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|  | |  |  |  | | --- | --- | --- | |  |  |  | | Министерство образования и науки РОССИЙСКОЙ ФЕДЕРАЦИИ | | |  Федеральное государственное бюджетное образовательное учреждение высшего образования  **«МИРЭА – Российский технологический университет»** | |
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ИНСТИТУТ КИБЕРНЕТИКИ

КАФЕДРА ВЫСШЕЙ МАТЕМАТИКИ

Лабораторная работа 3

 по курсу «**Случайные процессы**»

Тема: **Процесс роста и мутации популяции**

Выполнил:

Студент 4-го курса

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Группа: КМБО-03-17

МОСКВА 2020

**Задание**

В популяции могут находиться объекты двух видов: N-объекты и M-объекты.

**Дано:**

− время жизни каждого N-объекта является случайной величиной, имеющей показательное распределение с параметром (где − время рождения объекта,N(− число N-объектов до момента времени , M(− число M-объектов до момента времени , − заданный коэффициент);

− время жизни каждого M-объекта является случайной величиной, имеющей показательное распределение с параметром (где − время рождения объекта, N(− число N-объектов до момента времени , M( − число M-объектов до момента времени , − заданный коэффициент);

− по окончании времени жизни каждый N-объект порождает с вероятностью один N-объект (событие ), с вероятностью два N-объекта (событие ), с вероятностью один N-объект и один M-объект (событие );

− по окончании времени жизни каждый M-объект порождает с вероятностью один M-объект (событие ), ничего не порождает с вероятностью

(событие );

− до начального момента *t=0* не было объектов, в начальный момент происходит событие и появляется первый объект: N-объект.

Состояние системы в момент времени *t* характеризуется параметрами

(N(t), M(t)), где N(t) − число N-объектов, M(t) − число M-объектов. Событием в развитии системы называется момент окончания жизни (исчезновения) любого из объектов и (одновременно) появления новых объектов. События могут быть 5 типов: , , , , . При появлении каждого нового объекта случайным образом в соответствии с заданным законом распределения определяется время его жизни. Считать для первого события: момент наступления события ; тип события .

**Требуется:**

1. Провести моделирование первых 100 событий в развитии системы
2. Составить следующие таблицы:

Таблица 1 с данными о событиях:

- номер события *i*

- момент наступления события

- тип события *Type(i)*

- время жизни появившихся новых объектов (2 столбца)

- состояние системы после события *C(i)*

- время ожидания до следующего события

- номер объекта , у которого раньше закончится жизнь

- вид этого исчезающего объекта

Таблица 2 с данными об объектах:

- номер объекта *j*

- вид объекта *Gen(j)* (N или M)

- момент появления (рождения) объекта

- время жизни объекта

- момент исчезновения объекта

- номера объектов-потомков (2 столбца)

Таблица 3 с данными о типах событий.

Таблица 4 с данными о видах объектов.

Таблица 5 с данными о состояниях.

**Вычисления и вывод проводить с точностью до 0.00001.**



**Краткие теоретические сведения**

Процессом гибели и размножения называется марковский процесс с непрерывным временем, граф состояний которого:

… …

При этом число состояний может быть и бесконечным. Плотности называются «интенсивностями размножения», а плотности – «интенсивностями гибели».

Система уравнений Колмогорова для вероятностей состояний процесса гибели и размножения имеет вид:

*,*

*…*

*, (k=1, 2, …, n-1)*

*…*

*.*

Если число состояний n конечно, то для любого момента времени выполняется нормировочное условие

В том случае, когда у процесса гибели и размножения все интенсивности положительны , *i*=0, 1, …, n-1, , а число состояний n конечно, процесс является эргодическим и существуют предельные вероятности состояний :

, являющиеся также и стационарными вероятностями.

