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# Finite-Element Analysis of Magnetic Field and the Flow of Magnetic Fluid in the Core of Magnetic-Fluid Seal for Rotational Shaft

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**Abstract**

There has been developed a mathematical model for analyzing the magnetic field in the core of magnetic-fluid seal (MFS) with account of nonlinear magnetic properties of the materials for magnetic core and shaft. To provide for the numerical realization of this model, there was used a finite-element method and Comsol software package.



*Keywords*: Magnetic-fluid seal; finite element analysis; magnetic field; sealing; Maxwell's equation; Navier-Stokes equation.

## 1. Introduction

As for sealing level in the field of their application, the magnetic-fluid seals (MFS) for rotational shafts exceed all known types of seals and represent a new independent class of technical devices with permanent magnets using magnetic fluid [1]. The principle of their operation is based on the interaction of the magnetic fluid with magnetic field of the magnetic system of the seal, wherein there is provided for the sealing zone between the inner area of the seal and the shaft. The standard MFS design is shown in Fig. 1 and includes a magnetic system with permanent magnets magnetized in the axial direction. In the clearance between the seal body and the shaft there is a magnetic fluid being kept under the influence of

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magnetic forces and providing for sealing an internal medium at a certain pressure drop $\Delta p$ in the axial direction? The concentrators of magnetic flux are of toothed structure to obtain sharply inhomogeneous magnetic field.

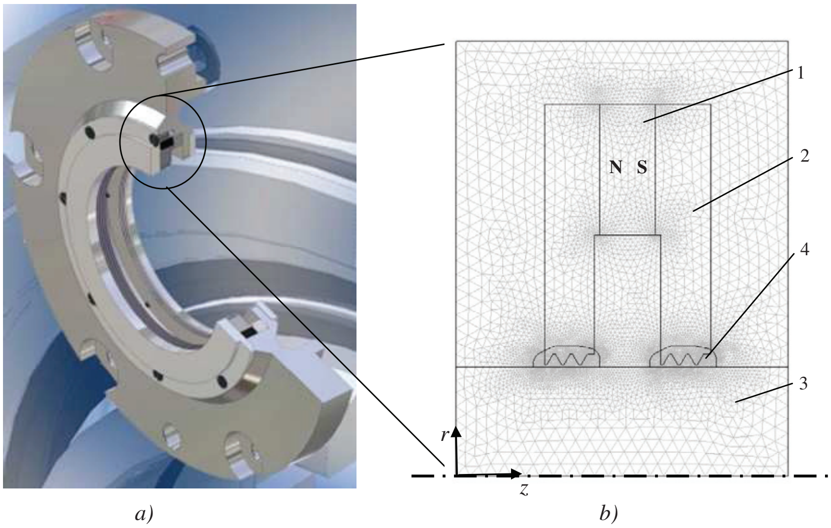



Fig. 1. General view of a MFS standard design (*a*) and the core estimation area with the applied finite element mesh (*b*)

Investigation of the basic laws of the magnetic field distribution and nature of the magnetic fluid flow in the core of the seal is an important scientific and practical problem associated with the creation of new MFS designs with extremely high performances. To solve this problem, it is expediently along with appropriate methods of physical modeling to use the modern methods of computer simulation.

The problems of analysis of MFS magnetic and hydrodynamic processes are the focus for a number of papers, see, for example, [2-5, 11, 12], which are devoted to using both analytical and numerical methods. Thus, the hydrodynamic problem is usually intended for considering the distribution of only a single azimuth component of a fluid motion velocity. Because of the nonlinearity of the magnetic properties of the materials for the case of the magnetic problem, as well as the nonlinearity of the Navier-Stokes equations for the hydrodynamic problem, the use of numerical methods and modern software packages allows to investigate more realistic MFS models and to take into account the distribution of three components of the magnetic fluid motion in the MFS clearance.

