The Land Redevelopment Problem

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Abstract

- **The land redevelopment problem**: Quite common in the real lives.
- **Methodology**: Mechanism design, modelling and simulation(MATLAB).
- **Application**: Real world land related problems.(Especially the land redevelopment problem).
- **Keywords**: Mechanism design, auction, implementation, non-convex optimization.

Literature Review

- **Main textbooks**: Mechanism Design: A Linear Programming Approach and An Introduction to the Theory of Mechanism Design.
- **Previous presentation paper**: A Conic Approach to the Implementation of Reduced-Form Allocation Rules, working paper, 2019.
- **The statement of the problem and feasible mechanisms**: The land redevelopment problem, 2017.

Preliminary

- A set of agents $N = \{1, ..., n\}$, each owns a separate plot of land.
- The value to agent i of his plot, vi, is private information, with distribution F_i on the support $[\underline{v}, \overline{v}]$. We assume that v_i is independently distributed across owners.
- The redevelopment will yield a payoff of W to a land developer.
- We assume that $W \in (n\underline{v}, n\overline{v})$ is common knowledge among all market participants.
- Consider a direct mechanism $\mathcal{M} = \{\rho, t_1 ..., t_n\}$.

Admissible mechanism requirements

1. *Dominant-strategy incentive compatibility constraint* **(DIC)**

$t_i(v) - \rho(v)v_i \geqslant t_i\left(v_i', v_{-i}\right) - \rho\left(v_i', v_{-i}\right)v_i.$

2. *No naked expropriation* **(NNE)**

$\rho(v) = 0 \Longrightarrow t_i(v) = 0.$

3. *IR constraints* **(IR)**

IR(v)=$\{i : t_i(v) - \rho(v)v_i \geqslant 0\}$, $\#IR(v) \geqslant m.$

4. *Adequate compensation* **(AC)**

$t_j(v) \geqslant \frac{1}{\#IR(v)} \sum_{i \in IR(v)} t_i(v).$

Admissible mechanism requirements

5. *Ex-post budget balance* **(EPBB)**

$\sum_i t_i(v) \leqslant W.$

6. *Ex-ante budget balance* **(EABB)**

$E[\rho(v) \left(\sum_i t_i(v) - W\right)] \leqslant 0.$

We say that a mechanism is admissible if it satisfies DIC, NNE, IR-m, AC, and EPBB.

Definition

Here, we set $n = 3$ and $m = 2$.

For any v, define (note: not sure about sup or max)

$$f_i(v) = max\{v'_i : \rho(v'_i) = 1, \forall i\}$$

$V^\star$=$\{v : \rho(v) = 1\}$

$$V_0^* = \{v \in V^* : f_i(v) < 1, \forall i\}$$
$$V_i^* = \{v \in V^* : f_i(v) = 1, f_j(v) < 1, j \neq i\}$$
$$V_{i,j}^* = \{v \in V^* : f_k(v) = 1, k = i, j; f_k(v) < 1, k \neq i, j\}$$