**Результаты расчетов**

Вариант 53

Таблица 1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *№* |  | *Type* |  |  | *С* |  |  |  |
| 1 | 0 | Sn(1) | 0.206189 | -1 | [1, 0] | 0.206189 | 1 | N |
| 2 | 0.206189 | Sn(2) | 0.054377 | 0.161204 | [2, 0] | 0.054377 | 2 | N |
| 3 | 0.260566 | Sn(1) | 0.085872 | -1 | [2, 0] | 0.085872 | 4 | N |
| 4 | 0.346437 | Sn(3) | 0.008643 | 3.361671 | [2, 1] | 0.008643 | 5 | N |
| 5 | 0.35508 | Sn(3) | 0.107597 | 1.394866 | [2, 2] | 0.012313 | 3 | N |
| 6 | 0.367393 | Sn(2) | 2.411342 | 2.992401 | [3, 2] | 0.095284 | 7 | N |
| 7 | 0.462677 | Sn(2) | 0.155501 | 1.143727 | [4, 2] | 0.155501 | 11 | N |
| 8 | 0.618178 | Sn(2) | 0.106061 | 0.135872 | [5, 2] | 0.106061 | 13 | N |
| 9 | 0.724239 | Sn(3) | 0.358739 | 5.489655 | [5, 3] | 0.02981 | 14 | N |
| 10 | 0.75405 | Sn(2) | 1.533305 | 9.356468 | [6, 3] | 0.328928 | 15 | N |
| 11 | 1.082978 | Sn(3) | 3.669077 | 12.266043 | [6, 4] | 0.523426 | 12 | N |
| 12 | 1.606404 | Sn(3) | 1.340193 | 3.596615 | [6, 5] | 0.143542 | 8 | M |
| 13 | 1.749946 | Sm(1) | 56.056971 | -1 | [6, 5] | 0.537409 | 17 | N |
| 14 | 2.287355 | Sn(2) | 3.298303 | 14.097377 | [7, 5] | 0.49138 | 9 | N |
| 15 | 2.778735 | Sn(1) | 0.054385 | -1 | [7, 5] | 0.054385 | 26 | N |
| 16 | 2.833119 | Sn(2) | 2.71224 | 8.025404 | [8, 5] | 0.113477 | 21 | N |
| 17 | 2.946597 | Sn(3) | 1.646076 | 13.357011 | [8, 6] | 0.413197 | 10 | N |
| 18 | 3.359793 | Sn(1) | 7.192626 | -1 | [8, 6] | 0.348315 | 6 | M |
| 19 | 3.708108 | Sm(0) | -1 | -1 | [8, 5] | 0.884564 | 29 | N |
| 20 | 4.592672 | Sn(2) | 1.579859 | 4.221517 | [9, 5] | 0.159382 | 19 | N |
| 21 | 4.752055 | Sn(1) | 1.868301 | -1 | [9, 5] | 0.450964 | 22 | M |
| 22 | 5.203019 | Sm(0) | -1 | -1 | [9, 4] | 0.34234 | 27 | N |
| 23 | 5.545359 | Sn(1) | 1.863382 | -1 | [9, 4] | 0.040298 | 24 | N |
| 24 | 5.585658 | Sn(2) | 2.5524 | 10.430205 | [10, 4] | 0.586874 | 32 | N |
| 25 | 6.172532 | Sn(2) | 0.212213 | 1.172475 | [11, 4] | 0.041362 | 16 | M |
| 26 | 6.213894 | Sm(0) | -1 | -1 | [11, 3] | 0.170852 | 38 | N |
| 27 | 6.384745 | Sn(2) | 0.244626 | 1.036747 | [12, 3] | 0.23561 | 34 | N |
| 28 | 6.620356 | Sn(3) | 0.879072 | 10.847415 | [12, 4] | 0.009016 | 40 | N |
| 29 | 6.629372 | Sn(2) | 3.602696 | 5.558645 | [13, 4] | 0.715635 | 39 | N |
| 30 | 7.345007 | Sn(2) | 7.473098 | 9.412245 | [14, 4] | 0.063735 | 35 | N |
| 31 | 7.408741 | Sn(1) | 3.986151 | -1 | [14, 4] | 0.012751 | 41 | N |
| 32 | 7.421492 | Sn(3) | 14.612164 | 42.901971 | [14, 5] | 0.077936 | 42 | N |
| 33 | 7.499428 | Sn(2) | 1.88375 | 10.929955 | [15, 5] | 0.63863 | 36 | N |
| 34 | 8.138058 | Sn(2) | 0.265984 | 8.452041 | [16, 5] | 0.265984 | 53 | N |
| 35 | 8.404041 | Sn(3) | 1.359773 | 15.820679 | [16, 6] | 0.410148 | 33 | N |
| 36 | 8.814189 | Sn(2) | 2.04182 | 4.825417 | [17, 6] | 0.568989 | 51 | N |
| 37 | 9.383178 | Sn(3) | 10.439818 | 11.941484 | [17, 7] | 0.380636 | 55 | N |
| 38 | 9.763814 | Sn(1) | 19.070684 | -1 | [17, 7] | 0.346704 | 18 | N |
| 39 | 10.110518 | Sn(3) | 1.025829 | 6.797376 | [17, 8] | 0.121549 | 44 | N |
| 40 | 10.232067 | Sn(2) | 2.183925 | 2.88984 | [18, 8] | 0.320352 | 31 | N |
| 41 | 10.552419 | Sn(2) | 3.77837 | 11.27552 | [19, 8] | 0.30359 | 57 | N |
| 42 | 10.856009 | Sn(2) | 1.097945 | 11.448161 | [20, 8] | 0.002514 | 28 | N |
| 43 | 10.858523 | Sn(2) | 3.319197 | 10.290397 | [21, 8] | 0.277824 | 62 | N |
| 44 | 11.136346 | Sn(2) | 9.883221 | 34.454999 | [22, 8] | 0.258546 | 48 | N |
| 45 | 11.394892 | Sn(2) | 1.420704 | 15.247175 | [23, 8] | 0.559061 | 68 | N |
| 46 | 11.953954 | Sn(1) | 12.213208 | -1 | [23, 8] | 0.234063 | 45 | N |
| 47 | 12.188016 | Sn(2) | 0.064408 | 6.14989 | [24, 8] | 0.064408 | 77 | N |
| 48 | 12.252425 | Sn(2) | 1.069754 | 30.780368 | [25, 8] | 0.163567 | 64 | N |
| 49 | 12.415992 | Sn(1) | 4.503503 | -1 | [25, 8] | 0.399604 | 74 | N |
| 50 | 12.815596 | Sn(2) | 0.588826 | 1.778053 | [26, 8] | 0.306311 | 65 | N |
