Aerodynamics Computational Assignment #4: Compressible Aerodynamics

Assigned Date: April 11, 2023
Due Date: April 28, 2023

Collaboration Policy:

Collaboration is permitted on the computational labs. You may discuss the means and methods for formulating and solving problems and even compare answers, but you are not free to copy someone else's work. *Copying material from any resource (including solutions manuals) and submitting it as one's own is considered plagiarism and is an Honor Code violation.*

Matlab Code Policy:

Computational codes must be written individually and are expected to be written in MATLAB. If you have collaborated with others while writing your code be sure to acknowledge them in the header of your code, otherwise you may receive a zero for plagiarism. All code files required to successfully run the computational assignment driver script along with a pdf of your code and its execution (i.e. printed comments and figures) should be submitted via the course website by 11:59pm on the due date. Code files will not be accepted after the given due date.

Reflection Questions:

In this assignment, there are multiple reflection questions. These reflection questions are provided to help you review the functionality of your code, help you analyze and understand your results, and to test your understanding of the concepts being studied.

Learning Outcomes:

1. Understand the oblique shock-wave ($\theta - \beta - M$) relation and how to use it to predict oblique shock-wave angles.
2. Understand how to analyze flow properties through normal shock-waves, oblique shock-waves, and expansion fans.
3. Practice using the Gas Dynamics Functions in the MATLAB Aerospace Engineering Toolbox to explicitly compute compressible flow properties.
4. Practice implementing Shock-Expansion Theory for predicting the aerodynamic performance of generic thin diamond airfoil sections in supersonic flow.
5. Observe the practical differences between aerodynamic performance predictions from Linearized Supersonic Flow and that from Shock-Expansion Theory.

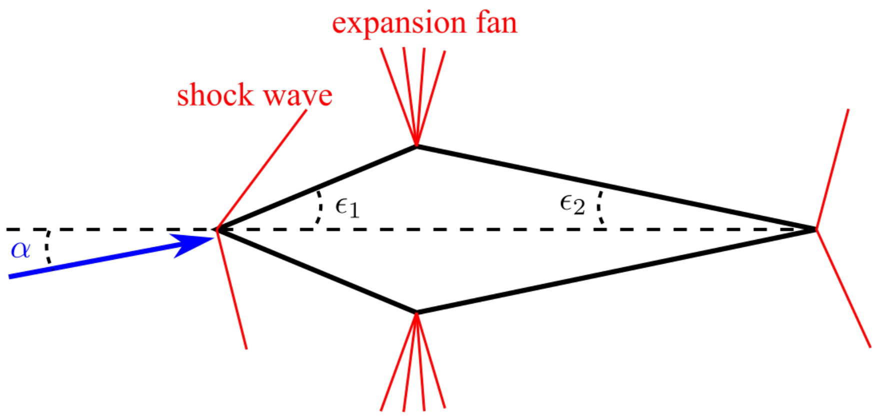
Problem #1: Validation of the $\theta - \beta - M$ Diagram

Using the provided function `ObliqueShockBeta` recreate the $\theta - \beta - M$ diagram or oblique shock-wave chart which is reprinted in Fig. 9.9 of Anderson's *Fundamental's of Aerodynamics* from Chart 2 in NACA Technical Report 1135; available directly from NASA here: `https://ntrs.nasa.gov/citations/19930091059`. You do not have to include the sonic limit line nor the maximum oblique shock angle line in your plot.

Note: Anderson has altered the notation from the original NACA technical report. Specifically, in NACA-TR-1135 the wedge angle is defined as δ, while the shock angle is defined as θ, whereas Anderson defines these as θ and β, respectively.

Problem #2: Computational of Lift and Drag for a Diamond-Wedge Airfoil

Using the provided function `ObliqueShockBeta` in addition to the built-in *Gas Dynamics* functions from the `Aerospace Toolbox` in MATLAB (i.e. `flowisentropic`, `flownormalshock`, `flowprandtlmeyer`), write a MATLAB function which uses shock-expansion theory to solve for the sectional lift and wave-drag coefficients for a diamond-wedge airfoil. Your function should be general enough to work for any arbitrary diamond-wedge airfoil as sketched in the figure below.



Consequently, your function should take the form:

```
function [c_l,c_dw] = DiamondAirfoil(M, alpha, epsilon1, epsilon2)
```

where `c_L` is the sectional coefficient of lift (to be computed and returned), `c_Dw` is the sectional coefficient of wave-drag (to be computed and returned), `alpha` is the angle of attack in degrees, `epsilon1` is the leading edge half-angle in degrees, `epsilon2` is the trailing edge half-angle in degrees.

Specifically, compute the sectional lift and wave-drag coefficients for a diamond airfoil at an angle of attack of $\alpha = 10$ degrees and a Mach number of $M = 3$ where the leading edge half-angle is $\epsilon_1 = 7.5$ degrees and a trailing edge half-angle is $\epsilon_2 = 5$ degrees. Print these values to the command window in MATLAB.

Note: If shock-expansion theory indicates that a given flow configuration produces a bow shock, then simply report as such and do not calculate the corresponding lift and wave-drag coefficients.

Reflection: How do the leading and trailing edge half-angles influence the lift and wave-drag coefficients? Specifically, how do asymmetric leading and trailing edge half-angles change the aerodynamic performance of the wing compared to symmetric (i.e. equal) leading and trailing edge half angles?

Problem #3: Impact of Angle of Attack and Mach Number

For the airfoil geometry defined in Problem #2 analyze the impact of Angle of Attack, α, and Mach number, M. Specifically, generate two plots: 1) sectional lift coefficient, c_l, and 2) sectional wave-drag coefficient, $c_{d,w}$, versus angle of attack, α, for four different Mach numbers, M = 2, 3, 4, and 5. Within each of these figures you should also compare the results from shock-expansion theory to that predicted by linearized supersonic flow:

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$c_{d,w} = \frac{2}{\sqrt{M_\infty^2 - 1}} \left(2\alpha^2 + g_l^2 + g_u^2\right)$$

where:

$$g_l^2 = \frac{1}{c} \int_0^c \left(\frac{dy_l}{dx}\right)^2 dx$$

$$g_u^2 = \frac{1}{c} \int_0^c \left(\frac{dy_u}{dx}\right)^2 dx$$

where $y_l = y_l(x)$ and $y_u = y_u(x)$ define the vertical locations of the upper and lower surfaces, respectively, as a function of x, the distance along the chord length.

Reflection: How do the predictions of lift and drag for a diamond-wedge airfoil vary between shock-expansion theory and linearized compressible flow theory? How do they compare with thin airfoil theory for incompressible (subsonic) flow? How do changes in the Mach number alter the aerodynamic performance (i.e. lift and wave-drag) of a diamond-wedge airfoil?