The aim of this work is a computer simulation of the interconnected nonlinear magnetic and hydrodynamic processes in the MFS core by applying the numerical finite element method with the use of the software package Comsol [7] as a basis for recommendations to improve the MFS design. The calculation is performed in two stages. At the first stage, there is calculated static magnetic field distribution in the seal core with the account of the magnetic core nonlinear characteristics and with the assumption of the magnetic fluid saturation - $M = M_s$. At the same time, there is determined the boundary of the magnetic fluid position in static, this position is limited by isobars $p = M_s B =$ const., where - $B = |\mathbf{B}|$ is modulus of magnetic induction. Then, at the same stage, there are calculated the hydrodynamic processes in a magnetic fluid for the case of a rotational shaft with the assumption that the boundaries of the fluid coincide with the boundaries of the fluid in static. In doing so, there is taken into account nonlinear empirical dependence of the magnetic fluid viscosity on the magnetic field value and the nonlinear properties of the Navier-Stokes equations. Thus, these two above said problems are considered as loosely coupled ones (per classification [6]) that makes it is possible to perform their sequential solution. Further in such a sequence there is described a solution of the above mentioned problems.

## 2. Numerical calculation of the magnetic field in the mfs core in static

The examined MFS is characterized by the axial symmetry hence the field problem can be solved in two-dimensional formulation in the cylindrical coordinate system in $r0z$ plane. The computational domain for the magnetic field analysis is shown in Fig. 1 b and contains the regions with the magnetic materials of three types: the permanent magnets, the magnets that magnetized in the axial direction 1, the ferromagnetic material of the magnetic system poles 2, and the rotational shaft 3, as well as the region occupied by the ferrofluid 4. Magnetization characteristics of these materials are discussed below.

The field problem is considered as the magnetostatic one and solved in the axisymmetric formulation in cylindrical coordinates in *r0z* plane for vector magnetic potential, which has a single $\varphi$-component, i.e. $\mathbf{A} = (0, A_\varphi, 0)$.

From the system of Maxwell differential equations for a stationary magnetic field

$$\nabla \times \mathbf{H} = 0, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad \nabla \cdot \mathbf{A} = 0, \tag{1}$$

and the equation of the magnetic material state written for the general case as

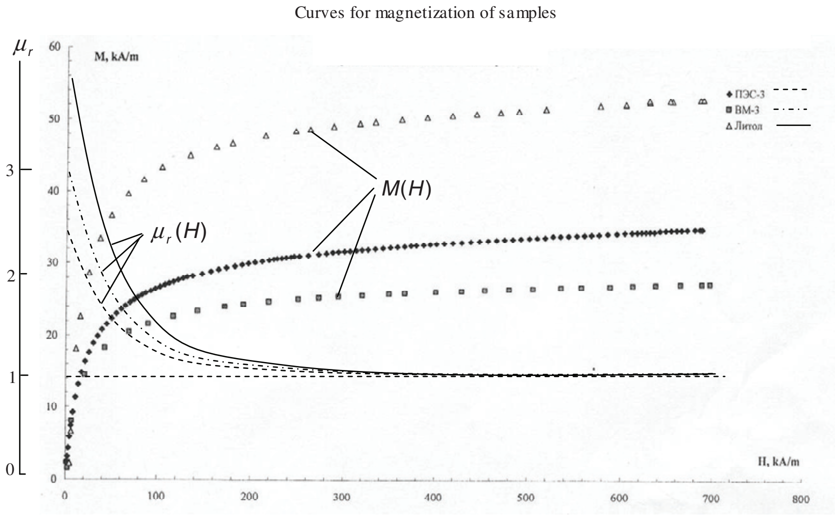
$$\mathbf{B} = \mu_0 \mu_r \mathbf{H} + \mathbf{B}_r, \tag{2}$$

it is obtained the following differential equation for the vector potential

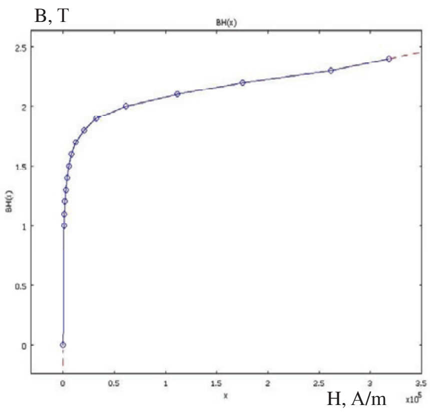
$$\nabla \times [(\mu_0 \mu_r)^{-1} \nabla \times \mathbf{A} - (\mu_0 \mu_r)^{-1} \mathbf{B}_r] = 0. \tag{3}$$

Here $\mathbf{H}$ is magnetic field strength; $\mathbf{B}$ is magnetic induction; $\mathbf{B}_r$ is the residual induction characterizing the permanent magnet, this residual induction is defined in the region occupied by this magnet; $\mu_0$ is magnetic permeability of vacuum; $\mu_r(|\mathbf{B}|)$ is the relative value of the permeability (scalar) for the magnetic material, which depends on the module of the magnetic induction vector.

