

Let  $L = \{\langle M \rangle \mid \exists w \in \Sigma^* \text{ such that when } M \text{ runs on } w, M \text{ prints a blank on a non-blank cell}\}$

WTS:  $L$  is undecidable

$\implies HALT \leq_m L$

Consider the following TM  $F$  and the reduction it computes:

$F =$  “On input  $\langle M, w \rangle$ :

1. Construct a TM  $M_1$  as follows:

$M_1 =$  “On input  $\langle M, w \rangle$ :

1. for  $s = 1$  to  $\infty$
2.     run  $M$  on  $w$  for  $s$  steps
3.     if  $M$  writes a blank on a non-blank cell then accept”

2. Construct a TM  $M_2$  as follows:

$M_2 =$  “On input  $x$ :

1.  $\triangleright$  empty Part1
2. run  $M_1$  on  $\langle M, w \rangle$
3. accept”

3. return  $\langle M_2 \rangle$ ”

We argue that  $\langle M, w \rangle \in HALT \iff \langle M_2 \rangle \in L$

( $\implies$ )

Suppose  $\langle M, w \rangle \in HALT$

$M$  halts on  $w$

[definition of  $HALT$ ]

$\implies M_2$  accepts  $\langle M, w \rangle$  such that  $M$  prints a blank in a non-blank cell

[description of  $M_2$ ]

$\implies \mathcal{L}(M_2) = \{\langle M \rangle \mid \exists w \in \Sigma^* \text{ such that when } M \text{ runs on } w, M \text{ prints a blank on a non-blank cell}\}$

$\implies \langle M_2 \rangle \in L$

[definition of  $L$ ]

as wanted.

( $\iff$ )

Suppose  $\langle M, w \rangle \notin HALT$

$M$  loops on  $w$

[definition of  $HALT$ ]

$\implies M_2$  loops on every input

[description of  $M_2$ ]

$\implies \mathcal{L}(M_2) \neq \{\langle M \rangle \mid \exists w \in \Sigma^* \text{ such that when } M \text{ runs on } w, M \text{ prints a blank on a non-blank cell}\}$

$\implies \langle M_2 \rangle \notin L$

[definition of  $L$ ]

as wanted.