

2021-09-07

## Questions

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### Q1.

Consider the experiment in which we record  $M(t)$ , the number of active calls at a telephone switch at time  $t$ , for each second over an interval of 15 minutes. Determine the state space and the index set of the stochastic process  $\{M(t) : t \geq 0\}$

**Answer** State space  $S_X = \{0, 1, 2, \dots, 900\}$   
Index set  $T_X = \{0, 1, 2, \dots\}$

### Q2.

6 green balls and 4 white balls are placed in two boxes  $A$  and  $B$  such that each box has 5 balls. At each stage, a ball is drawn at random from each box and two balls are interchanged.

(a) Let  $X_n$  denote the number of white balls in box  $A$  after the  $n^{\text{th}}$  draw. Find the state space and the index set of the stochastic process  $\{X_n\}$

(b) Let  $Y_n$  denote the number of green balls in box  $A$  after the  $n^{\text{th}}$  draw. Find the state space and the index set of the stochastic process  $\{Y_n\}$

**Answer**

**part (a)** State space  $S_X = \{0, 1, 2, 3, 4\}$   
Index set  $T_X = \{0, 1, 2, \dots\}$

**part (b)** State space  $S_Y = \{0, 1, 2, 3, 4, 5\}$   
Index set  $T_Y = \{0, 1, 2, \dots\}$

**Q3.**

A box contains 3 black and 7 white balls. At each trial, a ball is drawn randomly from the box. If it is white, it is put back into the box and if it is black, it is kept outside the box. Let  $X_n$  denote the number of black balls taken out of the box after the  $n^{\text{th}}$  trial. Find the state space and the index of the stochastic process  $\{X_n\}$ .

**Answer** State space  $S_X = \{0, 1, 2, 3\}$   
Index set  $T_X = \{0, 1, 2, \dots\}$