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Question 1.

The most-significant digit of the sum is a # in the b^3 column. However, we are only adding two 2-digit numbers, which cannot be more than $2 \times b^3$
This means that $\# = 1$.

So we have

$$\begin{array}{cccc} & & 1 & * \\ + & & 1 & * \\ \hline = & 1 & \Diamond & 1 \end{array}$$

In the b^1 column, two identical digits adding up to one must mean that the sum carries over, so
$* + * = b + 1$
This also means that b is odd, since $* + *$ must be even.

In the b^2 column, we have a carry over from the b^1 column. So
$1 + 1 + 1 = b + \Diamond$
Since LS is odd, and b is odd, this means $\Diamond$ is even.
Since $1 + 1 + 1$ carries over, this means that $b <= 3$. And since b is odd, then $b = 3 \implies \Diamond = 0$

Answer:

$$\begin{array}{cccc} & & 1 & 2 \\ + & & 1 & 2 \\ \hline = & 1 & 0 & 1 \end{array}$$

Where $b = 3$

Question 2.

$(0.1)_{10} = ()_2$?

Multiplier	Base	Product	Integral	Fraction
0.1	2	0.2	0	0.2
0.2	2	0.4	0	0.4
0.4	2	0.8	0	0.8
0.8	2	1.6	1	0.6
0.6	2	1.2	1	0.2

$(0.1)_{10} = ()_3$?

Multiplier	Base	Product	Integral	Fraction
0.1	3	0.3	0	0.3
0.3	3	0.9	0	0.9
0.9	3	2.7	2	0.7
0.7	3	2.1	2	0.1
0.1	3	0.3	0	0.3

$(0.1)_{10} = ()_4$?

Multiplier	Base	Product	Integral	Fraction
0.1	4	0.4	0	0.4
0.4	4	1.6	1	0.6
0.6	4	2.4	2	0.4

$(0.1)_{10} = ()_5$?

Multiplier	Base	Product	Integral	Fraction
0.1	5	0.5	0	0.5
0.5	5	2.5	2	0.5

$(0.1)_{10} = ()_6$?

Multiplier	Base	Product	Integral	Fraction
0.1	6	0.6	0	0.6
0.6	6	3.6	3	0.6

$(0.1)_{10} = ()_7$?

Multiplier	Base	Product	Integral	Fraction
0.1	7	0.7	0	0.7
0.7	7	4.9	4	0.9
0.9	7	6.3	6	0.3
0.3	7	2.1	2	0.1
0.1	7	0.7	0	0.7

$(0.1)_{10} = ()_8$?

Multiplier	Base	Product	Integral	Fraction
0.1	8	0.8	0	0.8
0.8	8	6.4	6	0.4
0.4	8	3.2	3	0.2
0.2	8	1.6	1	0.6
0.6	8	4.8	4	0.8

$(0.1)_{10} = ()_9$?

Multiplier	Base	Product	Integral	Fraction
0.1	9	0.9	0	0.9
0.9	9	8.1	8	0.1
0.1	9	0.9	0	0.9

In every base from 2 to 9, the decimal expansion loops. $\therefore (0.1)_{10}$ cannot be represented exactly with a finite mantissa.

Question 3.

$$\delta = \frac{x - fl(x)}{x}$$

To find an upper bound, we need to see what's the maximum value of $x - fl(x)$
We want to prove that δ is bounded above by

$$\epsilon = \begin{cases} b^{1-t} & \text{chopping} \\ \frac{1}{2}b^{1-t} & \text{rounding} \end{cases}$$

$=$

$=$

Let $x = \pm\ d_k \times b^k + d_{k-1} \times b^{k-1} + d_{k-2} \times b^{k-2} + \ldots$

Then $fl(x) = \pm\ \overbrace{d_k \times b^k + d_{k-1} \times b^{k-1} + d_{k-2} \times b^{k-2} + \ldots + d_{k-t-1} \times b^{k-t-1}}^{t \text{ values}}$

$x - fl(x) = d_{k-t-2} \times b^{k-t-2} + d_{k-t-3} \times b^{k-t-3} + \ldots$
Which is bounded above by

$$\begin{cases} 1 \times b^{k-t-1} & \text{chopping} & \text{[max value is 1 decimal place above]} \\ \frac{1}{2} \times b^{k-t-1} & \text{rounding} & \text{[since adding } \frac{1}{2} \text{ reduces reduces RRO]} \end{cases}$$

if t is the mantissa length, then the amount of significant digits b^{k-t-1} has is $b^1 - t$. So

$$\epsilon = \begin{cases} 1 \times b^{k-t-1} & \text{chopping} \\ \frac{1}{2} \times b^{k-t-1} & \text{rounding} \end{cases}$$

Question 4.

Question 5.