2021-09-07

Questions

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Q1.

Consider the experiment in which we record $M(t)$, the number of active calls at a telephone switch at time t, for each second over an interval of 15 minutes. Determine the state space and the index set of the stochastic process $\{M(t) : t \geq 0\}$.

Answer State space $S_X = \{0, 1, 2, ..., 900\}$
Index set $T_X = \{0, 1, 2, ...\}$

Q2.

6 green balls and 4 white balls are placed in two boxes A and B such that each box has 5 balls. At each stage, a ball is drawn at random from each box and two balls are interchanged.

(a) Let X_n denote the number of white balls in box A after the n^{th} draw. Find the state space and the index set of the stochastic process $\{X_n\}$.

(b) Let Y_n denote the number of green balls in box A after the n^{th} draw. Find the state space and the index set of the stochastic process $\{Y_n\}$.

Answer

part (a) State space $S_X = \{0, 1, 2, 3, 4\}$
Index set $T_X = \{0, 1, 2, ...\}$

part (b) State space $S_Y = \{0, 1, 2, 3, 4, 5\}$
Index set $T_Y = \{0, 1, 2, ...\}$

Q3.

A box contains 3 black and 7 white balls. At each trial, a ball is drawn randomly from the box. If it is white, it is put back into the box and if it is black, it is kept outside the box. Let X_n denote the number of black balls taken out of the box after the n^{th} trial. Find the state space and the index of the stochastic process $\{X_n\}$.

Answer State space $S_X = \{0, 1, 2, 3\}$
Index set $T_X = \{0, 1, 2, ...\}$

Q4.

Suppose a coin is tossed 15 times. Let X_n denote the total number of "heads" which appear up to the n^{th} toss. Find the state space and the index set of the stochastic process $\{X_n\}$.

Answer State space $S_X = \{0, 1, 2, ..., 15\}$
Index set $T_X = \{0, 1, 2, ..., 15\}$