CALIFORNIA STATE UNIVERSITY SAN MARCOS
DR. DOMINGUEZ, PHYSICS 490

Astrophysics H.W. I

Josh Lucas

September 5, 2018

1 WEIGHING THE EARTH AND THE SUN

$$\vec{F} = \frac{GMm}{r^2}\hat{r} \tag{1.1}$$

a) What are the units of the universal gravitational constant G?

We can find the units of the Universal Gravitaional constant by solving for G and analyzing the coresponding units.

$$m\vec{a} = \frac{GMm}{r^2}\hat{r} \rightarrow G = \frac{m\vec{a}r^2}{Mm} = \frac{\vec{a}r^2}{M} = \frac{meters^3}{kilograms * seconds^2}$$

(b) Using the radius of the earth,(R_E), and empirical observations that the acceleration of an object near the Earth is g ($9.8ms^-2$) and G is Newton's Universal gravitational constant, find an expression for the mass of the Earth and use it to estimate the Earth's mass.

We know that the acceleration of an object near the Earth is $\vec{a} = 9.8\frac{m}{s^2}$, the force of the Earth's gravity on an object is equal to the mass of the ogbject multiplied by the acceleration, $\vec{F}_g = m\vec{a}$. Using the radius of the earth and Newton's Universal gravitation constant we can solve for the mass of the earth.

$$m\vec{a} = G\frac{M_E m}{R_E^2}\hat{r} \rightarrow M_E = \frac{\vec{a}R_E^2}{G} = \frac{(9.8\frac{m}{s^2})\ 6.378\times10^6 m}{6.674\times10^-11\frac{m^3}{kg*s^2}} = 5.97\times10^{24}$$

2 WEIGHING THE SUN

(a) An astronomical unit (AU) is defined as the average distance between the Earth and the center of the Sun. Assuming that it take about 8.5 minutes for light from the Sun to reach the Earth, derive an expression using the speed of light that represents 1 AU.

Distance is equal to velocity multiplied by time

$$C * t = d \rightarrow 2.99 \times 10^8 \frac{m}{s} * 8.5min(\frac{60sec}{1min}) = 1.5 \times 10^8 km \approx 1AU$$

Velocity has units of meters per second which if multiplied by time and we get a distance. We can use a slightly closer approximation by using 500 seconds multiplied by the speed of light for 1 AU

(b)Using the fact that it takes 1 year for the Earth to complete one revolution around the Sun, please derive an expression for the mass of the Sun and calculate this mass.

The average distance of the radius of the Earth orbit is one astronomical unit which it completes in one years time. From this relation we can find the velocity of the earth and the force holding the earth in orbit. The Earth's velocity is equal to the circumference of the orbit over the time,

$$circumfrence = \int r d\theta = 2\pi r = 2\pi(1AU)$$

$$\frac{distance}{time} = velocity = \frac{2\pi(1AU)}{\pi \times 10^7 sec} = 30000\frac{m}{s}$$

We can now solve for the center force on the earth tethering it to the sun,

$$\vec{F}_c = m\frac{v^2}{r} = 5.97 \times 10^2 4kg \frac{(30000\frac{m}{s})^2}{6.378 \times 10^6} = 8.434 \times 10^{26} joules$$

We can now relate the center force to the Newtons equation for gravity,

$$\frac{mv^2}{r} = G\frac{Mm}{r^2}\hat{r} \rightarrow M = \frac{v^2 r}{G} = \frac{(30000\frac{m}{s})^2 6.378 \times 10^6}{1.5 \times 10^{11} \frac{m^3}{kg * s^2}} \approx 2 \times 10^{30} kg$$

(c) Using your expression for Problem 1 part (b) and your answer from part (b) of this problem, derive an expression for the ratio of Earth's mass to that of the Sun.

3 ESCAPE VELOCITIES

The escape velocity is defined as the velocity initially needed by an object to permanently leave the gravitational pull of a planet, star, etc. (a) Derive a general expression for the

escape velocity for an object of mass m leaving the gravitational in influence of a more massive object with mass M.
(b) Using your expressions from the previous problems, derive an escape velocity for (1) Earth and (2) The Sun. You can assume that for Earth, the initial position is RE and for the Sun the initial position is at 1 AU.
(c) Provide numerical answers to part (b) in units of (1) meter and seconds (2) miles and hours.
(d)Imagine an object with Earth's mass (mE) what is so compact that the escape velocity is equal to the speed of light c. Derive an expression, using your answer from part (a), for the radius of this object. What is this value numerically?

4 KEPPLER ORBITS 1/2

(a) Derive an expression for the orbital velocity of Earth assuming it has a circular orbit of radius r_{E-S}.

$$\vec{V_{orbit}} = \frac{\text{circumfrence of orbit}}{timeofonerevolution} = \frac{2\pi 1.5 \times 10^8 km}{\pi \times 10^7 sec} \approx \tag{4.1}$$

We can also use the relation of the center force and gravitational force and solve for velocity

$$\frac{mv^2}{r} = G\frac{Mm}{r^2} \rightarrow v^2 = \frac{GM}{r}$$

(b) Using your expression from part (a), derive an expression for the total energy of Earth (= KE + PE) and comment on whether it is less than or greater than zero.

$$T = \frac{1}{2}mv^2$$

replacing v for terms of gravitational force,

$$T = \frac{1}{2}m(\frac{GM}{r})$$

$$U = -\int \vec{F} \cdot d\vec{r} \rightarrow U = -\int -GMm\frac{1}{r^2}d\vec{r} = -GMm\frac{1}{2r}$$

5 KEPPLER ORBITS 2/2

(a) Run the Matlab code and make sure that it works. If it doesn't, you might have to modify the code to de
ne the mass of the Sun. You should not need to de
ne the mass of the object (Why?). What is the shape of the orbit?
(b) increase the initial velocity of the satellite by 10the orbit?
(c) increase the initial velocity of the object by 20orbit?
(d) Decrease the velocity by 20%. What happens to the shape of the orbit?
(e) Set the magnitude of the satellite's initial velocity equal to the escape velocity. Comment and sketch the shape of the orbit.

6 FREE FALLING

(a)Consider a mass m initially at a distance R from a much more massive mass M. Derive an expression for the time it would take for this mass to crash into the much more massive mass. You can assume that the radius of both masses if much smaller than R. Check the units of your answer.
(b) Calculate what this free-fall timescale is if R is 1 AU.
(c) Extra-credit: Check your answer in part (b) by using the numerical orbit simulator (or write your own code) used in our previous problem.