Business Analytics Group Project

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A. Interview Summary

1. Organizational Overview

- **Company**: PrecisionLink

- **Interviewee**: Michael Chen, Supply Chain Director

  PrecisionLink is a global manufacturing company that produces consumer electronics components. The company operates in five key markets: USA, Germany, Japan, Brazil, and India. Michael's team is responsible for designing and optimizing the company's global supply chain network to efficiently serve these markets while minimizing total costs.

2. Organizational Objectives

The primary objective is to minimize the total cost of production and distribution while ensuring that market demand is fully met. This involves:

- **Cost Minimization**: Reducing the combination of fixed costs (factory setup), variable production costs, and transportation costs.

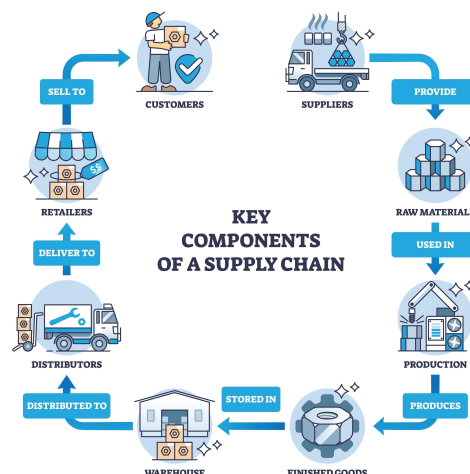- **Fulfilling the Demands**: Ensuring all market demand is satisfied without stockouts.

Figure 1: Key Components of a Supply Chain

3. Decision Variables

Michael's team makes two key sets of decisions:

- **Factory Location and Sizing**: Determining in which countries (USA, Germany, Japan, Brazil, and India) to establish factories and whether to build low-capacity (500k units/month) or high-capacity (1.5M units/month) facilities.

- **Production Allocation**: Deciding which factories serve which markets and in what quantities to minimize total delivered cost while fulfilling all the demands.

4. Risks

The organization faces several uncertainties:

- **Demand Volatility**: Market demand can fluctuate by ±5% due to seasonal trends and economic conditions.

- **Capacity Constraints**: Existing factories have fixed maximum capacities that cannot be easily modified in the short term.

5. Trade-offs

Several competing factors must be balanced:

- **Labor Cost vs. Logistics Cost**: Cheaper labor locations often mean higher transportation costs to major markets.

- **Robustness vs. Efficiency**: A network optimized for average demand may struggle during demand spikes, while an overly robust network may have underutilized capacity during normal conditions.

6. Constraints

The supply chain design is subject to several hard constraints:

- **Production Capacity**: Each factory type (low or high capacity) at different countries has different maximum monthly output limits. Production quantity cannot exceed capacity limits.

- **Demand Fulfillment**: Total production must meet or exceed market demand in each region.

- **Binary Decisions**: Factory opening decisions are yes/no choices. Partial factories are not needed to be built in a certain demand scenario.

7. Tools

Currently, the organization uses:

- **Excel Spreadsheets**: For storing cost data, demand forecasts, and performing basic calculations.

- **Manual Analysis**: For evaluating different network scenarios and comparing total costs.

- **Basic Optimization**: Simple linear programming approaches for production allocation decisions.

- **Experience-Based Judgment**: Heavy reliance on managerial experience for factory location decisions.

The current tools lack the capability to systematically evaluate demand uncertainty or perform robust scenario analysis, which limits the organization's ability to design supply chains that perform well under varying conditions.

B. Proposed Solution

1. Optimization

1.1 The Goal of Optimization

There are quantifiable trade-offs in this basic decision problem. Optimization is suitable since it can simultaneously select shipping allocations and facility locations/capabilities to reduce total delivered costs under certain constraints.

1.2 Decision Variables

- **Factory location & sizing** (binary decisions): for each candidate country, whether to open a Low-capacity or High-capacity factory (or open none).

- **Production / shipment allocation** (continuous decisions): how many units to produce in each opened factory and ship from each origin to each market.

