Assignment 1

Stochastic Processes

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Q1.

Consider the experiment in which we record $M(t)$, the number of active calls at a telephone switch at time t, for each second over an interval of 15 minutes. Determine the state space and the index set of the stochastic process $\{M(t) : t \geq 0\}$.

Answer State space $S_X = \{0, 1, 2, ...\}$
Index set $T_X = \{0, 1, 2, ...\}$

Q2.

6 green balls and 4 white balls are placed in two boxes A and B such that each box has 5 balls. At each stage, a ball is drawn at random from each box and two balls are interchanged.

(a) Let X_n denote the number of white balls in box A after the n^{th} draw. Find the state space and the index set of the stochastic process $\{X_n\}$.

(b) Let Y_n denote the number of green balls in box A after the n^{th} draw. Find the state space and the index set of the stochastic process $\{Y_n\}$.

Answer

part (a) State space $S_X = \{0, 1, 2, 3, 4\}$
Index set $T_X = \{0, 1, 2, ...\}$

part (b) State space $S_Y = \{0, 1, 2, 3, 4, 5\}$
Index set $T_Y = \{0, 1, 2, ...\}$

Q3.

A box contains 3 black and 7 white balls. At each trial, a ball is drawn randomly from the box. If it is white, it is put back into the box and if it is black, it is kept outside the box. Let X_n denote the number of black balls taken out of the box after the n^{th} trial. Find the state space and the index of the stochastic process $\{X_n\}$.

Answer State space $S_X = \{0, 1, 2, 3\}$
Index set $T_X = \{0, 1, 2, ...\}$

Q4.

Suppose a coin is tossed 15 times. Let X_n denote the total number of "heads" which appear up to the n^{th} toss. Find the state space and the index set of the stochastic process $\{X_n\}$.

Answer State space $S_X = \{0, 1, 2, ..., 15\}$
Index set $T_X = \{0, 1, 2, ..., 15\}$

Q5.

Consider an acceptance sampling plan. Let D_n denote the number of defective items after inspecting n items. Find the state space and the index set of the stochastic process $\{D_n\}$.

Answer State space $S_D = \{0, 1, 2, ...\}$
Index set $T_D = \{0, 1, 2, ...\}$

Q6.

M coins are placed in a row on a table. At each stage, a coin is selected at random and turned over. Let X_n denote the total number of "heads" out of the M coins after the n^{th} trial. Find the state space and the index set of the stochastic process $\{X_n\}$.

Answer State space $S_X = \{0, 1, 2, ..., M\}$
Index set $T_X = \{0, 1, 2, ...\}$

Q7.

2 white balls and 5 green balls are placed in two boxes A and B so that box A contains 4 balls and box B contains 3 balls. At each stage, one ball is selected at random from each box and the two balls are interchanged. Let Y_n denote the number of white balls in box A at the n^{th} trial. Find the state space and the index set of the stochastic process $\{Y_n\}$.

Answer State space $S_Y = \{0, 1, 2\}$
Index set $T_Y = \{0, 1, 2, ...\}$

Q8.

John and Alice start a game with \$3 each as their capital. At the end of every game, the loser pays \$1 to the winner. With every game played, the probability of John winning is 0.6 and the probability of Alice winning is 0.4. They quit playing when one of them loses all his/her capital. Let C_n be the balance of John after n games. Find the state space and the index set of the stochastic process $\{C_n\}$.

Answer State space $S_C = \{0, 1, 2, 3, 4, 5, 6\}$
Index set $T_C = \{0, 1, 2, ...\}$