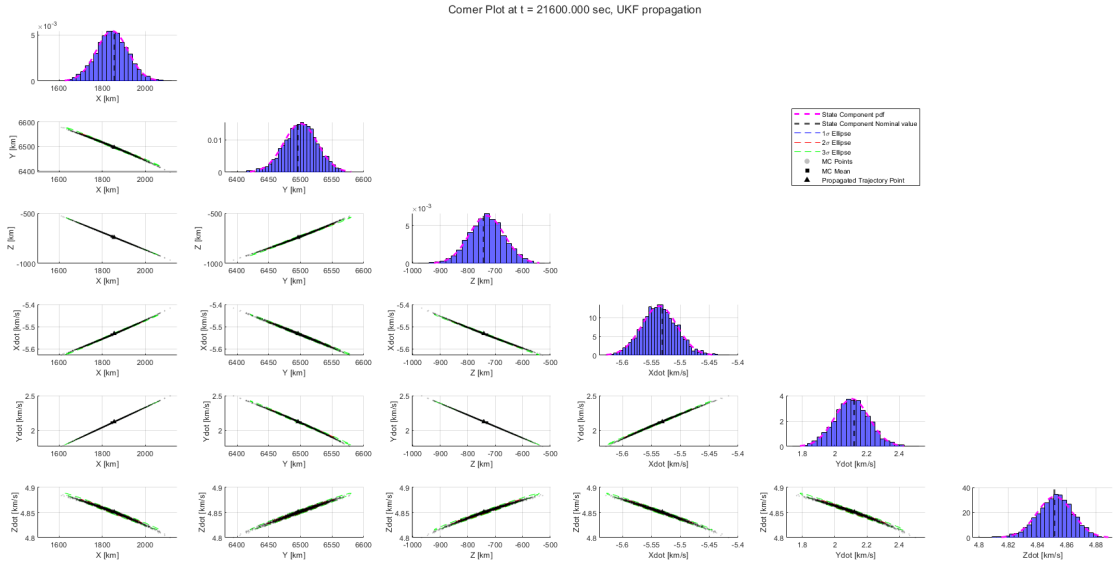


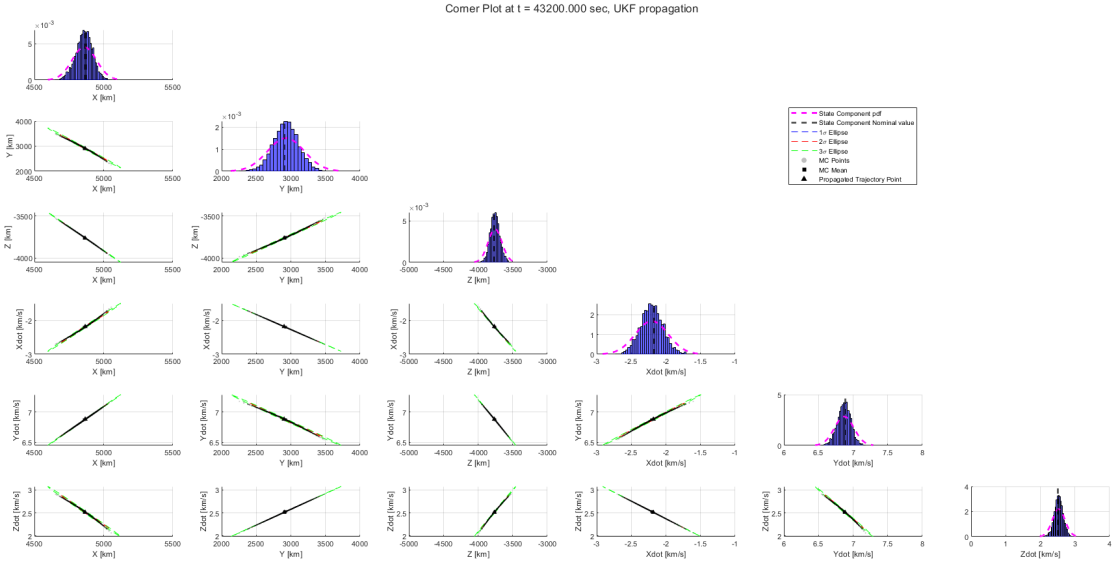
Part 3. Propagate Uncertainty using UKF

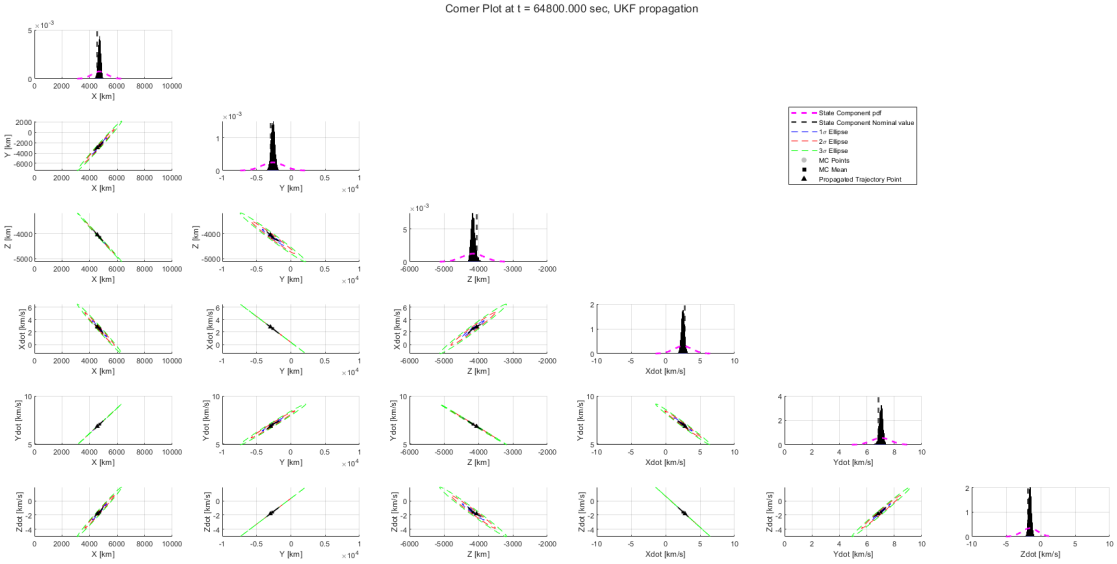
```
Analyzing Monte Carlo Data via UKF Propagation...

    Summary at t = 0.000, UKF prop:
        State component standard deviations:
            X: 1.000,    Y: 1.000,    Z: 1.000,    Xdot: 0.001,    Ydot:
0.001,    Zdot: 0.001
        State component means:
            X: -2390.538,    Y: 5705.687,    Z: 2806.776,    Xdot:
-5.284,    Ydot: -4.070,    Zdot: 3.774
        State component propagated UKF values:
```

