CALIFORNIA STATE UNIVERSITY SAN MARCOS
DR. DE LEONE, PHYSICS 323

H.W. 4

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PROBLEM 1.6

A beam of spin-1/2 particles is prepared in the state

$$|\psi\rangle = \frac{2}{\sqrt{13}}|+\rangle_x + i\frac{3}{\sqrt{13}}|-\rangle_x$$

a) What are the possible results of a measurement of the spin component S_z, and with what probabilities would they occur?

We know that the relation for state vectors in the x direction is $|\pm\rangle_x = \frac{1}{\sqrt{2}}\left[\,|+\rangle \pm |-\rangle\,\right]$ which we can substitute in our probability equation,

$$\begin{aligned}
\mathcal{P}_+ &= \left|\langle +|\left\{\tfrac{2}{\sqrt{13}}\left(\tfrac{1}{\sqrt{2}}\left[\,|+\rangle + |-\rangle\,\right]\right) + \tfrac{3}{\sqrt{13}}\left(\tfrac{1}{\sqrt{2}}\left[\,|+\rangle + |-\rangle\,\right]\right)\right\}\right|^2 \\
&= \left|\tfrac{2}{\sqrt{26}}\langle +|+\rangle + i\tfrac{3}{\sqrt{26}}\langle +|+\rangle\right|^2 \\
&= \left|\tfrac{2}{\sqrt{26}} + i\tfrac{3}{\sqrt{26}}\right|^2 \\
\mathcal{P}_+ &= \tfrac{13}{26} = \tfrac{1}{2}
\end{aligned}$$

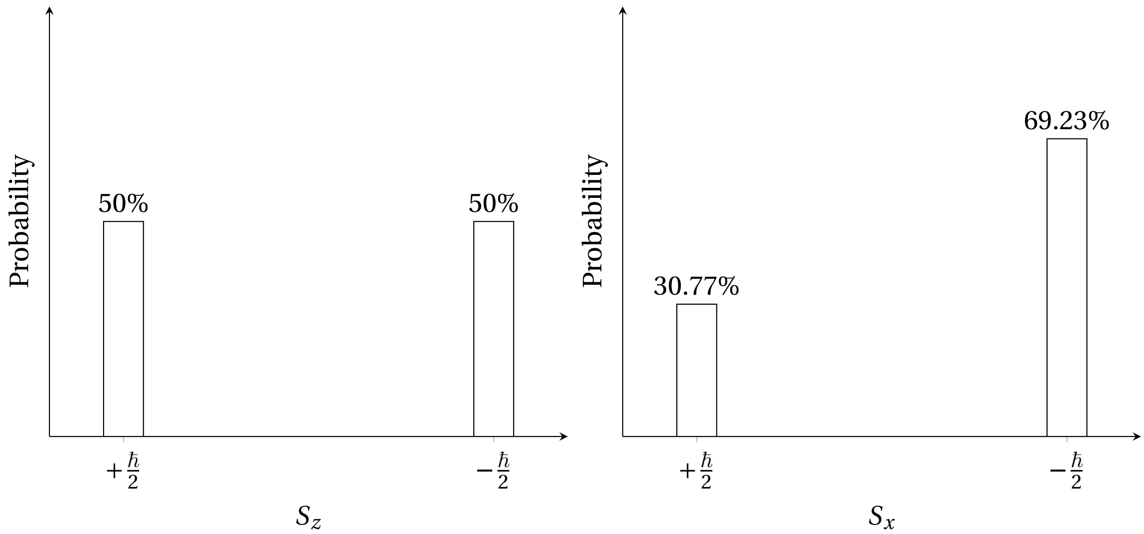
$$\begin{aligned}
\mathcal{P}_- &= \left|\langle -|\left\{\tfrac{2}{\sqrt{13}}\left(\tfrac{1}{\sqrt{2}}\left[\,|+\rangle - |-\rangle\,\right]\right) + \tfrac{3}{\sqrt{13}}\left(\tfrac{1}{\sqrt{2}}\left[\,|+\rangle - |-\rangle\,\right]\right)\right\}\right|^2 \\
&= \left|\tfrac{2}{\sqrt{26}}\langle -|-\rangle - i\tfrac{3}{\sqrt{26}}\langle -|-\rangle\right|^2 \\
&= \left|\tfrac{2}{\sqrt{26}} - i\tfrac{3}{\sqrt{26}}\right|^2 \\
\mathcal{P}_- &= \tfrac{13}{26} = \tfrac{1}{2}
\end{aligned}$$

b) What are the possible results of a measurement of the spin component S_x, and with what probabilities would they occur?

