Shewart's Control Charts.

Successive units produced are independent.

Chart 1: p-chart (Fraction defective chart).

Subgroups each of size n $\Rightarrow$ $1, 2, \ldots, i, \ldots m$

NB: # Defective items (d) $\Rightarrow$ $d_1, d_2, \ldots d_i, \ldots d_m$.

Fraction defectives (p) $\Rightarrow$ $p_1 = \frac{d_1}{n}$; $p_2 = \frac{d_2}{n}, \ldots, p_i = \frac{d_i}{n}, \ldots, p_m = \frac{d_m}{n}$

Sample fraction non conforming 'p' is defined as numbers of non-conforming units in a sample to the total sample size (n).

$$d \overset{Bin}{\sim} (n, p)$$

$$E(d) = np$$

$$Var(d) = np(1-p)$$

If we choose a statistic $T = p$ ← Fraction Defective

$$\therefore E(T) = E(p) = E\left(\frac{d}{n}\right) = \frac{1}{n} \cdot np = p.$$

$$\text{Var}(T) = \text{var}(p) = \text{var}\left(\frac{d}{n}\right)$$

$$= \frac{np(1-p)}{n^2} = \frac{p(1-p)}{n}$$

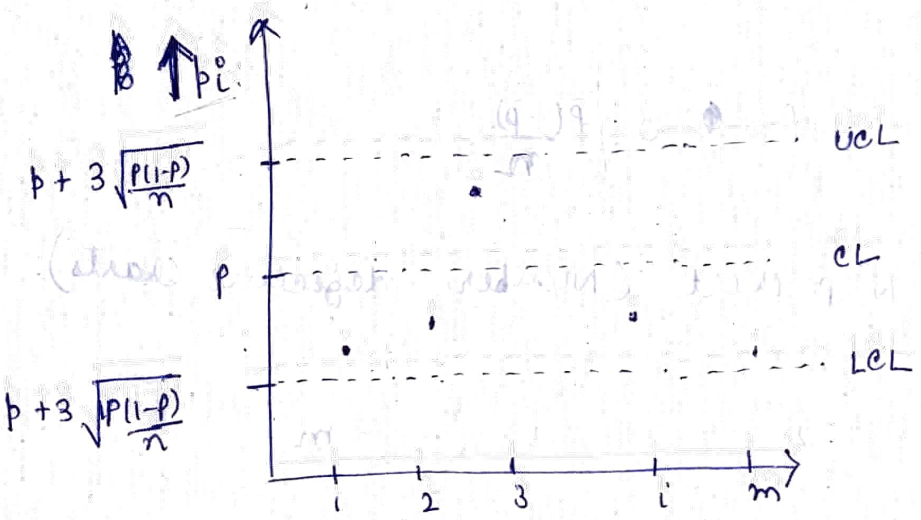
The proposed control chart is based on the following three control lines :-

$$LCL = \mu_T - 3\sigma_T = p - 3\sqrt{\frac{p(1-p)}{n}}$$

$$CL = \mu_T = p$$

$$UCL = \mu_T + 3\sigma_T = p + 3\sqrt{\frac{p(1-p)}{n}}$$

~~The~~ control chart (p chart).



Case 1 : Standard given :-

Let $p = p'$. Then the control chart is based on the following three lines ~~UCL,~~

$$UCL = p' - 3\sqrt{\frac{p'(1-p')}{n}}$$

$$CL = p'$$

$$LCL = p' + 3\sqrt{\frac{p'(1-p')}{n}}$$

Case II :- Standard not given:

The value of p is to be estimated from

$$\bar{p} = \frac{1}{m} \sum_{i=1}^{m} p_i$$

$$= \frac{1}{m} \sum_{i=1}^{m} \frac{d_i}{n}$$

$$= \frac{1}{nm} \sum_{i=1}^{m} d_i$$

Then the control chart is based on the follo
three lines :-

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$CL = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

art 2 :- N-p chart (Number defective charts).

Subgroups : $1, 2, \ldots, i, \ldots, m$

No of defective items : $d_1, d_2, \ldots, d_i, \ldots, d_m$.

Here our statistic is $T = d$.

No of defectives : $d \sim Bin(n, p)$