案例：出口商品总额对国内生产总值的影响

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一、提出问题

在之前二三章研究的线性回归模型中，我们做出了同方差性的假定，即要求多所有的$i(i=1,2,\ldots,n)$都有

$$Var(u_i)=\sigma^2$$

因为方差是度量被解释变量Y的观测值围绕回归线$E(Y_i)=\beta_1+\beta_2X_{2i}+\beta_3X_{3i}+\ldots+\beta_kX_{ki}$的分散程度，因此同方差性指的是所有观测值的分散程度相同。

设模型为

$$Y_i=\beta_1+\beta_2X_{2i}+\beta_3X_{3i}+\ldots+\beta_kX_{ki}+u_i \quad (i=1,2,\ldots,n)$$

如果其他假定不变，但模型中随机误差项u_i的方差为

$$Var(u_i)=\sigma_i^2 \quad (i=1,2,\ldots,n)$$

则称u_i具有异方差性。更进一步，如果把异方差看作是由于某个解释变量的变化而引起的，则

$$Var(u_i)=\sigma_i^2=\sigma^2f(X_i)$$

通常，模型设定的误差（忽略了某些重要的解释变量或模型函数形式不正确）、测量误差的变化和截面数据中总体各单位的差异通常都会带来异方差。如果模型存在异方差，则对模型参数估计式的统计特性、模型的假设检验和模型的预测都会产生影响。所以，对模型异方差性的检验和消除成为了一个重要的课题。

为此，我们通过一个具体的案例来研究这一流程。一国的出口商品总额常常是经济发展水平的重要指标，也会一定程度上对国内生产总值产生影响。为此，更真实可靠地探究中国出口商品总额对GDP的影响，就显得尤为关键。

二、模型设定

在之后的模型中，我们令X代表出口商品总额（亿元），Y代表GDP（亿元）。我们首先读入数据，并观察其特点。

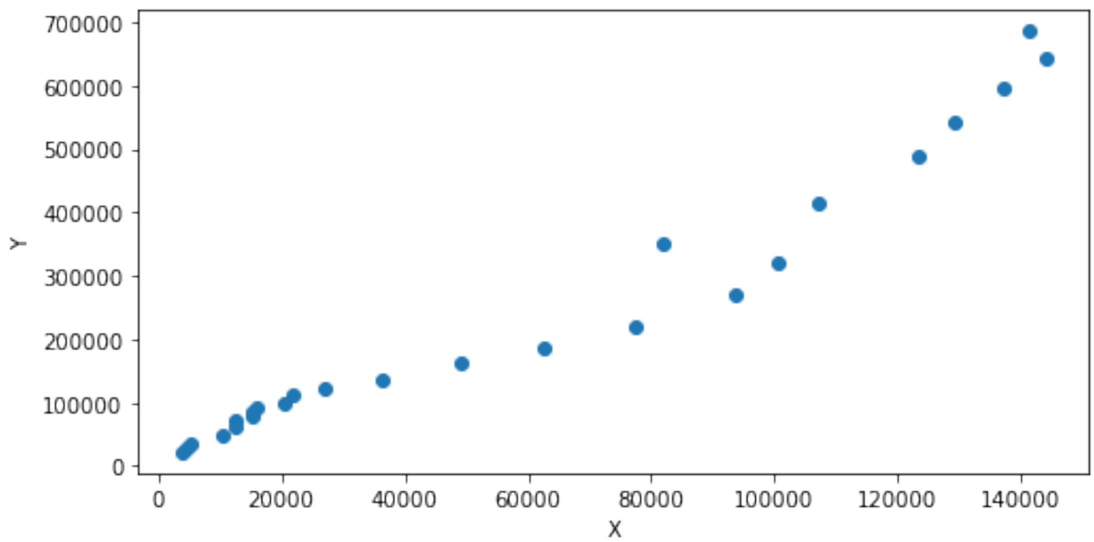
```python
import pandas as pd
import matplotlib.pyplot as plt

# 将数据读入为DataFrame
df = pd.read_excel('data.xlsx',header=0)
df.rename(columns = {'GDP':'Y','EXPORT':'X'},inplace = True)
print(df)

plt.figure(figsize = (8,4))
plt.scatter(df['X'],df['Y'])
plt.xlabel("X")
plt.ylabel("Y")
```

```
       时间          X          Y
0   1991     3827.1   22005.6
1   1992     4676.3   27194.5
2   1993     5284.8   35673.2
3   1994    10421.8   48637.5
4   1995    12451.8   61339.9
5   1996    12576.4   71813.6
6   1997    15160.7   79715.0
7   1998    15223.6   85195.5
8   1999    16159.8   90564.4
9   2000    20634.4  100280.1
10  2001    22024.4  110863.1
11  2002    26947.9  121717.4
12  2003    36287.9  137422.0
13  2004    49103.3  161840.2
14  2005    62648.1  187318.9
15  2006    77597.2  219438.5
16  2007    93627.1  270232.3
17  2008   100394.9  319515.5
18  2009    82029.7  349081.4
19  2010   107022.8  413030.3
20  2011   123240.6  489300.6
21  2012   129359.3  540367.4
22  2013   137131.4  595244.4
23  2014   143883.7  643974.0
24  2015   141166.8  685505.8
```

```
Text(0, 0.5, 'Y')
```



