CIS 12A Final

6)

1. Binary, octal, hex

a) $5.75_{10} \rightarrow$ Binary

5 .75

$5/2 = 2 \; r1$
$2/2 = 1 \; r0$
$1/2 = 0 \; r1$ $\rightarrow 101_2$

	bit	keep
$.75 \cdot 2 = 1.5 \rightarrow$	1	.5
$.5 \cdot 2 = 1 \rightarrow$	1	0.0

$.11_2$

5.75_{10}

$\boxed{101.11_2}$

$5 < 8 \rightarrow 5_8$ so

.75 $\Rightarrow$ octal

$.75 \cdot 8 = 6 \; r0$

$= \boxed{5.6_8}$

Hex

$5 < 16_{10} = 5_{16}$

$.75 \cdot 16 = 12 = C \; r0$

$\boxed{5.C_{16}}$

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b) $.9_{10}$

	bit	keep
$.9 \cdot 2 = 1.8$	1	.8
$.8 \cdot 2 = 1.6$	1	.6
$.6 \cdot 2 = 1.2$	1	.2
$.2 \cdot 2 = .4$	0	.4
$.4 \cdot 2 = .8$	0	.8

Repeat

$\rightarrow .9_{10} = .\overline{1110}_2$

Octal

	digit	keep
$.9 \cdot 8 = 7.2 \rightarrow$	7	.2
$.2 \cdot 8 = 1.6$	1	.6
$.6 \cdot 8 = 4.8$	4	.8
$.8 \cdot 8 = 6.4$	6	.4
$.4 \cdot 8 = 3.2$	3	.2
$.2 \cdot 8 = 1.6$	1	.6

Repeat

$.9_{10} = .71463146\overline{31}\ldots_8$

Hex

$.9 \cdot 16 = 14.4 \rightarrow E$
$.4 \cdot 16 = 6.4 \rightarrow 6$

keep
.4
.4
Repeat

$\rightarrow .9_{10} = .E\overline{66}\ldots$

c) $99.7 \rightarrow$ Binary

$99/2 = 49\ r1$
$49/2 = 24\ r1$
$24/2 = 12\ r0$
$12/2 = 6\ r0$
$6/2 = 3\ r0$
$3/2 = 1\ r1$
$1/2 = 0\ r1$

$\Rightarrow 99_{10} = 1100011_2$

	bit	keep
$.7 \cdot 2 = 1.4$	1	.4
$.4 \cdot 2 = .8$	0	.8
$.8 \cdot 2 = 1.6$	1	.6
$.6 \cdot 2 = 1.2$	1	.2
$.2 \cdot 2 = .4$	0	.4

Repeat

$$\boxed{99.7_{10} = 1100011.1011\overline{0011}_2}$$

$.7_{10} = .1011\overline{0011}_2$

Octal

$99/8 = 12\ r3$
$12/8 = 1\ r4$
$1/8 = 0\ r1$

143_8

	digit	keep
$.7 \cdot 8 = 5.6$	$\rightarrow 5$	.6
$.6 \cdot 8 = 4.8$	$\rightarrow 4$	.8
$.8 \cdot 8 = 6.4$	$\rightarrow 6$	.4
$.4 \cdot 8 = 3.2$	$\rightarrow 3$	.2
$.2 \cdot 8 = 1.6$	$\rightarrow 1$	.6

Repeat from 4

$$\boxed{99.7 = 143.5\overline{4631}_8}$$

$.7_{10} = .5\overline{4631}_8$

Hex

$99/16 = 6\ r3 \Rightarrow 63_{16}$

	digit	keep
$.7 \cdot 16 = 11.2$	$\rightarrow 11$	.2
$.2 \cdot 16 = 3.2$	$\rightarrow 3$	.2

Repeat

$$\boxed{99.7 = 63.B\overline{33}\ldots_{16}}$$

$.7_{10} = .B\overline{33}\ldots_{16}$

2) NASA – Hex Float (32 Bit)

a)

