The Land Redevelopment Problem

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Abstract

- **The land redevelopment problem**: Quite common in the real lives.
- **Methodology**: Mechanism design, modelling and simulation(MATLAB).
- **Application**: Real world land related problems.(Especially the land redevelopment problem).
- **Keywords**: Mechanism design, auction, implementation, non-convex optimization.

Literature Review

- **Main textbooks**: Mechanism Design: A Linear Programming Approach and An Introduction to the Theory of Mechanism Design.
- **Previous presentation paper**: A Conic Approach to the Implementation of Reduced-Form Allocation Rules, working paper, 2019.
- **The statement of the problem and feasible mechanisms**: The land redevelopment problem, 2017.

Preliminary

- A set of agents $N = \{1, ..., n\}$, each owns a separate plot of land.
- The value to agent i of his plot, vi, is private information, with distribution F_i on the support $[\underline{v}, \bar{v}]$. We assume that v_i is independently distributed across owners.
- The redevelopment will yield a payoff of W to a land developer.
- We assume that $W \in (n\underline{v}, n\bar{v})$ is common knowledge among all market participants.
- Consider a direct mechanism $\mathcal{M} = \{\rho, t_1 ..., t_n\}$.

Admissible mechanism requirements

1. *Dominant-strategy incentive compatibility constraint* (DIC)

$$t_i(v) - \rho(v)v_i \geqslant t_i\left(v_i', v_{-i}\right) - \rho\left(v_i', v_{-i}\right) v_i.$$

2. *No naked expropriation* (NNE)

$$\rho(v) = 0 \Longrightarrow t_i(v) = 0.$$

3. *IR constraints* (IR)

$$\text{IR}(v)=\{i : t_i(v) - \rho(v)v_i \geqslant 0\}, \#\text{IR}(v) \geqslant m.$$

4. *Adequate compensation* (AC)

$$t_j(v) \geqslant \frac{1}{\#\text{IR}(v)} \sum_{i \in \text{IR}(v)} t_i(v).$$

Admissible mechanism requirements

5. *Ex-post budget balance* **(EPBB)**

$\sum_i t_i(v) \leqslant W.$

6. *Ex-ante budget balance* **(EABB)**

$E[\rho(v)\left(\sum_i t_i(v) - W\right)] \leqslant 0.$

We say that a mechanism is admissible if it satisfies DIC, NNE, IR-m, AC, and EPBB.

Definition

Here, we set $n = 3$ and $m = 2$.

For any v, define (note: not sure about sup or max)

$$f_i(v) = max\{v'_i : \rho(v'_i) = 1, \forall i\}$$

$V^\star$=$\{v : \rho(v) = 1\}$

$$V_0^* = \{v \in V^* : f_i(v) < 1, \forall i\}$$
$$V_i^* = \{v \in V^* : f_i(v) = 1, f_j(v) < 1, j \neq i\}$$
$$V_{i,j}^* = \{v \in V^* : f_k(v) = 1, k = i, j; f_k(v) < 1, k \neq i, j\}$$

Venn illustration

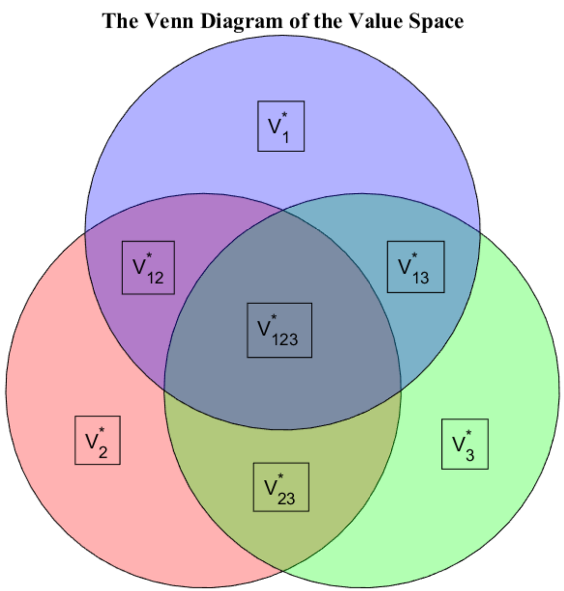

Figure 1: Venn diagram

Triple extension

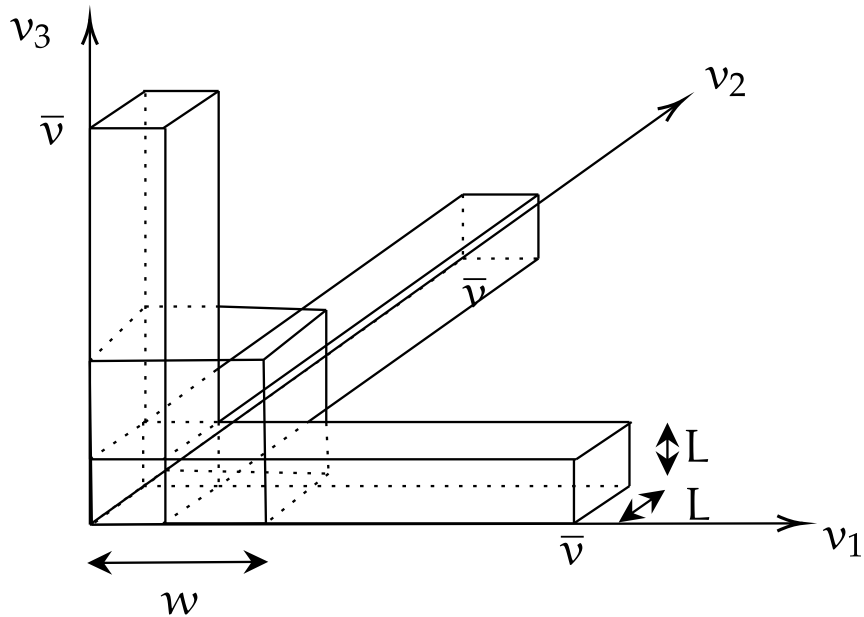

Figure 2: Conceptual graph of triple extension case

Triple extension

- As we all known, the function $\phi(v)$ is the social surplus function, which means $\phi(v) = 3w - v_1 - v_2 - v_3$. And here we set $w = 0.2$ as a constant.

