Dynamic Hedging Under Expected Utility and Kreps-Porteus Preference Models

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Motivation

Literature Review

1. Arrow proved in 1971 that if absolute risk aversion Arrow-Pratt decreases as income increases, risky assets are a normal commodity (its demand increases with income or wealth).

$$\mathrm{n} > 0 \Leftrightarrow E\tilde{R} > R_f$$

2. Arrow also proves that when the relative risk aversion is increasing, the sufficient condition of the demand income elasticity of risk-free assets is greater than 1.

3. Aura, Diamond, and Geanakoplos (2002) demonstrate that both risk assets and risk-free assets are common commodities when both conditions are satisfied.

4. Kubler, Selden and Wei (2013) demonstrate that in both cases, risk-free assets may be inferior goods.

Our innovation

- In the utility function, the Kreps-Porteus model, which is a model of risk and time preference separation, is used to conduct asset allocation research. It is generally assumed that the investment distribution is independent and consistent.
- At present, research on the correlation of asset return distribution is still very scarce. From special to general research is also more difficult.

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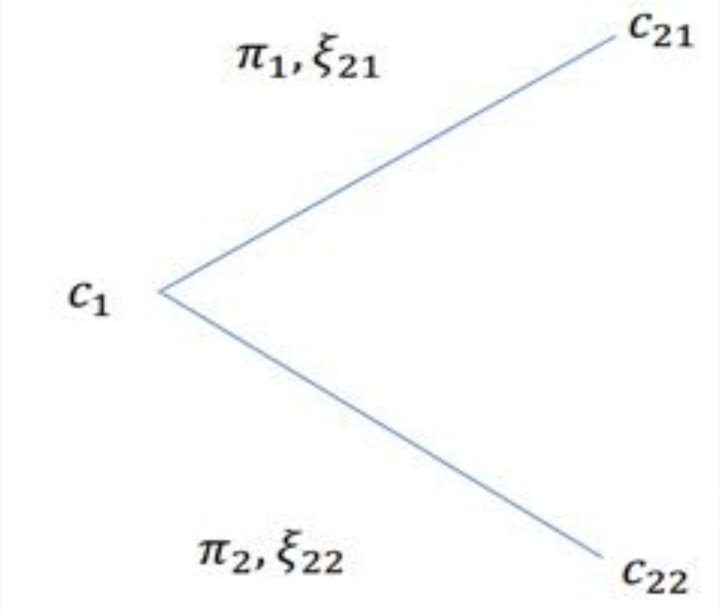
Progress

Two-stage Model



$$\max \mathrm{u}(c_1) + \beta[\pi_{21}\mathrm{u}(c_{21}) + \pi_{22}\mathrm{u}(c_{22})], \quad \text{where } \mathrm{u(c)} = -\frac{c^{-\delta}}{\delta}$$