Venn illustration

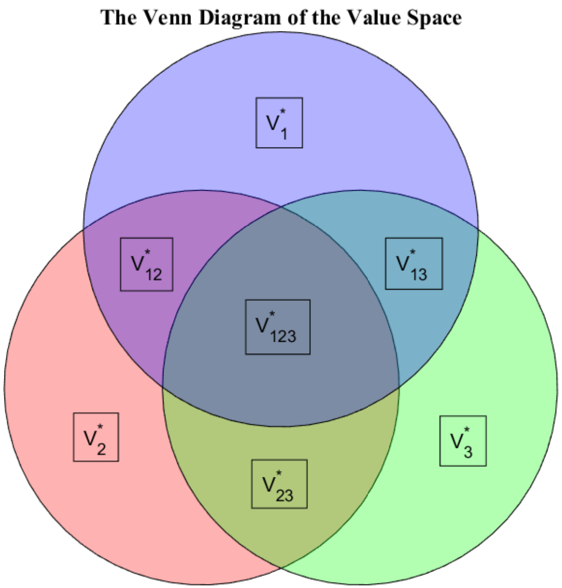

Figure 1: Venn diagram

Triple extension

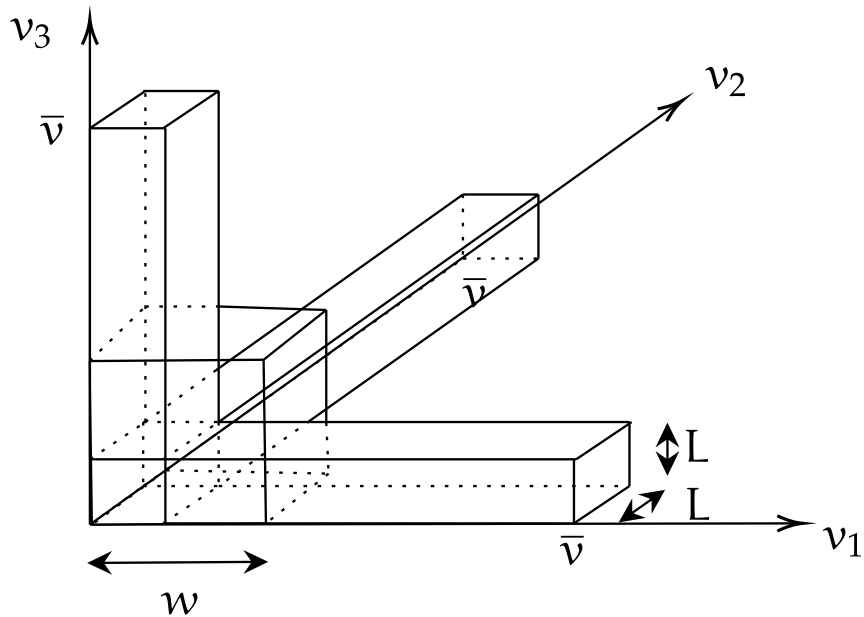

Figure 2: Conceptual graph of triple extension case

Triple extension

- As we all known, the function $\phi(v)$ is the social surplus function, which means $\phi(v) = 3w - v_1 - v_2 - v_3$. And here we set $w = 0.2$ as a constant.

$$M = \int_0^w \int_0^w \int_0^w \phi(v)\, dv_3\, dv_2\, dv_1 + 3 \times \int_w^1 \int_0^L \int_0^L \phi(v)\, dv_3\, dv_2\, dv_1 \tag{1}$$

Simulation of triple extension

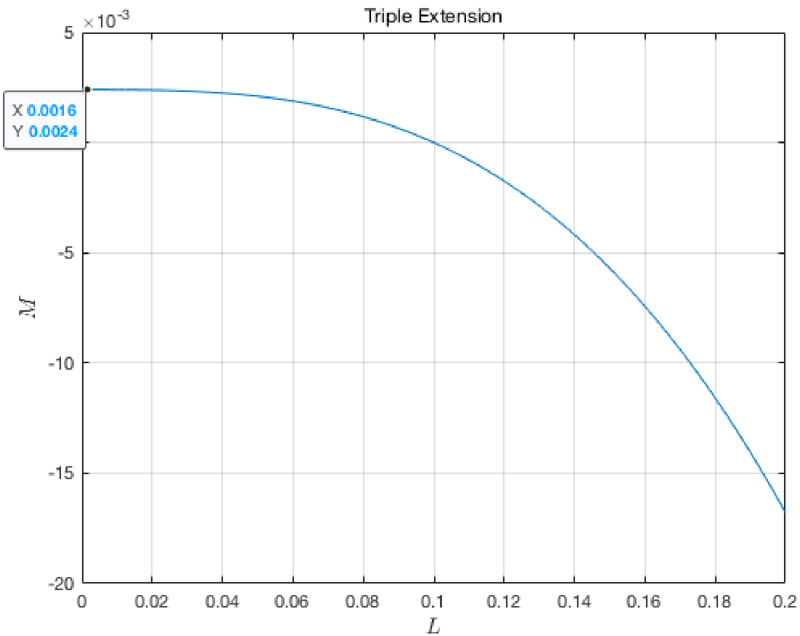

Figure 3: Simulation Results - Triple

Middle addition

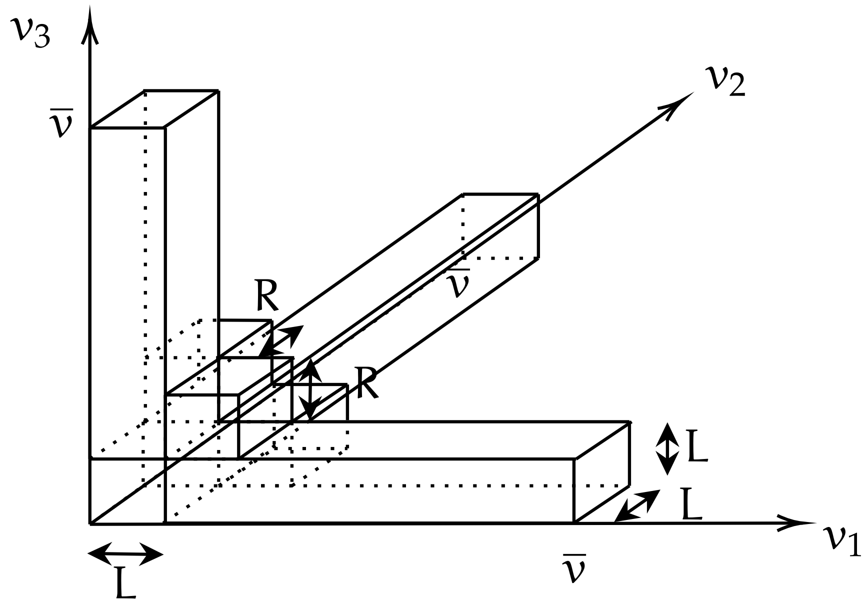

Figure 4: Conceptual graph of middle addition case

Middle addition

$$M = \int_0^L \int_0^L \int_0^L f(v)\, dv_3\, dv_2\, dv_1 \; + 3 \times \int_L^1 \int_0^L \int_0^L f(v)\, dv_3\, dv_2\, dv_1$$
$$+ 3 \times \int_0^L \int_L^{L+R} \int_L^{L+R} f(v)\, dv_3\, dv_2\, dv_1 \tag{2}$$

