1. Intro

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Stochastic Process

A stochastic process is a family of random varibles $X(t)$ parameterized by an index t belonging to an index set T.

e.g. let $T = \{1, 2, ...\}$
then the collection $\{X(t) : t \in T\}$ is a stochastic process.

Index Set

The index set is the set T for which a stochastic process is defined as $\{X(t) : t \in T\}$.

If the set T is discrete (countable) then the stochastic process $\{X(t) : t \in T\}$ is called a **discrete time stochastic process**. In such cases we use n instead of t to denote the index.

If T is continous (uncountable) then $\{X(t) : t \in T\}$ is called **continous time stochastic process**.

State Space

Any possible value which the random variable $X(t)$ can take is called a **state** and the collection of all possible states of $X(t)$ is called the **state space** of the stochastic process $\{X(t) : t \in T\}$, denoted by S.

If the state space of $\{X(t) : t \in T\}$ is discrete then $\{X(t) : t \in T\}$ is called **discrete state stochastic process**. If the state space of $\{X(t) : t \in T\}$ is continous then $\{X(t) : t \in T\}$ is called **continous state stochastic process**.

Properties of Stochastic Processes

Types of stochastic processes characterized by dependence relationships among $X(t)$.

Process with independent increments

For a stochastic process $\{X(t)\}$. If the random variables $X(t_2)-X(t_1), X(t_3)-X(t_2), ...X(t_n)-X(t_{n-1})$, are independent for all $t_1, t_2, ...t_n$, satisfying $t_1 < t_2 < ... < t_n$, then $\{X(t)\}$ is a stochastic process with **independent increments**.

Process with Markovian property

$P($ future | present , past $) = P($ future | present $)$

For a stochastic process $\{X(t)\}$ with the **Markovian property**, future states $X(t_{i+1})$ depend only on the present state $X(t_i)$ and not the past states $X(t_{i-1}), X(t_{i-2}), ...X(t_0)$. A stochastic process is called a **Markov process** if it has the Markovian property. If the state space of a Markov process is discrete, it's called a **Markov chain**.

It can be seen that the stochastic process $\{X(t)\}$ has the Markovian property if, given the value of X_t, the values of X_s for $s > t$ are not affected by the values of X_u for $u < t$.

Formally, a stochastic process $\{X(t)\}$ is a Markov processes if

$$P(X_{n+1} = k_{n+1}|X_n = k_n, X_{n-1} = k_{n-1}, ...X_1 = k_1) = P(X_{n+1} = k_{n+1}|X_n = k_n)$$

Process with stationary increment

A stochastic process $\{X(t) : t \in T\}$ is said to have **stationary incerement** if the distribution of the increment $X(t_1 + h) - X(t_1)$ depends only on the length h of the interval and not on the time t.

For a stochastic process $\{X(t)\}$ with stationary increments, the distribution of $X(t)$ is the same for all t.