$$\text{s.t. } c_1 + pn + \mathrm{p}_f\, n_f = I$$

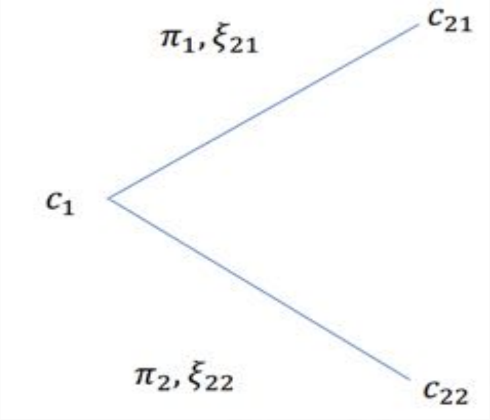
Two-stage Model

- Results：

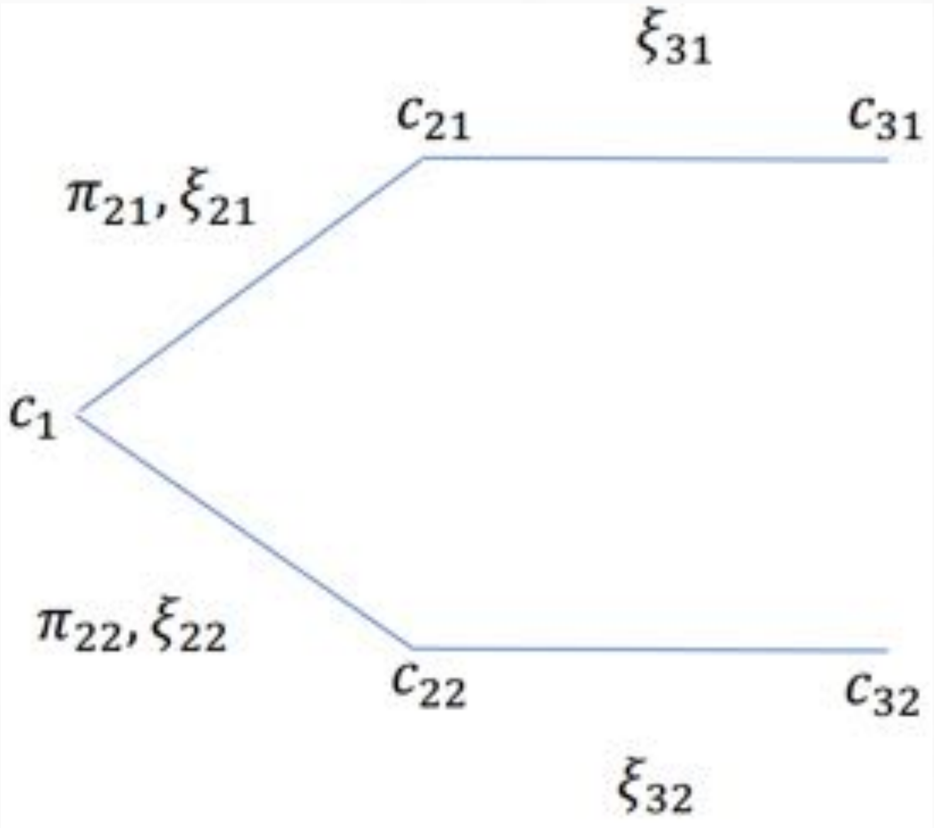
$$\frac{\pi_1\xi_{21}+\pi_2\xi_{22}}{p} > \frac{\xi_f}{p_f} \Leftrightarrow \mathrm{n} > 0$$

- Arrow's results：

$$\frac{\pi_1\xi_{21}+\pi_2\xi_{22}}{p} > \frac{\xi_f}{p_f} \Leftrightarrow E\tilde{R} > R_f \Leftrightarrow \mathrm{n} > 0$$



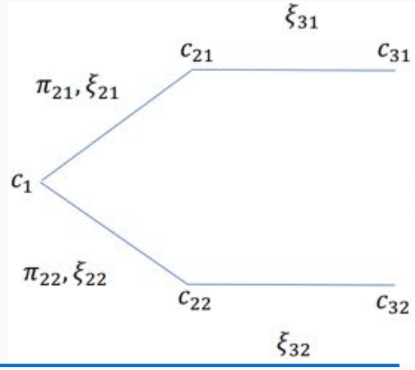
Dynamic



- Innovation: multi-period; asset return distribution is relevant
- Meaning of Correlation
- How to offset some risks ：s.t. $n < 0$

$$I_{21} = \xi_{21} n + \xi_f n_f$$
$$I_{22} = \xi_{22} n + \xi_f n_f$$

When they are positively correlated, although the rate of return is high, the investment is less.



Dynamic

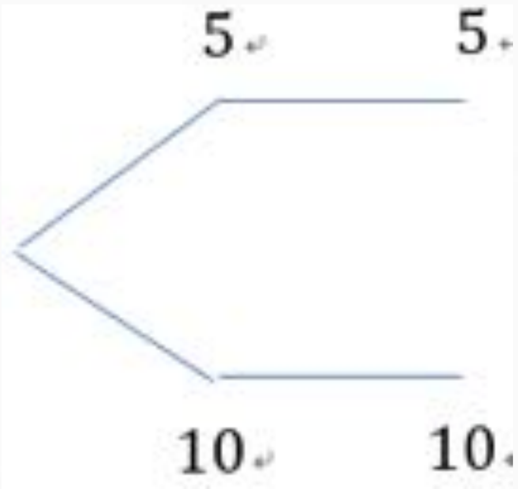
Results： (recall that $u(c) = -\frac{c^{-\delta}}{\delta}$, and assume that $\xi_{21} > \xi_{22}$)

