1. Let $L = \{\langle M_1,\ M_2\rangle \mid M_1,\ M_2 \text{ is a TM and } |\mathcal{L}(M_1)| < |\mathcal{L}(M_2)| < \infty\}$

2. L is recognizable

3. L is not co-recognizable

WTS: L is not co-recognizable
$\implies HALT \leq_{\mathrm{m}} \overline{L}$

Consider the following TM F and the reduction it computes:
$F =$ "On input $\langle M,\ w\rangle$:
1. Construct a TM M_1 as follows:
$M_1 =$ "On input x:
1. $\triangleright$ empty Part1
2. run M on w
3. if $x = 0$ then accept else loop"
2. Construct a TM M_2 as follows:
$M_2 =$ "On input x:
1. $\triangleright$ empty Part1
2. run M on w
3. if $x = 0$ or $x = 1$ then accept else loop"
3. return $\langle M_1,\ M_2\rangle$"

We argue that $\langle M,\ w\rangle \in HALT \iff \langle M_1,\ M_2\rangle \in L$

($\implies$)
Suppose $\langle M,\ w\rangle \in HALT$

M halts on w	[definition of $HALT$]
$\implies$ M_1 accepts only the string 0	[description of M_1]
M_2 accepts only the strings 0 and 1	[description of M_2]
$\implies$ $|\mathcal{L}(M_1)| < |\mathcal{L}(M_2)| < \infty$	[$\mathcal{L}(M_1) = \{0\}$ so $|\mathcal{L}(M_1)| = 1$ and $\mathcal{L}(M_2) = \{0, 1\}$ so $|\mathcal{L}(M_2)| = 2$]
$\implies$ $\langle M_1,\ M_2\rangle \in L$	[definition of L]

as wanted.

($\impliedby$)
Suppose $\langle M,\ w\rangle \notin HALT$

M loops on w	[definition of $HALT$]
$\implies$ M_1 loops on every input	[description of M_1]
M_2 loops on every input	[description of M_2]
$\implies$ $|\mathcal{L}(M_1)| = |\mathcal{L}(M_2)|$	[$\mathcal{L}(M_1) = \emptyset$ so $|\mathcal{L}(M_1)| = 0$ and $\mathcal{L}(M_2) = \emptyset$ so $|\mathcal{L}(M_2)| = 0$]
$\implies$ $\langle M_1,\ M_2\rangle \notin L$	[definition of L]

as wanted.