CALIFORNIA STATE UNIVERSITY SAN MARCOS
DR. DE LEONE, PHYSICS 323

H.W. 3

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1 STATE VECTORS

Consider the following state vectors:

$$|\psi_1\rangle = 2\,|+\rangle + 3\,|-\rangle\,; \quad |\psi_2\rangle = -3i\,|+\rangle + 2\,|-\rangle\,; \quad |\psi_3\rangle = |+\rangle + e^{i\frac{\pi}{4}}\,|-\rangle\,;$$

a) Calculate the inner product of $\langle\psi_2|\ \ with\ |\psi_1\rangle$

$$\begin{aligned}
\langle\psi_2|\psi_1\rangle &= -3i(2)\,\langle+|+\rangle + 2(3)\,\langle-|-\rangle \\
&= -6i + 6 \\
\langle\psi_2|\psi_2\rangle &= 6 - 6i
\end{aligned}$$

b) Normalize each state vector

$$\begin{aligned}
1 &= \langle\psi_1|\psi_1\rangle \\
&= C^*\big\{\,2\,\langle+| + 3\,\langle-|\,\big\}\,C\big\{\,2\,|+\rangle + 3\,|-\rangle\big\} \\
&= C^*C\big\{2^2\,\langle+|+\rangle + 2(3)\,\langle+|-\rangle + 3(2)\,\langle-|+\rangle + 3^2\,\langle-|-\rangle\big\} \\
1 &= 13|C|^2 \\
|C_{\psi_1}| &= \frac{1}{\sqrt{13}} \\
|\psi_1\rangle &= \frac{1}{\sqrt{13}}\big(2\,|+\rangle + 3\,|-\rangle\big)
\end{aligned}$$

$$
\begin{aligned}
1 &= \langle\psi_2|\psi_2\rangle \\
&= C^*\Big\{\, 3i\,\langle+| + 2\,\langle-|\ \Big\}\ C\Big\{\ -3i\,|+\rangle + 2\,|-\rangle\Big\} \\
&= C^*C\Big\{-9(i)^2\,\langle+|+\rangle + 3i(2)\,\langle+|-\rangle + 2(-3i)\,\langle-|+\rangle + 2^2\,\langle-|-\rangle\Big\} \\
&= C^*C\Big\{9+4\Big\} \\
1 &= 13|C|^2 \\
|C_{\psi_2}| &= \frac{1}{\sqrt{13}} \\
|\psi_2\rangle &= \frac{1}{\sqrt{13}}\Big(-3i\,|+\rangle + 2\,|-\rangle\Big)
\end{aligned}
$$

$$
\begin{aligned}
1 &= \langle\psi_3|\psi_3\rangle \\
&= C^*\Big\{\langle+| + e^{-i\frac{\pi}{4}}\,\langle-|\Big\}C\Big\{|+\rangle + e^{i\frac{\pi}{4}}\,|-\rangle\Big\} \\
&= C^*C\Big\{\langle+|+\rangle + e^{i\frac{\pi}{4}-i\frac{\pi}{4}}\,\langle-|-\rangle\Big\} \\
&= C^*C\Big\{1+1\Big\} \\
1 &= 2|C|^2 \\
|C_{\psi_3}| &= \frac{1}{\sqrt{2}} \\
|\psi_3\rangle &= \frac{1}{\sqrt{2}}\Big(|+\rangle + e^{i\frac{\pi}{4}}\,|-\rangle\Big)
\end{aligned}
$$

c) For each normalized state vector, use Postulate 4 to calculate the probability that the spin-component is up or down along the z-axis.