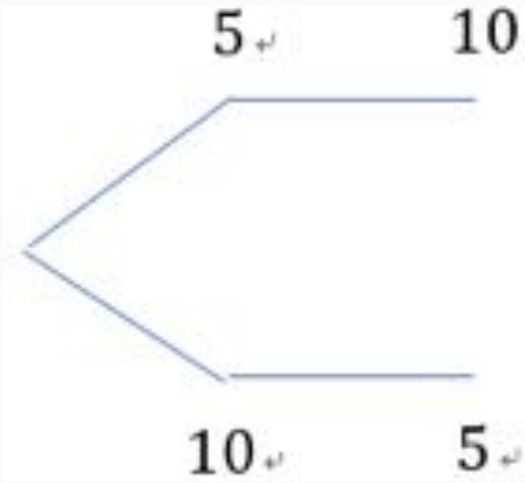
- When $\xi_{31} = \xi_{32}$, it equals to two-stage case:

$$n < 0 \Leftrightarrow \frac{\pi_1\xi_{21} + \pi_2\xi_{22}}{p} < \frac{\xi_f}{p_f} \Leftrightarrow E\tilde{R} < R_f$$

- When $\xi_{31} < \xi_{32}$ (negatively correlated) , $n < 0 \Leftrightarrow \delta \geq 0$
- When $\xi_{31} > \xi_{32}$ (positively correlated) , $n < 0 \Leftrightarrow -1 < \delta \leq 0$

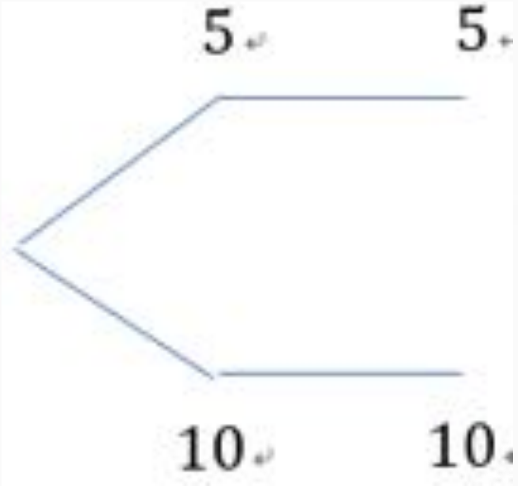
Dynamic



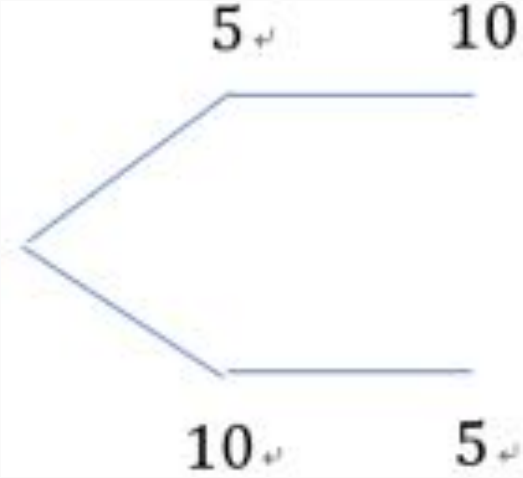


- Are they equivalent?