可以看出，Y与X成正相关，且基本符合线性变化，因此设定如下模型：

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

三、参数估计

和前几章一样，我们使用OLS对模型的参数进行估计。

```python
import statsmodels.formula.api as smf

est1 = smf.ols(formula='Y ~ X', data=df).fit()

# 打印系数
print(est1.params)
# 打印回归结果
print(est1.summary())
```

```
Intercept   -673.086253
X              4.061131
dtype: float64
                            OLS Regression Results
==============================================================================
Dep. Variable:                      Y   R-squared:                       0.946
Model:                            OLS   Adj. R-squared:                  0.944
Method:                 Least Squares   F-statistic:                     405.5
Date:                Sat, 18 Dec 2021   Prob (F-statistic):           4.17e-16
Time:                        02:00:25   Log-Likelihood:                -304.82
No. Observations:                  25   AIC:                             613.6
Df Residuals:                      23   BIC:                             616.1
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept   -673.0863   1.54e+04     -0.044      0.965   -3.24e+04    3.11e+04
X              4.0611      0.202     20.137      0.000       3.644       4.478
==============================================================================
Omnibus:                        3.211   Durbin-Watson:                   0.366
Prob(Omnibus):                  0.201   Jarque-Bera (JB):                1.639
Skew:                          -0.512   Prob(JB):                        0.441
Kurtosis:                       3.725   Cond. No.                     1.17e+05
==============================================================================


Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.17e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

从中我们可以得到

$\hat{Y}_i = -673.086253 + 4.061131 * X_i$

$SE = (1.54e+04)\ \ (0.202)$

$t = (-0.044)\ \ (20.137)$

$R^2 = 0.946 \quad \overline{R^2} = 0.944 \quad F = 405.5 \quad df = 23$

然而，这样的结论显然是不可靠的：出口商品总额每增加1亿元，GDP就能增加4.061131亿元，出口对GDP的拉动作用也太可怕了！

其实从上面的Y-X散点图中可以看出，随着X的增大，Y的离散程度有逐渐增大的趋势，这表明模型可能存在一定的异方差性。如果是这样的话，那OLS估计方法的效果就不能保证，同时t检验也失去了意义。

为了用更充分的理由说明这个结论的荒谬性，并寻找更真实的结论，我们就需要对模型的异方差性进行检验，并对此加以修正。

四、异方差性检验

4.1 White检验

White检验不需要关于异方差的任何先验信息，只需要在大样本的情况下，将OLS估计后的残差平方对常数、解释变量、解释变量的平方及其交叉乘积等构成辅助回归，利用辅助回归建立相应的检验统计量来判断异方差性。

White检验的步骤如下：

1. 用OLS估计原模型$Y_i = \beta_1 + \beta_2 X_i + u_i$，计算残差$e_t = Y_t - \hat{Y}_t$，并求其平方。

```
est2 = smf.ols(formula='Y ~ X', data=df).fit()
df['e2'] = (df['Y'] - est2.predict(df['X']))**2
print(df)
```