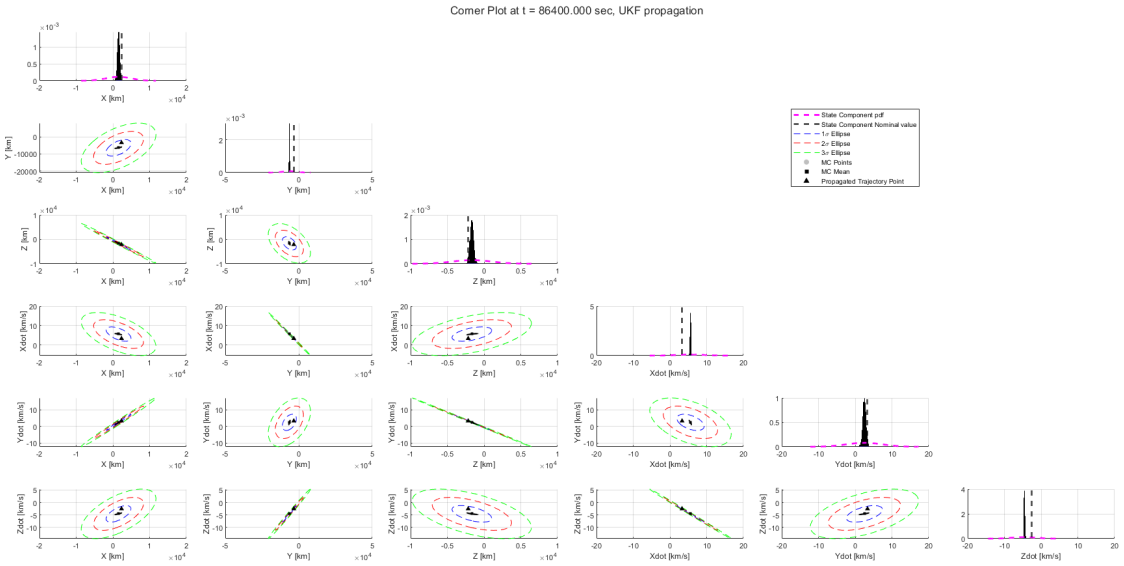
```
            X: -2390.579,    Y: 5705.820,    Z: 2806.713,    Xdot:
-5.283,    Ydot: -4.070,    Zdot: 3.776

    Summary at t = 21600.000, UKF prop:
        State component standard deviations:
            X: 73.099,    Y: 26.598,    Z: 63.812,    Xdot: 0.031,    Ydot:
0.107,    Zdot: 0.012
        State component means:
            X: 1845.015,    Y: 6500.322,    Z: -730.712,    Xdot: -5.536,
Ydot: 2.106,    Zdot: 4.852
        State component propagated UKF values:
            X: 1854.592,    Y: 6496.344,    Z: -741.007,    Xdot: -5.531,
Ydot: 2.121,    Zdot: 4.851

    Summary at t = 43200.000, UKF prop:
        State component standard deviations:
            X: 87.903,    Y: 265.693,    Z: 99.814,    Xdot: 0.237,    Ydot:
0.140,    Zdot: 0.182
        State component means:
            X: 4859.834,    Y: 2928.602,    Z: -3754.975,    Xdot:
-2.205,    Ydot: 6.871,    Zdot: 2.530
        State component propagated UKF values:
            X: 4866.352,    Y: 2902.018,    Z: -3765.397,    Xdot:
-2.181,    Ydot: 6.884,    Zdot: 2.512

    Summary at t = 64800.000, UKF prop:
        State component standard deviations:
            X: 536.841,    Y: 1595.508,    Z: 323.270,    Xdot: 1.327,
Ydot: 0.709,    Zdot: 1.160
        State component means:
            X: 4719.974,    Y: -2612.107,    Z: -4152.838,    Xdot:
2.522,    Ydot: 7.037,    Zdot: -1.550
        State component propagated UKF values:
            X: 4563.870,    Y: -2978.506,    Z: -4052.754,    Xdot:
2.822,    Ydot: 6.828,    Zdot: -1.819

    Summary at t = 86400.000, UKF prop:
        State component standard deviations:
            X: 3392.515,    Y: 4785.963,    Z: 2701.518,    Xdot: 3.699,
Ydot: 4.911,    Zdot: 3.301
        State component means:
            X: 1543.133,    Y: -6414.883,    Z: -1657.647,    Xdot:
5.592,    Ydot: 2.513,    Zdot: -4.536
        State component propagated UKF values:
            X: 2342.993,    Y: -3416.550,    Z: -2157.150,    Xdot:
3.302,    Ydot: 3.351,    Zdot: -2.522
```


Corner Plot at t = 0.000 sec, UKF propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]


Corner Plot at t = 21600.000 sec, UKF propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]


Corner Plot at t = 43200.000 sec, UKF propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]

