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## Question 1.

(a) 
$$A(n) = n^{2}A(n-1)$$

$$A(n-1) = (n-1)^{2}A(n-2)$$

$$A(n-2) = (n-2)^{2}A(n-3)$$

$$\vdots$$

$$A(2) = (2)^{2}A(1)$$

$$A(1) = 1$$

So by Repeated Back Substitution, we have:

$$A(n) = n^{2}(n-1)^{2}(n-2)^{2} \cdots (2)^{2}(1)^{2}$$

$$= (n(n-1)(n-2) \cdots (2)(1))^{2}$$

$$= (n!)^{2}$$

$$\therefore A(n) \in \Theta((n!)^{2})$$

(b)

$$C(n) = \begin{cases} 8C \left\lfloor \frac{n}{2} \right\rfloor + 2n^3 + 4n & \text{if } n > 0 \\ 6 & \text{if } n = 0 \end{cases}$$

We can calculate C(1) by plugging it in.

$$C(1) = 8C \left\lfloor \frac{1}{2} \right\rfloor + 2(1)^3 + 4(1)$$
$$= 8C(0) + 2 + 4$$
$$= 8(6) + 6$$
$$= 54$$

Now we can rewrite C(n) to...

$$C(n) = \begin{cases} 8C \left\lfloor \frac{n}{2} \right\rfloor + 2n^3 + 4n & \text{if } n > 1 \\ 54 & \text{if } n = 1 \end{cases}$$

We can see that this sastisfies the Generalized Master Theorem, where

$$a = 8$$

$$b=2$$

$$c = 3$$

$$d = 54$$

$$f(n) = 2n^3 + 4n$$

So we know that  $f(n) \in \Theta(n^3)$ , and  $\log_2 8 = 3$   $\Longrightarrow \log_b a = c$ 

By part (b) of **Generalized Master Theorem**, we have

$$T(n) = \Theta(n^{\log_b a} \log_b n)$$
$$= \Theta(n^3 \log_2 n)$$

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### Problem 2.

Suppose...

$$f(n) \in \mathcal{O}(g(n))$$

$$f(n) \ge 1$$

$$\log (g(n)) \ge 1$$

 $\forall n \in \mathbb{N}$ 

WTS: 
$$\log (f(n)) \in \mathcal{O}(\log (g(n)))$$
 OR  
 $\exists c' \in \mathbb{R}^+, \ \exists n'_0 \in \mathbb{N}, \ \forall n > n'_0 \implies \log (f(n)) \leq c' \cdot \log (g(n))$  (\*\*)

Since  $f(n) \in \mathcal{O}(g(n))$  we have...

$$\exists c \in \mathbb{R}^+, \ \exists n_0 \in \mathbb{N}, \ \forall n > n_0 \implies f(n) \le c \cdot g(n)$$

$$f(n) \leq c \cdot g(n)$$
 [by given] 
$$\log (f(n)) \leq \log (c \cdot g(n))$$
 [logging both sides since both sides  $\geq 1$ ] 
$$\leq \log(c) + \log (g(n))$$
 [by log laws] 
$$\leq \log(c) \log (g(n)) + \log (g(n))$$
 [since  $\log (g(n)) \geq 1$ ] 
$$\leq (\log(c) + 1) \log (g(n))$$
 [by factoring]

So choose:

$$c' = \log(c) + 1 \qquad n_0' = n_0$$

Then the predicate  $(\bigstar)$  holds

### Problem 3.

#### Implementation:

for RECENT, I would use an AVL Tree and a Doubly Linked List. I would also keep track of size, and the head and tail of the Doubly Linked List.

In the AVL Tree, the nodes would have an extra parameter: target: a pointer to its node in the Doubly Linked List.

For every ACCESS(x), it would insert x into both the AVL Tree and the Doubly Linked List. If x is already in the AVL Tree, then use target to delete that node in the Doubly Linked List. Then re-insert it back into the head of the Doubly Linked List by using head.

If size > m, then delete both tail, and its corresponding node in the AVL Tree. Then continue to insert x.

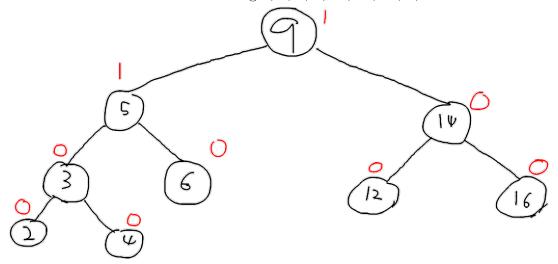
#### **Justification:**

This data structure uses  $\mathcal{O}(m \log n)$  space since we have 2 data structures of size m. This means that we will only have about 2m nodes to store, which according to this piazza post, takes about  $2(m \log n) = \mathcal{O}(m \log n)$  space

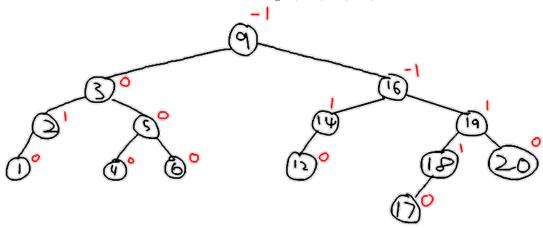
This data structure takes worst case  $\mathcal{O}(\log m)$  time complexity. This is because for every ACCESS(x), we need to search the AVL Tree which takes  $\mathcal{O}(\log m)$  time. It could also delete and insert in the Doubly Linked List after searching, which would take  $\mathcal{O}(1)$  since we keep a pointer to size, head, and tail. Lastly, we would also need to delete from both the AVL Tree and the Doubly Linked List if size gets too large, which would take  $\mathcal{O}(\log m)$  and  $\mathcal{O}(1)$  time respectively. Overall, the worst case time complexity is  $\mathcal{O}(\log m)$ .

## Problem 4.

Tree after inserting 5, 4, 6, 9, 12, 16, 14, 2, 3



Tree after inserting 1, 20, 19, 18, 17



Tree after deleting 1, 2, 3, 4, 5, 12, 6

