

1. Finite Difference Operators

Finite Differences

Given the function $y = f(x)$, x is called an **argument** and y is called an **entry**.

Here values of arguments are given at equal intervals

$$a, a + h, a + 2h, \dots, a + nh$$

Corresponding values of y are:

$$f(a), f(a + h), f(a + 2h), \dots, f(a + nh)$$

So we can write:

$$f(a + h) - f(a), f(a + 2h) - f(a + h), \dots, f(a + nh) - f(a + (n - 1)h)$$

such a representation is called **finite differences**

Finite Difference Operators

There are five difference operators, namely:

1. Forward Difference Operator Δ
2. Backward Difference Operator ∇
3. The Shifting Operator E
4. Central Difference Operator δ
5. The Averaging Operator μ

Forward Difference Operator Δ

Consider the function $y = f(x)$

Given the values of the function at points

$$x_0, x_1 = x_0 + h, x_2 = x_0 + 2h, \dots, x_n = x_0 + nh.$$

$$\text{Let } y_0 = f(x_0), y_1 = f(x_1), \dots, y_n = f(x_n).$$

We define

$$\Delta[f(x)] = f(x + h) - f(x)$$

Thus $\Delta y_0 = f(x_0 + h) - f(x_0) = f(x_1) - f(x_0) = y_1 - y_0 = \Delta f(x_0)$.

$$\therefore \Delta y_0 = y_1 - y_0$$

Further, x_0, x_1, \dots, x_n are called arguments. The corresponding $f(x)$ values are called entries and h is the interval of differencing.

Backward Difference Operator ∇

The Shifting Operator E

Central Difference Operator δ

The Averaging Operator μ