| 51 | 13.121908 | Sn(3) | 20.089594 | 28.976933 | [26, 9] | 0.200271 | 79 | N |
| 52 | 13.322179 | Sn(1) | 8.695361 | -1 | [26, 9] | 0.026842 | 20 | M |
| 53 | 13.349021 | Sm(0) | -1 | -1 | [26, 8] | 0.055402 | 82 | N |
| 54 | 13.404423 | Sn(3) | 14.391749 | 36.186058 | [26, 9] | 0.235183 | 58 | N |
| 55 | 13.639606 | Sn(2) | 1.491109 | 3.701856 | [27, 9] | 0.538114 | 70 | N |
| 56 | 14.17772 | Sn(2) | 12.630432 | 15.003907 | [28, 9] | 0.153069 | 66 | N |
| 57 | 14.330789 | Sn(2) | 1.682794 | 13.128178 | [29, 9] | 0.26286 | 83 | N |
| 58 | 14.59365 | Sn(3) | 20.363959 | 83.862028 | [29, 10] | 0.224456 | 46 | N |
| 59 | 14.818105 | Sn(2) | 14.853872 | 20.923917 | [30, 10] | 0.312609 | 89 | N |
| 60 | 15.130715 | Sn(2) | 9.759567 | 32.574186 | [31, 10] | 0.882868 | 93 | N |
| 61 | 16.013583 | Sn(3) | 18.467709 | 84.283359 | [31, 11] | 0.00228 | 37 | N |
| 62 | 16.015863 | Sn(3) | 2.07544 | 7.526701 | [31, 12] | 0.287745 | 30 | M |
| 63 | 16.303607 | Sm(1) | 29.911288 | -1 | [31, 12] | 0.081125 | 25 | N |
| 64 | 16.384732 | Sn(1) | 9.602625 | -1 | [31, 12] | 0.205367 | 54 | N |
| 65 | 16.590099 | Sn(3) | 0.076722 | 52.108137 | [31, 13] | 0.076722 | 107 | M |
| 66 | 16.666821 | Sm(0) | -1 | -1 | [31, 12] | 0.090431 | 47 | N |
| 67 | 16.757252 | Sn(2) | 3.508321 | 14.488681 | [32, 12] | 0.150642 | 63 | M |
| 68 | 16.907894 | Sm(1) | 22.454861 | -1 | [32, 12] | 0.011601 | 81 | N |
| 69 | 16.919494 | Sn(3) | 6.284991 | 37.864476 | [32, 13] | 0.421967 | 90 | N |
| 70 | 17.341461 | Sn(2) | 2.419711 | 3.933621 | [33, 13] | 0.12631 | 43 | M |
| 71 | 17.467771 | Sm(1) | 62.637864 | -1 | [33, 13] | 0.623531 | 103 | M |
| 72 | 18.091302 | Sm(1) | 123.565407 | -1 | [33, 13] | 0.246604 | 78 | N |
| 73 | 18.337906 | Sn(2) | 3.295673 | 6.438511 | [34, 13] | 0.091477 | 52 | N |
| 74 | 18.429383 | Sn(2) | 3.390115 | 5.469626 | [35, 13] | 1.33179 | 114 | N |
| 75 | 19.761172 | Sn(1) | 1.564432 | -1 | [35, 13] | 0.061824 | 59 | M |
| 76 | 19.822996 | Sm(1) | 101.014452 | -1 | [35, 13] | 0.442577 | 109 | N |
| 77 | 20.265573 | Sn(3) | 0.768168 | 66.725741 | [35, 14] | 0.753995 | 72 | N |
| 78 | 21.019568 | Sn(2) | 6.961696 | 29.942515 | [36, 14] | 0.014173 | 124 | N |
| 79 | 21.033741 | Sn(3) | 5.362173 | 46.856619 | [36, 15] | 0.115179 | 71 | N |
| 80 | 21.14892 | Sn(3) | 58.926534 | 101.97426 | [36, 16] | 0.126163 | 115 | N |
| 81 | 21.275083 | Sn(2) | 1.245539 | 5.452167 | [37, 16] | 0.049579 | 60 | N |
| 82 | 21.324662 | Sn(2) | 0.903126 | 32.143433 | [38, 16] | 0.000943 | 122 | N |
| 83 | 21.325604 | Sn(1) | 2.772199 | -1 | [38, 16] | 0.307975 | 118 | N |
| 84 | 21.63358 | Sn(2) | 5.26717 | 7.794674 | [39, 16] | 0.185918 | 120 | N |
| 85 | 21.819498 | Sn(1) | 18.723819 | -1 | [39, 16] | 0.008441 | 67 | N |
| 86 | 21.827939 | Sn(2) | 4.24532 | 19.819851 | [40, 16] | 0.189602 | 86 | N |
| 87 | 22.01754 | Sn(2) | 4.873996 | 46.845234 | [41, 16] | 0.016115 | 49 | N |
| 88 | 22.033656 | Sn(1) | 20.704848 | -1 | [41, 16] | 0.194131 | 134 | N |
| 89 | 22.227787 | Sn(2) | 6.993927 | 10.041391 | [42, 16] | 0.076383 | 69 | N |
| 90 | 22.30417 | Sn(2) | 8.265428 | 41.964225 | [43, 16] | 0.216452 | 132 | N |
| 91 | 22.520621 | Sn(2) | 26.982387 | 31.805485 | [44, 16] | 0.683864 | 112 | N |
| 92 | 23.204485 | Sn(2) | 1.231218 | 42.695504 | [45, 16] | 0.338079 | 104 | N |
| 93 | 23.542564 | Sn(3) | 27.549088 | 95.271574 | [45, 17] | 0.356445 | 121 | N |
| 94 | 23.899009 | Sn(3) | 7.07852 | 156.19363 | [45, 18] | 0.198794 | 136 | N |
| 95 | 24.097803 | Sn(3) | 6.167858 | 20.226408 | [45, 19] | 0.069359 | 76 | N |
| 96 | 24.167162 | Sn(2) | 21.989575 | 25.408794 | [46, 19] | 0.057559 | 56 | M |
| 97 | 24.22472 | Sm(1) | 10.839951 | -1 | [46, 19] | 0.210983 | 151 | N |
| 98 | 24.435703 | Sn(2) | 2.022141 | 55.923219 | [47, 19] | 0.340714 | 119 | N |
| 99 | 24.776417 | Sn(2) | 18.186956 | 29.948346 | [48, 19] | 0.113864 | 99 | N |
| 100 | 24.890281 | Sn(2) | 41.618401 | 52.111989 | [49, 19] | 1.097075 | 106 | N |

