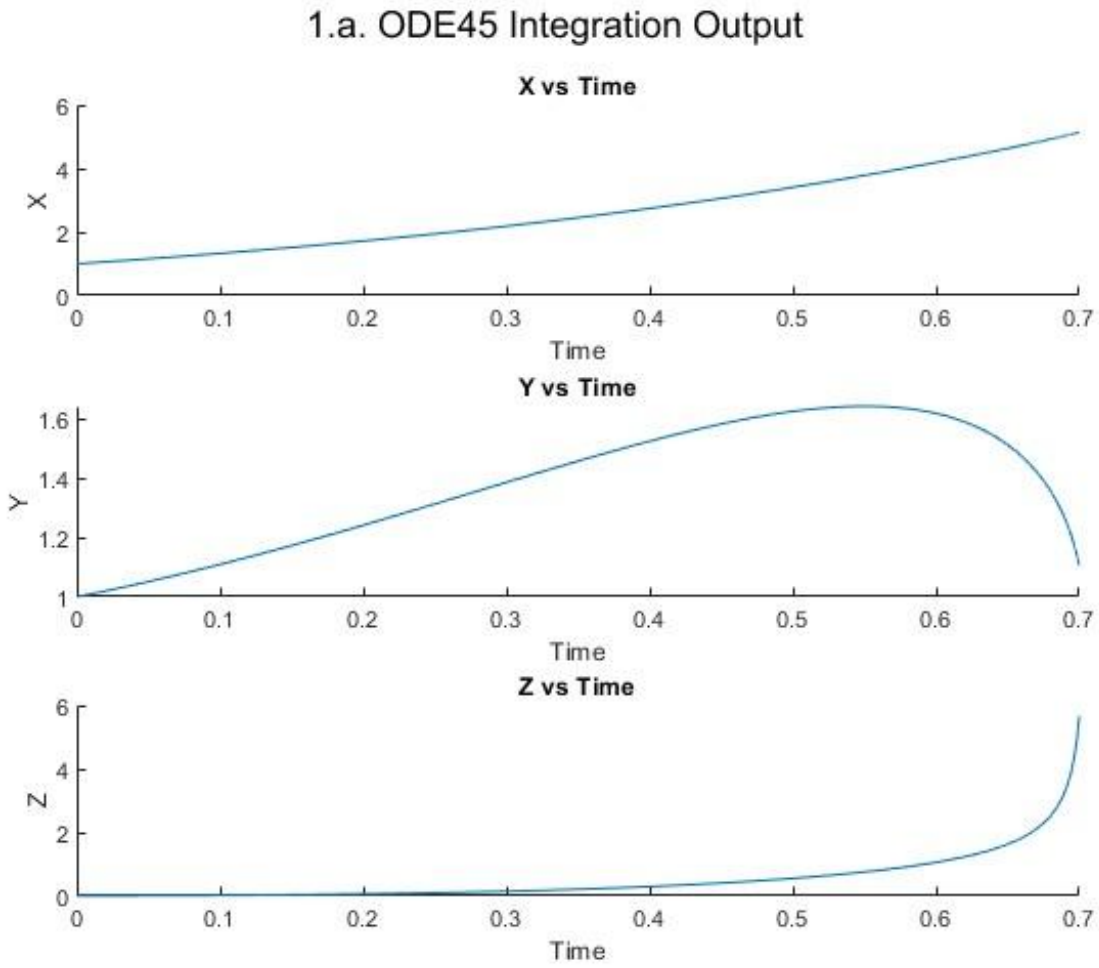
# ASEN 3128 Lab 1 Assignment

Section 011 - Ian Faber, Ashton Miner, Teegan Oatley, Chaney Sullivan

**Problem 1.** Nonlinear dynamic system

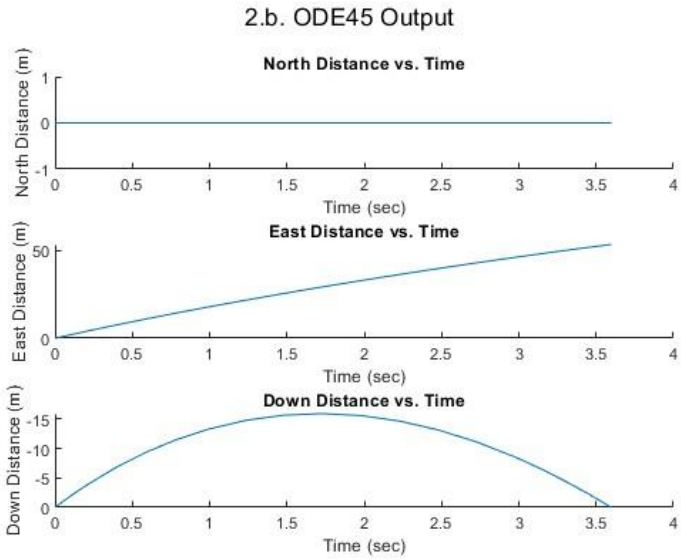


**Problem 2.** Golf ball trajectory simulation with drag and wind

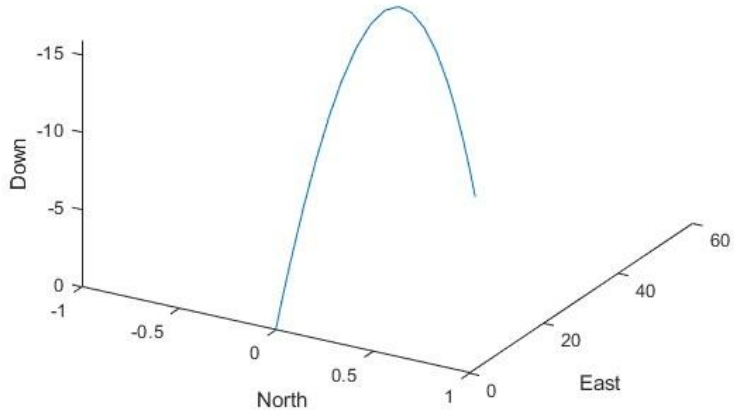
**2.a.** ObjectEOM function

```
function xdot = ObjectEOM(t,x,rho,Cd,A,m,g,wind)
%
% Inputs:   t = Time vector (sec)
%           x = State vector (m, m/s)
%             = [x; y; z; vx; vy; vz]
%           rho = Air density (kg/m^3)
%           Cd = Coefficient of drag
%           A = Cross-sectional area of golf ball relative to motion (m^2)
%           m = Mass of golf ball (kg)
%           g = Freefall acceleration due to gravity (m/s^2)
%           wind = Wind vector (m/s)
%                = [windx; windy; windz]
%
% Outputs: xdot = rate of change of state vector (m/s, m/s^2)
%               = [vx; vy; vz; ax; ay; az]
%
% Methodology: Function used by ode45 to calculate golf ball trajectory,
%              problem 2 part a
% Extract inertial velocity and calculate wind-relative velocity
Ve = x(4:6);
V = Ve - wind;
% Position rate of change is inertial velocity
pdot = Ve;
% Calculate speed and the wind-relative velocity unit vector
mag = norm(V);
unitV = V/mag;
% Calculate drag and weight forces
Fdrag = (-0.5*rho*(mag^2)*Cd*A) * unitV;
Fgrav = m*g;
% Combine forces into total force vector
F = Fdrag + Fgrav;
% Velocity rate of change is a = F/m (F = ma)
vdot = F/m;
% Format output vector
xdot = [pdot; vdot];
end
```

**2.b.** Base trajectory, no wind




**2.b. Ball Trajectory with No Wind**




This trajectory makes sense for multiple reasons. The first is that since there is no North wind in this simulation, there should be no displacement in the North axis, which is clearly shown to be true in the North Distance vs. Time plot. The second reason is that we can see a slight curvature in the East displacement on the East Distance vs. Time plot, which indicates the presence of drag. Without drag, the displacement curve on the East axis would be a straight line. Finally, given the common aircraft body frame of North-East-Down, we should expect the "down" displacement to get more negative ("gain more height") up to a maximum, then decrease until reaching 0 (the ball hitting the ground) like any other ballistic trajectory. This is exactly what is shown in the Down Distance vs. Time plot. Furthermore, the plot also shows that it takes slightly longer for the ball to fall than rise, meaning drag has slowed the ball down.

