

## Question 4

- a) Define the domain  $D = \{g_1, g_2, g_3, g_4\}$   
let  $g_1$  be beautiful, and calls  $g_2$   
let  $g_2$  be ugly, and calls  $g_3$   
let  $g_3$  be ugly, and calls  $g_4$   
let  $g_4$  be ugly, and calls  $g_2$

Let  $d = g_1$

Then  $(F_1 \wedge F_2 \wedge F_2)$  is satisfied.

This is because  $d = g_1$  is beautiful, so  $F_1$  is satisfied.

$g_1$  calls  $g_2$ , which is ugly.  
 $g_2$  calls  $g_3$ , which is ugly.  
 $g_3$  calls  $g_4$ , which is ugly.  
 $g_4$  calls  $g_2$ , which is ugly.  
So  $F_2$  is satisfied

Lastly, no 2 functions call each other, so  $F_3$  is satisfied

$\therefore (F_1 \wedge F_2 \wedge F_2)$  is satisfied.

b) No, there does not exist a domain  $\{h_1, h_2, h_3\}$  such that  $(F_1 \wedge F_2 \wedge F_3)$  is satisfied.

Suppose to the contrary that there exist a domain  $\{h_1, h_2, h_3\}$  such that  $(F_1 \wedge F_2 \wedge F_3)$  is satisfied. In order for  $F_3$  to be satisfied, then no 2 functions call each other, so we have either



Where a function points to the function it calls.

In order to satisfy  $F_2$ , then all functions must be ugly since all functions are called.

However,  $F_3$  cannot be satisfied since no functions are beautiful. All functions are called because  $F_3$  is satisfied.

Therefore our supposition was wrong, and there does not exist a domain  $\{h_1, h_2, h_3\}$  such that  $(F_1 \wedge F_2 \wedge F_3)$  is satisfied.