Таблица 2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *№* | *Gen* |  |  |  |  |  |
| 1 | N | 0 | 0.206189 | 0.206189 | 2 | 3 |
| 2 | N | 0.206189 | 0.054377 | 0.260566 | 4 | -1 |
| 3 | N | 0.206189 | 0.161204 | 0.367393 | 9 | 10 |
| 4 | N | 0.260566 | 0.085872 | 0.346437 | 5 | 6 |
| 5 | N | 0.346437 | 0.008643 | 0.35508 | 7 | 8 |
| 6 | M | 0.346437 | 3.361671 | 3.708108 | -1 | -1 |
| 7 | N | 0.35508 | 0.107597 | 0.462677 | 11 | 12 |
| 8 | M | 0.35508 | 1.394866 | 1.749946 | 23 | -1 |
| 9 | N | 0.367393 | 2.411342 | 2.778735 | 26 | -1 |
| 10 | N | 0.367393 | 2.992401 | 3.359793 | 31 | -1 |
| 11 | N | 0.462677 | 0.155501 | 0.618178 | 13 | 14 |
| 12 | N | 0.462677 | 1.143727 | 1.606404 | 21 | 22 |
| 13 | N | 0.618178 | 0.106061 | 0.724239 | 15 | 16 |
| 14 | N | 0.618178 | 0.135872 | 0.75405 | 17 | 18 |
| 15 | N | 0.724239 | 0.358739 | 1.082978 | 19 | 20 |
| 16 | M | 0.724239 | 5.489655 | 6.213894 | -1 | -1 |
| 17 | N | 0.75405 | 1.533305 | 2.287355 | 24 | 25 |
| 18 | N | 0.75405 | 9.356468 | 10.110518 | 62 | 63 |
| 19 | N | 1.082978 | 3.669077 | 4.752055 | 34 | -1 |
| 20 | M | 1.082978 | 12.266043 | 13.349021 | -1 | -1 |
| 21 | N | 1.606404 | 1.340193 | 2.946597 | 29 | 30 |
| 22 | M | 1.606404 | 3.596615 | 5.203019 | -1 | -1 |
| 23 | M | 1.749946 | 56.056971 | 57.806917 | -1 | -1 |
| 24 | N | 2.287355 | 3.298303 | 5.585658 | 36 | 37 |
| 25 | N | 2.287355 | 14.097377 | 16.384732 | 106 | -1 |
| 26 | N | 2.778735 | 0.054385 | 2.833119 | 27 | 28 |
| 27 | N | 2.833119 | 2.71224 | 5.545359 | 35 | -1 |
| 28 | N | 2.833119 | 8.025404 | 10.858523 | 70 | 71 |
| 29 | N | 2.946597 | 1.646076 | 4.592672 | 32 | 33 |
| 30 | M | 2.946597 | 13.357011 | 16.303607 | 105 | -1 |
| 31 | N | 3.359793 | 7.192626 | 10.552419 | 66 | 67 |
| 32 | N | 4.592672 | 1.579859 | 6.172532 | 38 | 39 |
| 33 | N | 4.592672 | 4.221517 | 8.814189 | 57 | 58 |
| 34 | N | 4.752055 | 1.868301 | 6.620356 | 42 | 43 |
| 35 | N | 5.545359 | 1.863382 | 7.408741 | 48 | -1 |
| 36 | N | 5.585658 | 2.5524 | 8.138058 | 53 | 54 |
| 37 | N | 5.585658 | 10.430205 | 16.015863 | 103 | 104 |
| 38 | N | 6.172532 | 0.212213 | 6.384745 | 40 | 41 |
| 39 | N | 6.172532 | 1.172475 | 7.345007 | 46 | 47 |
| 40 | N | 6.384745 | 0.244626 | 6.629372 | 44 | 45 |
| 41 | N | 6.384745 | 1.036747 | 7.421492 | 49 | 50 |
| 42 | N | 6.620356 | 0.879072 | 7.499428 | 51 | 52 |
| 43 | M | 6.620356 | 10.847415 | 17.467771 | 116 | -1 |
| 44 | N | 6.629372 | 3.602696 | 10.232067 | 64 | 65 |
| 45 | N | 6.629372 | 5.558645 | 12.188016 | 77 | 78 |
| 46 | N | 7.345007 | 7.473098 | 14.818105 | 97 | 98 |
| 47 | N | 7.345007 | 9.412245 | 16.757252 | 109 | 110 |
| 48 | N | 7.408741 | 3.986151 | 11.394892 | 74 | 75 |
| 49 | N | 7.421492 | 14.612164 | 22.033656 | 144 | -1 |
| 50 | M | 7.421492 | 42.901971 | 50.323463 | -1 | -1 |
| 51 | N | 7.499428 | 1.88375 | 9.383178 | 59 | 60 |
| 52 | N | 7.499428 | 10.929955 | 18.429383 | 120 | 121 |
| 53 | N | 8.138058 | 0.265984 | 8.404041 | 55 | 56 |
| 54 | N | 8.138058 | 8.452041 | 16.590099 | 107 | 108 |
| 55 | N | 8.404041 | 1.359773 | 9.763814 | 61 | -1 |
| 56 | M | 8.404041 | 15.820679 | 24.22472 | 161 | -1 |
| 57 | N | 8.814189 | 2.04182 | 10.856009 | 68 | 69 |
| 58 | N | 8.814189 | 4.825417 | 13.639606 | 89 | 90 |
| 59 | M | 9.383178 | 10.439818 | 19.822996 | 123 | -1 |
| 60 | N | 9.383178 | 11.941484 | 21.324662 | 134 | 135 |
| 61 | N | 9.763814 | 19.070684 | 28.834499 | -1 | -1 |
| 62 | N | 10.110518 | 1.025829 | 11.136346 | 72 | 73 |
| 63 | M | 10.110518 | 6.797376 | 16.907894 | 111 | -1 |
| 64 | N | 10.232067 | 2.183925 | 12.415992 | 81 | -1 |
| 65 | N | 10.232067 | 2.88984 | 13.121908 | 84 | 85 |
| 66 | N | 10.552419 | 3.77837 | 14.330789 | 93 | 94 |
| 67 | N | 10.552419 | 11.27552 | 21.827939 | 140 | 141 |
| 68 | N | 10.856009 | 1.097945 | 11.953954 | 76 | -1 |
| 69 | N | 10.856009 | 11.448161 | 22.30417 | 147 | 148 |
| 70 | N | 10.858523 | 3.319197 | 14.17772 | 91 | 92 |
| 71 | N | 10.858523 | 10.290397 | 21.14892 | 130 | 131 |
| 72 | N | 11.136346 | 9.883221 | 21.019568 | 126 | 127 |
| 73 | N | 11.136346 | 34.454999 | 45.591345 | -1 | -1 |
| 74 | N | 11.394892 | 1.420704 | 12.815596 | 82 | 83 |
| 75 | N | 11.394892 | 15.247175 | 26.642067 | -1 | -1 |
| 76 | N | 11.953954 | 12.213208 | 24.167162 | 159 | 160 |
| 77 | N | 12.188016 | 0.064408 | 12.252425 | 79 | 80 |
| 78 | N | 12.188016 | 6.14989 | 18.337906 | 118 | 119 |
| 79 | N | 12.252425 | 1.069754 | 13.322179 | 86 | -1 |
| 80 | N | 12.252425 | 30.780368 | 43.032793 | -1 | -1 |
| 81 | N | 12.415992 | 4.503503 | 16.919494 | 112 | 113 |
| 82 | N | 12.815596 | 0.588826 | 13.404423 | 87 | 88 |
| 83 | N | 12.815596 | 1.778053 | 14.59365 | 95 | 96 |
| 84 | N | 13.121908 | 20.089594 | 33.211502 | -1 | -1 |
| 85 | M | 13.121908 | 28.976933 | 42.098841 | -1 | -1 |