Dynamic



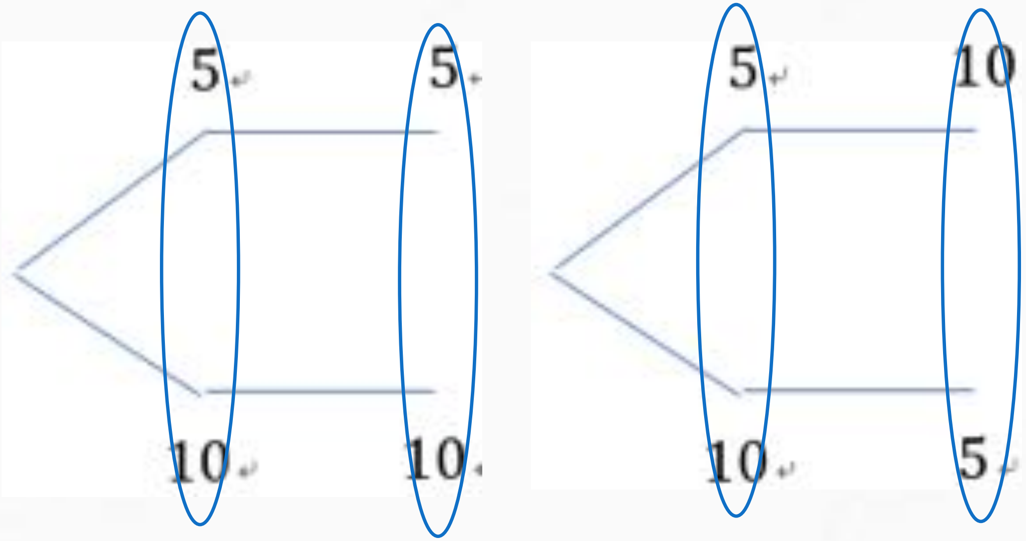
Consumption smoothing



Less risk

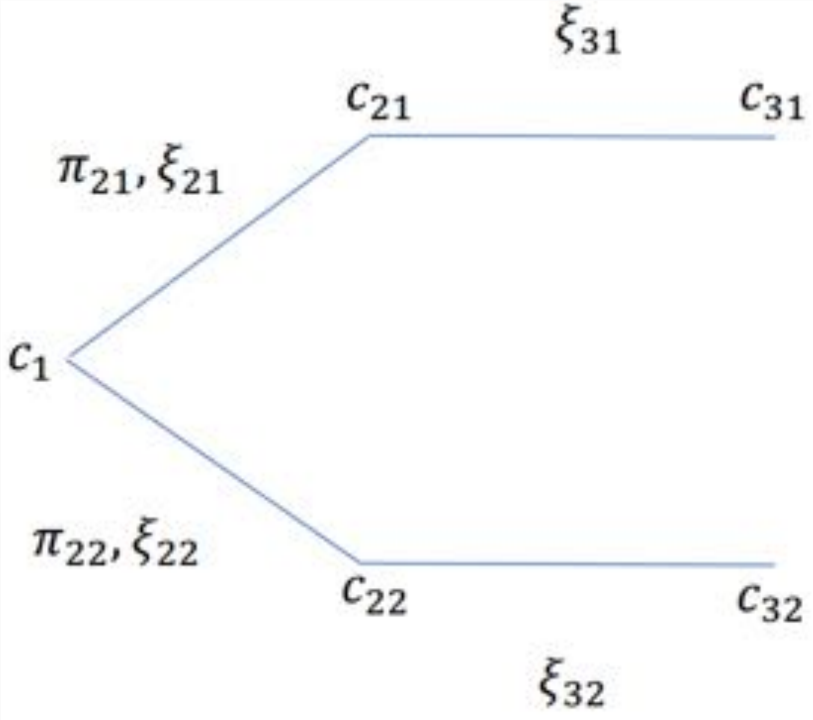
- Are they equivalent?
- No.

Dynamic

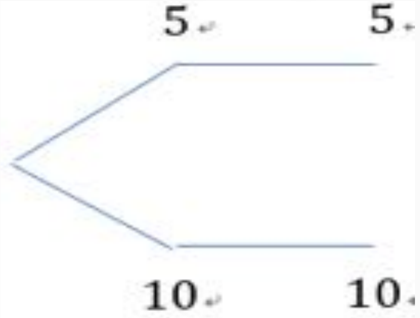


- Are they equivalent?
- $u(c_1) + \beta[\pi_{21}u(c_{21}) + \pi_{22}u(c_{22})]$
- Equivalent.

Explain the three-stage conclusions



- When $\xi_{31} < \xi_{32}$ (negatively correlated) , $n < 0 \Leftrightarrow \delta \geq 0$
- Consumers would rather take more risk and smooth the two periods of consumption.



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Future Work

Future Work

- First, we should solve the problem that risk appetite and time preference are not separated.
- Then, two generalizations :

  Utility function from special to general

  Generalization of the tree

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Reference

Reference

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- [5] Multi-period portfolio choice and the intertemporal hedging demands for stocks and bonds: International evidence. David E. Rapach, Mark E. Wohar
- [6] What Are Asset Demand Tests of Expected Utility Really Testing: Economic Journal, 127(601), 784-808 (2017). Kubler, Felix, Larry Selden and Xiao Wei

How to solve：Backward induction

$$\max \quad u(c_{21}) + \beta u(c_{31})$$
$$s.t. \quad c_{31} = (\xi_{21} n + \xi_f n_f - c_{21}) R_{31} \,.$$

