

Problem 4.

(a)

x	y	z	$(\neg x \rightarrow (y \wedge z))$			\wedge	$(\neg y \rightarrow (x \wedge z))$		
0	0	0	1	0	0	0	1	0	0
0	0	1	1	0	0	0	1	0	0
0	1	0	1	0	0	0	0	1	0
0	1	1	1	1	1	1	0	1	0
1	0	0	0	1	0	0	1	0	0
1	0	1	0	1	0	1	1	1	1
1	1	0	0	1	0	1	0	1	0
1	1	1	0	1	1	1	0	1	1

(b)

To get the DNF form, we highlight all rows that evaluate to true.

x	y	z	$(\neg x \rightarrow (y \wedge z)) \wedge (\neg y \rightarrow (x \wedge z))$						
0	0	0	1	0	0	0	1	0	0
0	0	1	1	0	0	0	1	0	0
0	1	0	1	0	0	0	0	1	0
0	1	1	1	1	1	1	0	1	0
1	0	0	0	1	0	0	1	0	0
1	0	1	0	1	0	1	1	1	1
1	1	0	0	1	0	1	0	1	0
1	1	1	0	1	1	1	0	1	1

So we end up with:

$$(\neg x \wedge y \wedge z) \vee (x \wedge \neg y \wedge z) \vee (x \wedge y \wedge \neg z) \vee (x \wedge y \wedge z)$$

(c)

To get the CNF form, we negate our original expression.

x	y	z	$\neg((\neg x \rightarrow (y \wedge z)) \wedge (\neg y \rightarrow (x \wedge z)))$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Now negate the DNF for this truth table.

$$\neg((\neg x \wedge \neg y \wedge \neg z) \vee (\neg x \wedge \neg y \wedge z) \vee (\neg x \wedge y \wedge \neg z) \vee (x \wedge \neg y \wedge \neg z))$$

$$\text{LEQV } \neg(\neg x \wedge \neg y \wedge \neg z) \wedge \neg(\neg x \wedge \neg y \wedge z) \wedge \neg(\neg x \wedge y \wedge \neg z) \wedge \neg(x \wedge \neg y \wedge \neg z)$$

$$\text{LEQV } (\neg\neg x \vee \neg\neg y \vee \neg\neg z) \wedge (\neg\neg x \vee \neg\neg y \vee \neg z) \wedge (\neg\neg x \vee \neg y \vee \neg\neg z) \wedge (\neg \vee \neg\neg y \vee \neg\neg z)$$

$$\text{LEQV } (x \vee y \vee z) \wedge (x \vee y \vee \neg z) \wedge (x \vee \neg y \vee z) \wedge (\neg \vee y \vee z)$$

Eaiser way: (I watched this part of the lecture after I did this already D:)

Highlight all rows that evaluate to false. Then negate all of the variables.

x	y	z	$(\neg x \rightarrow (y \wedge z)) \wedge (\neg y \rightarrow (x \wedge z))$						
0	0	0	1	0	0	0	1	0	0
0	0	1	1	0	0	0	1	0	0
0	1	0	1	0	0	0	0	1	0
0	1	1	1	1	1	1	0	1	0
1	0	0	0	1	0	0	1	0	0
1	0	1	0	1	0	1	1	1	1
1	1	0	0	1	0	1	0	1	0
1	1	1	0	1	1	1	0	1	1

So we end up with:

$$(x \vee y \vee z) \wedge (x \vee y \vee \neg z) \wedge (x \vee \neg y \vee z) \wedge (\neg \vee y \vee z)$$

$$\begin{array}{ll}
\text{(d)} & \\
(\neg x \rightarrow (y \wedge z)) \wedge (\neg y \rightarrow (x \wedge z)) & \\
\text{LEQV } (\neg\neg x \vee (y \wedge z)) \wedge (\neg\neg y \vee (x \wedge z)) & [\rightarrow \text{ law}] \\
\text{LEQV } (x \vee (y \wedge z)) \wedge (y \vee (x \wedge z)) & [\text{double negation}] \\
\text{LEQV } ((x \vee y) \wedge (x \vee z)) \wedge ((y \vee x) \wedge (y \vee z)) & [\text{distributive laws}] \\
\text{LEQV } (x \vee y) \wedge (x \vee z) \wedge (y \vee x) \wedge (y \vee z) & [\text{associative laws}] \\
\text{LEQV } ((x \vee y) \wedge (y \vee x)) \wedge (x \vee z) \wedge (y \vee z) & [\text{by reordering}] \\
\text{LEQV } (x \vee y) \wedge (x \vee z) \wedge (y \vee z) & [\text{idempotency laws}]
\end{array}$$