1.3 Key inputs and assumptions

- **Demand** (units/month):

  - USA: **2,800,000**

  - Germany: **90,000**

  – Japan: **1,700,000**

  – Brazil: **145,000**

  – India: **160,000**

- **Fixed costs** ($): The amount of money needed to establish a manufacturing facility in a certain nation, either one-time or over time.

Variable Costs	Fixed Cost ($)	
	Low	High
USA	$ 6,500	$ 9,500
Germany	$ 4,980	$ 7,270
Japan	$ 6,230	$ 9,100
Brazil	$ 3,230	$ 4,730
India	$ 2,110	$ 6,160

Figure 2: Fixed Costs

- **Variable costs** ($/unit): Cost of manufacturing per unit when one more unit is produced in a particular nation.

Variable Costs	Fixed Cost ($)		Variable Cost ($/units)				
	Low	High	USA	Germany	Japan	Brazil	India
USA	$ 6,500	$ 9,500	$ 12.00	$ 24.25	$ 13.10	$ 28.10	$ 20.78
Germany	$ 4,980	$ 7,270	$ 26.34	$ 13.00	$ 21.62	$ 33.24	$ 23.07
Japan	$ 6,230	$ 9,100	$ 25.40	$ 32.75	$ 10.00	$ 53.61	$ 24.35
Brazil	$ 3,230	$ 4,730	$ 24.45	$ 30.05	$ 36.00	$ 8.00	$ 37.75
India	$ 2,110	$ 6,160	$ 18.65	$ 20.40	$ 29.50	$ 34.40	$ 5.00

Figure 3: Variable Costs

- **Freight Costs** ($/container): Divided by 1,000 units per container to determine the transportation cost per unit.

Freight Costs ($/Container)	USA	Germany	Japan	Brazil	India
USA	0	12250	1100	16100	8778
Germany	13335	0	8617	20244	10073
Japan	15400	22750	0	43610	14350
Brazil	16450	22050	28000	0	29750
India	13650	15400	24500	29400	0

Figure 4: Freight Costs

1.4 Constraints

- **Demand**: For each demand market (USA, Germany, Japan, Brazil, India), the total inbound shipments from all production countries must equal the market demand.

Decision Variable							
	Open?		Variable Cost				
Amount	Low	High	USA	Germany	Japan	Brazil	India
USA	0	1	1300000	0	200000	0	0
Germany	0	0	0	0	0	0	0
Japan	0	1	0	0	1500000	0	0
Brazil	1	0	0	0	0	145000	0
India	0	1	1500000	90000	0	0	160000
Supply			2800000	90000	1700000	145000	160000
			=	=	=	=	=
Demand			2800000	90000	1700000	145000	160000

Figure 5: Constraint for Demand

- **Capacity**: For each potential production country, total production (i.e., total shipments leaving country across all markets) cannot exceed the available capacity for that country.

Decision Variable								Production Constraints		
	Open?		Variable Cost							
Amount	Low	High	USA	Germany	Japan	Brazil	India	Production	Operator	Capacity
USA	0	1	1300000	0	200000	0	0	1500000	<=	1500000
Germany	0	0	0	0	0	0	0	0	<=	0
Japan	0	1	0	0	1500000	0	0	1500000	<=	1500000
Brazil	1	0	0	0	0	145000	0	145000	<=	500000
India	0	1	1500000	90000	0	0	160000	1750000	<=	3000000
Supply			2800000	90000	1700000	145000	160000			
			=	=	=	=	=			
Demand			2800000	90000	1700000	145000	160000			

Figure 6: Constraint for Capacity

- **Facility-size exclusivity**: A maximum of one facility size (Low or High) may be chosen by each nation. If neither is chosen, the capacity of that nation is zero, and there must be no shipments from it.