```
      时间        X         Y            e2
0   1991   3827.1   22005.6  5.092725e+07
1   1992   4676.3   27194.5  7.879263e+07
2   1993   5284.8   35673.2  2.215341e+08
3   1994  10421.8   48637.5  4.880832e+07
4   1995  12451.8   61339.9  1.309789e+08
5   1996  12576.4   71813.6  4.584859e+08
6   1997  15160.7   79715.0  3.541361e+08
7   1998  15223.6   85195.5  5.780927e+08
8   1999  16159.8   90564.4  6.558940e+08
9   2000  20634.4  100280.1  2.942663e+08
10  2001  22024.4  110863.1  4.880662e+08
11  2002  26947.9  121717.4  1.677425e+08
12  2003  36287.9  137422.0  8.602221e+07
13  2004  49103.3  161840.2  1.361730e+09
14  2005  62648.1  187318.9  4.412962e+09
15  2006  77597.2  219438.5  9.028949e+09
16  2007  93627.1  270232.3  1.195228e+10
```

```
17  2008  100394.9  319515.5  7.661189e+09
18  2009   82029.7  349081.4  2.762629e+08
19  2010  107022.8  413030.3  4.380726e+08
20  2011  123240.6  489300.6  1.107228e+08
21  2012  129359.3  540367.4  2.463478e+08
22  2013  137131.4  595244.4  1.521699e+09
23  2014  143883.7  643974.0  3.638090e+09
24  2015  141166.8  685505.8  1.274236e+10
```

2. 用残差平方e_t^2作为异方差σ_t^2的估计，做辅助函数$e_t^2=\alpha_0+\alpha_1 x_t+\alpha_2 x_t^2+v_t$，并用OLS对其做估计。

```python
df['X2'] = df['X']**2
est3 = smf.ols(formula='e2 ~ X + X2', data=df).fit()

# 打印回归结果
print(est3.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     e2   R-squared:                       0.290
Model:                            OLS   Adj. R-squared:                  0.225
Method:                 Least Squares   F-statistic:                     4.493
Date:                Sat, 18 Dec 2021   Prob (F-statistic):             0.0231
Time:                        02:00:25   Log-Likelihood:                -582.41
No. Observations:                  25   AIC:                             1171.
Df Residuals:                      22   BIC:                             1174.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept  -1.005e+09   1.43e+09     -0.700      0.491   -3.98e+09    1.97e+09
X           1.022e+05   6.07e+04      1.685      0.106   -2.36e+04    2.28e+05
X2            -0.4554      0.421     -1.082      0.291      -1.328       0.418
==============================================================================
Omnibus:                        6.978   Durbin-Watson:                   0.750
Prob(Omnibus):                  0.031   Jarque-Bera (JB):                4.893
Skew:                           0.969   Prob(JB):                       0.0866
Kurtosis:                       3.972   Cond. No.                     1.95e+10
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.95e+10. This might indicate that there are
strong multicollinearity or other numerical problems.
```

3. 计算统计量$nR^2=7.25$。

4. 假设$H_0: \alpha_1 = \alpha 2 = 0$，则可证明$nR^2$渐进服从自由度为$p$的$\chi^2$分布，$p = 2$。$\alpha = 0.05$时查表得$\chi^2_{0.05}(2) = 5.9915$，因为$nR^2 = 7.25 > \chi^2_{0.05}(2) = 5.9915$，所以拒绝原假设，即表明模型存在异方差。

4.2 ARCH检验

ARCH检验是一种针对时间序列的异方差检验方法，其思想是，在时间序列数据中，可认为存在的异方差性为ARCH过程，并通过检验这一过程是否成立去判断时间序列有无异方差性。

设二阶的ARCH过程为

$$\sigma_t^2 = \alpha_0 + \alpha_1\sigma_{t-1}^2 + \alpha_2\sigma_{t-2}^2 + v_t$$

并且$\alpha_0 > 0, \alpha_i \geq 0 \ \ (i = 1, 2)$

ARCH检验的步骤如下：

1. 提出假设$H_0: \alpha_1 = \alpha_2 = 0$。
2. 对原模型做OLS估计，求出残差e_t，并计算残差平方序列$e_t^2, e_{t-1}^2, e_{t-2}^2$，分别作为对$\sigma_t^2, \sigma_{t-1}^2, \sigma_{t-2}^2$的估计。

```python
import numpy as np

est4 = smf.ols(formula='Y ~ X', data=df).fit()
df['e2t'] = (df['Y'] - est4.predict(df['X']))**2