**2.c.** Landing location sensitivity to north wind

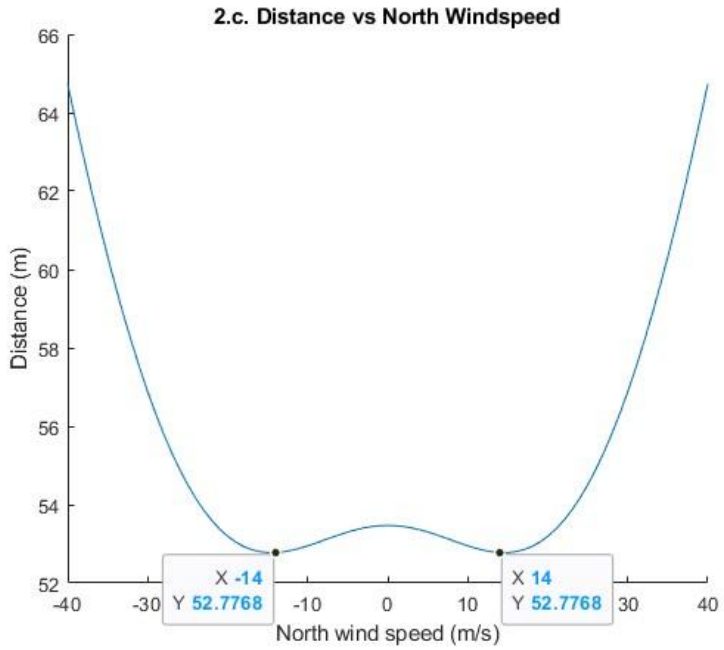



**Figure 2.c.1. Plot of the relationship between North wind speed and the distance from the origin to the landing location of the golf ball**

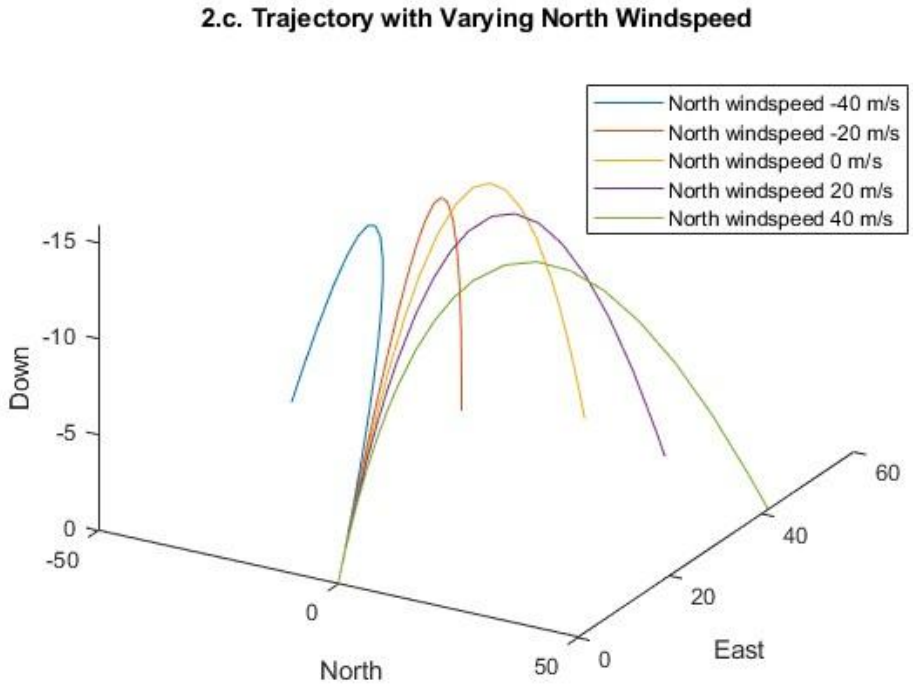



**Figure 2.c.2. 3-D plot of the trajectory of the golf ball with a subset (every 20 m/s) of the tested North windspeeds**