$$
\begin{aligned}
\mathcal{P}_+ &= |\,\langle+|\psi_1\rangle\,|^2 \\
&= |\,\langle+|\,\big[\tfrac{2}{\sqrt{13}}\,|+\rangle + \tfrac{3}{\sqrt{13}}\,|-\rangle\,\big]|^2 \\
&= |\,\tfrac{2}{\sqrt{13}}\,\langle+|+\rangle + \tfrac{3}{\sqrt{13}}\,\langle+|-\rangle\,|^2 \\
&= \Big|\tfrac{2}{\sqrt{13}}\Big|^2 \\
\mathcal{P}_+ &= \frac{4}{13}
\end{aligned}
\qquad
\begin{aligned}
\mathcal{P}_- &= |\,\langle-|\psi_1\rangle\,|^2 \\
&= |\,\langle-|\,\big[\tfrac{2}{\sqrt{13}}\,|+\rangle + \tfrac{3}{\sqrt{13}}\,|-\rangle\,\big]|^2 \\
&= |\,\tfrac{2}{\sqrt{13}}\,\langle-|+\rangle + \tfrac{3}{\sqrt{13}}\,\langle-|-\rangle\,|^2 \\
&= \Big|\tfrac{3}{\sqrt{13}}\Big|^2 \\
\mathcal{P}_- &= \frac{9}{13}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_+ &= |\,\langle+|\psi_2\rangle\,|^2 \\
&= |\,\langle+|\,\big[\tfrac{-3i}{\sqrt{13}}\,|+\rangle + \tfrac{2}{\sqrt{13}}\,|-\rangle\,\big]|^2 \\
&= |\,\tfrac{-3i}{\sqrt{13}}\,\langle+|+\rangle + \tfrac{2}{\sqrt{13}}\,\langle+|-\rangle\,|^2 \\
&= \Big|\tfrac{-3i}{\sqrt{13}}\Big|^2 \\
\mathcal{P}_+ &= \frac{9}{13}
\end{aligned}
\qquad
\begin{aligned}
\mathcal{P}_- &= |\,\langle-|\psi_2\rangle\,|^2 \\
&= |\,\langle-|\,\big[\tfrac{-3i}{\sqrt{13}}\,|+\rangle + \tfrac{2}{\sqrt{13}}\,|-\rangle\,\big]|^2 \\
&= |\,\tfrac{-3i}{\sqrt{13}}\,\langle-|+\rangle + \tfrac{2}{\sqrt{13}}\,\langle-|-\rangle\,|^2 \\
&= \Big|\tfrac{2}{\sqrt{13}}\Big|^2 \\
\mathcal{P}_- &= \frac{4}{13}
\end{aligned}
$$

$$\begin{aligned}
\mathcal{P}_+ &= |\langle+|\psi_3\rangle|^2 \\
&= |\langle+|\left[\tfrac{1}{\sqrt{2}}|+\rangle + \tfrac{e^{i\frac{\pi}{4}}}{\sqrt{2}}|-\rangle\right]|^2 \\
&= |\tfrac{1}{\sqrt{2}}\langle+|+\rangle + \tfrac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\langle+|-\rangle|^2 \\
&= \left|\frac{1}{\sqrt{2}}\right|^2 \\
\mathcal{P}_+ &= \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_- &= |\langle-|\psi_3\rangle|^2 \\
&= |\langle-|\left[\tfrac{1}{\sqrt{2}}|+\rangle + \tfrac{e^{i\frac{\pi}{4}}}{\sqrt{2}}|-\rangle\right]|^2 \\
&= |\tfrac{1}{\sqrt{2}}\langle-|+\rangle + \tfrac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\langle-|-\rangle|^2 \\
&= \left|\frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\right|^2 \\
\mathcal{P}_- &= \frac{1}{2}
\end{aligned}$$

d) Would you expect to find the same probabilities for the measured spin-components along the x- and y- axes?

We would not expect to find the same probabilities for the other axes. The probabilities in the orthogonal directions are independent of each other.

2 Phase of Quantum State Vector

Show that a change in the overall phase of a quantum state vector does not change the probability of obtaining a particular result in a measurement.

The phase of the quantum state vector is not physically measurable only the difference in the phases. The complex conjugate will ensure the magnitude is the same.

$$\begin{aligned}
\mathcal{P}_\pm &= |\langle\pm|\psi\rangle|^2 \\
\mathcal{P}_{\psi_\alpha} &= |\langle\pm|e^{i\alpha}\psi\rangle|^2 \\
&= |e^{i\alpha}\langle\pm|\psi\rangle|^2 \\
&= (e^{i\alpha}\langle\pm|\psi\rangle)(e^{-i\alpha}\langle\pm|\psi\rangle) \\
&= |(1)\langle\pm|\psi\rangle|^2 \\
\mathcal{P}_\pm &= \psi_\alpha \\
|\langle\pm|\psi\rangle|^2 &= |\langle\pm|\psi\rangle|^2
\end{aligned}$$