

## Problem 4.

(a)

| $x$ | $y$ | $z$ | $(\neg x \rightarrow (y \wedge z))$ |   |   | $\wedge$ | $(\neg y \rightarrow (x \wedge z))$ |   |   |
|-----|-----|-----|-------------------------------------|---|---|----------|-------------------------------------|---|---|
| 0   | 0   | 0   | 1                                   | 0 | 0 | 0        | 1                                   | 0 | 0 |
| 0   | 0   | 1   | 1                                   | 0 | 0 | 0        | 1                                   | 0 | 0 |
| 0   | 1   | 0   | 1                                   | 0 | 0 | 0        | 0                                   | 1 | 0 |
| 0   | 1   | 1   | 1                                   | 1 | 1 | 1        | 0                                   | 1 | 0 |
| 1   | 0   | 0   | 0                                   | 1 | 0 | 0        | 1                                   | 0 | 0 |
| 1   | 0   | 1   | 0                                   | 1 | 0 | 1        | 1                                   | 1 | 1 |
| 1   | 1   | 0   | 0                                   | 1 | 0 | 1        | 0                                   | 1 | 0 |
| 1   | 1   | 1   | 0                                   | 1 | 1 | 1        | 0                                   | 1 | 1 |

(b)

To get the DNF form, we highlight all rows that evaluate to true.

| $x$ | $y$ | $z$ | $(\neg x \rightarrow (y \wedge z)) \wedge (\neg y \rightarrow (x \wedge z))$ |   |   |   |   |   |   |
|-----|-----|-----|--|---|---|---|---|---|---|
| 0   | 0   | 0   | 1  | 0 | 0 | 0 | 1 | 0 | 0 |
| 0   | 0   | 1   | 1  | 0 | 0 | 0 | 1 | 0 | 0 |
| 0   | 1   | 0   | 1  | 0 | 0 | 0 | 0 | 1 | 0 |
| 0   | 1   | 1   | 1  | 1 | 1 | 1 | 0 | 1 | 0 |
| 1   | 0   | 0   | 0  | 1 | 0 | 0 | 1 | 0 | 0 |
| 1   | 0   | 1   | 0  | 1 | 0 | 1 | 1 | 1 | 1 |
| 1   | 1   | 0   | 0  | 1 | 0 | 1 | 0 | 1 | 0 |
| 1   | 1   | 1   | 0  | 1 | 1 | 1 | 0 | 1 | 1 |

So we end up with:

$$(\neg x \wedge y \wedge z) \vee (x \wedge \neg y \wedge z) \vee (x \wedge y \wedge \neg z) \vee (x \wedge y \wedge z)$$

(c)

To get the CNF form, we negate our original expression.

| $x$ | $y$ | $z$ | $\neg((\neg x \rightarrow (y \wedge z)) \wedge (\neg y \rightarrow (x \wedge z)))$ |
|-----|-----|-----|--|
| 0   | 0   | 0   | 1  |
| 0   | 0   | 1   | 1  |
| 0   | 1   | 0   | 1  |
| 0   | 1   | 1   | 0  |
| 1   | 0   | 0   | 1  |
| 1   | 0   | 1   | 0  |
| 1   | 1   | 0   | 0  |
| 1   | 1   | 1   | 0  |

Now negate the DNF for this truth table.

$$\neg((\neg x \wedge \neg y \wedge \neg z) \vee (\neg x \wedge \neg y \wedge z) \vee (\neg x \wedge y \wedge \neg z) \vee (x \wedge \neg y \wedge \neg z))$$

$$\text{LEQV } \neg(\neg x \wedge \neg y \wedge \neg z) \wedge \neg(\neg x \wedge \neg y \wedge z) \wedge \neg(\neg x \wedge y \wedge \neg z) \wedge \neg(x \wedge \neg y \wedge \neg z)$$

$$\text{LEQV } (\neg\neg x \vee \neg\neg y \vee \neg\neg z) \wedge (\neg\neg x \vee \neg\neg y \vee \neg z) \wedge (\neg\neg x \vee \neg y \vee \neg\neg z) \wedge (\neg \vee \neg\neg y \vee \neg\neg z)$$

$$\text{LEQV } (x \vee y \vee z) \wedge (x \vee y \vee \neg z) \wedge (x \vee \neg y \vee z) \wedge (\neg \vee y \vee z)$$

**Eaiser way:** (I watched this part of the lecture after I did this already D:)

Highlight all rows that evaluate to false. Then negate all of the variables.

| $x$ | $y$ | $z$ | $(\neg x \rightarrow (y \wedge z)) \wedge (\neg y \rightarrow (x \wedge z))$ |   |   |   |   |   |   |
|-----|-----|-----|--|---|---|---|---|---|---|
| 0   | 0   | 0   | 1  | 0 | 0 | 0 | 1 | 0 | 0 |
| 0   | 0   | 1   | 1  | 0 | 0 | 0 | 1 | 0 | 0 |
| 0   | 1   | 0   | 1  | 0 | 0 | 0 | 0 | 1 | 0 |
| 0   | 1   | 1   | 1  | 1 | 1 | 1 | 0 | 1 | 0 |
| 1   | 0   | 0   | 0  | 1 | 0 | 0 | 1 | 0 | 0 |
| 1   | 0   | 1   | 0  | 1 | 0 | 1 | 1 | 1 | 1 |
| 1   | 1   | 0   | 0  | 1 | 0 | 1 | 0 | 1 | 0 |
| 1   | 1   | 1   | 0  | 1 | 1 | 1 | 0 | 1 | 1 |

So we end up with:

$$(x \vee y \vee z) \wedge (x \vee y \vee \neg z) \wedge (x \vee \neg y \vee z) \wedge (\neg \vee y \vee z)$$

$$\begin{array}{ll}
\text{(d)} & \\
(\neg x \rightarrow (y \wedge z)) \wedge (\neg y \rightarrow (x \wedge z)) & \\
\text{LEQV} \quad (\neg \neg x \vee (y \wedge z)) \wedge (\neg \neg y \vee (x \wedge z)) & [\rightarrow \text{ law}] \\
\text{LEQV} \quad (x \vee (y \wedge z)) \wedge (y \vee (x \wedge z)) & [\text{double negation}] \\
\text{LEQV} \quad ((x \vee y) \wedge (x \vee z)) \wedge ((y \vee x) \wedge (y \vee z)) & [\text{distributive laws}] \\
\text{LEQV} \quad (x \vee y) \wedge (x \vee z) \wedge (y \vee x) \wedge (y \vee z) & [\text{associative laws}] \\
\text{LEQV} \quad ((x \vee y) \wedge (y \vee x)) \wedge (x \vee z) \wedge (y \vee z) & [\text{by reordering}] \\
\text{LEQV} \quad (x \vee y) \wedge (x \vee z) \wedge (y \vee z) & [\text{idempotency laws}]
\end{array}$$