2. Finite Difference Operators

Finite Diferences

Given the function $y = f(x)$, x is called called an **argument** and y is called an **entry**.

Here values of arguments are given at equal intervals
$a, a+h, a+2h, ..., a+nh$

Corresponding values of y are:
$f(a), f(a+h), f(a+2h), ..., f(a+nh)$

So we can write:
$f(a+h) - a, f(a+2h) - f(a+h), ..., f(a+nh) - f(a+(n-1)h)$

such a representation is called **fnite diferences**

Finite diference operators

There are five differnce operators, namely:

1. Forward Difference Operator Δ
2. Backward Difference Operator ∇
3. The Shifting Opetator E
4. Central Difference Operator δ
5. The Averaging Operator μ

Forward diference operator Δ

Consider the function $y = f(x)$

Given the values of the function at points
$x_0, x_1 = x_0 + h, x_2 = x_0 + 2h, ..., x_n = x_0 + nh$.

Let $y_0 = f(x_0), y_1 = f(x_1), ..., y_n = f(y_n)$.

We define

$$\Delta[f(x)] = f(x+h) - f(x)$$

Thus $\Delta y_0 = f(x_0 + h) - f(x_0) = f(x_1) - f(x_0) = y_1 - y_0 = \Delta f(x_0)$.

$$\therefore \Delta y_0 = y_1 - y_0$$

Further, $x_0, x_1, ..., x_n$ are called arguments. The corresponding $f(x)$ values are called entries and h is the interval of differencing.

Similarly,

$$\begin{gathered}\Delta y_0 = y_1 - y_0 \\ . \\ . \\ . \\ \Delta y_n = y_{n+1} - y_n\end{gathered}$$

Δ is known as the **forward dif erence operator** and $\Delta y_0, \Delta y_1, ..., \Delta y_{n-1}$ are called the **frst forward dif erence** of the function $y = f(x)$.

The second order differences of the function are defined by

$$\begin{gathered}\Delta^2 y_0 = \Delta y_1 - \Delta y_0 \\ \Delta^2 y_1 = \Delta y_2 - \Delta y_1 \\ . \\ . \\ . \\ \Delta^2 y_{n-1} = \Delta y_n - \Delta y_{n-1}\end{gathered}$$

Similarly, in general the n^{th} order differences are defined as

$$\Delta^n y_i = \Delta^{n-1} y_{i+1} - \Delta^{n-1} y_i$$

The differences of the function can be systematically represented in the form of a table called the **forward dif erence table**.

Backward dif erence operator ∇

Consider the function $y = f(x)$

Given the values of the function at points
$x_0, x_1 = x_0 + h, x_2 = x_0 + 2h, ..., x_n = x_0 + nh$.

Let $y_0 = f(x_0), y_1 = f(x_1), ..., y_n = f(y_n)$.

We define

$$\nabla[f(x)] = f(x) - f(x-h)$$

Thus $\nabla y_1 = f(x_1) - f(x_1 - h) = f(x_1) - f(x_0) = y_1 - y_0 = \nabla f(x_1)$.

$$\therefore \nabla y_1 = y_1 - y_0$$

Further, $x_0, x_1, ..., x_n$ are called arguments. The corresponding $f(x)$ values are called entries and h is the interval of differencing.

Similarly,

$$\begin{array}{c} \nabla y_2 = y_2 - y_1 \\ . \\ . \\ . \\ \nabla y_n = y_n - y_{n-1} \end{array}$$

∇ is known as the **backward diference operator** and $\nabla y_1, \nabla y_1, ..., \Delta y_n$ are called the **frst backward diference** of the function $y = f(x)$.

The second order differences of the function are defined by

$$\begin{array}{c} \nabla^2 y_2 = \nabla y_2 - \nabla y_1 \\ \nabla^2 y_3 = \nabla y_3 - \nabla y_2 \\ . \\ . \\ . \\ \nabla^2 y_n = \nabla y_n - \nabla y_{n-1} \end{array}$$

Similarly, in general the n^{th} order differences are defined as

$$\nabla^n y_i = \nabla^{n-1} y_i - \nabla^{n-1} y_{i-1}$$

The differences of the function can be systematically represented in the form of a table called the **backward diference table**.

Remark : The relation between the two difference operators is given by

$$\nabla[f(x+h)] = \Delta f(x)$$

Similarly, $\nabla^2[f(x+2h)] = \nabla[f(x+2h) - f(x+h)]$

The shifting opetator E

Central diference operator δ

The averaging operator μ