| 86 | N | 13.322179 | 8.695361 | 22.01754 | 142 | 143 |
| 87 | N | 13.404423 | 14.391749 | 27.796171 | -1 | -1 |
| 88 | M | 13.404423 | 36.186058 | 49.590481 | -1 | -1 |
| 89 | N | 13.639606 | 1.491109 | 15.130715 | 99 | 100 |
| 90 | N | 13.639606 | 3.701856 | 17.341461 | 114 | 115 |
| 91 | N | 14.17772 | 12.630432 | 26.808152 | -1 | -1 |
| 92 | N | 14.17772 | 15.003907 | 29.181627 | -1 | -1 |
| 93 | N | 14.330789 | 1.682794 | 16.013583 | 101 | 102 |
| 94 | N | 14.330789 | 13.128178 | 27.458967 | -1 | -1 |
| 95 | N | 14.59365 | 20.363959 | 34.957608 | -1 | -1 |
| 96 | M | 14.59365 | 83.862028 | 98.455678 | -1 | -1 |
| 97 | N | 14.818105 | 14.853872 | 29.671977 | -1 | -1 |
| 98 | N | 14.818105 | 20.923917 | 35.742023 | -1 | -1 |
| 99 | N | 15.130715 | 9.759567 | 24.890281 | 166 | 167 |
| 100 | N | 15.130715 | 32.574186 | 47.704901 | -1 | -1 |
| 101 | M | 16.013583 | 18.467709 | 34.481292 | -1 | -1 |
| 102 | N | 16.013583 | 84.283359 | 100.296942 | -1 | -1 |
| 103 | M | 16.015863 | 2.07544 | 18.091302 | 117 | -1 |
| 104 | N | 16.015863 | 7.526701 | 23.542564 | 153 | 154 |
| 105 | M | 16.303607 | 29.911288 | 46.214895 | -1 | -1 |
| 106 | N | 16.384732 | 9.602625 | 25.987357 | -1 | -1 |
| 107 | M | 16.590099 | 0.076722 | 16.666821 | -1 | -1 |
| 108 | N | 16.590099 | 52.108137 | 68.698236 | -1 | -1 |
| 109 | N | 16.757252 | 3.508321 | 20.265573 | 124 | 125 |
| 110 | N | 16.757252 | 14.488681 | 31.245933 | -1 | -1 |
| 111 | M | 16.907894 | 22.454861 | 39.362755 | -1 | -1 |
| 112 | N | 16.919494 | 6.284991 | 23.204485 | 151 | 152 |
| 113 | M | 16.919494 | 37.864476 | 54.783971 | -1 | -1 |
| 114 | N | 17.341461 | 2.419711 | 19.761172 | 122 | -1 |
| 115 | N | 17.341461 | 3.933621 | 21.275083 | 132 | 133 |
| 116 | M | 17.467771 | 62.637864 | 80.105636 | -1 | -1 |
| 117 | M | 18.091302 | 123.565407 | 141.65671 | -1 | -1 |
| 118 | N | 18.337906 | 3.295673 | 21.63358 | 137 | 138 |
| 119 | N | 18.337906 | 6.438511 | 24.776417 | 164 | 165 |
| 120 | N | 18.429383 | 3.390115 | 21.819498 | 139 | -1 |
| 121 | N | 18.429383 | 5.469626 | 23.899009 | 155 | 156 |
| 122 | N | 19.761172 | 1.564432 | 21.325604 | 136 | -1 |
| 123 | M | 19.822996 | 101.014452 | 120.837449 | -1 | -1 |
| 124 | N | 20.265573 | 0.768168 | 21.033741 | 128 | 129 |
| 125 | M | 20.265573 | 66.725741 | 86.991314 | -1 | -1 |
| 126 | N | 21.019568 | 6.961696 | 27.981264 | -1 | -1 |
| 127 | N | 21.019568 | 29.942515 | 50.962083 | -1 | -1 |
| 128 | N | 21.033741 | 5.362173 | 26.395914 | -1 | -1 |
| 129 | M | 21.033741 | 46.856619 | 67.89036 | -1 | -1 |
| 130 | N | 21.14892 | 58.926534 | 80.075454 | -1 | -1 |
| 131 | M | 21.14892 | 101.97426 | 123.12318 | -1 | -1 |
| 132 | N | 21.275083 | 1.245539 | 22.520621 | 149 | 150 |
| 133 | N | 21.275083 | 5.452167 | 26.72725 | -1 | -1 |
| 134 | N | 21.324662 | 0.903126 | 22.227787 | 145 | 146 |
| 135 | N | 21.324662 | 32.143433 | 53.468094 | -1 | -1 |
| 136 | N | 21.325604 | 2.772199 | 24.097803 | 157 | 158 |
| 137 | N | 21.63358 | 5.26717 | 26.900749 | -1 | -1 |
| 138 | N | 21.63358 | 7.794674 | 29.428254 | -1 | -1 |
| 139 | N | 21.819498 | 18.723819 | 40.543316 | -1 | -1 |
| 140 | N | 21.827939 | 4.24532 | 26.073259 | -1 | -1 |
| 141 | N | 21.827939 | 19.819851 | 41.647789 | -1 | -1 |
| 142 | N | 22.01754 | 4.873996 | 26.891537 | -1 | -1 |
| 143 | N | 22.01754 | 46.845234 | 68.862775 | -1 | -1 |
| 144 | N | 22.033656 | 20.704848 | 42.738504 | -1 | -1 |
| 145 | N | 22.227787 | 6.993927 | 29.221714 | -1 | -1 |
| 146 | N | 22.227787 | 10.041391 | 32.269178 | -1 | -1 |
| 147 | N | 22.30417 | 8.265428 | 30.569598 | -1 | -1 |
| 148 | N | 22.30417 | 41.964225 | 64.268395 | -1 | -1 |
| 149 | N | 22.520621 | 26.982387 | 49.503009 | -1 | -1 |
| 150 | N | 22.520621 | 31.805485 | 54.326107 | -1 | -1 |
| 151 | N | 23.204485 | 1.231218 | 24.435703 | 162 | 163 |
| 152 | N | 23.204485 | 42.695504 | 65.899989 | -1 | -1 |
| 153 | N | 23.542564 | 27.549088 | 51.091652 | -1 | -1 |
| 154 | M | 23.542564 | 95.271574 | 118.814138 | -1 | -1 |
| 155 | N | 23.899009 | 7.07852 | 30.977529 | -1 | -1 |
| 156 | M | 23.899009 | 156.19363 | 180.092638 | -1 | -1 |
| 157 | N | 24.097803 | 6.167858 | 30.265661 | -1 | -1 |
| 158 | M | 24.097803 | 20.226408 | 44.324211 | -1 | -1 |
| 159 | N | 24.167162 | 21.989575 | 46.156737 | -1 | -1 |
| 160 | N | 24.167162 | 25.408794 | 49.575955 | -1 | -1 |
| 161 | M | 24.22472 | 10.839951 | 35.064671 | -1 | -1 |
| 162 | N | 24.435703 | 2.022141 | 26.457844 | -1 | -1 |
| 163 | N | 24.435703 | 55.923219 | 80.358922 | -1 | -1 |
| 164 | N | 24.776417 | 18.186956 | 42.963374 | -1 | -1 |
| 165 | N | 24.776417 | 29.948346 | 54.724763 | -1 | -1 |
| 166 | N | 24.890281 | 41.618401 | 66.508682 | -1 | -1 |
| 167 | N | 24.890281 | 52.111989 | 77.002271 | -1 | -1 |