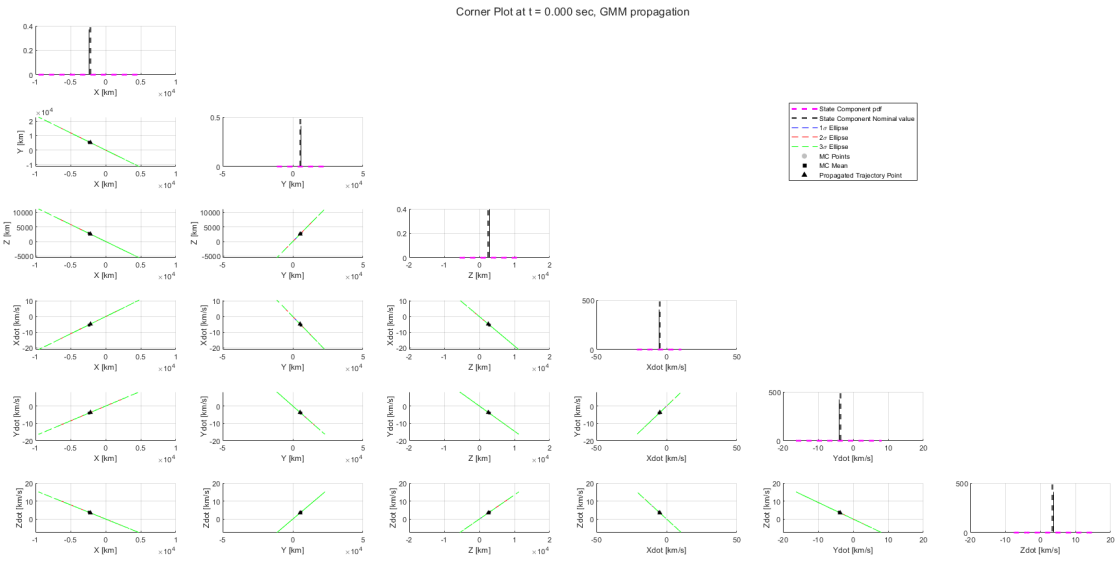

Corner Plot at t = 64800.000 sec, UKF propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]

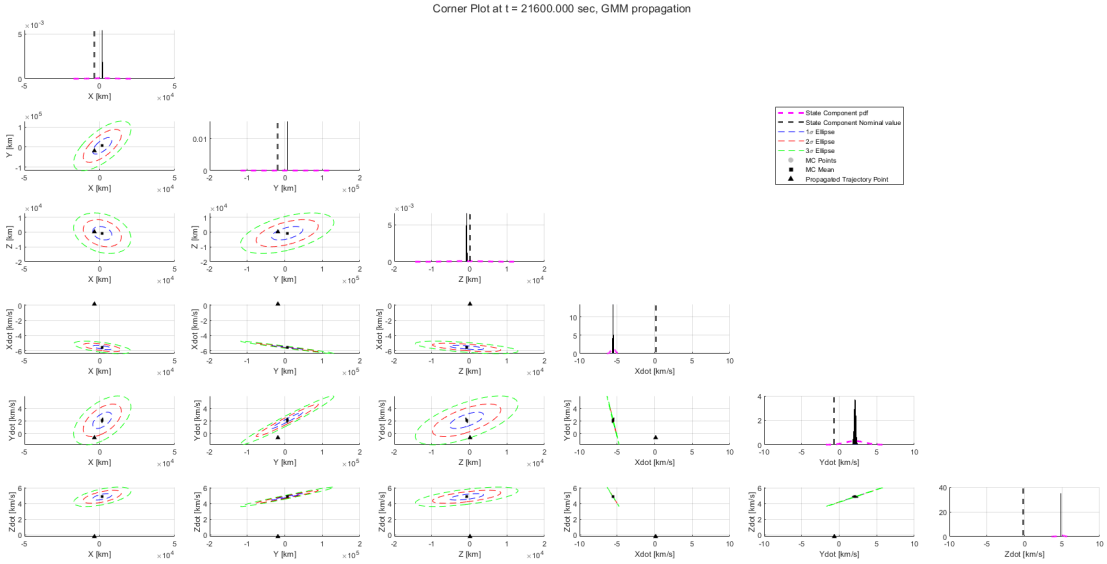


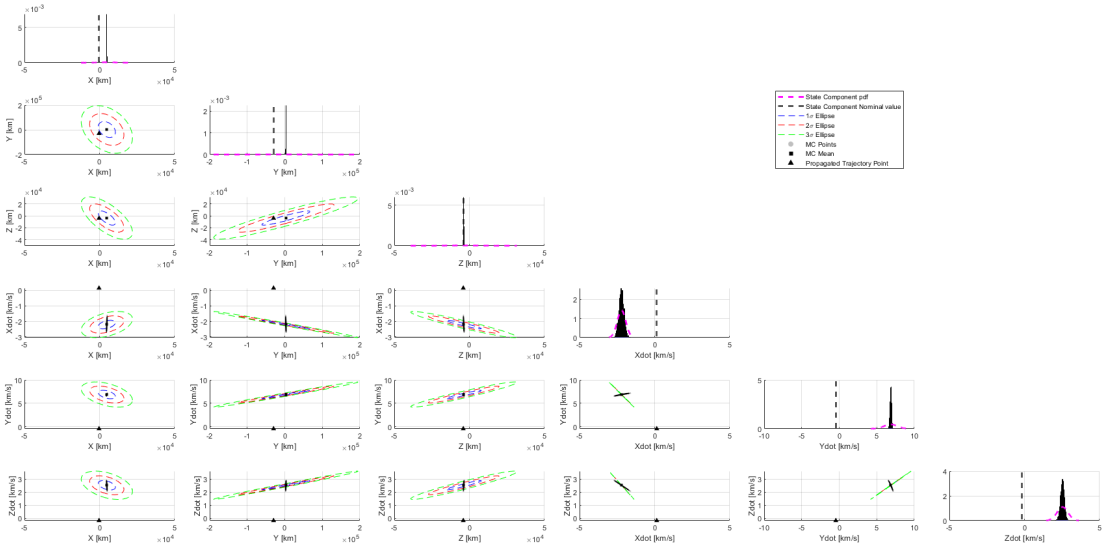
Part 4. Propagate Uncertainty using Gaussian Sums

