

Question 4

- a) Define the domain $D = \{g_1, g_2, g_3, g_4\}$
let g_1 be beautiful, and calls g_2
let g_2 be ugly, and calls g_3
let g_3 be ugly, and calls g_4
let g_4 be ugly, and calls g_2

Let $d = g_1$

Then $(F_1 \wedge F_2 \wedge F_2)$ is satisfied.

This is because $d = g_1$ is beautiful, so F_1 is satisfied.

g_1 calls g_2 , which is ugly.
 g_2 calls g_3 , which is ugly.
 g_3 calls g_4 , which is ugly.
 g_4 calls g_2 , which is ugly.
So F_2 is satisfied

Lastly, no 2 functions call each other, so F_3 is satisfied

$\therefore (F_1 \wedge F_2 \wedge F_2)$ is satisfied.

b) No, there does not exist a domain $\{h_1, h_2, h_3\}$ such that $(F_1 \wedge F_2 \wedge F_2)$ is satisfied.

Suppose to the contrary that there exist a domain $\{h_1, h_2, h_3\}$ such that $(F_1 \wedge F_2 \wedge F_2)$ is satisfied. In order for F_3 to be satisfied, then no 2 functions call each other, so we have either



Where a function points to the function it calls.

In order to satisfy F_2 , then all functions must be ugly since all functions are called.

However, F_3 cannot be satisfied since no functions are beautiful. All functions are called because F_3 is satisfied.

Therefore our supposition was wrong, and there does not exist a domain $\{h_1, h_2, h_3\}$ such that $(F_1 \wedge F_2 \wedge F_2)$ is satisfied.