**Анализ результатов**

Таблица 3

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Тип события |  |  |  |  |  |  |
| Число событий | 16 | 48 | 24 | 5 | 7 | 100 |
| Относительная частота | 0.16 | 0.48 | 0.24 | 0.05 | 0.07 | 1.0 |

Таблица 4

|  |  |  |
| --- | --- | --- |
| Вид объекта | Число появившихся объектов за время [0, ] | Число объектов в момент |
| *N* | 136 | 49 |
| *M* | 31 | 19 |

Таблица 5

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Состояние |  |  |  |  |
| [1, 0] | 1 | 0.01 | 0.206189 | 0.008284 |
| [2, 0] | 2 | 0.02 | 0.140248 | 0.005635 |
| [2, 1] | 1 | 0.01 | 0.008643 | 0.000347 |
| [2, 2] | 1 | 0.01 | 0.012313 | 0.000495 |
| [3, 2] | 1 | 0.01 | 0.095284 | 0.003828 |
| [4, 2] | 1 | 0.01 | 0.155501 | 0.006247 |
| [5, 2] | 1 | 0.01 | 0.106061 | 0.004261 |
| [5, 3] | 1 | 0.01 | 0.02981 | 0.001198 |
| [6, 3] | 1 | 0.01 | 0.328928 | 0.013215 |
| [6, 4] | 1 | 0.01 | 0.523426 | 0.021029 |
| [6, 5] | 2 | 0.02 | 0.680951 | 0.027358 |
| [7, 5] | 2 | 0.02 | 0.545764 | 0.021927 |
| [8, 5] | 2 | 0.02 | 0.998042 | 0.040098 |
| [8, 6] | 2 | 0.02 | 0.761512 | 0.030595 |
| [9, 5] | 2 | 0.02 | 0.610346 | 0.024521 |
| [9, 4] | 2 | 0.02 | 0.382639 | 0.015373 |
| [10, 4] | 1 | 0.01 | 0.586874 | 0.023578 |
| [11, 4] | 1 | 0.01 | 0.041362 | 0.001662 |
| [11, 3] | 1 | 0.01 | 0.170852 | 0.006864 |
| [12, 3] | 1 | 0.01 | 0.23561 | 0.009466 |
| [12, 4] | 1 | 0.01 | 0.009016 | 0.000362 |
| [13, 4] | 1 | 0.01 | 0.715635 | 0.028752 |
| [14, 4] | 2 | 0.02 | 0.076485 | 0.003073 |
| [14, 5] | 1 | 0.01 | 0.077936 | 0.003131 |
| [15, 5] | 1 | 0.01 | 0.63863 | 0.025658 |
| [16, 5] | 1 | 0.01 | 0.265984 | 0.010686 |
| [16, 6] | 1 | 0.01 | 0.410148 | 0.016478 |
| [17, 6] | 1 | 0.01 | 0.568989 | 0.02286 |
| [17, 7] | 2 | 0.02 | 0.72734 | 0.029222 |
| [17, 8] | 1 | 0.01 | 0.121549 | 0.004883 |
| [18, 8] | 1 | 0.01 | 0.320352 | 0.012871 |
| [19, 8] | 1 | 0.01 | 0.30359 | 0.012197 |
| [20, 8] | 1 | 0.01 | 0.002514 | 0.000101 |
| [21, 8] | 1 | 0.01 | 0.277824 | 0.011162 |
| [22, 8] | 1 | 0.01 | 0.258546 | 0.010387 |
| [23, 8] | 2 | 0.02 | 0.793124 | 0.031865 |
| [24, 8] | 1 | 0.01 | 0.064408 | 0.002588 |
| [25, 8] | 2 | 0.02 | 0.563171 | 0.022626 |
| [26, 8] | 2 | 0.02 | 0.361713 | 0.014532 |
| [26, 9] | 3 | 0.03 | 0.462297 | 0.018573 |
| [27, 9] | 1 | 0.01 | 0.538114 | 0.021619 |
| [28, 9] | 1 | 0.01 | 0.153069 | 0.00615 |
| [29, 9] | 1 | 0.01 | 0.26286 | 0.010561 |
| [29, 10] | 1 | 0.01 | 0.224456 | 0.009018 |
| [30, 10] | 1 | 0.01 | 0.312609 | 0.012559 |
| [31, 10] | 1 | 0.01 | 0.882868 | 0.03547 |
| [31, 11] | 1 | 0.01 | 0.00228 | 0.000092 |
| [31, 12] | 4 | 0.04 | 0.664667 | 0.026704 |
| [31, 13] | 1 | 0.01 | 0.076722 | 0.003082 |
| [32, 12] | 2 | 0.02 | 0.162243 | 0.006518 |
| [32, 13] | 1 | 0.01 | 0.421967 | 0.016953 |
| [33, 13] | 3 | 0.03 | 0.996445 | 0.040033 |
| [34, 13] | 1 | 0.01 | 0.091477 | 0.003675 |
| [35, 13] | 3 | 0.03 | 1.83619 | 0.073771 |
| [35, 14] | 1 | 0.01 | 0.753995 | 0.030293 |
| [36, 14] | 1 | 0.01 | 0.014173 | 0.000569 |
| [36, 15] | 1 | 0.01 | 0.115179 | 0.004627 |
| [36, 16] | 1 | 0.01 | 0.126163 | 0.005069 |
| [37, 16] | 1 | 0.01 | 0.049579 | 0.001992 |