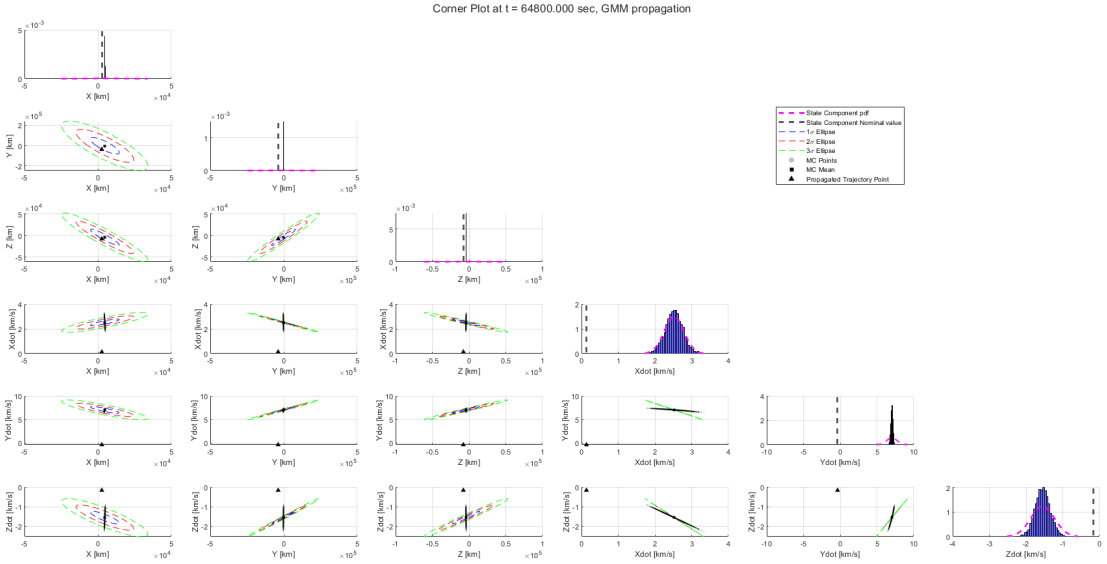
```
Analyzing Monte Carlo Data via Gaussian Sums...

    Summary at t = 0.000, GMM prop:
        State component standard deviations:
            X: 0.500,    Y: 0.500,    Z: 0.500,    Xdot: 0.001,    Ydot:
0.001,    Zdot: 0.001
        State component means:
            X: -2390.538,    Y: 5705.687,    Z: 2806.776,    Xdot:
-5.284,    Ydot: -4.070,    Zdot: 3.774
        State component propagated LKF values:
            X: -2166.462,    Y: 5170.899,    Z: 2543.583,    Xdot:
-4.788,    Ydot: -3.689,    Zdot: 3.422

    Summary at t = 21600.000, GMM prop:
        State component standard deviations:
            X: 22.917,    Y: 25.583,    Z: 15.753,    Xdot: 0.001,    Ydot:
0.002,    Zdot: 0.001
        State component means:
            X: 1845.015,    Y: 6500.322,    Z: -730.712,    Xdot: -5.536,
Ydot: 2.106,    Zdot: 4.852
        State component propagated LKF values:
            X: -3404.221,    Y: -18284.143,    Z: 170.016,    Xdot:
0.139,    Ydot: -0.651,    Zdot: -0.209

    Summary at t = 43200.000, GMM prop:
        State component standard deviations:
            X: 41.927,    Y: 73.891,    Z: 24.937,    Xdot: 0.001,    Ydot:
```