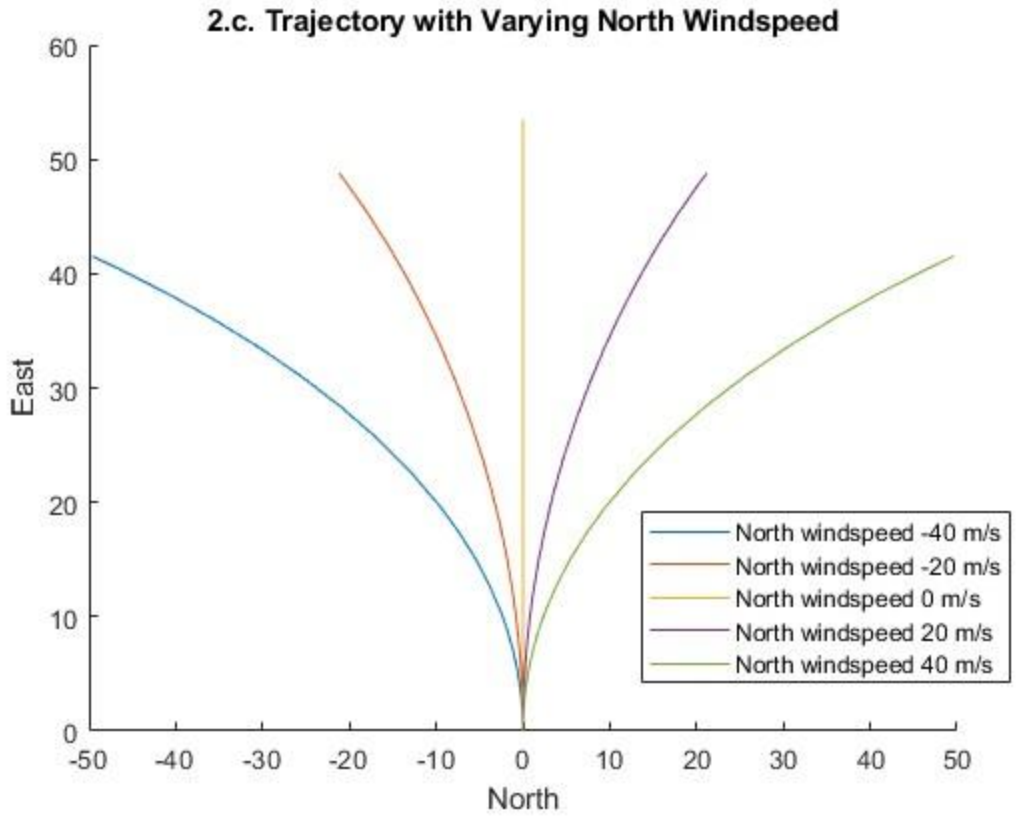



**Figure 2.c.3. Top view of the trajectory of the golf ball under a subset (every 20 m/s) of the tested North windspeeds**

Based on Figure 2.c.1, we concluded that the landing location of the golf ball is very sensitive to windspeeds higher than ~3 m/s. After that, the distance from the origin decreases to a minimum at a windspeed of ~14 m/s, then increases rapidly with any windspeed greater than 14 m/s. In terms of coordinates, Figure 2.c.3 shows that any amount of wind will decrease the magnitude of the East coordinate of the landing location and increase the magnitude of the North coordinate of the landing location. The sign of the North coordinate matches the sign of the windspeed vector's North component, and the sign of the East coordinate is always positive. Wind will also cause the maximum height of the golf ball to decrease, as seen in Figure 2.c.2, but that has little to no effect on the ball's final landing location.

**2.d.** Limited kinetic energy, ball mass variation

To examine the effect different golf ball masses had on the landing location, we first needed to calculate the amount of kinetic energy available to the golf player. To do this, we found the speed of the golf ball in part 2.a, then substituted that value into the equation for kinetic energy.

$$V_E = \begin{bmatrix} 0 \\ 20 \\ -20 \end{bmatrix}_E$$
$$\|V\| = \sqrt{0^2 + 20^2 + (-20)^2} = 20\sqrt{2}\, m/s$$
$$KE = \frac{1}{2} m_{ball} \|V\|^2 = \frac{1}{2}(0.02)\left(20\sqrt{2}\right)^2 = 12\, J$$

The next thing we needed to do was derive an equation for speed in relation to the mass of the ball, given the constant kinetic energy of 12 J.

$$KE = \frac{1}{2} m_{ball} \|V\|^2$$
$$\frac{2 \cdot KE}{m_{ball}} = \|V\|^2$$
$$\|V\| = \sqrt{\frac{2 \cdot KE}{m_{ball}}}$$
$$= \sqrt{\frac{24}{m_{ball}}}$$

With this equation relating the speed of the ball to any given ball mass, we were able to simulate the trajectory of the golf ball with a set of different ball masses and obtain a relationship between the distance from the origin to the landing location and the mass of the golf ball, given a limited kinetic energy.