| [38, 16] | 2 | 0.02 | 0.308918 | 0.012411 |
| [39, 16] | 2 | 0.02 | 0.194359 | 0.007809 |
| [40, 16] | 1 | 0.01 | 0.189602 | 0.007618 |
| [41, 16] | 2 | 0.02 | 0.210247 | 0.008447 |
| [42, 16] | 1 | 0.01 | 0.076383 | 0.003069 |
| [43, 16] | 1 | 0.01 | 0.216452 | 0.008696 |
| [44, 16] | 1 | 0.01 | 0.683864 | 0.027475 |
| [45, 16] | 1 | 0.01 | 0.338079 | 0.013583 |
| [45, 17] | 1 | 0.01 | 0.356445 | 0.014321 |
| [45, 18] | 1 | 0.01 | 0.198794 | 0.007987 |
| [45, 19] | 1 | 0.01 | 0.069359 | 0.002787 |
| [46, 19] | 2 | 0.02 | 0.268541 | 0.010789 |
| [47, 19] | 1 | 0.01 | 0.340714 | 0.013689 |
| [48, 19] | 1 | 0.01 | 0.113864 | 0.004575 |
| [49, 19] | 1 | 0.01 | 1.097075 | 0.044076 |
|  | 100 | 1.0 | 25.987357 | 1.044076 |

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**Приложение**

**import** copy

**import** math

**import** numpy

**def** get\_next\_obj(table2, prev\_event\_time, deaths\_nums):

num, typee, death = None, None, None

**for** obj\_num, obj\_type, birth\_time, live\_time, death\_time, des1, des2 **in** table2:

**if** **not**(obj\_num **in** deaths\_nums) **and** death\_time > prev\_event\_time:

**if** (death **is** None) **or** death\_time < death:

num, typee, death = obj\_num, obj\_type, death\_time

**return** death, num, typee

pn1, pn2, pm1, gamma1, gamma2 = 0.223, 0.521, 0.443, 0.21, 0.5

pn11 = 1 - pn1 - pn2

pm0 = 1 - pm1

table1, table2, table3, table4, table5 = [], [], [], [], []

events\_num = 1

event\_time = 0

system\_state = [1, 0]

max\_object\_num = 1

event\_type = "Sn(1)"

object\_live\_time = numpy.random.exponential(0.1)

wait\_time = object\_live\_time

table1.append([events\_num, event\_time, event\_type, object\_live\_time, -1, str(system\_state), wait\_time, 1, "N"])

table2.append([max\_object\_num, "N", 0, object\_live\_time, object\_live\_time, -1, -1])

deaths = []

**while** events\_num < 100:

events\_num += 1

event\_time += table1[-1][-3]

prev\_event\_type = table1[-1][2]

death\_object\_num = table1[-1][-2]

death\_object\_type = table1[-1][-1]

deaths.append(death\_object\_num)

**if** death\_object\_type == "N":

w = numpy.random.random\_sample()

event\_type = 0

**if** w < pn1:

event\_type = "Sn(1)"

**elif** w < pn1 + pn2:

event\_type = "Sn(2)"

**else**:

event\_type = "Sn(3)"

**if** event\_type == "Sn(3)":

max\_object\_num += 2

tn = numpy.random.exponential(gamma1 \* system\_state[0] + 2 \* gamma1 \* system\_state[1] + 0.1)

tm = numpy.random.exponential(3 \* gamma2 \* system\_state[0] + gamma2 \* system\_state[1])

system\_state[1] += 1

table2[death\_object\_num-1][-2] = max\_object\_num - 1

table2[death\_object\_num-1][-1] = max\_object\_num

**if** tm < tn:

table2.append([max\_object\_num - 1, "M", event\_time, tm, event\_time + tm, -1, -1])

table2.append([max\_object\_num, "N", event\_time, tn, event\_time + tn, -1, -1])

next\_event\_time, next\_obj\_num, next\_obj\_type = get\_next\_obj(table2, event\_time, deaths)

table1.append([events\_num, event\_time, "Sn(3)", tm, tn, str(system\_state), next\_event\_time - event\_time, next\_obj\_num, next\_obj\_type])