MFS permanent magnet is made of NdFeB, grade 38SH, material, characterized by the residual induction $B_r = 1{,}26$ T and coercive force $H_s = 950$ kA/m. Hence, for the equation of the magnet state, after substituting these values, it can be obtained $\mu_r = 1{,}06$ for the region of the permanent magnet from expression (2). The magnetic fluids being used in MFS, are characterized by the magnetization curves shown in Fig. 2a. Further, in the calculations there would be used BM-3 fluid, which is characterized by saturation magnetization $M_s = 30$ kA/m. As can be seen from Fig. 2a, at the magnetic field, wherein $H$ > 200 kA/m, the magnetic permeability of the fluid can be assumed equal to $\mu_r \approx 1$. The poles of the magnetic system and the rotational shaft are characterized by the magnetization curve given in Fig. 2b. The characteristics of this curve in the form of tabular data were entered into the program Comsol.




*a)*




*b)*

Fig. 2. Magnetic characteristics for three magnetic fluids (*a*) and materials for the magnetic system, that is, poles and rotational shaft (*b*) used at calculating magnetic field

As boundary condition, there was used a symmetry condition on the axis of the shaft rotation and the condition of magnetic confinement $\mathbf{B} \cdot \mathbf{n} = 0$ on the side and top surfaces. To perform the numerical solution of the differential equations involving the partial derivatives (3) with the specified boundary conditions, there was used a finite element method realized in the software package Comsol [7].

The distribution of the magnetic field lines (isolines $A_\varphi r$ ), as well as magnetic induction (in color and with arrows) in the MFS magnetic system is represented in Fig. 3a. Fig. 3 b shows the distribution of the radial component of magnetic induction on the shaft surface. As can be seen in these figures, the presence of the toothed structure on the surface of the poles creates a strongly non-uniform field in the clearance with maximum value $B = 2.25$ T on the surface of the shaft in the toothed region.

It is well known (see e.g. [8, 9]) that the pressure applied to the magnetic fluid in static in the magnetic field is determined by the following expression:

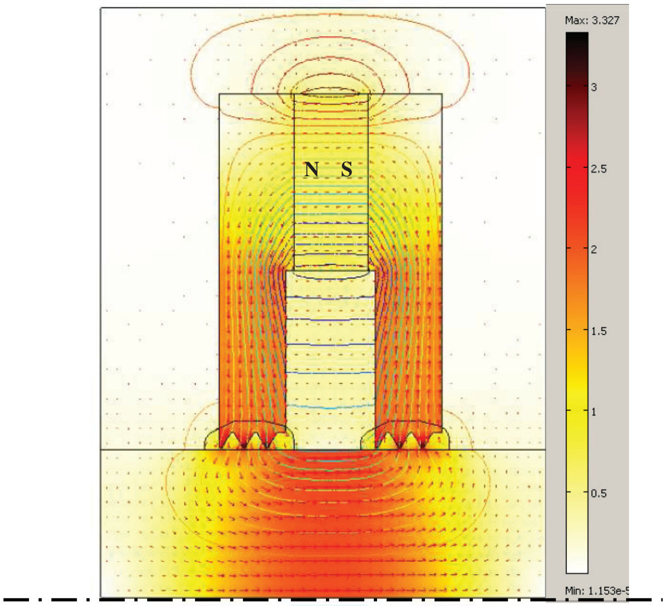
$$p = \int_0^B MdB + C, \tag{4}$$

where $M$ is a magnetic fluid magnetization, and $B = |\mathbf{B}|$ and $C$ is a constant determined by the boundary conditions.
Assuming that the fluid saturated, i.e. $M = M_s$, there can be obtained from expression (4) as follows:

$$p = M_s B + C. \tag{5}$$

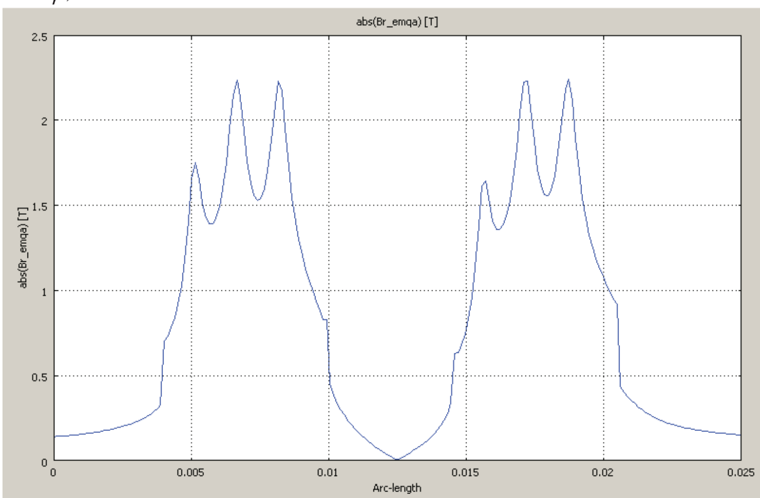
This shows that the lines of the equal pressures (isobars) under the assumed conditions coincide with the lines of the equal values $B$.

Fig. 4 shows distribution $B$ over the surface of the shaft at one pole in the region occupied (as expected) with a magnetic fluid. It is clear that the maximum field being equal to 2.25 T is localized in the subsurface region of the middle and right teeth. The field under the leftmost tooth is somewhat weakened (it makes 1.65 T) due to exposing the edge effect associated with the bulging lines at the edges of the poles. Distributing isolines $B$ (or isobars $M_s B$ that are coincident with them) allows approximate determining the configuration of the magnetic fluid in the presence of the axial external pressure drop $\Delta p$, for example, see [1].




*a)*

$B_r$ , T




Distance, m

*b)*

Fig. 3. Distribution of magnetic field lines (isolines $A_\varphi r$ ), as well as magnetic induction (in color and with arrows) in MFS core (*a*) and distribution of radial component of magnetic induction on the surface of the shaft (*b)*

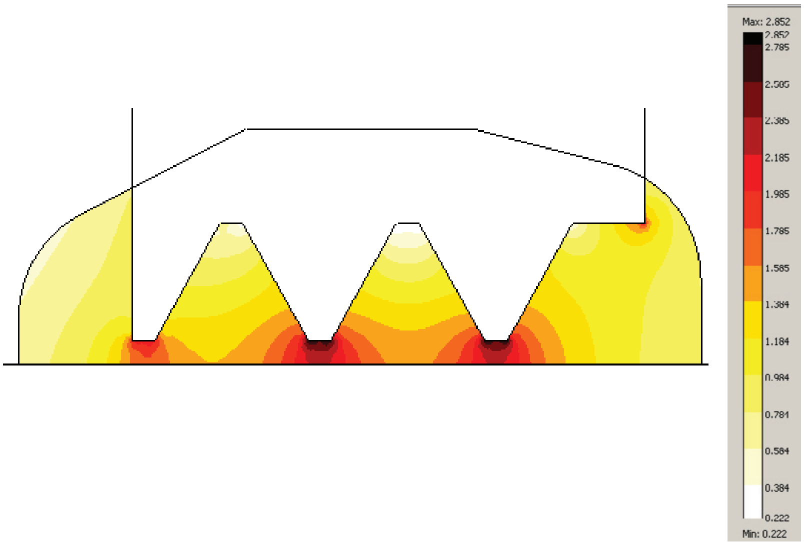



Fig. 4. Distribution of magnetic induction isolines $|\mathbf{B}|$ T in magnetic fluid under a pole

In the MFS clearance, free surface of the magnetic fluid in static state (the case of a stationary shaft) will be limited to the lines of the equal $B$ values. And, depending on the volume of the fluid introduced into the clearance, the region occupied by the fluid will be different, but there it will always be limited to the isolines of different $B$ values. Fig. 5 shows the design region occupied by the fluid in static and at the absence of axial pressure drop in the MFS for the various volumes of the fluid. In the case of the small volume (Fig. 5 a), the region with the fluid will be limited to the isolines of the significant value characterizing the field (1.8 T in the figure) and it will be localized in the clearance arranged just below the two inner teeth. In the process of increasing the volume of the fluid, this region will be increased too, occupying all the clearance for the case corresponding $B$ =0.8 T, see fig. 5 *e*.