Simulation of middle addition

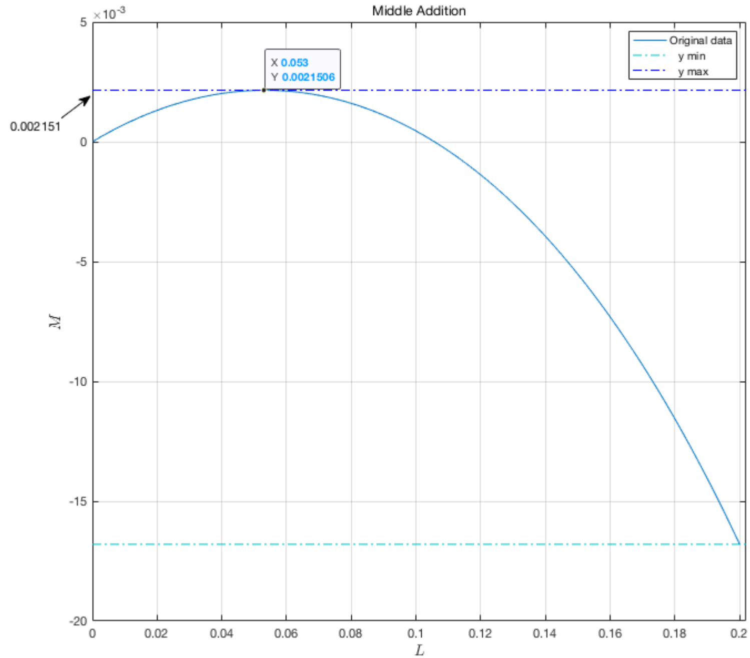

Figure 5: Simulation Results - Middle

Merged simulation

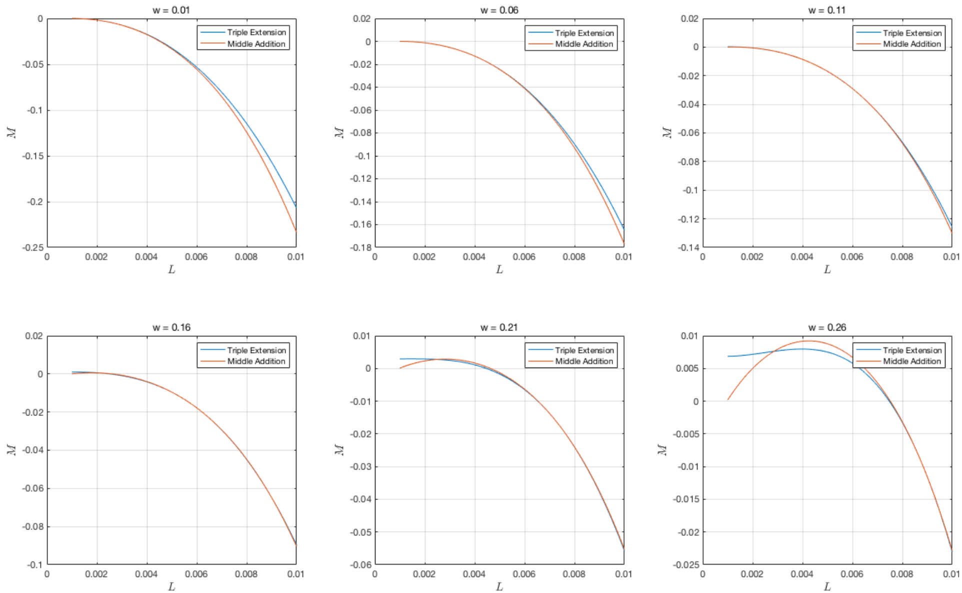

Figure 6: Simulation Results (Merged)