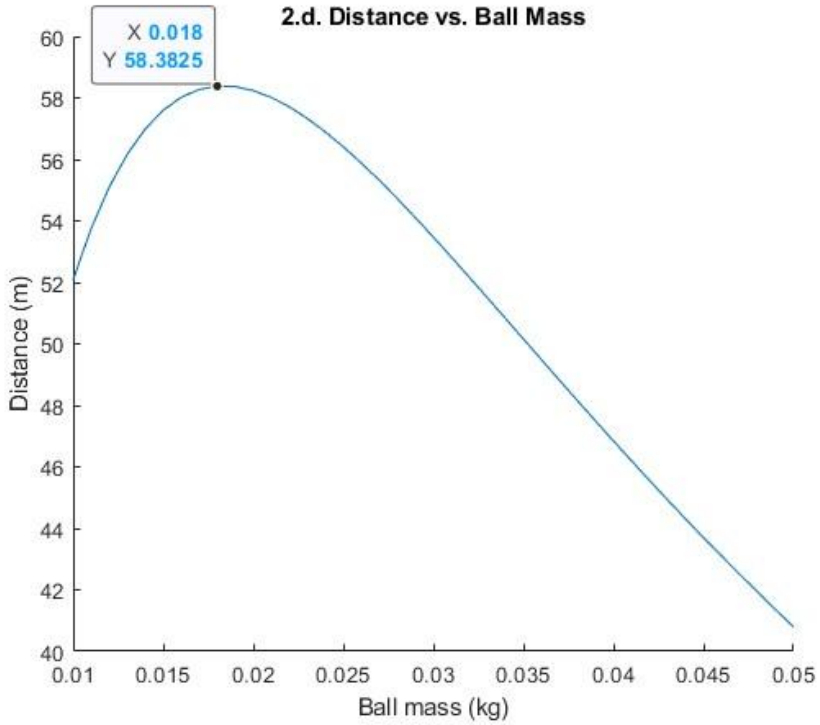



**Figure 2.d.1. Plot of the relationship between golf ball mass and distance from the origin to the landing location of the golf ball**