The magnetic force acting on volume unit of the magnetic fluid in the non-uniform magnetic field is determined by the following expression [7, 8]:

$$\mathbf{f}_m = \mu_0 (\mathbf{M} \cdot \nabla)\mathbf{H} . \tag{6}$$

Assuming that the vectors $\mathbf{M}$ and $\mathbf{H}$ are parallel, i.e. it is fair the equality $\mathbf{M} = M\mathbf{H}/|\mathbf{H}|$, and using the vector identity [10] $\nabla(\mathbf{H}\cdot\mathbf{H}) = 2(\mathbf{H}\cdot\nabla)\mathbf{H}$, the expression is transformed for force (6) to the form

$$\mathbf{f}_m = \mu_0 M \nabla H \text{ , where } M = |\mathbf{M}|, \quad H = |\mathbf{H}| . \tag{7}$$

Distributing the vector for $\mathbf{f}_m$, calculated on the basis of expression (7), in the clearance of the magnetic system under the assumption that all the clearance is filled by the magnetic fluid with the saturation magnetization $M_s$ = 30 kA/m, is shown in Fig. 6. It is clear that the magnetic force tends to draw the fluid into a small area below the teeth of the magnetic poles. The maximum value of this force is $4{\cdot}10^8$ N/m$^3$. The obtained distribution of the magnetic forces volume density will be used later in solving the hydrodynamic problem.

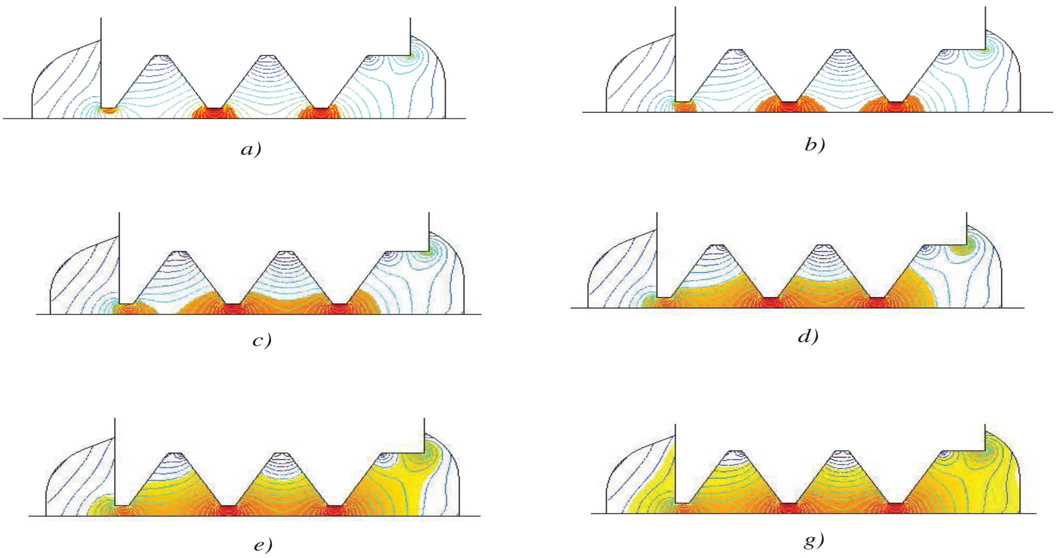



Fig. 5. Different volume occupied by magnetic fluid under a pole at axial pressure drop $\Delta p = 0$ , limited by isolines of the magnetic induction B = 1.8 T (*a*), 1.6 T (*b*), 1.4 T (*c*), 1.2 T (*d*), 1.0 T (*e*), 0.8 T (*g*)

