Question 4

a) Define the domain D = $\{g_1,\ g_2,\ g_3,\ g_4\}$

let g_1 be beautiful, and calls g_2
let g_2 be ugly, and calls g_3
let g_3 be ugly, and calls g_4
let g_4 be ugly, and calls g_2

Let $d = g_1$

Then $(F_1 \wedge F_2 \wedge F_2)$ is satisfied.

This is because $d = g_1$ is beautiful, so F_1 is satisfied.

g_1 calls g_2, which is ugly.
g_2 calls g_3, which is ugly.
g_3 calls g_4, which is ugly.
g_4 calls g_2, which is ugly.
So F_2 is satisfied

Lastly, no 2 functions call each other, so F_3 is satisfied

$\therefore (F_1 \wedge F_2 \wedge F_2)$ is satisfied.

b) No, there does not exist a domain $\{h_1,\ h_2,\ h_3\}$ such that $(F_1 \wedge F_2 \wedge F_2)$ is satisfied.

Suppose to the contrary that there exist a domain $\{h_1,\ h_2,\ h_3\}$ such that $(F_1 \wedge F_2 \wedge F_2)$ is satisfied. In order for F_3 to be satisfied, then no 2 functions call each other, so we have either

$$\begin{tikzcd} & h_1 \arrow[dr] & \\ h_2 \arrow[ur] & & h_3 \arrow[ll] \end{tikzcd} \qquad \begin{tikzcd} & h_1 \arrow[dl] & \\ h_2 \arrow[rr] & & h_3 \arrow[ul] \end{tikzcd}$$

Where a function points to the function it calls.

In order to sastify F_2, then all functions must be ugly since all functions are called.

However, F_3 cannot be satisfied since no functions are beautiful. All functions are called because F_3 is satisfied.

Therefore our supposition was wrong, and there does not exist a domain $\{h_1,\ h_2,\ h_3\}$ such that $(F_1 \wedge F_2 \wedge F_2)$ is satisfied.