Question 2

[step 1]
Define the set $\mathcal{G}$ of formulas that uoc $\{\neg, \wedge\}$

Let $\mathcal{G}$ be the smallest set such that

BASIS: If x is a propositional variable, then $x \in \mathcal{G}$

INDUCTION STEP: If $F_1, F_2 \in \mathcal{G}$, then $\neg F, (F_1 \wedge F_2) \in G$

[step 2]
WTS: For all $F \in \mathcal{G}$ there exists F' such that
F' uoc from $\{\underline{1}, \ll\}$

BASIS: Let $F = x$, where x is a propositional variable.

Let $F' = x$

Then F' uoc $\{\underline{1}, \ll\}$

and F' LEQV F, as wanted

INDUCTION STEP: Let $F_1, F_2 \in \mathcal{G}$

Suppose F_1', F_2' uoc $\{\underline{1}, \ll\}$ and

F_1' LEQV F_1 and F_2' LEQV F_2 **[IH]**

Case 1: $F = \neg F_1$

Let $F' = (\underline{1}F_1 \ll \underline{1}F_1) \ll (F_1 \ll \underline{1}F_1)$

then F' uoc $\{\underline{1}, \ll\}$ and

$F' = (\underline{1}F_1' \ll \underline{1}F_1') \ll (F_1' \ll \underline{1}F_1')$	[by given]
LEQV $(\underline{1}F_1 \ll \underline{1}F_1) \ll (F_1 \ll \underline{1}F_1)$	[by **IH**]
LEQV $(\underline{1}F_1 \ll \underline{1}F_1) \ll (\neg F_1)$	[since $F_1 \ll \underline{1}F_1$ sastified if F_1 falsified]
LEQV $(\underline{0}F_1) \ll (\neg F_1)$	[since $x \ll x$ always falsified]
LEQV $\neg F_1$	[since $\underline{0}F_1 \ll \neg F_1$ sastified if $\neg F_1$ sastified]
$= F$	[by given]

as wanted.

Case 2: $F = F_1 \wedge F_2$

Let $F' = (F_1 \ll \underline{1}F_1) \ll F_2$

Then F' uoc $\{\underline{1}, \ll\}$ and

$F' = (F_1' \ll \underline{1}F_1') \ll F_2'$	[by given]
LEQV $(F_1 \ll \underline{1}F_1) \ll F_2$	[by **IH**]
LEQV $(\neg F_1) \ll F_2$	[since $F_1 \ll \underline{1}F_1$ sastified if F_1 falsified]
LEQV $F_1 \wedge F_2$	[since they have the same truth tables]
$= F$	[by given]

as wanted.

[step 3]
Since $\{\neg, \wedge\}$ is complete, therefore $\{\underline{1}, \ll\}$ is also complete. ■