$$M = \int_0^w \int_0^w \int_0^w \phi(v)\, dv_3\, dv_2\, dv_1 + 3 \times \int_w^1 \int_0^L \int_0^L \phi(v)\, dv_3\, dv_2\, dv_1 \tag{1}$$

Simulation of triple extension

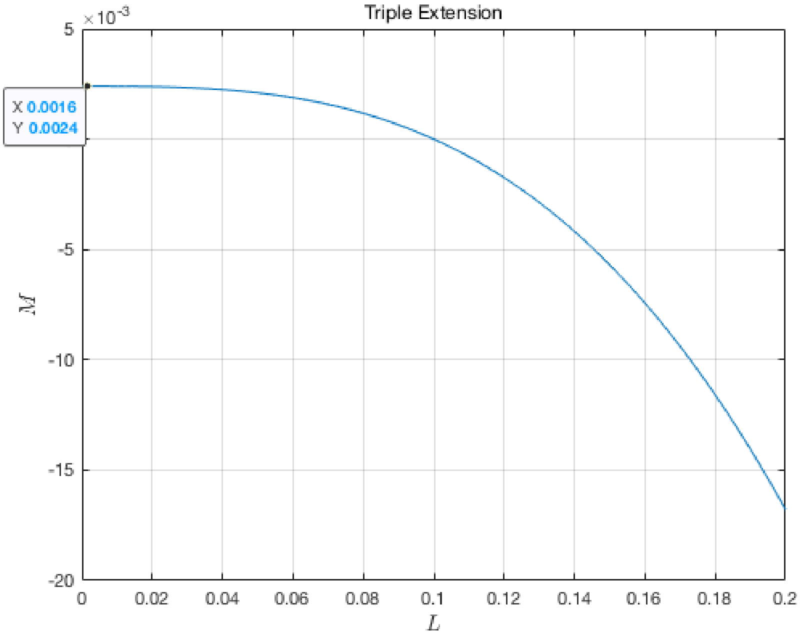

Figure 3: Simulation Results - Triple

Middle addition

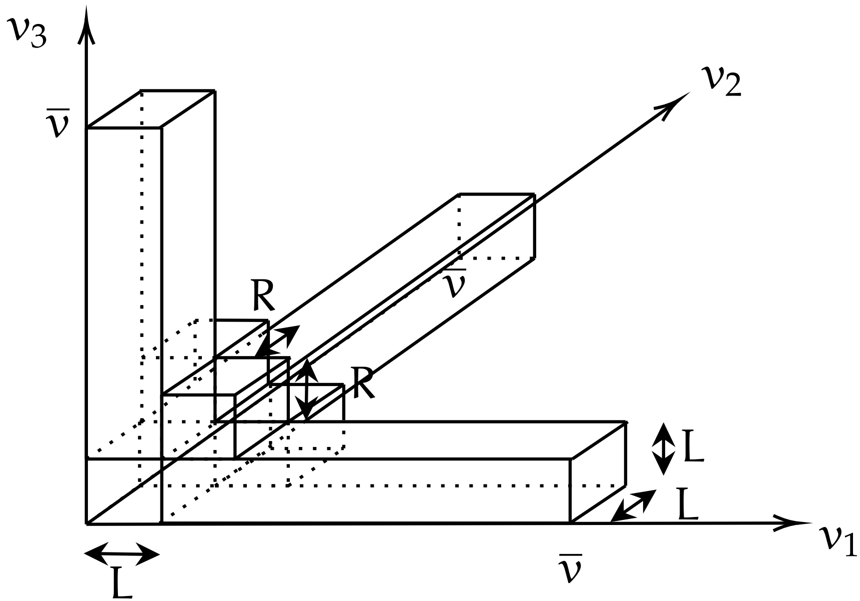

Figure 4: Conceptual graph of middle addition case

Middle addition

$$M = \int_0^L \int_0^L \int_0^L f(v)\,dv_3\,dv_2\,dv_1 + 3 \times \int_L^1 \int_0^L \int_0^L f(v)\,dv_3\,dv_2\,dv_1$$
$$+ 3 \times \int_0^L \int_L^{L+R} \int_L^{L+R} f(v)\,dv_3\,dv_2\,dv_1 \tag{2}$$