**else**:

table2.append([max\_object\_num - 1, "N", event\_time, tn, event\_time + tn, -1, -1])

table2.append([max\_object\_num, "M", event\_time, tm, event\_time + tm, -1, -1])

next\_event\_time, next\_obj\_num, next\_obj\_type = get\_next\_obj(table2, event\_time, deaths)

table1.append([events\_num, event\_time, "Sn(3)", tn, tm, str(system\_state), next\_event\_time - event\_time, next\_obj\_num, next\_obj\_type])

**elif** event\_type == "Sn(2)":

max\_object\_num += 2

tn1 = numpy.random.exponential(gamma1 \* system\_state[0] + 2 \* gamma1 \* system\_state[1] + 0.1)

tn2 = numpy.random.exponential(gamma1 \* system\_state[0] + 2 \* gamma1 \* system\_state[1] + 0.1)

system\_state[0] += 1

table2[death\_object\_num-1][-2] = max\_object\_num - 1

table2[death\_object\_num-1][-1] = max\_object\_num

**if** tn1 < tn2:

table2.append([max\_object\_num - 1, "N", event\_time, tn1, event\_time + tn1, -1, -1])

table2.append([max\_object\_num, "N", event\_time, tn2, event\_time + tn2, -1, -1])

next\_event\_time, next\_obj\_num, next\_obj\_type = get\_next\_obj(table2, event\_time, deaths)

table1.append([events\_num, event\_time, "Sn(2)", tn1, tn2, str(system\_state), next\_event\_time - event\_time, next\_obj\_num, next\_obj\_type])

**else**:

table2.append([max\_object\_num-1, "N", event\_time, tn2, event\_time + tn2, -1, -1])

table2.append([max\_object\_num, "N", event\_time, tn1, event\_time + tn1, -1, -1])

next\_event\_time, next\_obj\_num, next\_obj\_type = get\_next\_obj(table2, event\_time, deaths)

table1.append([events\_num, event\_time, "Sn(2)", tn2, tn1, str(system\_state), next\_event\_time - event\_time, next\_obj\_num, next\_obj\_type])

**else**:

max\_object\_num += 1

tn = numpy.random.exponential(gamma1 \* system\_state[0] + 2 \* gamma1 \* system\_state[1] + 0.1)

table2[death\_object\_num-1][-2] = max\_object\_num

table2.append([max\_object\_num, "N", event\_time, tn, event\_time + tn, -1, -1])

next\_event\_time, next\_obj\_num, next\_obj\_type = get\_next\_obj(table2, event\_time, deaths)

table1.append([events\_num, event\_time, "Sn(1)", tn, -1, str(system\_state), next\_event\_time - event\_time, next\_obj\_num, next\_obj\_type])

**else**:

w = numpy.random.random\_sample()

event\_type = 0

**if** w < pm1:

event\_type = "Sm(1)"

**else**:

event\_type = "Sm(0)"

**if** event\_type == "Sm(1)":

max\_object\_num += 1

tm = numpy.random.exponential(3 \* gamma2 \* system\_state[0] + gamma2 \* system\_state[1])

table2[death\_object\_num-1][-2] = max\_object\_num

table2.append([max\_object\_num, "M", event\_time, tm, event\_time + tm, -1, -1])

next\_event\_time, next\_obj\_num, next\_obj\_type = get\_next\_obj(table2, event\_time, deaths)

table1.append([events\_num, event\_time, "Sm(1)", tm, -1, str(system\_state), next\_event\_time - event\_time, next\_obj\_num, next\_obj\_type])

**else**:

system\_state[1] -= 1

next\_event\_time, next\_obj\_num, next\_obj\_type = get\_next\_obj(table2, event\_time, deaths)

table1.append([events\_num, event\_time, "Sm(0)", -1, -1, str(system\_state), next\_event\_time - event\_time, next\_obj\_num, next\_obj\_type])

# print(table1)

**for** i **in** table1:

**print**(i)

**print**("///////////////////////////////////////////////////////////////////////////////////////")

**for** i **in** table2:

**print**(i)

t1 = [i[1] **for** i **in** table1[1:]]

t2 = list(sorted([i[4] **for** i **in** table2]))

count = 0

**for** i **in** range(len(t1)):

**if** t1[i] == t2[i]:

count += 1

**print**(t1[i], ' -> ', t2[i])

**print**(count)

table3.append([len(list(filter(**lambda** x: x[2] == "Sn(1)", table1)))])

table3[-1].append(len(list(filter(**lambda** x: x[2] == "Sn(2)", table1))))

table3[-1].append(len(list(filter(**lambda** x: x[2] == "Sn(3)", table1))))

table3[-1].append(len(list(filter(**lambda** x: x[2] == "Sm(0)", table1))))

table3[-1].append(len(list(filter(**lambda** x: x[2] == "Sm(1)", table1))))

table3[-1].append(sum(table3[0][:]))

table3.append([round(i / table3[0][-1], 6) **for** i **in** table3[0][:-1]])

table3[-1].append(sum(table3[1][:]))

**print**(table3)

table4.append([len(list(filter(**lambda** x: x[1] == "N", table2))), system\_state[0]])

table4.append([len(list(filter(**lambda** x: x[1] == "M", table2))), system\_state[1]])

**print**(table4)

states = {}

**for** i **in** table1:

**if** i[5] **in** states.keys():

states[i[5]] += 1

**else**:

states[i[5]] = 1

**print**(states)

sm1, sm2, sm3, sm4 = 0, 0, 0, 0

**for** i **in** states:

time = 0

**for** \_, \_, \_, \_, \_, state, wait, \_, \_ **in** table1:

**if** state == i:

time += wait

table5.append([i, states[i], states[i] / 100, time])

table5[-1].append(table5[-1][-1] / table1[-1][1])

sm1 += table5[-1][1]

sm2 += table5[-1][2]

sm3 += table5[-1][3]

sm4 += table5[-1][4]

table5.append(["!", sm1, sm2, sm3, sm4])

**for** i **in** table5:

**print**(i)

**print**(table5)