$$\begin{aligned}
\mathcal{P}_+ &= \left|\langle +_x|\psi\rangle\right|^2 \\
&= \left|\langle +|\left\{\tfrac{2}{\sqrt{13}}|+\rangle + i\tfrac{3}{\sqrt{13}}|-\rangle\right\}\right|^2 \\
&= \left|\tfrac{2}{\sqrt{13}}\langle +|+\rangle\right|^2 \\
&= \left|\tfrac{2}{\sqrt{13}}\right|^2 \\
\mathcal{P}_+ &= \frac{4}{13}
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_- &= \left|\langle +_x|\psi\rangle\right|^2 \\
&= \left|\langle -|\left\{\tfrac{2}{\sqrt{13}}|+\rangle - i\tfrac{3}{\sqrt{13}}|-\rangle\right\}\right|^2 \\
&= \left|\tfrac{3i}{\sqrt{13}}\langle -|-\rangle\right|^2 \\
&= \left|\tfrac{3i}{\sqrt{13}}\right|^2 \\
\mathcal{P}_- &= \frac{9}{13}
\end{aligned}$$

c) Plot histograms of the predicted measurement results from parts (a) and (b).



PROBLEM 1.10

Consider the three quantum states:

$$\begin{aligned}
|\psi_1\rangle &= \tfrac{4}{5}|+\rangle + i\tfrac{3}{5}|-\rangle \\
|\psi_2\rangle &= \tfrac{4}{5}|+\rangle - i\tfrac{3}{5}|-\rangle \\
|\psi_1\rangle &= -\tfrac{4}{5}|+\rangle + i\tfrac{3}{5}|-\rangle
\end{aligned}$$

a) For each of the $|\psi\rangle$ above, calculate the probabilities of spin component measurements along the x-, y-, and z-axes.

$$
\begin{aligned}
\mathcal{P}_{+x} &= \left| \langle +_x|\psi_1\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle +| + \langle -| \left\{ \frac{4}{5} |+\rangle + i\frac{3}{5} |-\rangle \right\} \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} \langle +|+\rangle + i\frac{3}{5\sqrt{2}} \langle -|-\rangle \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} + i\frac{3}{5\sqrt{2}} \right|^2 \\
\mathcal{P}_{+x} &= \frac{25}{50} = \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_{-} &= \left| \langle -_x|\psi_1\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle +| - \langle -| \left\{ \frac{4}{5} |+\rangle + i\frac{3}{5} |-\rangle \right\} \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} \langle +|+\rangle - i\frac{3}{5\sqrt{2}} \langle -|-\rangle \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} - i\frac{3}{5\sqrt{2}} \right|^2 \\
\mathcal{P}_{-x} &= \frac{25}{50} = \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_{+y} &= \left| \langle +_y|\psi_1\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle +| - i\langle -| \left\{ \frac{4}{5} |+\rangle + i\frac{3}{5} |-\rangle \right\} \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} \langle +|+\rangle - i^2\frac{3}{5\sqrt{2}} \langle -|-\rangle \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} + \frac{3}{5\sqrt{2}} \right|^2 \\
\mathcal{P}_{+y} &= \frac{49}{50}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_{-Y} &= \left| \langle -_y|\psi_1\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle +| + i\langle -| \left\{ \frac{4}{5} |+\rangle + i\frac{3}{5} |-\rangle \right\} \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} \langle +|+\rangle + i^2\frac{3}{5\sqrt{2}} \langle -|-\rangle \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} - \frac{3}{5\sqrt{2}} \right|^2 \\
\mathcal{P}_{-y} &= \frac{1}{50}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_{+z} &= \left| \langle +_z|\psi_1\rangle \right|^2 \\
&= \left| \langle +| \left\{ \frac{4}{5} |+\rangle + i\frac{3}{5} |-\rangle \right\} \right|^2 \\
&= \left| \frac{4}{5} \langle +|+\rangle \right|^2 \\
&= \left| \frac{4}{5} \right|^2 \\
\mathcal{P}_{+z} &= \frac{16}{25}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_{-z} &= \left| \langle -_z|\psi_1\rangle \right|^2 \\
&= \left| \langle -| \left\{ \frac{4}{5} |+\rangle + i\frac{3}{5} |-\rangle \right\} \right|^2 \\
&= \left| i\frac{3}{5} \langle -|-\rangle \right|^2 \\
&= \left| i\frac{3}{5} \right|^2 \\
\mathcal{P}_{-z} &= \frac{9}{25}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_{+x} &= \left| \langle +_x|\psi_2\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle +| + \langle -| \left\{ \frac{4}{5} |+\rangle - i\frac{3}{5} |-\rangle \right\} \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} \langle +|+\rangle - i\frac{3}{5\sqrt{2}} \langle -|-\rangle \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} - i\frac{3}{5\sqrt{2}} \right|^2 \\
\mathcal{P}_{+x} &= \frac{25}{50} = \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_{-} &= \left| \langle -_x|\psi_2\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle +| - \langle -| \left\{ \frac{4}{5} |+\rangle - i\frac{3}{5} |-\rangle \right\} \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} \langle +|+\rangle + i\frac{3}{5\sqrt{2}} \langle -|-\rangle \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} + i\frac{3}{5\sqrt{2}} \right|^2 \\
\mathcal{P}_{-x} &= \frac{25}{50} = \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_{+y} &= \left| \langle +_y|\psi_2\rangle \right|^2 \\
&= \left| \frac{1}{\sqrt{2}} \langle +| - i\langle -| \left\{ \frac{4}{5} |+\rangle - i\frac{3}{5} |-\rangle \right\} \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} \langle +|+\rangle + \frac{3i^2}{5\sqrt{2}} \langle -|-\rangle \right|^2 \\
&= \left| \frac{4}{5\sqrt{2}} - \frac{3}{5\sqrt{2}} \right|^2 \\
\mathcal{P}_{+y} &= \frac{1}{50}
\end{aligned}
$$