Density function

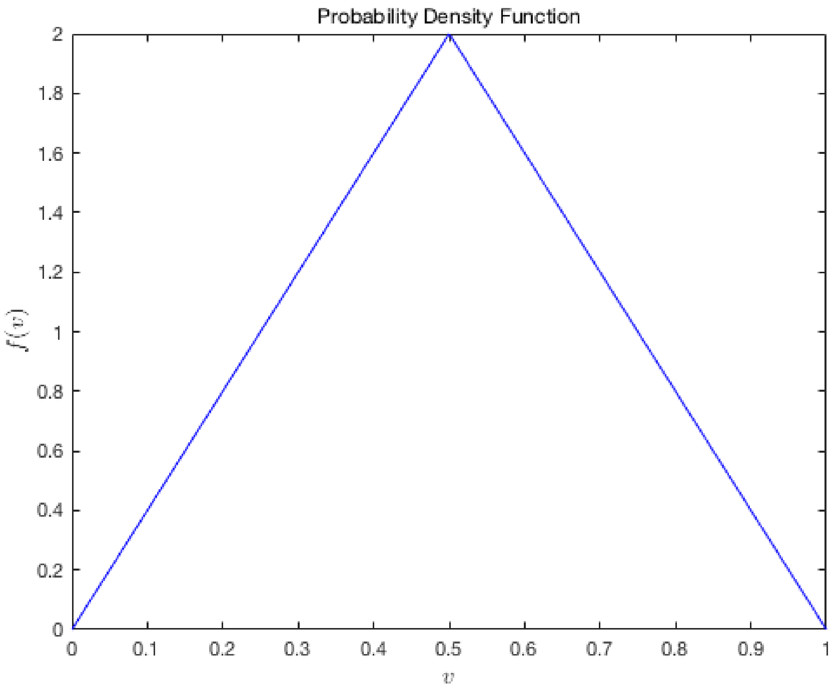

Figure 7: Probability density function

Density function

$$f_X(x) = \begin{cases} f_X^1(x) = 4 \times x, x \in [0, \frac{1}{2}] \\ f_X^2(x) = 4 - 4 \times x, x \in [\frac{1}{2}, 1] \end{cases} \tag{3}$$

- And we assume three variables (v_1, v_2, v_3) are i.i.d. random variables, which means $f_X(v_1, v_2, v_3) = f_X(v_1) \times f_X(v_2) \times f_X(v_3)$.
- We choose w from 0 to 0.2, and the integral can be divided into the following parts(W.L.G.)

Density function(triple extension)

$$
\begin{aligned}
M &= \int_0^w \int_0^w \int_0^w \phi(v) f_X(v_1, v_2, v_3)\, dv_3\, dv_2\, dv_1 \\
&\quad + 3 \times \int_w^{\frac{1}{2}} \int_0^L \int_0^L \phi(v) f_X(v_1, v_2, v_3)\, dv_3\, dv_2\, dv_1 \\
&\quad + 3 \times \int_{\frac{1}{2}}^1 \int_0^L \int_0^L \phi(v) f_X(v_1, v_2, v_3)\, dv_3\, dv_2\, dv_1 \\
&= \int_0^w \int_0^w \int_0^w \phi(v) f_X^1(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1 \\
&\quad + 3 \int_w^{\frac{1}{2}} \int_0^L \int_0^L \phi(v) f_X^1(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1 \\
&\quad + 3 \times \int_{0.5}^1 \int_0^L \int_0^L \phi(v) f_X^2(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1
\end{aligned} \tag{4}
$$

Density function(middle addition)

We choose w from 0 to 0.2, and the integral can be divided into the following parts(W.L.G.):

$$
\begin{aligned}
M' = & \int_0^L \int_0^L \int_0^L \phi(v) f_X^1(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1 \\
& + 3 \times \int_L^{\frac{1}{2}} \int_0^L \int_0^L \phi(v) f_X^1(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1 \\
& + 3 \times \int_{\frac{1}{2}}^1 \int_0^L \int_0^L \phi(v) f_X^2(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1 \\
& + 3 \times \int_0^L \int_L^{L+R} \int_L^{L+R} \phi(v) f_X^1(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1
\end{aligned}
\tag{5}
$$

