# 1. Intro

## Intro

### Stochastic Process

A stochastic process is a family of random varibles $X(t)$ parameterized by an index $t$ belonging to an index set $T$.

e.g. let $T = \{1, 2, ...\}$
then the collection $\{X(t) : t \in T\}$ is a stochastic process

### Index Set

The index set is the set $T$ for which a stochastic process is defined as $\{X(t) : t \in T\}$

If the set $T$ is discrete (countable) then the stochastic process $\{X(t) : t \in T\}$ is called a **discrete time stochastic process**. In such cases we use $n$ instead of $t$ to denote the index.

If $T$ is continous (uncountable) then $\{X(t) : t \in T\}$ is called **continous time stochastic process**.

### State Space

Any possible value which the random variable $X(t)$ can take is called a **state** and the collection of all possible states of $X(t)$ is called the **state space** of the stochastic process $\{X(t) : t \in T\}$, denoted by $S$

If the state space of $\{X(t) : t \in T\}$ is discrete then $\{X(t) : t \in T\}$ is called **discrete state stochastic process**. If the state space of $\{X(t) : t \in T\}$ is continous then $\{X(t) : t \in T\}$ is called **continous state stochastic process**.

### Properties of Stochastic Processes

Types of stochastic processes characterized by dependence relationships among $X(t)$.

### Process with independent increments

For a stochastic process $\{X(t)\}$. If the random variables $X(t_2) - X(t_1), X(t_3) - X(t_2), ...X(t_n) - X(t_{n-1})$, are independent for all $t_1, t_2, ...t_n$, satisfying $t_1 < t_2 < ... < t_n$, then $\{X(t)\}$ is a stochastic process with **independent increments**

### Process with Markovian property

$P(\text{ future } | \text{ present }, \text{ past }) = P(\text{ future } | \text{ present })$

For a stochastic process $\{X(t)\}$ with the Markovian property, future states $X(t_{i+1})$ depend only on the present state $X(t_i)$ and not the past states $X(t_{i-1}), X(t_{i-2}), ...X(t_0)$