# 计算残差平方序列
df.loc[0,'e2t1'] = 0
df.loc[1:,'e2t1'] = np.array(df.loc[:len(df)-2,'e2t'])
df.loc[0,'e2t2'] = 0
df.loc[1:,'e2t2'] = np.array(df.loc[:len(df)-2,'e2t1'])

print(df)
```

```
      时间        X         Y            e2            X2           e2t  \
0   1991   3827.1   22005.6  5.092725e+07  1.464669e+07  5.092725e+07
1   1992   4676.3   27194.5  7.879263e+07  2.186778e+07  7.879263e+07
2   1993   5284.8   35673.2  2.215341e+08  2.792911e+07  2.215341e+08
3   1994  10421.8   48637.5  4.880832e+07  1.086139e+08  4.880832e+07
4   1995  12451.8   61339.9  1.309789e+08  1.550473e+08  1.309789e+08
5   1996  12576.4   71813.6  4.584859e+08  1.581658e+08  4.584859e+08
6   1997  15160.7   79715.0  3.541361e+08  2.298468e+08  3.541361e+08
7   1998  15223.6   85195.5  5.780927e+08  2.317580e+08  5.780927e+08
8   1999  16159.8   90564.4  6.558940e+08  2.611391e+08  6.558940e+08
9   2000  20634.4  100280.1  2.942663e+08  4.257785e+08  2.942663e+08
10  2001  22024.4  110863.1  4.880662e+08  4.850742e+08  4.880662e+08
11  2002  26947.9  121717.4  1.677425e+08  7.261893e+08  1.677425e+08
12  2003  36287.9  137422.0  8.602221e+07  1.316812e+09  8.602221e+07
13  2004  49103.3  161840.2  1.361730e+09  2.411134e+09  1.361730e+09
14  2005  62648.1  187318.9  4.412962e+09  3.924784e+09  4.412962e+09
15  2006  77597.2  219438.5  9.028949e+09  6.021325e+09  9.028949e+09
```

```
16  2007   93627.1  270232.3  1.195228e+10  8.766034e+09  1.195228e+10
17  2008  100394.9  319515.5  7.661189e+09  1.007914e+10  7.661189e+09
18  2009   82029.7  349081.4  2.762629e+08  6.728872e+09  2.762629e+08
19  2010  107022.8  413030.3  4.380726e+08  1.145388e+10  4.380726e+08
20  2011  123240.6  489300.6  1.107228e+08  1.518825e+10  1.107228e+08
21  2012  129359.3  540367.4  2.463478e+08  1.673383e+10  2.463478e+08
22  2013  137131.4  595244.4  1.521699e+09  1.880502e+10  1.521699e+09
23  2014  143883.7  643974.0  3.638090e+09  2.070252e+10  3.638090e+09
24  2015  141166.8  685505.8  1.274236e+10  1.992807e+10  1.274236e+10

            e2t1          e2t2
0   0.000000e+00  0.000000e+00
1   5.092725e+07  0.000000e+00
2   7.879263e+07  5.092725e+07
3   2.215341e+08  7.879263e+07
4   4.880832e+07  2.215341e+08
5   1.309789e+08  4.880832e+07
6   4.584859e+08  1.309789e+08
7   3.541361e+08  4.584859e+08
8   5.780927e+08  3.541361e+08
9   6.558940e+08  5.780927e+08
10  2.942663e+08  6.558940e+08
11  4.880662e+08  2.942663e+08
12  1.677425e+08  4.880662e+08
13  8.602221e+07  1.677425e+08
14  1.361730e+09  8.602221e+07
15  4.412962e+09  1.361730e+09
16  9.028949e+09  4.412962e+09
17  1.195228e+10  9.028949e+09
18  7.661189e+09  1.195228e+10
19  2.762629e+08  7.661189e+09
20  4.380726e+08  2.762629e+08
21  1.107228e+08  4.380726e+08
22  2.463478e+08  1.107228e+08
23  1.521699e+09  2.463478e+08
24  3.638090e+09  1.521699e+09
```

3. 做辅助函数：$\hat{e_t}^2 = \hat{\alpha_0} + \hat{\alpha_1} e_{t-1}^2 + \hat{\alpha_2} e_{t-2}^2$，并用OLS对其做估计。

```python
est5 = smf.ols(formula='e2t ~ e2t1 + e2t2', data=df).fit()

# 打印回归结果
print(est5.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    e2t   R-squared:                       0.638
Model:                            OLS   Adj. R-squared:                  0.605
```