Merged simulation

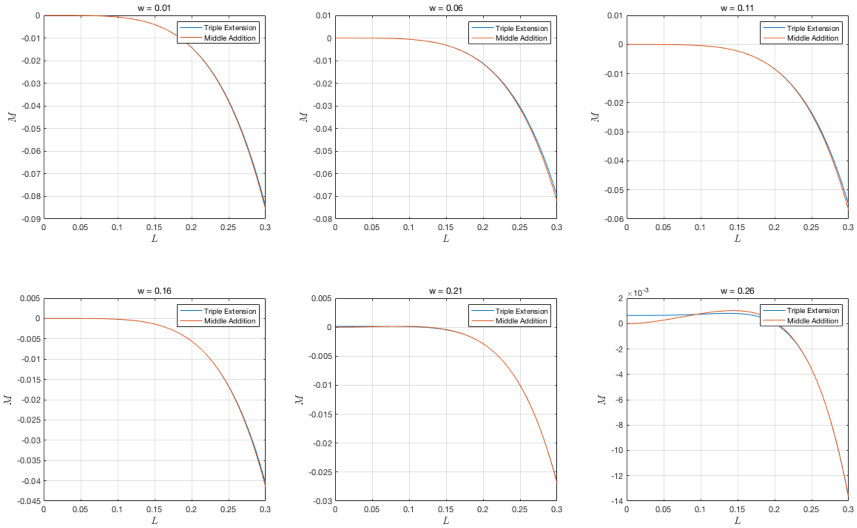

Figure 8: Plus probability density function

Review of previous results

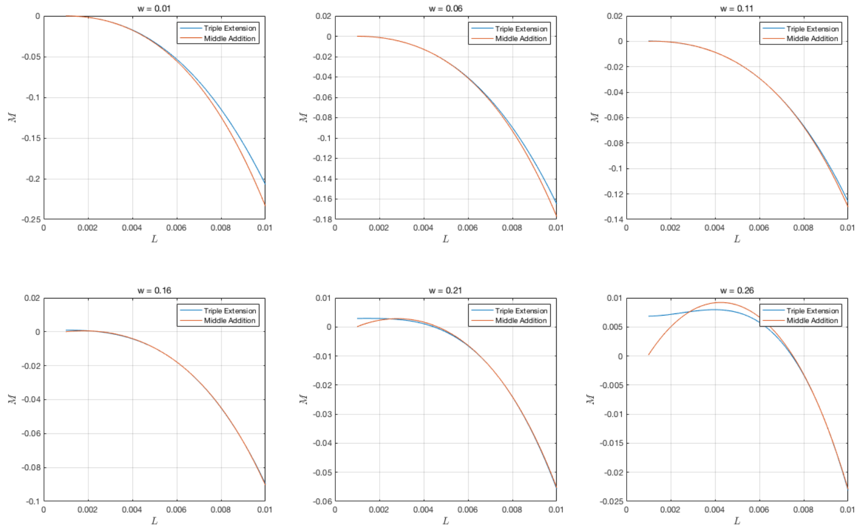

Figure 9: Simulation Results (Merged)

Cutting of triple extension

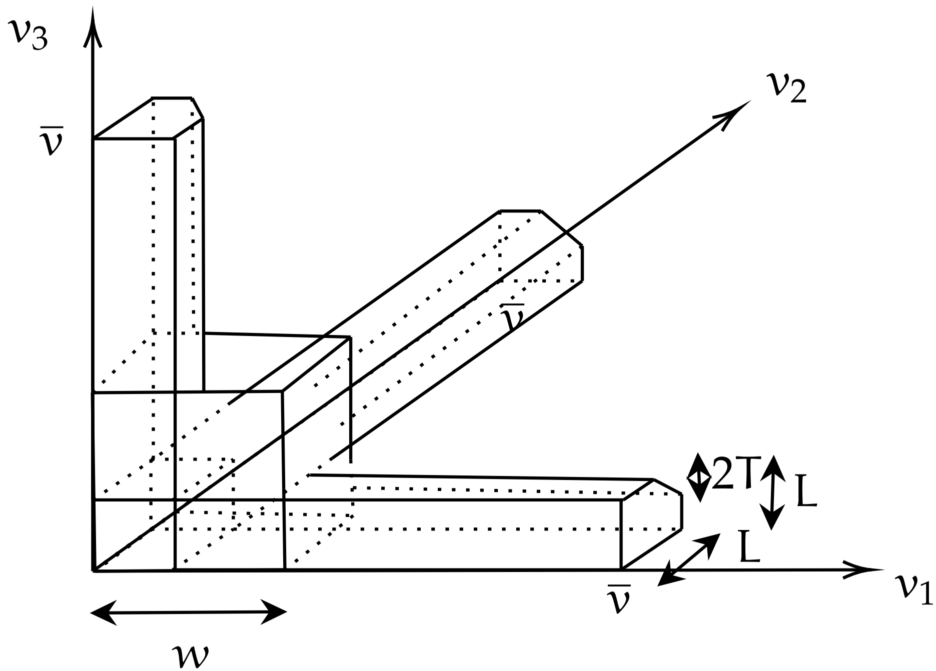

Figure 10: Conceptual graph of triple extension cutting case

Cutting of triple extension

The integral on this area is as followed:

$$M_3 = \int_0^w \int_0^w \int_0^w \phi(v)\, dv_3\, dv_2\, dv_1 + \int_0^L \int_0^L \int_w^1 \phi(v)\, dv_3\, dv_2\, dv_1 - \int_{L-2T}^{L} \int_{2L-2T-v_1}^{L} \int_w^1 \phi(v)\, dv_3\, dv_2\, dv_1 \tag{6}$$