Let $u(c) = -\frac{c^{-\delta}}{\delta}, u'(c) = c^{-\delta-1}$

F.O.C.: $\frac{\partial V}{\partial c_{21}} = c_{21}^{-\delta-1} - \beta R_{31}(\xi_{21} n + \xi_f n_f - c_{21})^{-\delta-1} R_{31}^{-\delta} = 0$

$$c_{21} = \frac{\xi_{21} n + \xi_f n_f}{(\beta R_{31}^{-\delta})^{\frac{1}{\delta+1}} + 1}$$

$$c_{31} = \frac{(\beta R_{31})^{\frac{1}{\delta+1}}}{(\beta R_{31}^{-\delta})^{\frac{1}{\delta+1}} + 1} (\xi_{21} n + \xi_f n_f)$$

How to solve：Backward induction

$$\begin{cases} x_{21} = \xi_{21} n + \xi_f n_f \\ x_{22} = \xi_{22} n + \xi_f n_f \end{cases}$$

$$\Rightarrow \begin{cases} n = \frac{x_{21}\xi_f - x_{22}\xi_f}{\xi_{21}\xi_f - \xi_{22}\xi_f} = \frac{x_{21} - x_{22}}{\xi_{21} - \xi_{22}} \\ n_f = \frac{x_{22}\xi_{21} - x_{21}\xi_{22}}{\xi_{21}\xi_f - \xi_{22}\xi_f} = \frac{x_{22}\xi_{21} - x_{21}\xi_{22}}{\xi_f(\xi_{21} - \xi_{22})} \end{cases}$$

How to solve：Backward induction

$$\begin{cases} x_{21} = \xi_{21} n + \xi_f n_f \\ x_{22} = \xi_{22} n + \xi_f n_f \end{cases}$$

$$\Rightarrow \begin{cases} n = \frac{x_{21}\xi_f - x_{22}\xi_f}{\xi_{21}\xi_f - \xi_{22}\xi_f} = \frac{x_{21} - x_{22}}{\xi_{21} - \xi_{22}} \\ n_f = \frac{x_{22}\xi_{21} - x_{21}\xi_{22}}{\xi_{21}\xi_f - \xi_{22}\xi_f} = \frac{x_{22}\xi_{21} - x_{21}\xi_{22}}{\xi_f(\xi_{21} - \xi_{22})} \end{cases}$$

Plug it into budget constraint

$$c_1 + pn + p_f n_f = I$$

$$c_1 + \frac{p\xi_f - p_f\xi_{22}}{\xi_{21}\xi_f - \xi_{22}\xi_f} x_{21} + \frac{p_f\xi_{21} - p\xi_f}{\xi_{21}\xi_f - \xi_{22}\xi_f} x_{22} = I$$

$$c_1 + p_1 x_{21} + p_2 x_{22} = I$$

How to solve：Backward induction

Using the Lagrange Method:

$$\mathcal{L} = -\frac{c_1^{-\delta}}{\delta} + \lambda_{21}\frac{x_{21}}{-\delta} + \lambda_{22}\frac{x_{22}}{-\delta} + \lambda(c_1 + p_1 x_{21} + p_2 x_{22} - I)$$

$$\frac{\partial \mathcal{L}}{\partial c_1} = c_1^{-\delta-1} + \lambda = 0 \quad \Rightarrow \lambda = -c_1^{-\delta-1}$$

$$\frac{\partial \mathcal{L}}{\partial x_{21}} = \lambda_{21} x_{21}^{-\delta-1} + \lambda p_1 = 0 \Rightarrow x_{21} = \left(\frac{p_1}{\lambda_{21}}\right)^{\frac{1}{-\delta-1}} c_1$$

$$\frac{\partial \mathcal{L}}{\partial x_{22}} = \lambda_{22} x_{22}^{-\delta-1} + \lambda p_2 = 0 \Rightarrow x_{22} = \left(\frac{p_2}{\lambda_{22}}\right)^{\frac{1}{-\delta-1}} c_1$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow c_1 + p_1 x_{21} + p_2 x_{22} = I$$

