1. Intro

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Stochastic Process

A stochastic process is a family of random varibles $X(t)$ parameterized by an index t belonging to an index set T.

e.g. let $T = \{1, 2, ...\}$
then the collection $\{X(t) : t \in T\}$ is a stochastic process

Index Set

The index set is the set T for which a stochastic process is defined as $\{X(t) : t \in T\}$

If the set T is discrete (countable) then the stochastic process $\{X(t) : t \in T\}$ is called a **discrete time stochastic process**. In such cases we use n instead of t to denote the index.

If T is continous (uncountable) then $\{X(t) : t \in T\}$ is called **continous time stochastic process**.

State Space

Any possible value which the random variable $X(t)$ can take is called a **state** and the collection of all possible states of $X(t)$ is called the **state space** of the stochastic process $\{X(t) : t \in T\}$, denoted by S

If the state space of $\{X(t) : t \in T\}$ is discrete then $\{X(t) : t \in T\}$ is called **discrete state stochastic process**. If the state space of $\{X(t) : t \in T\}$ is continous then $\{X(t) : t \in T\}$ is called **continous state stochastic process**.

Properties of Stochastic Processes

Types of stochastic processes characterized by dependence relationships among $X(t)$.

Process with independent increments

For a stochastic process $\{x(t)\}$. If the random variables $X(t_2) - X(t_1), X(t_3) - X(t_2), ...X(t_n) - X(t_{n-1})$, are independent for all $t_1, t_2, ...t_n$, satisfying $t_1 < t_2 < ... < t_n$, then $\{x(t)\}$ is a stochastic process with **independent increments**