1.5 Optimal solution from Excel Solver

Optimal *total cost* (objective value): **$92,981,000** per month. This total includes:

- Fixed costs: **$27,990,000**

- Variable costs + Freight costs: **$64,991,000**

The optimal *factory-opening decisions* include:

- USA: Open **High-capacity** factory

- Japan: Open **High-capacity** factory

- Brazil: Open **Low-capacity** factory

- India: Open **High-capacity** factory

- Germany: **No factory** opened

The optimal *shipping / production allocation* (units/month) results show as:

- Serve USA (2,800,000):

  - From USA: **1,300,000**

  - From India: **1,500,000**

- Serve Germany (90,000):

  - From India: **90,000**

- Serve Japan (1,700,000):

  - From Japan: **1,500,000**

  - From USA: **200,000**

- Serve Brazil (145,000):

  - From Brazil: **145,000**

- Serve India (160,000):

  - From India: **160,000**

The optimal *capacity utilization* results show as:

- USA (High): 1,500,000 / 1,500,000 = 100%

- Japan (High): 1,500,000 / 1,500,000 = 100%

- Brazil (Low): 145,000 / 500,000 = 29%

- India (High): 1,750,000 / 3,000,000 = 58.33%

- Total opened capacity is **6,500,000**, total demand is **4,895,000**. Therefore, there is approximately **1,605,000** units of excess capacity (slack). This surplus can provide a buffer when demand fluctuates.

Objective Coefficients							
	Fixed Cost ($)		Variable Cost ($/units)				
Variable Costs	Low	High	USA	Germany	Japan	Brazil	India
USA	$ 6,500	$ 9,500	$ 12.00	$ 24.25	$ 13.10	$ 28.10	$ 20.78
Germany	$ 4,980	$ 7,270	$ 26.34	$ 13.00	$ 21.62	$ 33.24	$ 23.07
Japan	$ 6,230	$ 9,100	$ 25.40	$ 32.75	$ 10.00	$ 53.61	$ 24.35
Brazil	$ 3,230	$ 4,730	$ 24.45	$ 30.05	$ 36.00	$ 8.00	$ 37.75
India	$ 2,110	$ 6,160	$ 18.65	$ 20.40	$ 29.50	$ 34.40	$ 5.00

Objective Value $ 92,981,000.00

Decision Variable										
	Open?		Variable Cost					Production Constraints		
Amount	Low	High	USA	Germany	Japan	Brazil	India	Production	Operator	Capacity
USA	0	1	1300000	0	200000	0	0	1500000	<=	1500000
Germany	0	0	0	0	0	0	0	0	<=	0
Japan	0	1	0	0	1500000	0	0	1500000	<=	1500000
Brazil	1	0	0	0	0	145000	0	145000	<=	500000
India	0	1	1500000	90000	0	0	160000	1750000	<=	3000000
Supply			2800000	90000	1700000	145000	160000			
			=	=	=	=	=			
Demand			2800000	90000	1700000	145000	160000			

Capacity (kUnits/month)	Low	High		Variable Costs ($/Unit)	USA	Germany	Japan	Brazil	India
USA	500	1500		USA	12	12	12	12	12
Germany	500	1500		Germany	13	13	13	13	13
Japan	500	1500		Japan	10	10	10	10	10
Brazil	500	1500		Brazil	8	8	8	8	8
India	500	3000		India	5	5	5	5	5

Freight Costs ($/Container)	USA	Germany	Japan	Brazil	India
USA	0	12250	1100	16100	8778
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India	13650	15400	24500	29400	0

Figure 7: Optimization Results

1.6 Managerial interpretation

Instead of setting up a German factory, Germany is served by imports (from India). Germany has a modest demand (90k). At that volume, opening a facility would result in significant fixed costs that are not necessary.

To meet the enormous demand in the USA, India serves as a cost-effective "swing" supplier to supplement the US facility. This illustrates the trade-off between labor and logistics: For some lanes, the low cost of production in India can offset the cost of transportation.

Japan relies primarily on domestic production, supplemented by a small amount from the United States: Japanese demand is very high (1.7 million units), making local high-capacity factories more economical; at the same time, the US supplies Japan with 200,000 units, indicating that the overall landed cost of this supply route is marginally competitive.

Brazil meets its demand using local, low-capacity factories: Brazil's demand is relatively small (145,000 units), and using low-capacity factories avoids unnecessary fixed costs while also reducing international transportation costs through local production.

The solution further emphasizes the tradeoff between efficiency and robustness: the deterministic optimum leaves significant slack capacity even if it reduces predicted cost for the baseline

demand.