Simulation of middle addition

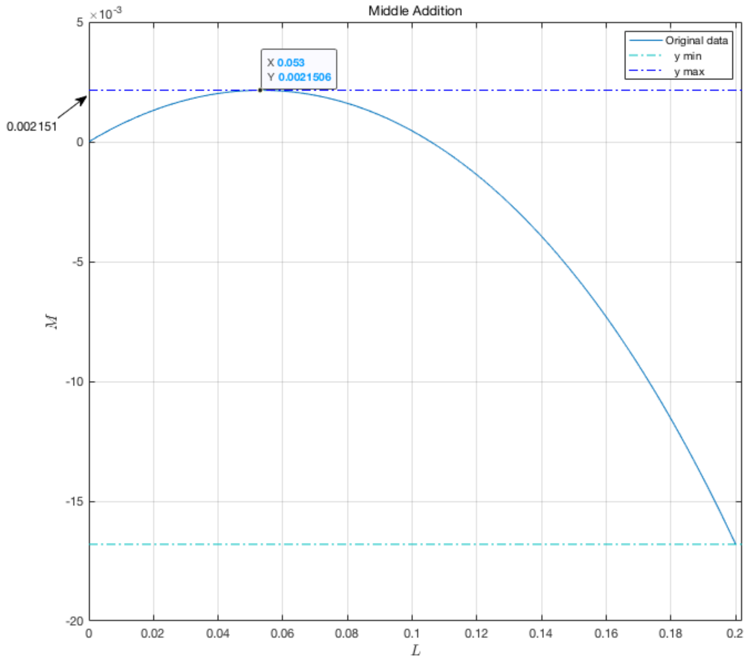

Figure 5: Simulation Results - Middle

Merged simulation

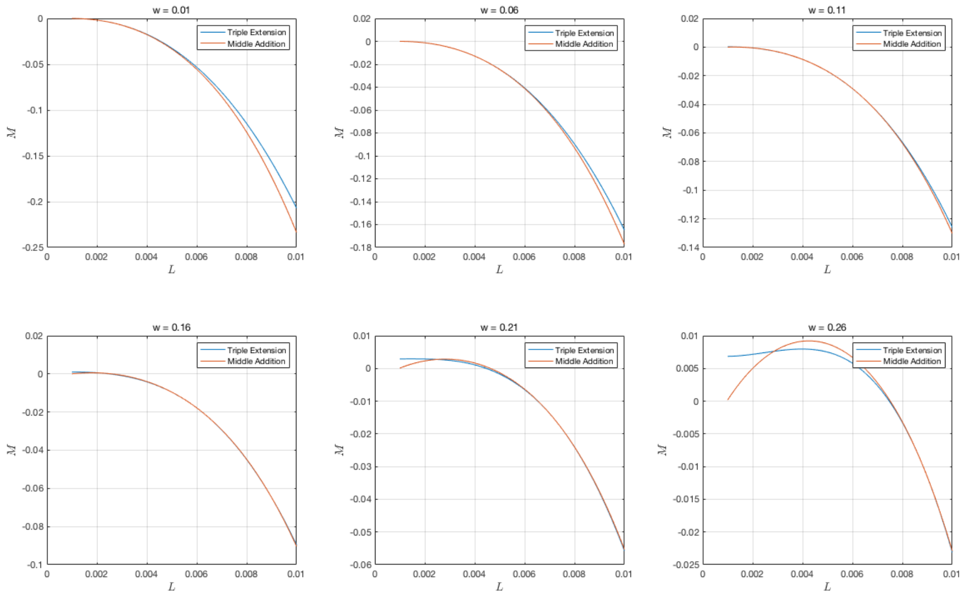

Figure 6: Simulation Results (Merged)

Density function

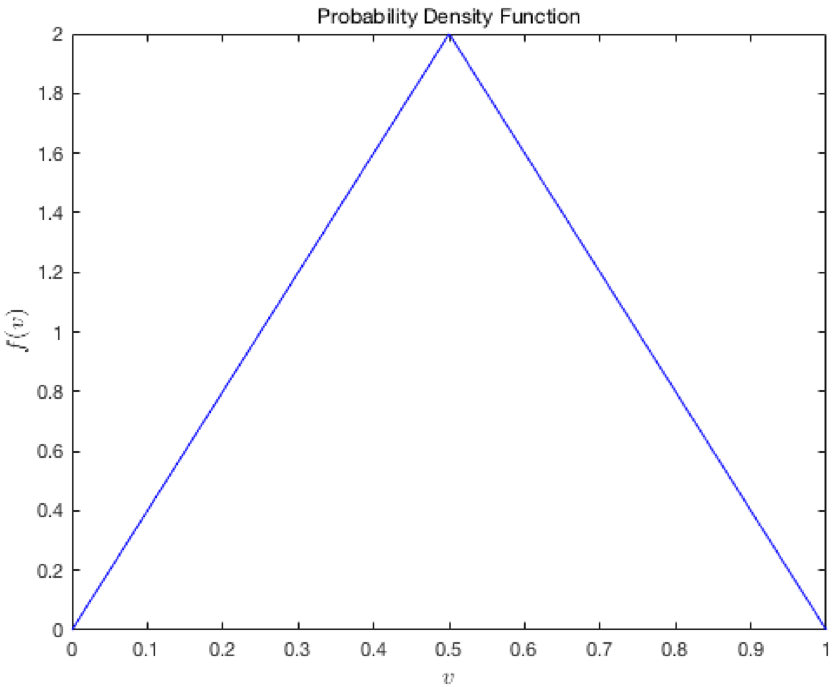

Figure 7: Probability density function

Density function

$$f_X(x) = \begin{cases} f_X^1(x) = 4 \times x, x \in [0, \frac{1}{2}] \\ f_X^2(x) = 4 - 4 \times x, x \in [\frac{1}{2}, 1] \end{cases} \tag{3}$$

- And we assume three variables (v_1, v_2, v_3) are i.i.d. random variables, which means $f_X(v_1, v_2, v_3) = f_X(v_1) \times f_X(v_2) \times f_X(v_3)$.
- We choose w from 0 to 0.2, and the integral can be divided into the following parts(W.L.G.)