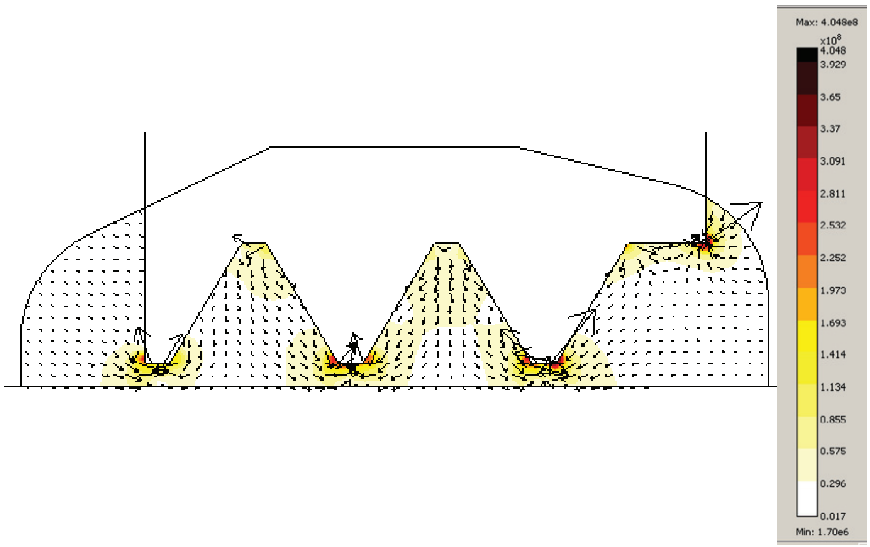



Fig. 6. Distributing the vector of volume density of the magnetic force $\mathbf{f}_m$ , N/m$^3$ in a magnetic fluid (shown by arrows and in color) at $M_s$ = 30 kA/m.

## 3. Numerical calculation of the magnetic fluid flow at shaft rotation

At solving the hydrodynamic problem of the magnetic fluid flow in the MFS clearance, there are used the following assumptions.

1. In the case of the shaft rotation, the position of the magnetic fluid free boundary in dynamics coincides with those boundaries in static. The boundaries were determined by distributing the isolines $B$ shown in Fig. 7 a for the situation, when the pressure drop under each tooth is $\Delta p = M_s(B_2 - B_1) =$ 30 (1,6-0,5) = 33 kPa. Thus obtained, the position of the fluid boundary is shown in Fig. 7 *b* and is further considered as a boundary of the computational domain for the hydrodynamic problem.

2. The magnetic field distribution in MFS clearance at the given position of the magnetic fluid boundary coincides with the field distribution in Figs. 3, 4. The validity of this position is due to a saturated state of the magnetic fluid in the clearance, whereat its magnetic permeability is approximately equal to $\mu_r \approx 1$ in accordance with Fig. 2 *a*. It should be noted that the field value in the clearance affects the nature of the fluid flow due to the strong dependence of its viscosity on the value of magnetic induction $B$ .

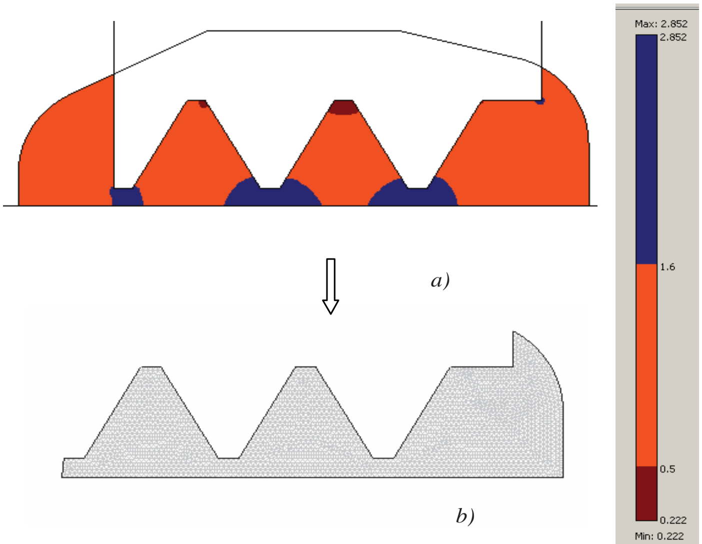



Fig. 7. The magnetic induction isolines in the MFS clearance with the values 0.5 T and 1.6 T, which are used for building the computational domain for the hydrodynamic problem (*a*) and the computational domain configuration covered with the finite elements (*b*)

Next, we herein consider the laminar condition for the flow of the magnetic fluid. The steady motion of the magnetic fluid in the clearance of the magnetic system is described by the system of the Navier-Stokes equation and continuity equation that have been written using a vector operator $\nabla$ in the form of

$$\begin{aligned} &\rho(\mathbf{u}\cdot\nabla)\mathbf{u} = \nabla\cdot[-p\mathbf{I} + \eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)] + \mathbf{f}_m, \\ &\nabla\cdot\mathbf{u} = 0, \end{aligned} \tag{8}$$

wherein $\rho$ is the density of the magnetic fluid; $\mathbf{u} = (u_r, u_\varphi, u_z)$ is the fluid velocity vector, which has radial, azimuth and axial components, respectively, directed along axis $r, \varphi$ and $p$ is a pressure; $\eta$ is a dynamic viscosity, which depends on the magnetic field value; $\mathbf{f}_m$ is the volume density of the magnetic force. To study the stationary problem characterized by axial symmetry, all the components of the velocity of the magnetic fluid, as well as the pressure do not depend on spatial coordinate $\varphi$, and the problem can be solved in two-dimensional formulation in the plane *r0z*.

