3. Discrete Time Markov Chain

Discrete Time Markov Chain

Let $\{X_n, n \geq 0\}$ be a stochastic process taking values in a state space S that has N states. such a stochastic process is a Markov processes if it satisfies a following property:

$$P(X_{n+1}=k_{n+1}|X_n=k_n,X_{n-1}=k_{n-1},...X_1=k_1)=P(X_{n+1}=k_{n+1}|X_n=k_n)$$

If the state space of a Markov process is discrete, it's called a Markov chain.

To understand the behaviour of this process, we will need to calculate probabilities like,

$$P[X_0 = i_0, X_1 = i_1, ..., X_n = i_n]$$

..(1)

 $:P(A,B) = P(A) \cdot P(B|A)$, this can be computed by multiplying conditional probabilities as follows.

$$=P(X_0=i_0)\cdot P(X_1=i_1|X_0=i_0)\cdot P(X_2=i_2|X_1=i_1,X_0=i_0)...$$

$$P(X_n=i_n|X_{n-1}=i_{n-1},X_{n-2}=i_{n-2},...,X_0=i_0)$$

..(2)