Density function(triple extension)

$$
\begin{aligned}
M &= \int_0^w \int_0^w \int_0^w \phi(v) f_X(v_1, v_2, v_3)\, dv_3\, dv_2\, dv_1 \\
&\quad + 3 \times \int_w^{\frac{1}{2}} \int_0^L \int_0^L \phi(v) f_X(v_1, v_2, v_3)\, dv_3\, dv_2\, dv_1 \\
&\quad + 3 \times \int_{\frac{1}{2}}^1 \int_0^L \int_0^L \phi(v) f_X(v_1, v_2, v_3)\, dv_3\, dv_2\, dv_1 \\
&= \int_0^w \int_0^w \int_0^w \phi(v) f_X^1(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1 \\
&\quad + 3 \int_w^{\frac{1}{2}} \int_0^L \int_0^L \phi(v) f_X^1(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1 \\
&\quad + 3 \times \int_{0.5}^1 \int_0^L \int_0^L \phi(v) f_X^2(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1
\end{aligned} \tag{4}
$$

Density function(middle addition)

We choose w from 0 to 0.2, and the integral can be divided into the following parts(W.L.G.):

$$
\begin{aligned}
M' = & \int_0^L \int_0^L \int_0^L \phi(v) f_X^1(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1 \\
& + 3 \times \int_L^{\frac{1}{2}} \int_0^L \int_0^L \phi(v) f_X^1(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1 \\
& + 3 \times \int_{\frac{1}{2}}^1 \int_0^L \int_0^L \phi(v) f_X^2(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1 \\
& + 3 \times \int_0^L \int_L^{L+R} \int_L^{L+R} \phi(v) f_X^1(v_1) f_X^1(v_2) f_X^1(v_3)\, dv_3\, dv_2\, dv_1
\end{aligned} \tag{5}
$$

Merged simulation

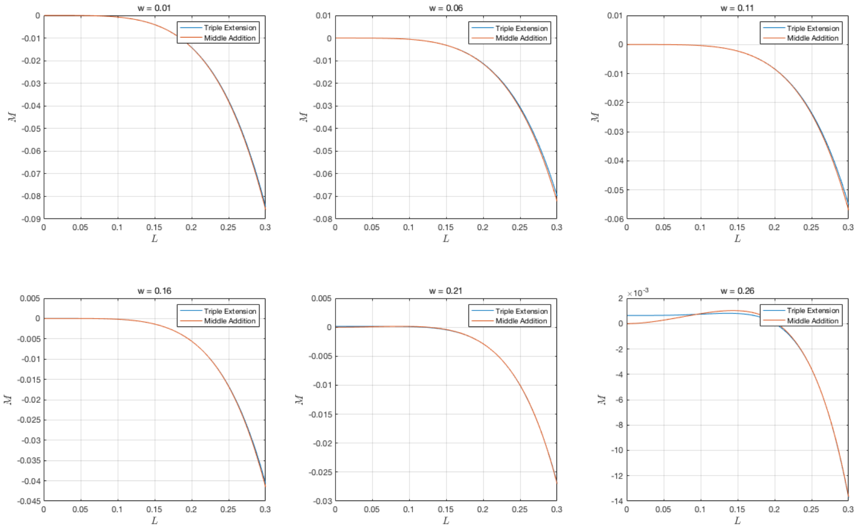

Figure 8: Plus probability density function

Review of previous results

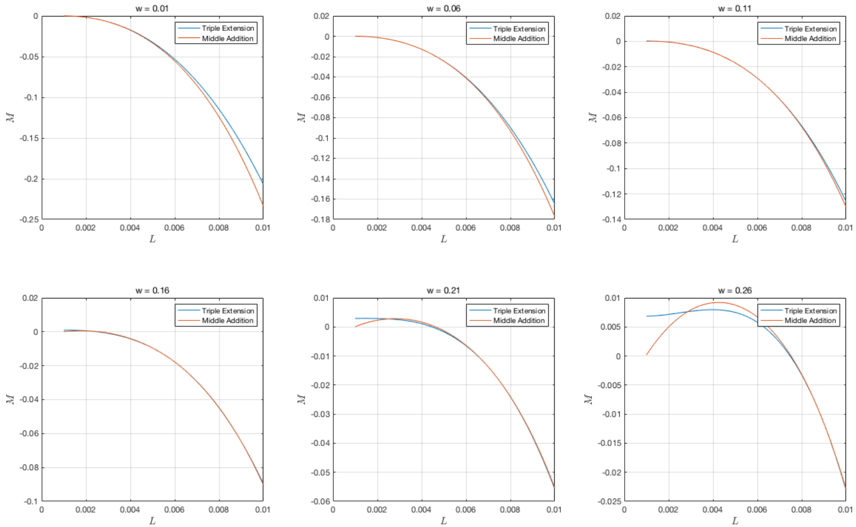

Figure 9: Simulation Results (Merged)