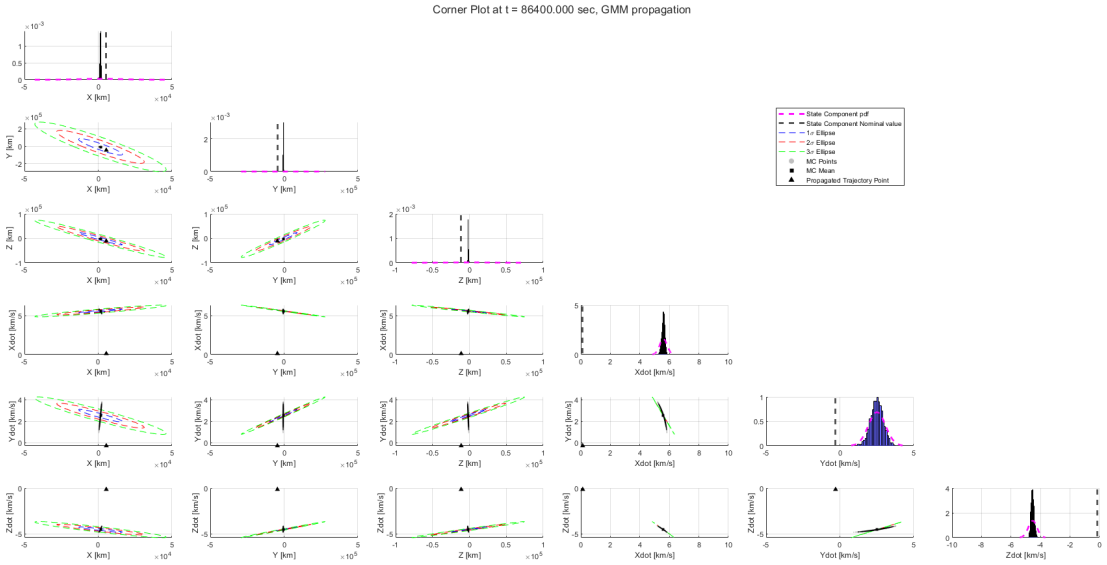
```
0.003,    Zdot: 0.000
        State component means:
            X: 4859.834,    Y: 2928.602,    Z: -3754.975,    Xdot:
-2.205,    Ydot: 6.871,    Zdot: 2.530
        State component propagated LKF values:
            X: -326.008,    Y: -29866.449,    Z: -4052.948,    Xdot:
0.142,    Ydot: -0.452,    Zdot: -0.183

    Summary at t = 64800.000, GMM prop:
        State component standard deviations:
            X: 56.677,    Y: 133.267,    Z: 29.215,    Xdot: 0.001,    Ydot:
0.003,    Zdot: 0.000
        State component means:
            X: 4719.974,    Y: -2612.107,    Z: -4152.838,    Xdot:
2.522,    Ydot: 7.037,    Zdot: -1.550
        State component propagated LKF values:
            X: 2660.748,    Y: -38481.522,    Z: -7770.541,    Xdot:
0.134,    Ydot: -0.354,    Zdot: -0.162

    Summary at t = 86400.000, GMM prop:
        State component standard deviations:
            X: 67.763,    Y: 201.159,    Z: 29.660,    Xdot: 0.000,    Ydot:
0.003,    Zdot: 0.000
        State component means:
            X: 1543.133,    Y: -6414.883,    Z: -1657.647,    Xdot:
5.592,    Ydot: 2.513,    Zdot: -4.536
        State component propagated LKF values:
            X: 5474.022,    Y: -45415.236,    Z: -11101.155,    Xdot:
0.126,    Ydot: -0.291,    Zdot: -0.147
```




Corner Plot at t = 21600.000 sec, GMM propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]


Corner Plot at t = 43200.000 sec, GMM propagation
State Component pdf
State Component Nominal value
1σ Ellipse
2σ Ellipse
3σ Ellipse
MC Points
MC Mean
Propagated Trajectory Point
X [km]
Y [km]
Z [km]
Xdot [km/s]
Ydot [km/s]
Zdot [km/s]





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