```
Method:                 Least Squares   F-statistic:                     19.37
Date:                Sat, 18 Dec 2021   Prob (F-statistic):           1.41e-05
Time:                        02:00:25   Log-Likelihood:                -574.00
No. Observations:                  25   AIC:                             1154.
Df Residuals:                      22   BIC:                             1158.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept   1.047e+09    5.6e+08      1.871      0.075   -1.14e+08    2.21e+09
e2t1           1.3961      0.235      5.934      0.000       0.908       1.884
e2t2          -0.7622      0.236     -3.234      0.004      -1.251      -0.273
==============================================================================
Omnibus:                       21.730   Durbin-Watson:                   1.365
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               29.751
Skew:                           1.907   Prob(JB):                     3.46e-07
Kurtosis:                       6.744   Cond. No.                     5.53e+09
==============================================================================


Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 5.53e+09. This might indicate that there are
strong multicollinearity or other numerical problems.
```

得到可决系数$R^2 = 0.638$，可证明$(n-p)R^2$渐进服从自由度为p的χ^2分布，$p=2$。$\alpha = 0.05$时查表得$\chi^2_{0.05}(2) = 5.9915$，因为$(n-p)R^2 = 14.674 > \chi^2_{0.05}(2) = 5.9915$，所以拒绝原假设，即表明模型存在异方差。

五、异方差性修正

对异方差进行修正一般有三种方法：

- 模型变换法：适用于已知异方差形式的情况
- 加权最小二乘法：最常用的修正方法，实际上与模型变换法是等价的
- 模型对数变换法：需要考虑模型的经济学意义，即变量间是否具备对数线性关系。

5.1 加权最小二乘法

在我们的模型

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

中，经检验存在异方差，且

$$Var(u_i) = \sigma_i^2 = \sigma^2 f(X_i)$$

加权最小二乘的想法是，对于不同的样本给予不同的权重，即对较小的e_i^2给予较大的权数，对较大的e_i^2给予较小的权数。通常可将权数取为$w_i = \frac{1}{\sigma_i^2} \ \ (i = 1, 2, \ldots, n)$，将权数与残差平方和相乘后再求和，得

$\sum w_i e_i^2 = \sum w_i(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2$

称为加权的残差平方和。根据最小二乘原理，即要使得

$min \sum w_i e_i^2 = min \sum w_i(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2$

可得

$\hat{\beta}_1^* = \overline{Y}^* - \hat{\beta}_2^* \overline{X}^*$

$\hat{\beta}_2^* = \frac{\sum w_i(X_i - \overline{X}^*)(Y_i - \overline{Y}^*)}{\sum w_i(X_i - \overline{X}^*)^2}$

其中

$\overline{X}^* = \frac{\sum w_i X_i}{\sum w_i}$

$\overline{Y}^* = \frac{\sum w_i Y_i}{\sum w_i}$

下面我们按照步骤实现一下。e_i^2在前面已经计算过了，直接用即可。据此计算每个样本的权重。

```
df['w'] = 1/df['e2']
```

计算$\overline{X}^*$和$\overline{Y}^*$。

```
X_star = (df.loc[:,'X'] * df.loc[:,'w']).sum() / df.loc[:,'w'].sum()
Y_star = (df.loc[:,'Y'] * df.loc[:,'w']).sum() / df.loc[:,'w'].sum()
print(X_star, Y_star)
```

```
31356.140304015065 133157.65306699194
```

计算$\hat{\beta}_2^*$和$\hat{\beta}_1^*$。

```
# 分子部分
beta2_star = (df.loc[:,'w']*(df.loc[:,'X']-X_star)*(df.loc[:,'Y']-Y_star)).sum()
# 除以分母
beta2_star /= (df.loc[:,'w']*(df.loc[:,'X']-X_star)**2).sum()

beta1_star = Y_star - beta2_star * X_star

print(beta1_star, beta2_star)
```

```
8819.842005272847 3.9653417115817025
```

得到的模型为

$\hat{Y}_i = 8819.842005272847 + 3.9653417115817025 * X_i$

以下我们用python自带的包进行验证。

```python
import statsmodels.api as sm

est6 = sm.WLS(df['Y'],sm.add_constant(df['X']),weights = df['w']).fit()
print(est6.summary())
```