Cutting of triple extension

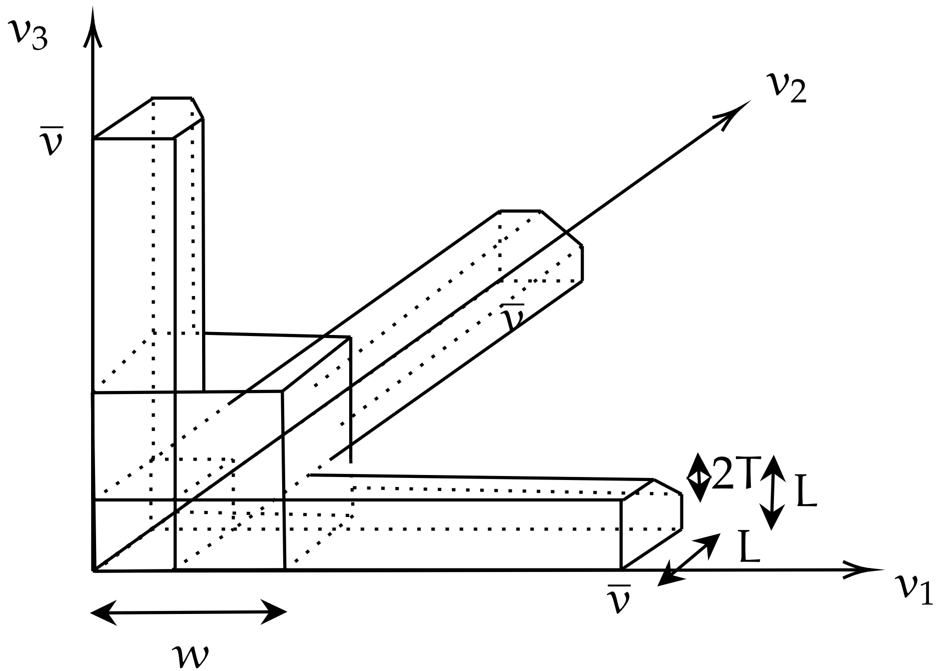

Figure 10: Conceptual graph of triple extension cutting case

Cutting of triple extension

The integral on this area is as followed:

$$M_3 = \int_0^w \int_0^w \int_0^w \phi(v)\, dv_3\, dv_2\, dv_1 + \int_0^L \int_0^L \int_w^1 \phi(v)\, dv_3\, dv_2\, dv_1 \\ - \int_{L-2T}^L \int_{2L-2T-v_1}^L \int_w^1 \phi(v)\, dv_3\, dv_2\, dv_1 \tag{6}$$

Cutting of middle addition

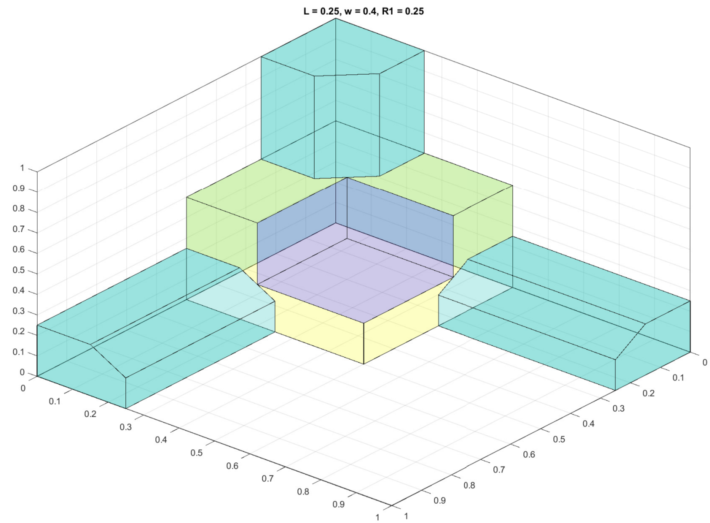

Figure 11: Conceptual graph of middle addition cutting case

Cutting of middle addition

$$
\begin{aligned}
M_4 =& 3 \times \int_{L-T}^{L+R'} \int_{0}^{L-T} \int_{L-T}^{L+R'} \phi(v)\, dv_3\, dv_2\, dv_1 \\
&+ 3 \times \int_{0}^{L} \int_{0}^{L} \int_{L+R'}^{1} \phi(v)\, dv_3\, dv_2\, dv_1 \\
&- 3 \times \int_{L-2T}^{L} \int_{2L-2T-v_1}^{L} \int_{L+R'}^{1} \phi(v)\, dv_3\, dv_2\, dv_1 \\
&+ \int_{0}^{L-T} \int_{0}^{L-T} \int_{0}^{L-T} \phi(v)\, dv_3\, dv_2\, dv_1 \\
&+ 3 \times \int_{L-T}^{L+R'} \int_{0}^{L-T} \int_{0}^{L-T} \phi(v)\, dv_3\, dv_2\, dv_1
\end{aligned}
\tag{7}
$$

Concrete simulation

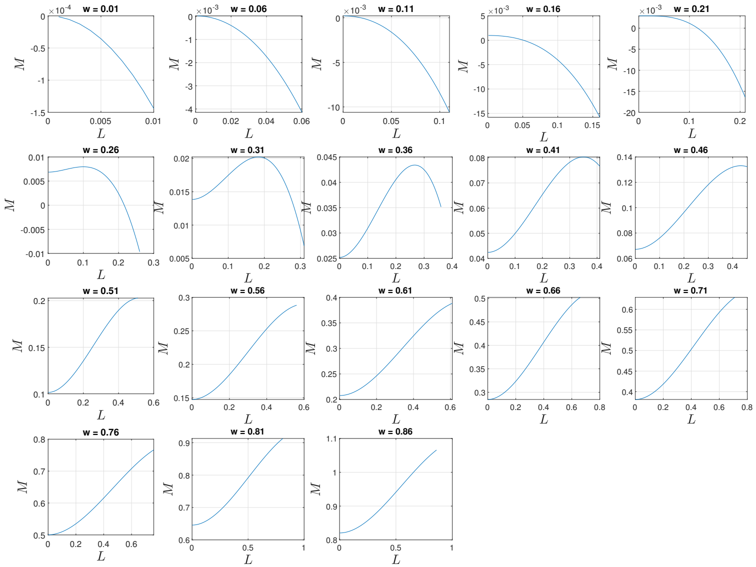

Figure 12: Simulation Results: Triple extension(Updated)

