For $A \in M^{n\times n}$, a matrix norm $\| \ \| : \mathbb{R}^{n\times n} \to \mathbb{R}$ satisfies the following properties:

1. $\|A\| > 0$ if $A \neq 0$
2. $\|\alpha A\| = |\alpha| \ \|A\|, \ \alpha \in \mathbb{R}$
3. $\|A + B\| \leq \|A\| + \|B\|$ for $A, \ B \in \mathbb{R}^{n\times n}$
4. $\|AB\| \leq \|A\|\|B\|$ for $A, \ B \in \mathbb{R}^{n\times n}$
5. $\|A\vec{x}\| \leq \|A\|\|\vec{x}\|$ for $A \in \mathbb{R}^{n\times n}, \ \vec{x} \in \mathbb{R}^n$

Example:

$\|A\| \stackrel{\text{def}}{\equiv} \overset{\max}{\text{j}} \ \Sigma_{i=1}^{n}|a_{ij}|$ (maximum absolute column sum)