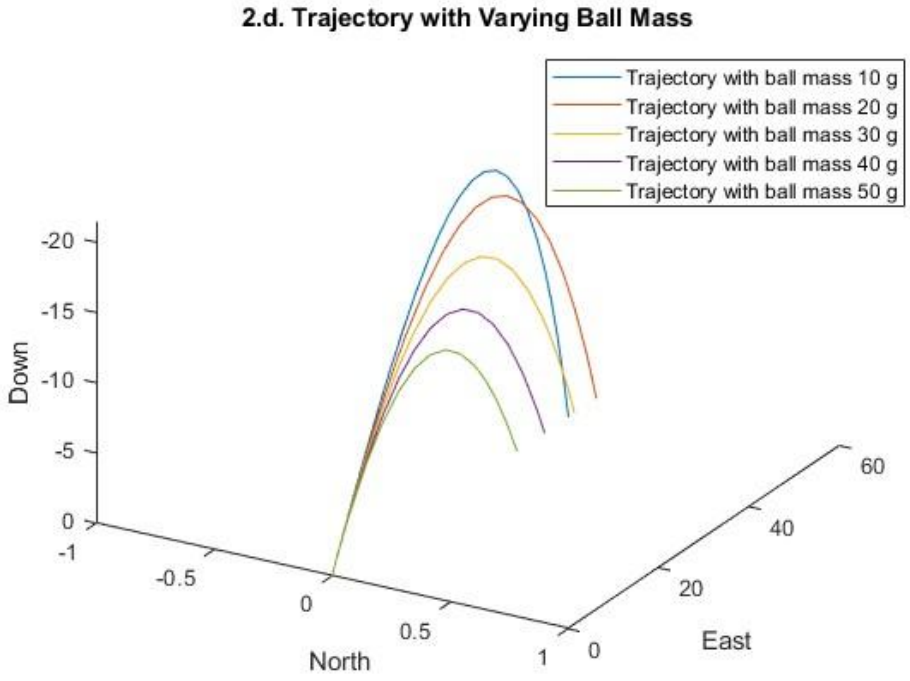



**Figure 2.d.2. 3-D plot of the trajectory of the golf ball with a subset (every 10 g) of the tested golf ball masses**

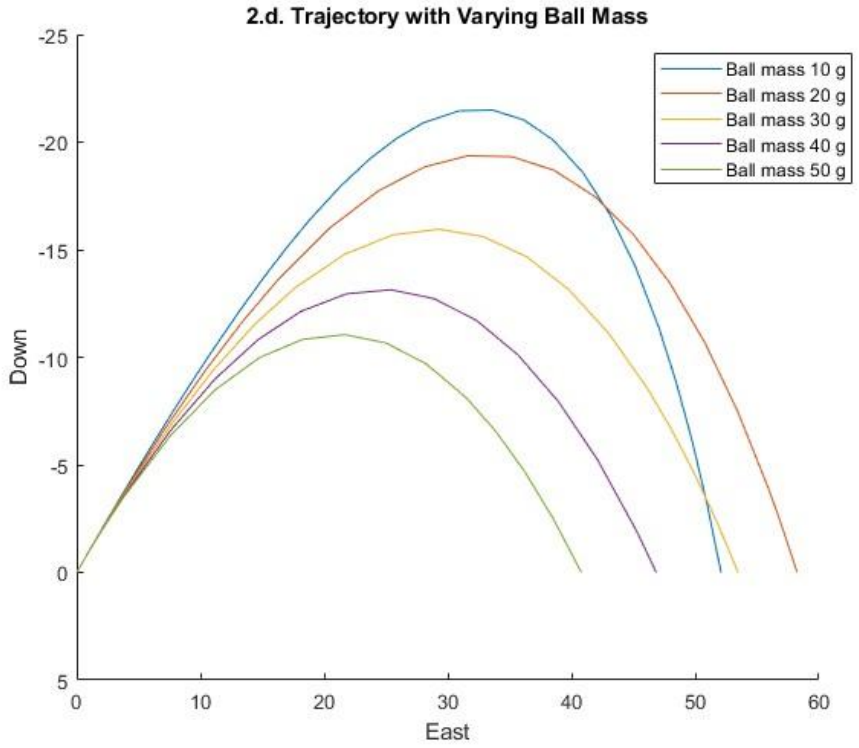



**Figure 2.d.3. Side view of the trajectory of the golf ball with a subset (every 10 g) of the tested golf ball masses**

Based on Figure 2.d.1, we concluded that there was an ideal golf ball mass at ~18 g, before and after which the distance from the origin to the landing location decreased. Thus, we concluded that a longer distance would be achieved by using a golf ball that was lighter than the original mass of 30 g, down to ~11 g. Figure 2.d.3 shows that a ball mass of 10 g does slightly worse than the original mass of 30 g, and a ball mass of 20 g outperforms all other masses plotted. This confirms our finding of an “ideal” ball mass for a given limited kinetic energy.

## Appendix

See attached PDFs for all code

# Table of Contents



```
% Ian Faber, Ashton Miner, Teegan Oatley, Chaney Sullivan
% ASEN 3128-011
% ASEN3128Lab1Problem1.m
% Created: 8/23/22
```

# Housekeeping

```
clc; clear; close all;
```

# ODE45 Setup

```
% Initial conditions
X0 = [1; 1; 0.01];

% Integration time span
tspan = [0 .7];

% Tolerance settings
setup = odeset('RelTol', 1e-12, 'AbsTol', 1e-12);
```

# ODE45 Call

```
[t, state] = ode45(@(t, state)EOM(t, state), tspan, X0, setup);
```

# Plotting

```
figure(1)
sgtitle("1.a. ODE45 Integration Output")
subplot(3,1,1)
hold on
title("X vs Time")
xlabel("Time")
ylabel("X")
plot(t, state(:,1))
hold off

subplot(3,1,2)
hold on
title("Y vs Time")
xlabel("Time")
```

```
ylabel("Y")
plot(t, state(:,2))
hold off

subplot(3,1,3)
hold on
title("Z vs Time")
xlabel("Time")
ylabel("Z")
plot(t, state(:,3))
hold off
```