2. Simulation

2.1. Excel Simulation

(a) Purpose

The primary purpose of this Excel-based simulation is to evaluate the operational robustness and reliability of the supply chain network we previously determined in the Optimization part. While our previous model determined a cost-minimizing strategy under static demands, we want to quantify how this optimized supply chain model performs under uncertainties. To be more precise, this part measures the effect of fluctuating demands on the total cost and calculates the probability that the supply chain network could fully satisfy the market demands.

(b) Simulation Design & Key Assumption

The simulation model is structured following the simulation GAP framework with 1,000 iterations. For the simulated stress-test, we make the following assumptions and constraints in addition to the optimization model:

- **Fixed Network Configuration**: Unlike in Python simulation which re-optimizes the structure, the Excel model takes factory locations and capacities as fixed inputs based on the optimized solution – high-capacity factories in the USA, Japan, and India, and a low-capacity factory in Brazil.

- **Fluctuating Demands** (New Assumption): We assume that the real demand in each market follows a Normal Distribution with mean of the forecasted demand (previous demand) and a standard deviation of 2.5% of the forecast.

- **Penalty Cost for Unsatisfied Demand** (New Assumption): To account for the risk of stockouts, a penalty mechanism was introduced. Any demand that exceeds the assigned factory's capacity is recorded as "Unmet Demand" and assigned a penalty cost of $100 per unit. This serves as a proxy for lost sales, expedited shipping, or reputational damage.

(c) Key Results: Performance Measures

Based on 1,000 iterations, the performance of the optimized network under a fixed allocation policy is summarized in Table 1:

Performance Metric	Statistical Value
Average Total Cost	$95,628,179.64
Minimum Total Cost	$88,011,930.20
Maximum Total Cost	$119,248,725.48
Standard Deviation	$5,294,713.35
Service Level	48.30%

Table 1: Excel Simulation Performance Metrics (N=1,000)

- **Service Level**: The service level is defined as the probability that all market demands are fully satisfied without stockouts. In our simulation, we derived this by calculating the frequency of iterations where the "*Total Penalty*" column equals to **0**.

- **Cost Comparison**: The average total cost we get (**$95.63M**) in this simulation is approximately **2.8%** higher than the optimal cost (**$92.98M**) in the fixed demand optimization. This difference is primarily due to the penalty cost in the scenarios that market demands exceed factory capacities.

(d) Distribution Analysis

From the table above, we observe a high standard deviation of over $5M, which indicates a high variability risk that total costs are sensitive to demand fluctuations under a fixed production configuration.

Meanwhile, in about 51.7% simulated cases, the supply chain model incurred penalty costs. This is because the rigid allocation does not allow for dynamic capacity usage that a more flexible system would provide. Even though the total global capacity exceeds average global demand, localized demand spikes can still trigger penalties if they exceed the capacity of a specific assigned facility. From Table 2, we notice that all stockout cases happen in the USA and Japan, and that is due to the high demands in those markets (see Table 3) – 2.8M in the US and 1.7M in Japan, which are significantly higher than the other three markets. Since the fluctuation is the same for all markets (roughly 5%), given the size, only the USA and Japan contribute to the stockout.

Countries	Probability of Stockout
USA	51.70%
Japan	51.70%
Brazil	0.00%
India	0.00%

Table 2: Probability of Stockout by Country

Countries	USA	Germany	Japan	Brazil	India
Demand	2,800,000	90,000	1,700,000	145,000	160,000

Table 3: Market Demand Overview

(e) Managerial Implications

From this Excel-based simulation, we see a need for operational flexibility to offer a dynamic production pattern under uncertainties. Although in the previous part we have determined the optimal strategy of factory allocation and capacity choice, the high proportion of unsatisfied demands indicates that the rigid shipping policy is insufficient. To fully satisfy demands from every market, the company has to develop a dynamic allocation strategy that allows factory with surplus supply to support markets experiencing unexpected demand surges.

In addition, the management team should account this uncertainty into budgeting. They should use the maximum total cost of $119.25M as a worst-case budgetary benchmark. This ensures a healthy cash flow for the company even if they encounter extreme market volatility that yields the peak penalty costs.