It is known that the viscosity of the magnetic fluid depends on the magnetic field value [1]. In this paper, to account for this dependence, there was used an empirical expression [11]:

$$\eta(B) = \eta_{|B=0}(1+50\sqrt{B}),\ \textit{where}\ \ B \textit{ is given in } T$$

The components of the magnetic force were calculated according to expression (7) and their distribution in the fluid is shown in Fig. 6.

As the boundary conditions, there were used as follows: on the surface of the magnetic poles, there was set zero velocity - $\mathbf{u} = (0,0,0)$, on the surface of the shaft - $\mathbf{u} = (0, u_{\varphi,0}, 0)$, where $u_{\varphi,0}$ is a linear velocity at the surface of the rotational shaft; on the axis of symmetry, there were set conditions of the axial symmetry; on the free boundaries of the fluid, as a condition of symmetry, there was equality to zero for the normal component of the velocity and the tangential component of the surface tension.

To perform a numerical solution of system (8) taking into account the specified boundary conditions, there was used the finite element method realized in software package Comsol [7].

At carrying out the calculations, the following parameters of the fluid were used: $\rho$ =1122 kg/m$^3$, $\eta_{|B=0}$ =0.85 Pa·s.

Distributing the azimuth component of velocity $u_\varphi$ in the clearance of the magnetic system is displayed in Fig. 8 at an angular frequency of the shaft rotation $\omega = 314$ rad/s that correspond to the linear velocity on the shaft surface of radius $R =$ of 0.105 m/s, equal $u_{\varphi,0} = 33$ m/s. According to the obtained results of the calculation, it is visible that the velocity is monotonically diminished from 33 m/s on the shaft surface to zero on the magnetic pole surface. The sharpest change of the velocity occurs immediately under the teeth of the magnetic system, i.e. in the regions of the minimum clearances.

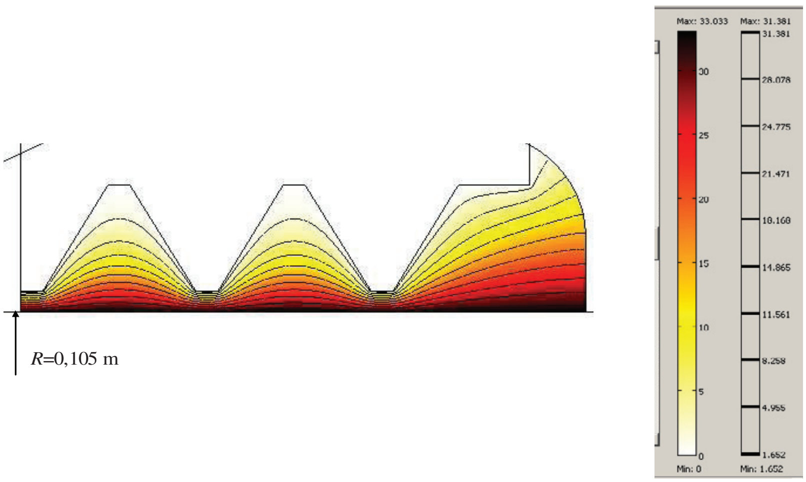



Fig. 8. Distributing the azimuth component of the magnetic fluid of the velocity in the magnetic system clearance at the surface velocity $v_0$ = 33 m/s

In Fig. 9 there are shown vortex rotational structures arising in the magnetic fluid and organized by radial $u_r$ and azimuth $u_z$ components of the velocity. As it is seen from these drawings, the structure with the greatest vortex rotation velocity being equal to 0.34 m/s (makes about 10 % from linear velocity on the surface of the rotational shaft) arises in the most right region. Such a situation is caused by the presence of rather sized free boundary of the fluid. In two other grooved regions, there are also vortex rotational structures; however their maximum velocity value makes about 0.1 m/s being caused by intensive breaking the fluid nearby the walls of the magnetic core.