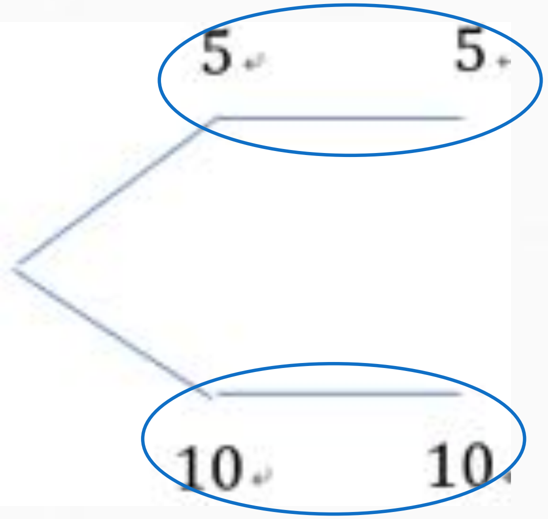
How to solve：Backward induction

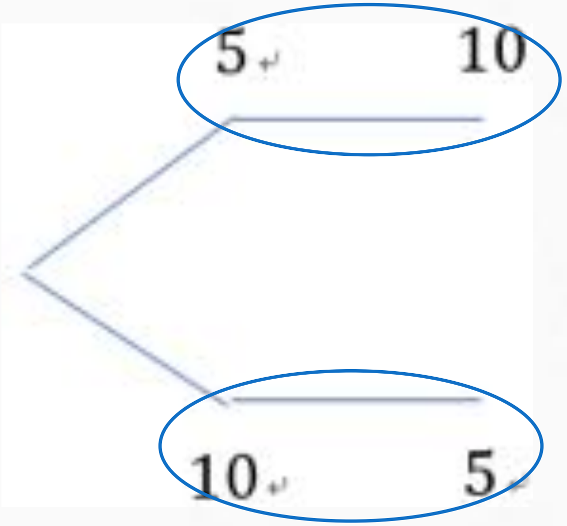
$$
\begin{cases}
c_1 = \dfrac{I}{p_1(\frac{p_1}{\lambda_{21}})^{\frac{1}{-\delta-1}} + p_2(\frac{p_2}{\lambda_{22}})^{\frac{1}{-\delta-1}} + 1} \\[2ex]
x_{21} = (\frac{p_1}{\lambda_{21}})^{\frac{1}{-\delta-1}} c_1 = \dfrac{1}{p_1 + p_2(\frac{p_2\lambda_{21}}{p_1\lambda_{22}})^{\frac{1}{-\delta-1}} + (\frac{p_1}{\lambda_{21}})^{\frac{1}{\delta+1}}} \\[2ex]
x_{22} = (\frac{p_2}{\lambda_{22}})^{\frac{1}{-\delta-1}} c_1 = \dfrac{1}{p_1(\frac{p_1\lambda_{22}}{p_2\lambda_{21}})^{\frac{1}{-\delta-1}} + p_2 + (\frac{p_1}{\lambda_{22}})^{\frac{1}{\delta+1}}}
\end{cases}
$$

$$
\begin{cases}
c_1 = \dfrac{I}{p_1(\frac{p_1}{\lambda_{21}})^{\frac{1}{-\delta-1}} + p_2(\frac{p_2}{\lambda_{22}})^{\frac{1}{-\delta-1}} + 1} \\[2ex]
n = \dfrac{(\frac{p_1}{\lambda_{21}})^{\frac{1}{-\delta-1}} - (\frac{p_2}{\lambda_{22}})^{\frac{1}{-\delta-1}}}{\xi_{21} - \xi_{22}} c_1 \\[2ex]
n_f = \dfrac{\xi_{21}(\frac{p_2}{\lambda_{22}})^{\frac{1}{-\delta-1}} - \xi_{22}(\frac{p_1}{\lambda_{21}})^{\frac{1}{-\delta-1}}}{\xi_{21}\xi_f - \xi_{22}\xi_f} c_1
\end{cases}
$$

$$
x_{21} < x_{22} \Rightarrow (\frac{p_1}{\lambda_{21}})^{\frac{1}{-\delta-1}} c_1 < (\frac{p_2}{\lambda_{22}})^{\frac{1}{-\delta-1}} c_1 \Rightarrow \frac{p_1}{p_2} < \frac{\lambda_{21}}{\lambda_{22}}
$$

How to separate two preferences - Kreps-Porteus model





- Take u_{21} and u_{22} as new consumption

How to separate two preferences - Kreps-Porteus model

- Take u_{21} and u_{22} as new consumption

$$u_{21} = (c_{21}^{-\delta_1} + \beta c_{31}^{-\delta_1})^{1/-\delta_1}$$
$$u_{22} = (c_{22}^{-\delta_1} + \beta c_{32}^{-\delta_1})^{1/-\delta_1}$$

- $\mathrm{U} = c_1^{-\delta_1} + \beta(\pi_{21} u_{21}^{-\delta_2} + \pi_{22} u_{22}^{-\delta_2})^{\delta_1/\delta_2}$

Thanks

Yi Cui, Jingyan Zhang