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Question 1.

The most-significant digit of the sum is a $\#$ in the b^3 column. However, we are only adding two 2-digit numbers, which cannot be more than $2 \times b^3$

This means that $\# = 1$.

So we have

$$\begin{array}{r} 1 \quad * \\ + \quad 1 \quad * \\ \hline = 1 \quad \Diamond \quad 1 \end{array}$$

In the b^1 column, two identical digits adding up to one must mean that the sum carries over, so $* + * = b + 1$

This also means that b is odd, since $* + *$ must be even.

In the b^2 column, we have a carry over from the b^1 column. So

$$1 + 1 + 1 = b + \Diamond$$

Since LS is odd, and b is odd, this means \Diamond is even.

Since $1 + 1 + 1$ carries over, this means that $b \leq 3$. And since b is odd, then $b = 3 \implies \Diamond = 0$

Answer:

$$\begin{array}{r} 1 \quad 2 \\ + \quad 1 \quad 2 \\ \hline = 1 \quad 0 \quad 1 \end{array}$$

Where $b = 3$

Question 2.

$$(0.1)_{10} = ()_2?$$

Multiplier	Base	Product	Integral	Fraction
0.1	2	0.2	0	0.2
0.2	2	0.4	0	0.4
0.4	2	0.8	0	0.8
0.8	2	1.6	1	0.6
0.6	2	1.2	1	0.2

$$(0.1)_{10} = ()_3?$$

Multiplier	Base	Product	Integral	Fraction
0.1	3	0.3	0	0.3
0.3	3	0.9	0	0.9
0.9	3	2.7	2	0.7
0.7	3	2.1	2	0.1
0.1	3	0.3	0	0.3

$$(0.1)_{10} = ()_4?$$

Multiplier	Base	Product	Integral	Fraction
0.1	4	0.4	0	0.4
0.4	4	1.6	1	0.6
0.6	4	2.4	2	0.4

$$(0.1)_{10} = ()_5?$$

Multiplier	Base	Product	Integral	Fraction
0.1	5	0.5	0	0.5
0.5	5	2.5	2	0.5

$$(0.1)_{10} = ()_6?$$

Multiplier	Base	Product	Integral	Fraction
0.1	6	0.6	0	0.6
0.6	6	3.6	3	0.6

$$(0.1)_{10} = ()_7?$$

Multiplier	Base	Product	Integral	Fraction
0.1	7	0.7	0	0.7
0.7	7	4.9	4	0.9
0.9	7	6.3	6	0.3
0.3	7	2.1	2	0.1
0.1	7	0.7	0	0.7

$$(0.1)_{10} = ()_8?$$

Multiplier	Base	Product	Integral	Fraction
0.1	8	0.8	0	0.8
0.8	8	6.4	6	0.4
0.4	8	3.2	3	0.2
0.2	8	1.6	1	0.6
0.6	8	4.8	4	0.8

$$(0.1)_{10} = ()_9?$$

Multiplier	Base	Product	Integral	Fraction
0.1	9	0.9	0	0.9
0.9	9	8.1	8	0.1
0.1	9	0.9	0	0.9

In every base from 2 to 9, the decimal expansion loops. $\therefore (0.1)_{10}$ cannot be represented exactly with a finite mantissa.

Question 3.

$$\delta = \frac{x - fl(x)}{x}$$

To find an upper bound, we need to see what's the maximum value of $x - fl(x)$

We want to prove that δ is bounded above by

$$\epsilon = \begin{cases} b^{1-t} & \text{chopping} \\ \frac{1}{2}b^{1-t} & \text{rounding} \end{cases}$$

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Let $x = \pm d_k \times b^k + d_{k-1} \times b^{k-1} + d_{k-2} \times b^{k-2} + \dots$

t values

Then $fl(x) = \pm \overbrace{d_k \times b^k + d_{k-1} \times b^{k-1} + d_{k-2} \times b^{k-2} + \dots + d_{k-t-1} \times b^{k-t-1}}^{t \text{ values}}$

$$x - fl(x) = d_{k-t-2} \times b^{k-t-2} + d_{k-t-3} \times b^{k-t-3} + \dots$$

Which is bounded above by

$$\begin{cases} 1 \times b^{k-t-1} & \text{chopping} & [\text{max value is 1 decimal place above}] \\ \frac{1}{2} \times b^{k-t-1} & \text{rounding} & [\text{since adding } \frac{1}{2} \text{ reduces reduces RRO}] \end{cases}$$

if t is the mantissa length, then the amount of significant digits b^{k-t-1} has is $b^1 - t$. So

$$\epsilon = \begin{cases} 1 \times b^{k-t-1} & \text{chopping} \\ \frac{1}{2} \times b^{k-t-1} & \text{rounding} \end{cases}$$

Question 4.

Question 5.