3. Discrete Time Markov Chain

Discrete Time Markov Chain

Let $\{X_n, n \geq 0\}$ be a stochastic process taking values in a state space S that has N states. such a stochastic process is a Markov processes if it satisfies a following property :

$$P(X_{n+1} = k_{n+1}|X_n = k_n, X_{n-1} = k_{n-1}, ...X_1 = k_1) = P(X_{n+1} = k_{n+1}|X_n = k_n)$$

If the state space of a Markov process is discrete, it's called a **Markov chain**.

To understand the behaviour of this process, we will need to calculate probabilities like,

$$P\left[X_0 = i_0, X_1 = i_1, ..., X_n = i_n\right]$$

..(1)

$\because P(A, B) = P(A) \cdot P(B|A)$, this can be computed by multiplying conditional probabilities as follows.

$$= P(X_0 = i_0) \cdot P(X_1 = i_1|X_0 = i_0) \cdot P(X_2 = i_2|X_1 = i_1, X_0 = i_0)...$$

$$P(X_n = i_n|X_{n-1} = i_{n-1}, X_{n-2} = i_{n-2}, ..., X_0 = i_0)$$

..(2)