Concrete simulation

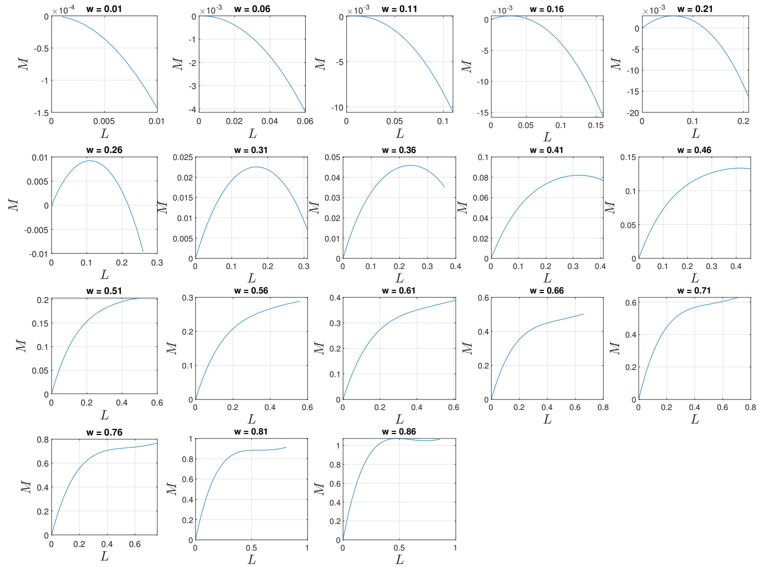

Figure 13: Simulation Results: Middle addition(Updated)

Four shapes: T=0.0025(T is exogenous variable)

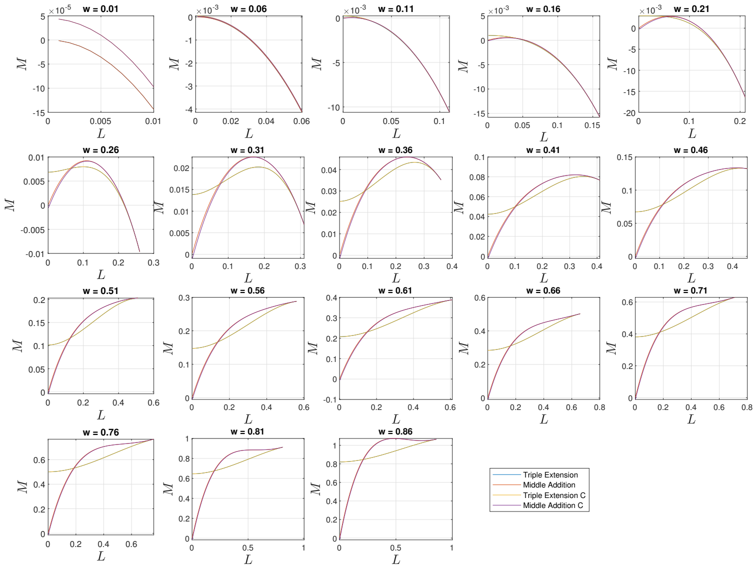

Figure 14: Simulation results: four shapes

Tabular for the results

Table 1: The max of the four shapes respectly.(T=0.0025)

	Different shapes			
	Triple	Middle	Triple C	Middle C
subfigure 5(w=0.21)	0.002923	0.002848	0.002923	0.002744
subfigure 6(w=0.26)	0.007965	0.009212	0.007968	0.009126
subfigure 7(w=0.31)	0.02023	0.02255	0.02024	0.02251
subfigure 8(w=0.36)	0.0434	0.04584	0.04341	0.04588
subfigure 18(w=0.86)	1.066	1.073	1.066	1.073

Triple extension: (T is endogenous variable)

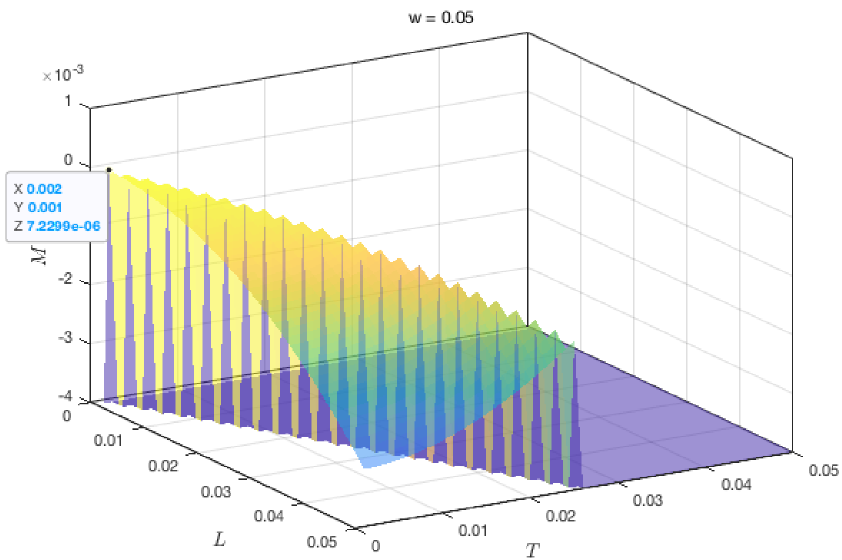

Figure 15: Cutting of triple extension

Middle addition: (T is endogenous variable)

