3. Method of Mathematical Curves

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1. Straight Line Trend (Linear Trend)

$$T_t = a + b \cdot t, \quad b \neq 0$$

2. Second Degree Polynomial (Parabolic Trend)

$$T_t = a + b \cdot t + c \cdot t^2, \quad c \neq 0$$

3. Exponential Curve

$$T_t = a \cdot b^t, \quad a > 0$$

4. Modified Exponential Curve

$$T_t = k + a \cdot b^t, \quad k > 0$$

5. Gompertz

$$T_t = k \cdot a^{b^t}, \quad k > 0$$

Linear Trend

Y_t = observed value of the time series at time t
$T_t = a + b \cdot t$ (Estimated from the graphical representation)

Method of Least Squares

$$S = \sum_t (Y_t - T_t)^2$$

[To be minimised w.r.t. a and b]

$$S = \sum_t (Y_t - a - b \cdot t)^2$$

$$\therefore \frac{\delta S}{\delta a} = 0$$

$$\Rightarrow -2\sum_t (Y_t - a - b \cdot t) = 0$$

$$\Rightarrow \sum_t Y_t = n \cdot a + b \sum_t t$$

..(1)

$$\therefore \frac{\delta S}{\delta b} = 0$$

$$\Rightarrow -2\sum_t (Y_t - a - b \cdot t) \cdot t = 0$$

$$\Rightarrow \sum_t t \cdot Y_t = a\sum_t t + b\sum_t t^2$$

..(2)

eq 1 anbd 2 are called the **normal equations**

Second Degree Polynomial (Parabolic Trend)

$$T_t = a + b \cdot t + c \cdot t^2$$

$$S = \sum_t (Y_t - a - b \cdot t - c \cdot t^2)^2$$

[To be minimised w.r.t. a, b and c]

Exponential Curve

Here the trend eq us given by

$$Y_t = a \cdot b^t, \quad a, b > 0$$

$\therefore \log Y_t = \log a + t \log b$
i.e. $\log Y_t$ is a linear function of t

Note that $\frac{Y_t}{Y_{t-1}} = b$, i.e. the exponential curve indicates a constant ratio of change

> **Note :** if $0 < b < 1$, the value of Y_t gradually decays but if $b > 1$, the value of Y_t gradually increases