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```
% Ian Faber, Ashton Miner, Teegan Oatley, Chaney Sullivan
% ASEN 3128-011
% EOM.m
% Created: 8/23/22

function [dX] = EOM(t,X)
%
% Inputs:   t = Time vector
%           X = State vector
%
% Outputs:  dX = change of state vector
%
% Methodology: Rate of change equations to be used in ode45 call

x = X(1);
y = X(2);
z = X(3);

xdot = x + 2*y + z;
ydot = x - 5*z;
zdot = x*y - y^2 + 3*(z^3);

dX = [xdot;ydot;zdot];

end
```

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```
% Ian Faber, Ashton Miner, Teegan Oatley, Chaney Sullivan
% ASEN 3128-011
% ASEN3128Lab1Problem2.m
% Created: 8/23/22
```

# Housekeeping

```
clc; clear; close all;
```

# Setup

```
mass = 0.03; % kg
diam = 0.03; % m
Cd = 0.6;
rho = 1.275; % kg/m^3
g = [0;0;9.81]; % m/s^2

A = pi*(diam/2)^2; % m^2

X0 = [0;0;0;0;20;-20]; % Initial conditions
tspan = [0 10]; % Integration time span
wind = [0;0;0]; % Wind vector

options = odeset('Events', @detectGround);
```

# Trajectory Calculation and plottinh, problem 2 part b

```
[t, state] = ode45(@(t, state)ObjectEOM(t, state, rho, Cd, A, mass, g, wind),
 tspan, X0, options);

figure(1)
sgtitle("2.b. ODE45 Output")
subplot(3,1,1)
hold on
title("North Distance vs. Time")
xlabel("Time (sec)")
ylabel("North Distance (m)")
```

```
plot(t, state(:,1));
hold off

subplot(3,1,2)
hold on
title("East Distance vs. Time")
xlabel("Time (sec)")
ylabel("East Distance (m)")
plot(t, state(:,2));
hold off

subplot(3,1,3)
hold on
title("Down Distance vs. Time")
xlabel("Time (sec)")
ylabel("Down Distance (m)")
set(gca, 'YDir', 'reverse')
plot(t, state(:,3));
hold off


figure(2)
hold on
title("2.b. Ball Trajectory with No Wind")
xlabel("North")
ylabel("East")
zlabel("Down")
set(gca, 'ZDir', 'reverse')
view([30 35])
plot3(state(:,1), state(:,2), state(:,3))
hold off

% Calculate distance from the origin with no wind for later comparison
distance0 = norm([state(end,1), state(end,2)]);
```

# Wind Sensitivity Analysis, problem 2 part c

```
% Test vector of north windspeeds
testWind = -40:1:40;

% Plot label index initialization
kk = 0;

figure(3)
hold on
title("2.c. Trajectory with Varying North Windspeed")
xlabel("North")
ylabel("East")
zlabel("Down")
set(gca, 'ZDir', 'reverse')
view([30 35])

for k = 1:length(testWind)
```

```
    % Update wind vector with values from test vector
    wind = [testWind(k); 0; 0];

    % Calculate trajectory with updated wind vector
    [t, state] = ode45(@(t, state)ObjectEOM(t, state, rho, Cd, A, mass, g,
 wind), tspan, X0, options);

    % Calculate distance from origin
    distance(k) = norm([state(end, 1), state(end, 2)]);

    % Plot every 20th trajectory (increments of 20 m/s, starting at -40)
    % with the updated wind vector
    if mod(k-1,20) == 0
        kk = kk + 1;
        plot3(state(:,1), state(:,2), state(:,3))
        % Create label vector for dynamic legend
        label(kk) = sprintf("North windspeed %.0f m/s", testWind(k));
    end

end
% Create legend
legend(label, 'Location', 'best')
hold off

figure(4)
hold on
title("2.c. Distance vs North Windspeed")
xlabel("North wind speed (m/s)")
ylabel("Distance (m)")
plot(testWind, distance)
hold off

% Calculate North axis deflection vector
deflection = distance - distance0;

% figure(5)
% hold on
% title("Deflection vs North Windspeed")
% xlabel("North wind speed (m/s)")
% ylabel("Deflection in North (m)")
% plot(testWind, deflection)
% hold off
```