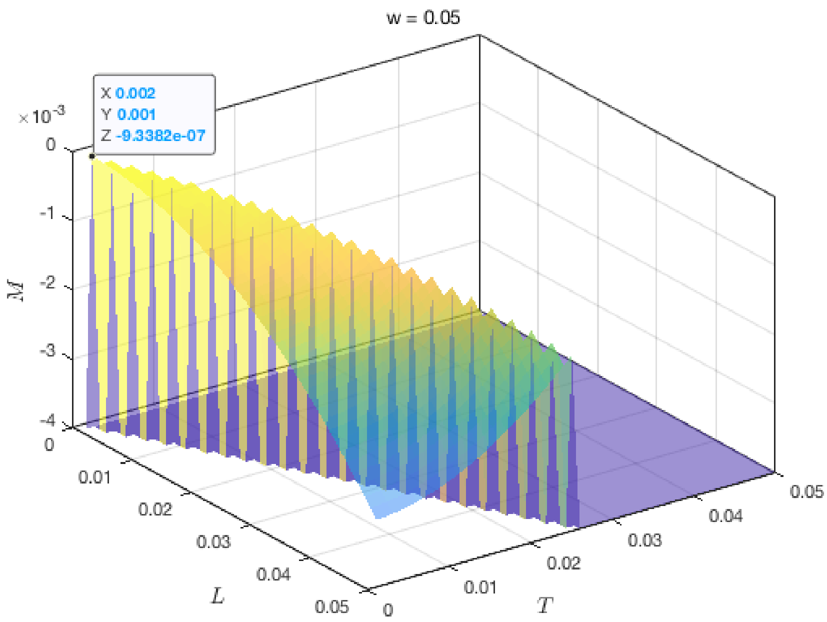

Figure 16: Cutting of middle addition

Merged forms: w=0.01

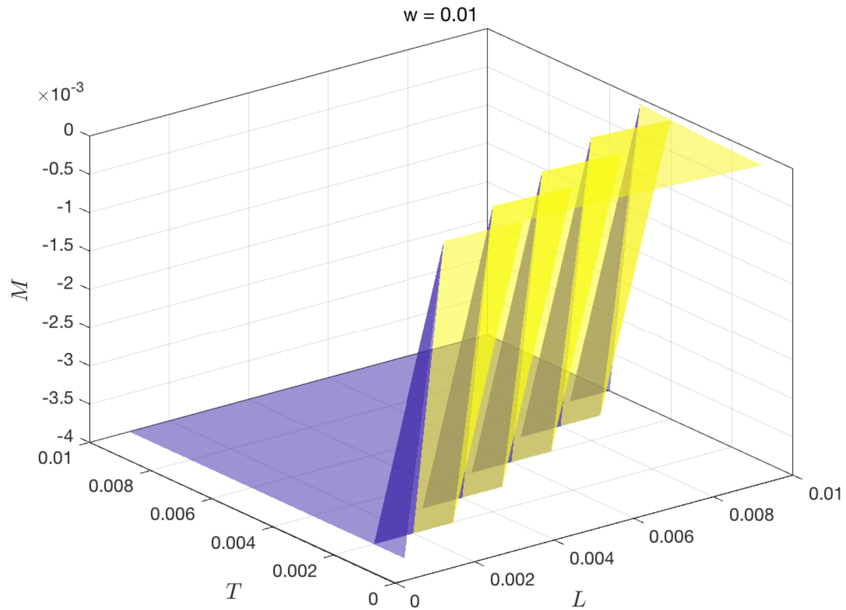

Figure 17: Merged forms: w=0.01

Merged forms: w=0.05

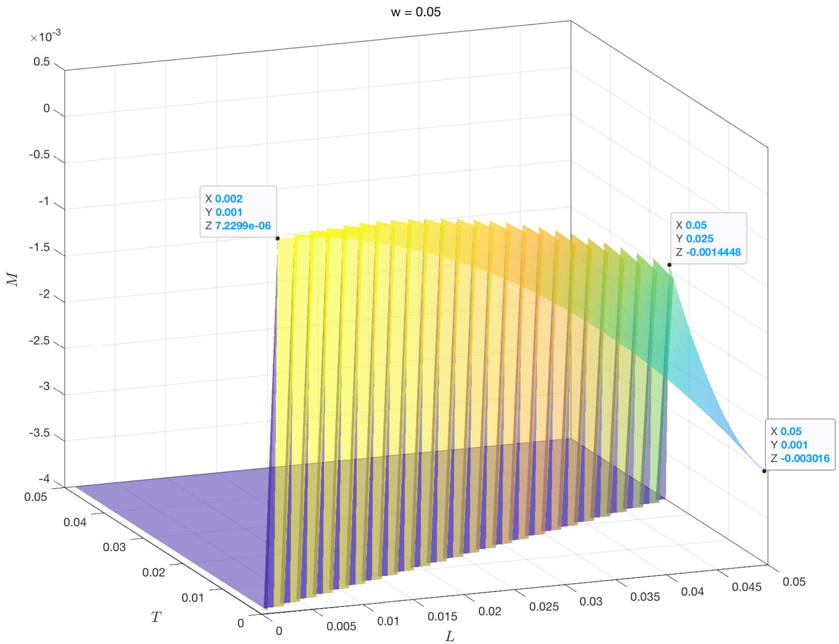

Figure 18: Merged forms: w=0.05

Merged forms: w=0.2

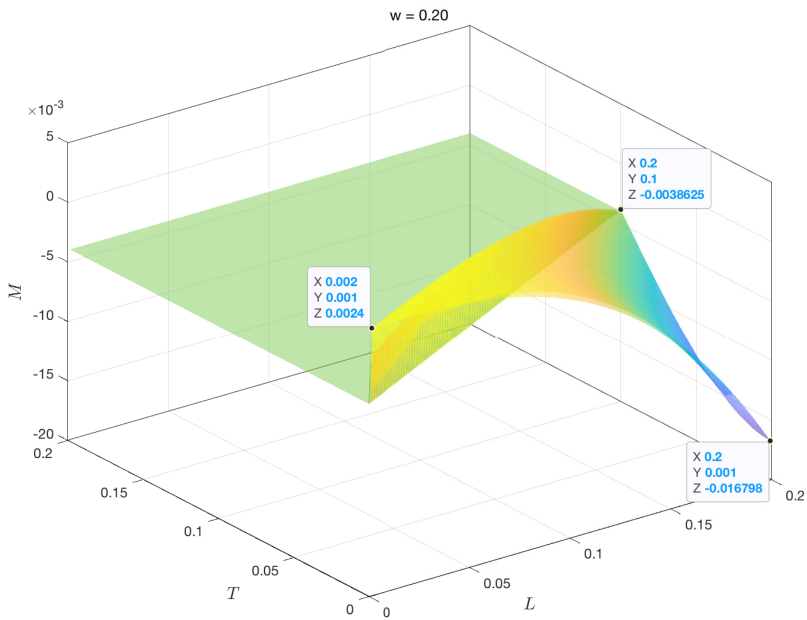

Figure 19: Merged forms: w=0.2

Merged forms: w=0.31

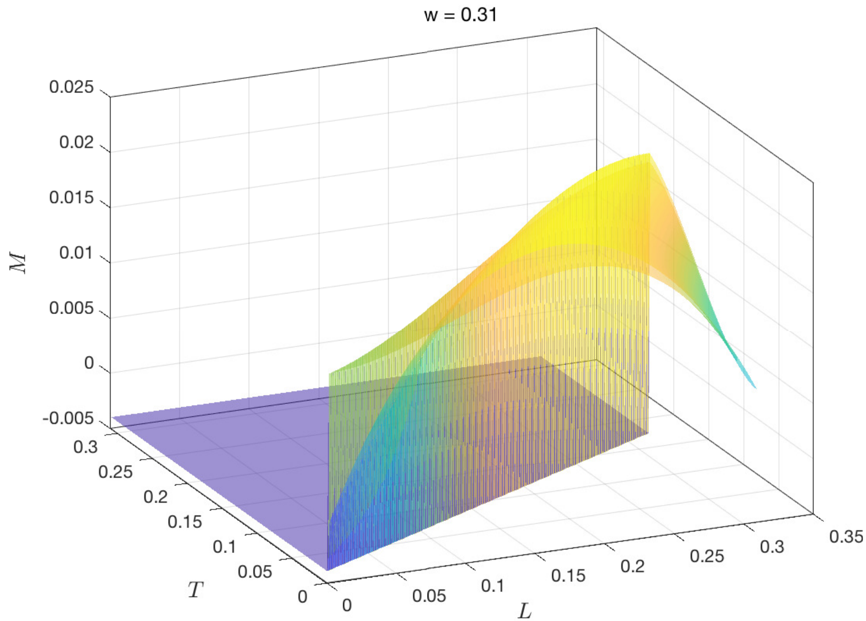

Figure 20: Merged forms: w=0.31

Merged forms: w=0.86

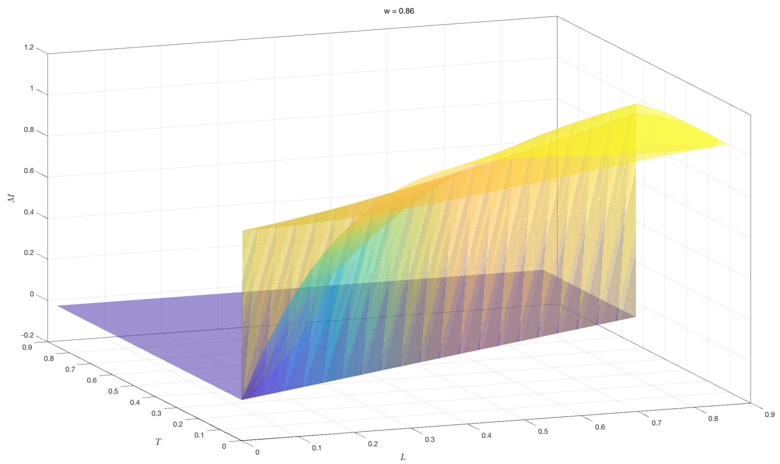

Figure 21: Merged forms: w=0.86

Tabular for the results

Table 2: The maximum of two shapes respectly.(T is an endogenous variable)

	Different shapes and according T and L					
	Triple C	T	L	Middle C	T	L
w=0.01	−0.000003	0.001	0.002	−0.000003	0.001	0.002
w=0.06	0.000002	0.001	0.002	0.0000001	0.001	0.002
w=0.11	0.000218	0.001	0.002	0.00004	0.001	0.008
w=0.31	0.021	0.136	0.274	0.0225	0.001	0.169
w=0.86	1.0659	0.035	0.86	1.0734	0.001	0.483

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