Cutting of middle addition

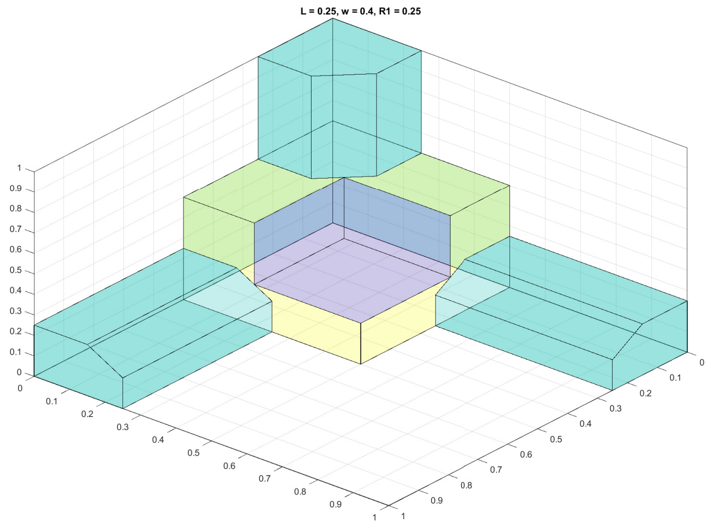

Figure 11: Conceptual graph of middle addition cutting case

Cutting of middle addition

$$
\begin{aligned}
M_4 =& 3 \times \int_{L-T}^{L+R'} \int_{0}^{L-T} \int_{L-T}^{L+R'} \phi(v)\, dv_3\, dv_2\, dv_1 \\
&+ 3 \times \int_{0}^{L} \int_{0}^{L} \int_{L+R'}^{1} \phi(v)\, dv_3\, dv_2\, dv_1 \\
&- 3 \times \int_{L-2T}^{L} \int_{2L-2T-v_1}^{L} \int_{L+R'}^{1} \phi(v)\, dv_3\, dv_2\, dv_1 \\
&+ \int_{0}^{L-T} \int_{0}^{L-T} \int_{0}^{L-T} \phi(v)\, dv_3\, dv_2\, dv_1 \\
&+ 3 \times \int_{L-T}^{L+R'} \int_{0}^{L-T} \int_{0}^{L-T} \phi(v)\, dv_3\, dv_2\, dv_1
\end{aligned}
\tag{7}
$$

Concrete simulation

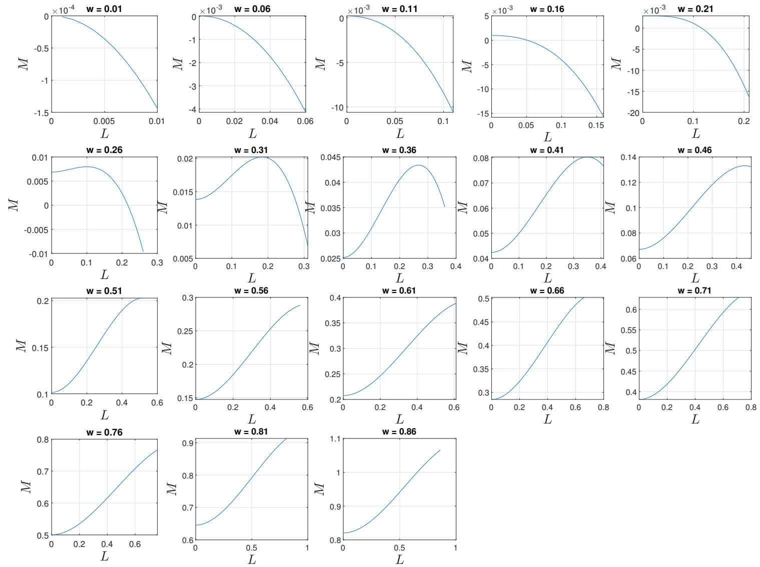

Figure 12: Simulation Results: Triple extension(Updated)

Concrete simulation

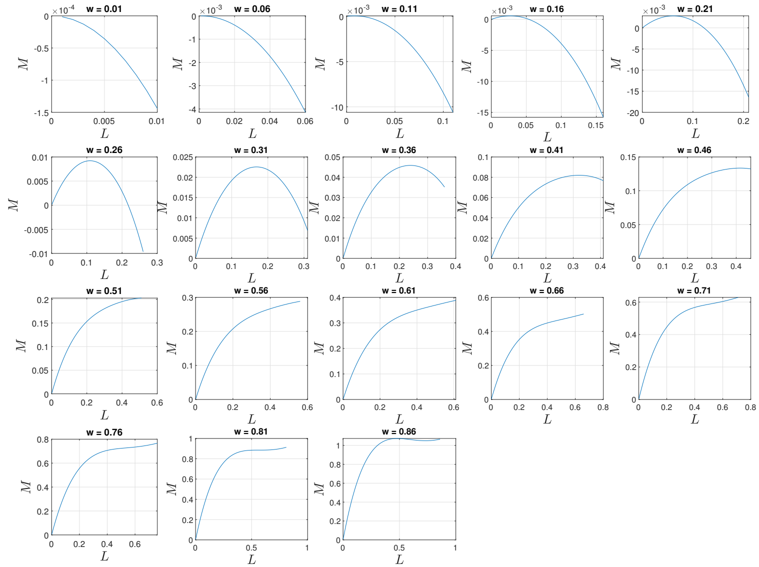

Figure 13: Simulation Results: Middle addition(Updated)

Four shapes: T=0.0025(T is exogenous variable)