# Kinetic Energy Limitation Analysis, problem 2 part d

```
% Extract initial velocity vector and calculate the corresponding limited
% kinetic energy
V = X0(4:6);
speed = norm(V);
unitV = V/speed;
kineticE = 0.5*mass*speed^2
```

```
% Set up ball mass vector in kg, reset wind vector and label index
ballMass = (10:1:50)/1000; % g to kg
wind = [0; 0; 0];
kk = 0;

figure(6)
hold on
title("2.d. Trajectory with Varying Ball Mass")
xlabel("North")
ylabel("East")
zlabel("Down")
set(gca, 'ZDir', 'reverse')
view([30 35])

for k = 1:length(ballMass)
    % Calculate initial ball speed based on available kinetic energy and
    % a mass from the mass vector
    ballSpeed = sqrt((2*kineticE)/ballMass(k));

    % Calculate initial velocity vector
    ballV = ballSpeed*unitV;

    % Update initial conditions vector
    X0 = [0;0;0;ballV];

    % Calculate trajectory with updated ball mass and initial velocity
    [t, state] = ode45(@(t, state)ObjectEOM(t, state, rho, Cd, A, ballMass(k),
 g, wind), tspan, X0, options);

    % Calculate distance from origin for each mass in the mass vector
    distanceMass(k) = norm([state(end, 1), state(end, 2)]);

    % Plot every 10th trajectory (increments of 10 g, starting at 10) with
    % the updated ball mass
    if mod(k-1, 10) == 0
        kk = kk + 1;
        plot3(state(:,1), state(:,2), state(:,3));
        % Create label vector for dynamic legend
        label(kk) = sprintf("Ball mass %.0f g", ballMass(k)*1000);
    end
end

% Create legend
legend(label, 'Location', 'best')
hold off

figure(7)
hold on
title("2.d. Distance vs. Ball Mass")
xlabel("Ball mass (kg)")
ylabel("Distance (m)")
plot(ballMass, distanceMass)
hold off
```

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```
% Ian Faber, Ashton Miner, Teegan Oatley, Chaney Sullivan
% ASEN 3128-011
% ObjectEOM.m
% Created: 8/23/22

function xdot = ObjectEOM(t,x,rho,Cd,A,m,g,wind)
%
% Inputs:   t = Time vector (sec)
%           x = State vector (m, m/s)
%             = [x; y; z; vx; vy; vz]
%           rho = Air density (kg/m^3)
%           Cd = Coefficient of drag
%           A = Cross-sectional area of golf ball relative to motion (m^2)
%           m = Mass of golf ball (kg)
%           g = Freefall acceleration due to gravity (m/s^2)
%           wind = Wind vector (m/s)
%                = [windx; windy; windz]
%
% Outputs: xdot = rate of change of state vector (m/s, m/s^2)
%               = [vx; vy; vz; ax; ay; az]
%
% Methodology: Function used by ode45 to calculate golf ball trajectory,
%              problem 2 part a

% Extract inertial velocity and calculate wind-relative velocity
Ve = x(4:6);
V = Ve - wind;

% Position rate of change is inertial velocity
pdot = Ve;

% Calculate speed and the wind-relative velocity unit vector
mag = norm(V);
unitV = V/mag;

% Calculate drag and weight forces
Fdrag = (-0.5*rho*(mag^2)*Cd*A) * unitV;
Fgrav = m*g;

% Combine forces into total force vector
F = Fdrag + Fgrav;

% Velocity rate of change is a = F/m (F = ma)
vdot = F/m;

% Format output vector
xdot = [pdot; vdot];

end
```

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```
% Ian Faber, Ashton Miner, Teegan Oatley, Chaney Sullivan
% ASEN 3128-011
% detectGround.m
% Created: 8/30/22

function [value, isterminal, direction] = detectGround(t, X)
%
%   Inputs:      t = Time vector
%                X = State vector
%                  = [x; y; z; vx; vy; vz]
%
%   Outputs:     value = Value of state vector z component
%                isterminal = Boolean used to stop or continue integration
%                direction = Indicator for detecting when a value occurs
%
%   Methodology: Function passed to odeset that detects when to stop
%                integration based on the value of a specified state vector
%                component

% Interested in detecting when z goes to 0
z = X(3);

value = z; % Look at the value of z
isterminal = 1; % 1: stop integration once z hits 0
direction = 1; % 1: Trigger when value goes to 0 from negative to positive
end
```

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