Let $L_1,\ L_2$ is recognizable
Let $M_1,\ M_2$ be TM's such that
$\mathcal{L}(M_1) = L_1$
$\mathcal{L}(M_2) = L_2$

(a)
Suppose $L = L_1 \cup L_2$
WTS: L is recognizable

Here is a TM M that recognizes L
M = "On input w
1. for $s = 1$ to ∞
2. run M_1 on w for s steps
3. run M_2 on w for s steps
4. if M_1 accepts w or M_2 accepts w
5. accept"

(b)
Suppose $L = L_1 \circ L_2$
WTS: L is recognizable

Create a new TM with the q_{reject} from L_1 replaced with a state going to q_1 from L_2

(c)
Suppose $L = {L_1}^*$
WTS: L is recognizable

(d)
Suppose $L = L_1 \cap L_2$
WTS: L is recognizable

Here is a TM M that recognizes L
M = "On input w
1. for $s = 1$ to ∞
2. run M_1 on w for s steps
3. run M_2 on w for s steps
4. if M_1 accepts w and M_2 accepts w
5. accept"

' (e')
Let f be a homomorphic function.
Suppose $L = f(L_1)$
WTS: L is recognizable