Let $L = \{\langle M,\ w\rangle \mid$ When M runs on w, M moves the head beyond the right-most cell of $w\}$

WTS: L is decidable

Here is a TM that decides L

$\mathcal{M} =$ "On input $\langle M,\ w\rangle$

1. run M on w for $\big(|Q| \times |w| \times |\Gamma|^{|w|} + 1\big)$ steps

2. if M moves the head beyond w then accept else reject"

For the number of possible configurations of M where the head is within w, there are $|Q|$ states, the head can be in $|w|$ positions, and every character in Γ can be in each of w cells.

So we have

$$\begin{aligned} n &= \text{Number of states} \times \text{Number of head positions} \times \text{Number of possible tapes} \\ &= |Q| \times |w| \times |\Gamma|^{|w|} \end{aligned}$$

To prove $\mathcal{M}$ recognizes L, we will look at 2 cases

Case 1: M's head stays within w for $n + 1$ steps

We run M for $n + 1$ steps, but there are only n possible configurations where M's head is within w. By the pigeon hole principle, a configuration must be visited at least twice. This implies that M loops on w with the head never reading beyond w. This leads to $\langle M,\ w\rangle \notin L$ which we reject as wanted.

Case 2: M's head moves beyond w within $n + 1$ steps

This implies $\langle M,\ w\rangle \in L$ which we accept as wanted.