2. Common Stochastic Processes

Common Stochastic Processes

A list of some special stochastic processes and their properties

The Bernoulli process

The stochastic process $\{X_n : n = 1, 2, ...\}$ is called a **Bernoulli process** with success probablity p if,

1. $X_1, X_2, ...$ are independent
2. $P(X_n = 1) = p.P(X_n = 0) = 1 - p = q.$ for all n

The Bernoulli process $\{X(n)\}$ has state space $S = \{0, 1\}$ and index set $T = \{1, 2, 3, ...\}$

The Binomial process

Let $\{X_n : n = 1, 2, ...\}$ be a Bernoulli process with success probability p and let

$$S = \begin{cases} 0, & n = 0 \\ X_1 + X_2 + ..., & n = 1, 2, ... \end{cases}$$

Then S_n is the number of successes in the first n Bernoulli trials. Thus,

$P\left(S_n = k\right) = \left(\frac{n}{k}\right) \cdot p^k \cdot (1-p)^{n-k}, \;\; k = 0, 1, ..., n$

The stochastic process $\{S_n : n = 1, 2, ...\}$ is called a **Binomial process**. The state space of the binomial process $\{S_n\}$ is $S = \{1, 2, ..., n\}$ while the index set is $T = \{1, 2, ...\}$

The Poisson process

A poisson process of intensity $\lambda > 0$ is an integer-valued stochastic process $\{N(t) : t \geq 0\}$ with the following properties:

1. $N(0) = 0$.
2. For $s > t, t > 0$, the random variable $N(t+s) - N(t)$ has a Poisson distribution with parameter λs.
3. $N(t)$ has independent increments.

The state space of the Poisson process $\{N(t) : t \geq 0\}$ is $S = \{n = 0, 1, 2, ...\}$. This is discrete and the index set T is continous. The Poisson process arises in many applications. especially as a model for the arrival at a service point, eg., arrival of customers to a bank, arrival of calls at a telephone exchange, etc.

Gaussian Process

A stochastic process $\{X(t) : t \geq 0\}$ is said to be Gaussian if the n-dimensional random variable $\{X(t_1), X(t_2), ..., X(t_n)\}$ has the multivariate normal distribution for all $n \geq 1$ and all $t_1, t_2, ..., t_n \in [0, \infty)$. This process has a continous state space and a continous index set. In electrical engineering, Gaussian process is used as a model for noise in recievers and noise voltages in resistors.

Weiner Process

A stochastic process $\{W(t) : t \geq 0\}$ is a **Weiner process** if it satisfies the following properties:

1. $W(0) = 0$.
2. $\{W(t)\}$ has stationary and independent increments.
3. For all $t > 0$, $W(t)$ is $N(0, c^2 t)$, where c is constant.

The process has continous state space and continous index set. It is sometimes called the **Brownian motion process** and has applications in quantum mechanics, diffusion phenomenon, economics, etc.