

Astrophysics H.W. I

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1 WEIGHING THE EARTH AND THE SUN

$$\vec{F} = \frac{GMm}{r^2} \hat{r} \quad (1.1)$$

a) What are the units of the universal gravitational constant G?

(b) Using the radius of the earth, (R_E), and empirical observations that the acceleration of an object near the Earth is g (9.8 m s^{-2}) and G is Newton's Universal gravitational constant, find an expression for the mass of the Earth and use it to estimate the Earth's mass.

2 WEIGHING THE SUN

(a) An astronomical unit (AU) is defined as the average distance between the Earth and the center of the Sun. Assuming that it takes about 8.5 minutes for light from the Sun to reach the Earth, derive an expression using the speed of light that represents 1 AU.

Velocity has units of meters per second which if multiplied by time and we get a distance.

$$c \times t = d \rightarrow 2.99 \times 10^8 \frac{\text{m}}{\text{s}} \left(\frac{60 \text{ sec}}{1 \text{ min}} (8.5 \text{ min}) \right) = 2.99 \times 10^8 \frac{\text{m}}{\text{s}} (510 \text{ sec}) =$$

We can use a slightly closer approximation by using 500 seconds multiplied by the speed of light for 1 AU

(b) Using the fact that it takes 1 year for the Earth to complete one revolution around the Sun, please derive an expression for the mass of the Sun and calculate this mass.

(c) Using your expression for Problem 1 part (b) and your answer from part (b) of this problem, derive an expression for the ratio of Earth's mass to that of the Sun.

3 ESCAPE VELOCITIES

The escape velocity is defined as the velocity initially needed by an object to permanently leave the gravitational pull of a planet, star, etc. (a) Derive a general expression for the escape velocity for an object of mass m leaving the gravitational influence of a more massive object with mass M .

(b) Using your expressions from the previous problems, derive an escape velocity for (1) Earth and (2) The Sun. You can assume that for Earth, the initial position is R_E and for the Sun the initial position is at 1 AU.

(c) Provide numerical answers to part (b) in units of (1) meter and seconds (2) miles and hours.

(d) Imagine an object with Earth's mass (m_E) what is so compact that the escape velocity is equal to the speed of light c . Derive an expression, using your answer from part (a), for the radius of this object. What is this value numerically?

4 KEPLER ORBITS 1/2

(a) Derive an expression for the orbital velocity of Earth assuming it has a circular orbit of radius r_{E-S} .

(b) Using your expression from part (a), derive an expression for the total energy of Earth ($= KE + PE$) and comment on whether it is less than or greater than zero.

5 KEPLER ORBITS 2/2

(a) Run the Matlab code and make sure that it works. If it doesn't, you might have to modify the code to de

ne the mass of the Sun. You should not need to de

ne the mass of the object (Why?). What is the shape of the orbit?

(b) increase the initial velocity of the satellite by 10% the orbit?

(c) increase the initial velocity of the object by 20% orbit?

(d) Decrease the velocity by 20%. What happens to the shape of the orbit?

(e) Set the magnitude of the satellite's initial velocity equal to the escape velocity. Comment and sketch the shape of the orbit.

6 FREE FALLING

(a) Consider a mass m initially at a distance R from a much more massive mass M . Derive an expression for the time it would take for this mass to crash into the much more massive mass. You can assume that the radius of both masses is much smaller than R . Check the

units of your answer.

(b) Calculate what this free-fall timescale is if R is 1 AU.

(c) Extra-credit: Check your answer in part (b) by using the numerical orbit simulator (or write your own code) used in our previous problem.