

## 2. Common Stochastic Processes

### Common Stochastic Processes

A list of some special stochastic processes and their properties

#### The Bernoulli process

The stochastic process  $\{X_n : n = 1, 2, \dots\}$  is called a **Bernoulli process** with success probability  $p$  if,

1.  $X_1, X_2, \dots$  are independent
2.  $P(X_n = 1) = p, P(X_n = 0) = 1 - p = q$ . for all  $n$

The Bernoulli process  $\{X(n)\}$  has state space  $S = \{0, 1\}$  and index set  $T = \{1, 2, 3, \dots\}$

#### The Binomial process

Let  $\{X_n : n = 1, 2, \dots\}$  be a Bernoulli process with success probability  $p$  and let

$$S = \begin{cases} 0, & n = 0 \\ X_1 + X_2 + \dots, & n = 1, 2, \dots \end{cases}$$

Then  $S_n$  is the number of successes in the first  $n$  Bernoulli trials. Thus,

$$P(S_n = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k}, \quad k = 0, 1, \dots, n$$

The stochastic process  $\{S_n : n = 1, 2, \dots\}$  is called a **binomial process**. The state space of the binomial process  $\{S_n\}$  is  $S = \{1, 2, \dots, n\}$  while the index set is  $T = \{1, 2, \dots\}$