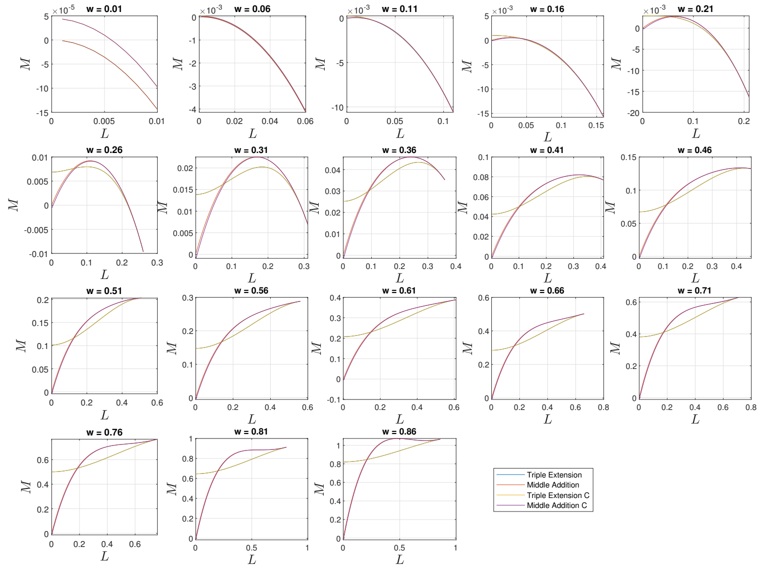

Figure 14: Simulation results: four shapes

Tabular for the results

Table 1: The max of the four shapes respectly.(T=0.0025)

	Different shapes			
	Triple	Middle	Triple C	Middle C
subfigure 5(w=0.21)	0.002923	0.002848	0.002923	0.002744
subfigure 6(w=0.26)	0.007965	0.009212	0.007968	0.009126
subfigure 7(w=0.31)	0.02023	0.02255	0.02024	0.02251
subfigure 8(w=0.36)	0.0434	0.04584	0.04341	0.04588
subfigure 18(w=0.86)	1.066	1.073	1.066	1.073

Four shapes: (T is endogenous variable)

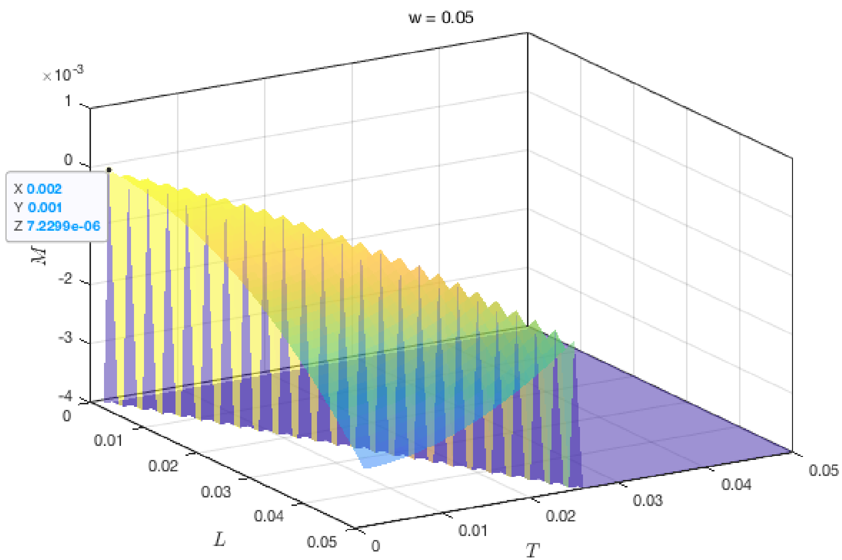

Figure 15: Cutting of triple extension

Four shapes: (T is endogenous variable)