```
                            WLS Regression Results
==============================================================================
Dep. Variable:                      Y   R-squared:                       0.994
Model:                            WLS   Adj. R-squared:                  0.994
Method:                 Least Squares   F-statistic:                     3818.
Date:                Sat, 18 Dec 2021   Prob (F-statistic):           4.53e-27
Time:                        02:00:26   Log-Likelihood:                -283.60
No. Observations:                  25   AIC:                             571.2
Df Residuals:                      23   BIC:                             573.6
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       8819.8420   3287.850      2.683      0.013    2018.407    1.56e+04
X              3.9653      0.064     61.792      0.000       3.833       4.098
==============================================================================
Omnibus:                        6.095   Durbin-Watson:                   0.919
Prob(Omnibus):                  0.047   Jarque-Bera (JB):                2.334
Skew:                          -0.398   Prob(JB):                        0.311
Kurtosis:                       1.732   Cond. No.                     6.48e+04
==============================================================================


Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 6.48e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

同样可以得到结果：

$$\hat{Y}_i = 8819.8420 + 3.9653 * X_i$$

$$SE = (3287.850) \ \ (0.064)$$

$$t = (2.683) \ \ (61.792)$$

$$R^2 = 0.994 \quad \overline{R^2} = 0.994 \quad F = 3818$$

然而这样的结果还是不够让人信服。一方面，经White检验发现其异方差性还是较大，另一方面从经济意义的角度，出口商品总额每增加1亿元，GDP就能增加3.9653亿元，也不是太合理。因此我们考虑对数变换法。

5.2 对数变换法

建立模型

$lnY_i = \beta_1 + \beta_2 lnX_i + u_i$

然后对模型参数进行估计：

```python
df['lnY'] = np.log(df['Y'])
df['lnX'] = np.log(df['X'])
est7 = smf.ols('lnY ~ lnX',data=df).fit()
print(est7.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    lnY   R-squared:                       0.972
Model:                            OLS   Adj. R-squared:                  0.971
Method:                 Least Squares   F-statistic:                     808.5
Date:                Sat, 18 Dec 2021   Prob (F-statistic):           2.01e-19
Time:                        02:00:26   Log-Likelihood:                 9.4830
No. Observations:                  25   AIC:                            -14.97
Df Residuals:                      23   BIC:                            -12.53
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      2.9897      0.316      9.456      0.000       2.336       3.644
lnX            0.8559      0.030     28.434      0.000       0.794       0.918
==============================================================================
Omnibus:                        1.449   Durbin-Watson:                   0.425
Prob(Omnibus):                  0.485   Jarque-Bera (JB):                1.321
Skew:                          -0.461   Prob(JB):                        0.517
Kurtosis:                       2.353   Cond. No.                         97.0
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

得到模型：

$ln\hat{Y_i} = 2.9897 + 0.8559 * lnX_i$

$SE = (0.316) \ \ (0.030)$

$t = (9.456) \ \ (28.434)$

$R^2 = 0.972 \quad \overline{R^2} = 0.971 \quad F = 808.5$

可以看出无论是t检验还是F检验模型都很显著。经济意义上表示出口商品总额每增加1%，GDP就能增加0.8559%，这也比较合理。

下面来通过White检验分析一下模型的异方差性。

```python
df['ee'] = (df['lnY'] - est7.predict(df['lnX']))**2
df['lnX2'] = df['lnX']**2
est8 = smf.ols('ee ~ lnX + lnX2',data=df).fit()
print(est8.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     ee   R-squared:                       0.244
Model:                            OLS   Adj. R-squared:                  0.175
Method:                 Least Squares   F-statistic:                     3.546
Date:                Sat, 18 Dec 2021   Prob (F-statistic):             0.0463
Time:                        02:00:26   Log-Likelihood:                 54.158
No. Observations:                  25   AIC:                            -102.3
Df Residuals:                      22   BIC:                            -98.66
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0100      0.549      0.018      0.986      -1.129       1.149
lnX           -0.0111      0.108     -0.103      0.919      -0.235       0.213
lnX2           0.0012      0.005      0.229      0.821      -0.010       0.012
==============================================================================
Omnibus:                        4.198   Durbin-Watson:                   0.546
Prob(Omnibus):                  0.123   Jarque-Bera (JB):                2.929
Skew:                           0.833   Prob(JB):                        0.231
Kurtosis:                       3.185   Cond. No.                     1.07e+04
==============================================================================


Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.07e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

计算统计量$nR^2=6.1$。假设$H_0:\alpha_1=\alpha 2=0$，则可证明nR^2渐进服从自由度为p的χ^2分布，$p=2$。$\alpha=0.05$时查表得$\chi^2_{0.05}(2)=5.9915$。$nR^2=6.1$虽然还是大于$\chi^2_{0.05}(2)=5.9915$，但相对于之前已经有了明显改进。