$$\begin{aligned}\mathcal{P}_{-Y} &= \left|\langle -_y|\psi_2\rangle\right|^2\\ &= \left|\tfrac{1}{\sqrt{2}}\langle +| + i\langle -|\left\{\tfrac{4}{5}|+\rangle - i\tfrac{3}{5}|-\rangle\right\}\right|^2\\ &= \left|\tfrac{4}{5\sqrt{2}}\langle +|+\rangle - \tfrac{3i^2}{5\sqrt{2}}\langle -|-\rangle\right|^2\\ &= \left|\tfrac{4}{5\sqrt{2}} + \tfrac{3}{5\sqrt{2}}\right|^2\\ \mathcal{P}_{-y} &= \tfrac{49}{50}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{+z} &= \left|\langle +_z|\psi_2\rangle\right|^2\\ &= \left|\langle +|\left\{\tfrac{4}{5}|+\rangle - i\tfrac{3}{5}|-\rangle\right\}\right|^2\\ &= \left|\tfrac{4}{5}\langle +|+\rangle\right|^2\\ &= \left|\tfrac{4}{5}\right|^2\\ \mathcal{P}_{+z} &= \tfrac{16}{25}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{-z} &= \left|\langle -_z|\psi_2\rangle\right|^2\\ &= \left|\langle -|\left\{\tfrac{4}{5}|+\rangle - i\tfrac{3}{5}|-\rangle\right\}\right|^2\\ &= \left|-i\tfrac{3}{5}\langle -|-\rangle\right|^2\\ &= \left|-i\tfrac{3}{5}\right|^2\\ \mathcal{P}_{-z} &= \tfrac{9}{25}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{+x} &= \left|\langle +_x|\psi_3\rangle\right|^2\\ &= \left|\tfrac{1}{\sqrt{2}}\langle +| + \langle -|\left\{-\tfrac{4}{5}|+\rangle + i\tfrac{3}{5}|-\rangle\right\}\right|^2\\ &= \left|-\tfrac{4}{5\sqrt{2}}\langle +|+\rangle + i\tfrac{3}{5\sqrt{2}}\langle -|-\rangle\right|^2\\ &= \left|-\tfrac{4}{5\sqrt{2}} + i\tfrac{3}{5\sqrt{2}}\right|^2\\ \mathcal{P}_{+x} &= \tfrac{25}{50} = \tfrac{1}{2}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{-} &= \left|\langle -_x|\psi_3\rangle\right|^2\\ &= \left|\tfrac{1}{\sqrt{2}}\langle +| - \langle -|\left\{-\tfrac{4}{5}|+\rangle + i\tfrac{3}{5}|-\rangle\right\}\right|^2\\ &= \left|-\tfrac{4}{5\sqrt{2}}\langle +|+\rangle - i\tfrac{3}{5\sqrt{2}}\langle -|-\rangle\right|^2\\ &= \left|-\tfrac{4}{5\sqrt{2}} - i\tfrac{3}{5\sqrt{2}}\right|^2\\ \mathcal{P}_{-x} &= \tfrac{25}{50} = \tfrac{1}{2}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{+y} &= \left|\langle +_y|\psi_3\rangle\right|^2\\ &= \left|\tfrac{1}{\sqrt{2}}\langle +| - i\langle -|\left\{-\tfrac{4}{5}|+\rangle + i\tfrac{3}{5}|-\rangle\right\}\right|^2\\ &= \left|-\tfrac{4}{5\sqrt{2}}\langle +|+\rangle - i^2\tfrac{3}{5\sqrt{2}}\langle -|-\rangle\right|^2\\ &= \left|-\tfrac{4}{5\sqrt{2}} + \tfrac{3}{5\sqrt{2}}\right|^2\\ \mathcal{P}_{+y} &= \tfrac{1}{50}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{-Y} &= \left|\langle -_y|\psi_3\rangle\right|^2\\ &= \left|\tfrac{1}{\sqrt{2}}\langle +| + i\langle -|\left\{-\tfrac{4}{5}|+\rangle + i\tfrac{3}{5}|-\rangle\right\}\right|^2\\ &= \left|-\tfrac{4}{5\sqrt{2}}\langle +|+\rangle + i^2\tfrac{3}{5\sqrt{2}}\langle -|-\rangle\right|^2\\ &= \left|-\tfrac{4}{5\sqrt{2}} - \tfrac{3}{5\sqrt{2}}\right|^2\\ \mathcal{P}_{-y} &= \tfrac{49}{50}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{+z} &= \left|\langle +_z|\psi_3\rangle\right|^2\\ &= \left|\langle +|\left\{-\tfrac{4}{5}|+\rangle + i\tfrac{3}{5}|-\rangle\right\}\right|^2\\ &= \left|-\tfrac{4}{5}\langle +|+\rangle\right|^2\\ &= \left|-\tfrac{4}{5}\right|^2\\ \mathcal{P}_{+z} &= \tfrac{16}{25}\end{aligned}$$