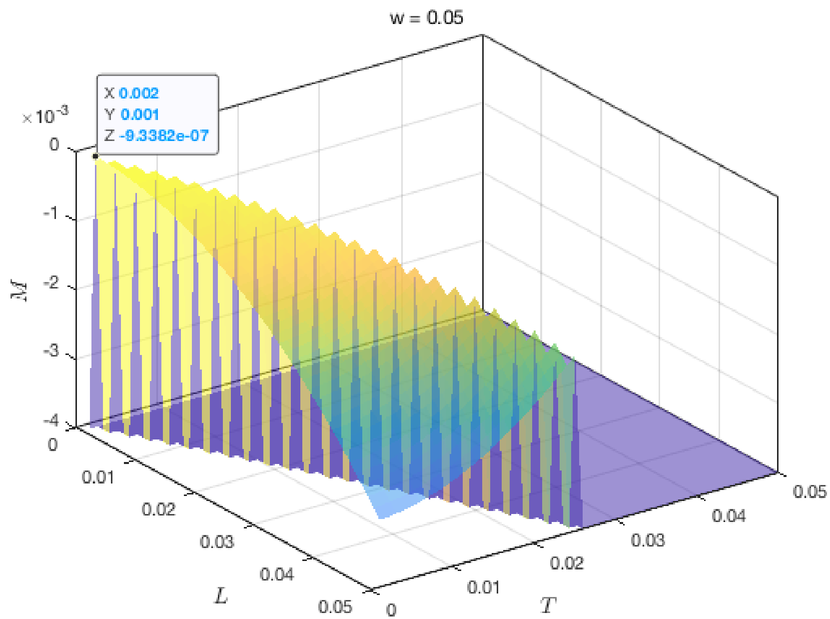

Figure 16: Cutting of middle addition

Tabular for the results

T	w	Triple	Middle	Triple C	Middle C
0.0025	0.01	-1.40E-06	-1.41E-06	1.62E-05	1.62E-05
	0.06	1.85E-05	1.13E-06	3.07E-05	7.99E-06
	0.11	0.000219012	5.37E-05	0.000226476	3.47E-05
	0.16	0.000982785	0.000566583	0.000985893	0.000516013
	0.21	0.002922694	0.002847669	0.002922862	0.002781278
	0.26	0.00796464	0.009212192	0.007965935	0.009157404
	0.31	0.02023046	0.022545649	0.020232729	0.022525776
	0.36	0.043398599	0.045844561	0.043401735	0.04586886
	0.41	0.08033079	0.081962165	0.080334588	0.082019334
	0.46	0.133071723	0.133532655	0.13307603	0.133583876
	0.51	0.20279997	0.20279997	0.202804411	0.202805017
	0.56	0.28826112	0.28826112	0.288264695	0.288265427
	0.61	0.38836077	0.38836077	0.388363573	0.388364442
	0.66	0.50233392	0.50233392	0.502336045	0.502337062
	0.71	0.62896557	0.62896557	0.628967111	0.628968288
	0.76	0.76659072	0.76659072	0.76659177	0.766593119
	0.81	0.91309437	0.91309437	0.913095023	0.913096556
	0.86	1.06591152	1.073484372	1.06591187	1.073190926

Figure 17: T=0.0025

Tabular for the results

T	w	Triple	Middle	Triple C	Middle C
0.005	~~0.01~~	~~-0.00000139872~~	~~-0.00000140540~~	~~0.00006844578~~	~~0.00006837755~~
	0.06	0.00001845018	0.00000112785	0.00006714218	0.00003845146
	0.11	0.00021901158	0.00005370348	0.00024842608	0.00003072976
	0.16	0.00098278548	0.00056658336	0.00099479748	0.00047365622
	0.21	0.00292269444	0.00284766895	0.00292310366	0.00271952198
	0.26	0.00796464000	0.00921219209	0.00796945000	0.00910494335
	0.31	0.02023046016	0.02254564865	0.02023920522	0.02250724986
	0.36	0.04339859904	0.04584456145	0.04341082304	0.04589497852
	0.41	0.08033079000	0.08196216505	0.08034568750	0.08208013675
	0.46	0.13307172306	0.13353265536	0.13308867906	0.13364292344
	0.51	0.20279997000	0.20279997000	0.20281748750	0.20281990242
	0.56	0.28826112000	0.28826112000	0.28827520000	0.28827811414
	0.61	0.38836077000	0.38836077000	0.38837178750	0.38837524773
	0.66	0.50233392000	0.50233392000	0.50234225000	0.50234630320
	0.71	0.62896557000	0.62896557000	0.62897158750	0.62897628055
	0.76	0.76659072000	0.76659072000	0.76659480000	0.76660017977
	0.81	0.91309437000	0.91309437000	0.91309688750	0.91310300086
	0.86	1.06591152000	1.07348437200	1.06591285000	1.07290198632

Figure 18: T=0.005

Tabular for the results

T	w	Triple	Middle	Triple C	Middle C
0.01	~~0.01~~	~~-0.00000139872~~	~~-0.00000140340~~	~~0.00027401928~~	~~0.00027383109~~
	~~0.06~~	~~0.00001845018~~	~~0.00000112785~~	~~0.00020945018~~	~~0.00016868700~~
	~~0.11~~	~~0.00021901158~~	~~0.00005370348~~	~~0.00033310958~~	~~0.00006423074~~
	~~0.16~~	~~0.00098278548~~	~~0.00056658336~~	~~0.00102770784~~	~~0.00041260814~~
	0.21	0.00292269444	0.00284766895	0.00292335646	0.00260834564
	0.26	0.00796464000	0.00921219209	0.00798147278	0.00900638732
	0.31	0.02023046016	0.02254564865	0.02026325172	0.02247347159
	0.36	0.04339859904	0.04584456145	0.04344493504	0.04595093760
	0.41	0.08033079000	0.08196216505	0.08038802000	0.08221104925
	0.46	0.13307172306	0.13353265536	0.13313768352	0.13378150592
	0.51	0.20279997000	0.20279997000	0.20286808000	0.20287764875
	0.56	0.28826112000	0.28826112000	0.28831568000	0.28832723625
	0.61	0.38836077000	0.38836077000	0.38840328000	0.38841701125
	0.66	0.50233392000	0.50233392000	0.50236588000	0.50238197375
	0.71	0.62896557000	0.62896557000	0.62898848000	0.62900712375
	0.76	0.76659072000	0.76659072000	0.76660608000	0.76662746125
	0.81	0.91309437000	0.91309437000	0.91310368000	0.91312798625
	0.86	1.06591152000	1.07348437200	1.06591628000	1.07233901131

Figure 19: T=0.01

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