

Let L_1, L_2 is recognizable

Let M_1, M_2 be TM's such that

$$\mathcal{L}(M_1) = L_1$$

$$\mathcal{L}(M_2) = L_2$$

(a)

Suppose $L = L_1 \cup L_2$

WTS: L is recognizable

Here is a TM M that recognizes L

$M =$ "On input w

1. for $s = 1$ to ∞
2. run M_1 on w for s steps
3. run M_2 on w for s steps
4. if M_1 accepts w or M_2 accepts w
5. accept"

(b)

Suppose $L = L_1 \circ L_2$

WTS: L is recognizable

Create a new TM with the q_{reject} from L_1 replaced with a state going to q_1 from L_2

(c)

Suppose $L = L_1^*$

WTS: L is recognizable

(d)

Suppose $L = L_1 \cap L_2$

WTS: L is recognizable

Here is a TM M that recognizes L

$M =$ "On input w

1. for $s = 1$ to ∞
2. run M_1 on w for s steps
3. run M_2 on w for s steps
4. if M_1 accepts w and M_2 accepts w
5. accept"

(e')

Let f be a homomorphic function.

Suppose $L = f(L_1)$

WTS: L is recognizable