$$
\begin{aligned}
\mathcal{P}_{-z} &= \left|\langle -_z|\psi_3\rangle\right|^2 \\
&= \left|\langle -|\left\{-\tfrac{4}{5}|+\rangle + i\tfrac{3}{5}|-\rangle\right\}\right|^2 \\
&= \left|i\tfrac{3}{5}\langle -|-\rangle\right|^2 \\
&= \left|i\tfrac{3}{5}\right|^2 \\
\mathcal{P}_{-z} &= \tfrac{9}{25}
\end{aligned}
$$

b) Use your results from (a) to comment on the importance of the overall phase and of the relative phases of the quantum state vector.

Because the same results were observed from state vectors in different phases we can say that the phases are not observable in our measurements.

Problem 1.11

A beam of spin-1/2 particles is prepared in the state

$$|\psi\rangle = \frac{3}{\sqrt{34}}|+\rangle + i\frac{5}{\sqrt{34}}|-\rangle$$

a) What are the possible results of a measurement of the spin component Sz, and with what probabilities would they occur?

$$
\begin{aligned}
\mathcal{P}_+ &= \left|\langle +|\left\{\tfrac{3}{\sqrt{34}}|+\rangle + i\tfrac{5}{\sqrt{34}}|-\rangle\right\}\right|^2 \\
&= \left|\tfrac{3}{\sqrt{34}}\right|^2 \\
\mathcal{P}_+ &= \tfrac{9}{34}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_- &= \left|\langle -|\left\{\tfrac{3}{\sqrt{34}}|+\rangle + i\tfrac{5}{\sqrt{34}}|-\rangle\right\}\right|^2 \\
&= \left|i\tfrac{5}{\sqrt{34}}\right|^2 \\
\mathcal{P}_- &= \tfrac{25}{34}
\end{aligned}
$$

b) Suppose that the S_z measurement yields the result Sz = -$\frac{\hbar}{2}$. Subsequent to that result a second measurement is performed to measure the spin component S_x. What are the possible results of that measurement, and with what probabilities would they occur?

If the measurement show that the system is prepared in the z down state then we would expect an even distribution in spins for the x direction as,

$$
\begin{aligned}
\mathcal{P}_+ &= \left|\tfrac{1}{\sqrt{2}}\langle +|\left\{\,|+\rangle + |-\rangle\right\}\right|^2 \\
&= \left|\tfrac{1}{\sqrt{2}}\right|^2 \\
\mathcal{P}_+ &= \tfrac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_- &= \left|\tfrac{1}{\sqrt{2}}\langle -|\left\{|+\rangle + |-\rangle\right\}\right|^2 \\
&= \left|\tfrac{1}{\sqrt{2}}\right|^2 \\
\mathcal{P}_- &= \tfrac{1}{2}
\end{aligned}
$$

c) Draw a schematic diagram depicting the successive measurements in parts (a) and (b).