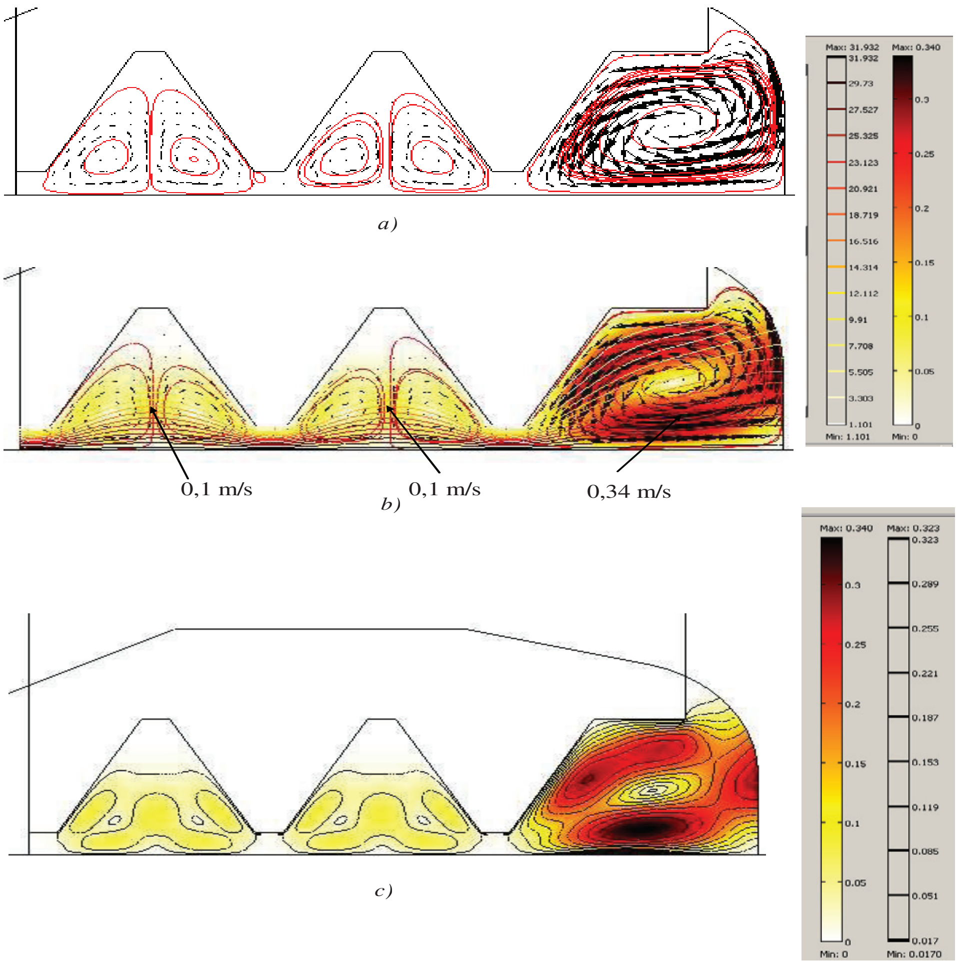



Fig. 9. Vortex rotational structures organized by radial $u_r$ and azimuth $u_z$ components of the velocity (*a*), absolute velocity value $\sqrt{u_r^2+u_z^2}$ that is shown in color (*b*), and isolines of equal values $\sqrt{u_r^2+u_z^2}$ (c)

## 4. Conclusion

There has been developed a mathematical model for analyzing the magnetic field in the core of magnetic-fluid seal (MFS) with account of nonlinear magnetic properties of the materials for magnetic core (magnetic conductor) and shaft. To provide for the numerical realization of this model, there was used a finite element method and Comsol software package. It has been displayed that the most non-uniform magnetic field takes place immediately under the teeth (middle and internal), located at a magnetic pole, and the maximum field on a shaft surface reaches 2.25 T. Under the most external tooth of the pole, the field is weakened (it makes 1.65 T) due to exposing the edge effect associated with the bulging the power lines of the field.

By results of the field calculation, there has been determined the configuration of the magnetic fluid free boundary corresponding to various values of the fluid volume and also defined a position of this boundary at axial pressure drop $\Delta p = 33$ kPa.

There has been developed a hydrodynamic model for calculation of laminar flow of magnetic fluid in the clearance of the magnetic system, which takes into account the fact of fluid viscosity dependence on a value of magnetic induction. To apply this model in practice, it is used the finite element method realized in software package Comsol. There are obtained the vortex rotational structures of the fluid movement in grooves of the magnetic poles, and it was exposed that the greatest vortex rotational velocity exists in the edge internal groove adjoining the free boundary of the fluid. The maximum value of the vortex rotational velocity makes about 10 % of the linear velocity on the surface of the rotational shaft.

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