$5.25_{10} = 101.11_2$ E = +3 Sign Bit = 0

$.10111_2 \cdot 2^3$

+3

↓

$00000011 = 03_{16}$

0 10111000000000000000000

5 C 0 0 0 0

$\boxed{5C000003}$

b) $.9_{10}$ E = 0

$.9_{10} = .1110011001\ldots_2$

E = 0 = 00000000 = 00

0 1110011001100110011

7 3 3 3 3 3

$\boxed{73333300}$

c) $99.7_{10} = 1100011.1011001\ldots_2$

$0.1100011101100110011\ldots_2 \cdot 2^7$

E = +7

↓

$00000111 = 07$

01100011101100110011

6 7 B 3 3 3

$\boxed{67B33307}$

3) Scaled Integer Binary

$N = \lfloor x \cdot 2^k \rfloor$

1 byte	255
2	65535
3	16777215

a) 5.75 with 1 unsigned byte (8 bits)

$5.75 \cdot 2^5 = 5.75 \cdot 32 = 184$ $\quad k = 5 \quad N = 184$

$5.75 \cdot 2^6 = 368 \not\leq 255$ $\quad 184_{10} = 10111000_2$

Scale $x = N \cdot 2^{-5}$

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b) .9 2 unsigned bytes (16 bits)

$.9 \cdot 2^k \leq 65535$

$k = 16 \quad N = \lfloor 58982.4 \rfloor = 58982_{10} = 1110011001100110_2$

$x \approx N \cdot 2^{-16}$

$.9 \cdot 2^{10} = 921.6$

$.9 \cdot 2^{15} = 29491$

$.9 \cdot 2^{16} = 58982.4 \leq 65535$ ✓

$.9 \cdot 2^{17} = 117964 \not\leq 65535$

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c) 99.7 with 3 unsigned bytes (24 bits ≤ 16777215

$99.7 \cdot 2^{10} = 102092.8$ $\quad k = 17$

$99.7 \cdot 2^{20} = 104543027.2$ $\quad N = \lfloor 13{,}067{,}878.4 \rfloor = 13{,}067{,}878$

$= 110001101001100110010_2$

$x \approx N \cdot 2^{-17}$

$99.7 \cdot 2^{15} = 3266969.6$

$99.7 \cdot 2^{16} = 6533939.2$

$99.7 \cdot 2^{17} = 13067878.4$ ✓

4) Multiply by 1 byte value 7

$x \approx N \cdot 2^{-k}$

$(N \cdot 7) \cdot 2^{-k}$ $\qquad \frac{N \cdot 7}{2^k}$

a) $N = 184$
$k = 5$

$184 \cdot 7 = \frac{1288}{2^5} = \frac{1288}{32} = 40.25 \Rightarrow 40$

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b) $N = 58982$
$k = 16$

$58982 \cdot 7 = \frac{412874}{2^{16}} = \frac{412874}{65536} = 6.2999 = \boxed{6}$

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c) $N = 13067828$
$k = 17$

$13067828 \cdot 7 = \frac{91475146}{2^{17}} = 697.89 = \boxed{698}$

5) IEEE754 (32-bit single-precision)

1 bit sign

8 bits E 127 $(-1)^S \cdot 1.f \cdot 2^{(E-127)}$

23 F bits after loading

a) 5.75_{10}

$5.75 = 101.11_2 = 1.011_2 \cdot 2^2$

$S = 0$

$p = 2 \rightarrow E = 2 + 127 = 129 = 10000001_2$

$f = 01110000000000000000000$

S	E	F
0	10000001	01110000000000000000000

4 0 B 8 0 0 0 0

hex: 0x40B80000

b) $.9_{10}$

$.9 = .1110011001100110\ldots_2 = 1.1100110011001100\ldots_2 \cdot 2^{-1}$

$S = 0$

$p = -1 \rightarrow E = -1 + 127 = 126 = 01111110_2$

$f = 11001100110\ldots$

S	E	F
0	01111110	11001100110011001100110

3 F 6 6 6 6 6 6

0x3F666666

c) $99.7 = 1100011.1011001100110\ldots_2$

$1.10001101100110\ldots_2 \cdot 2^6$

$S = 0$

$p = 6 \rightarrow E = 6 + 127 = 133 = 10000101_2$

$f = 10001101100110$

S	E	F
0	10000101	10001101100110011001100110

4 2 C 7 6 6 6 6

0x42C76666