2.2. Python Simulation

(a) Purpose

While the deterministic optimization model identifies a cost-minimizing supply chain configuration under forecasted demand, PrecisionLink's real operational challenge is demand uncertainty. To evaluate how stable our recommended network is when demand fluctuates, we implemented a Python-based Monte Carlo simulation with **Optimization-in-the-Loop**.

In each iteration, we randomly generate market demands (±5% volatility), then re-solve the optimization model to obtain the cost-minimizing factory-opening decisions and flows for that scenario. Repeating this process across many scenarios allows us to quantify robustness to find how often the same facilities are chosen when the world changes.

```
--- Probability of Opening Factories (Robustness) ---
Open_USA_High        1.000
Open_Japan_High      1.000
Open_Brazil_Low      0.924
Open_Brazil_High     0.006
Open_India_Low       0.006
Open_India_High      0.994
dtype: float64

=== Key Monte Carlo Simulation Statistics ===
Mean Total Cost: 92,614,252
Stddev of Total Cost: 6,006,979
Min Total Cost: 76,732,537
Max Total Cost: 110,995,936
Demand_USA Mean: 2,782,672  Stddev: 278,573
Demand_Germany Mean: 90,608  Stddev: 8,629
Demand_Japan Mean: 1,691,849  Stddev: 171,662
Demand_Brazil Mean: 144,094  Stddev: 14,666
Demand_India Mean: 159,946  Stddev: 15,744
```

Figure 8: Factory Opening Robustness and Monte Carlo Cost Summary

Figure 8 reports the factory opening probabilities across 500 Monte Carlo demand scenarios, along with summary statistics of total supply chain cost. The results quantify the robustness of each facility decision and illustrate the overall cost distribution under demand uncertainty.

(b) Simulation Design

- **Random variables**: Monthly demand in USA, Germany, Japan, Brazil, and India (modeled with ±5% fluctuations around forecast).

- **Decision variables** (re-optimized each iteration): Factory opening decisions (Low vs High capacity) and Production/shipping allocation decisions.

- **Performance metrics**:

    - Factory opening frequency (robustness probability)

    - Cost distribution (mean, standard deviation, min/max)

    - Drivers of Brazil Low opening (feature importance)

(c) Key Results: Robust Facility Choices (Robustness)

Across the simulated scenarios, the optimization repeatedly selects a consistent network structure:

- USA High is opened in 100% of scenarios

- Japan High is opened in 100% of scenarios

- India High is opened in 99.4% of scenarios

- Brazil Low is opened in 92.4% of scenarios

- Brazil High and India Low are almost never chosen ($\sim 0.6\%$)

This pattern indicates a highly stable "core backbone" of the network - USA High + Japan High + India High - which remains optimal even when demand shifts. Meanwhile, Brazil Low appears as a frequent supplemental choice, suggesting it acts as a flexible buffer to handle uncertainty without committing to a high fixed-cost facility.
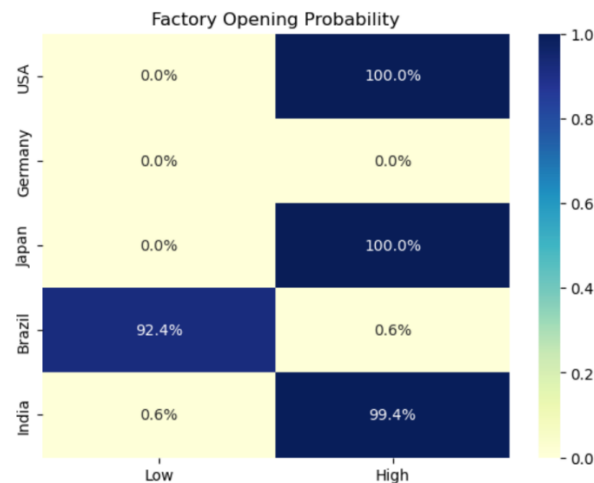

Figure 9: Factory Opening Probability Heatmap

This heatmap (*Figure 9*) visualizes the probability of opening low- and high-capacity factories in each country across all simulated demand scenarios. Darker colors indicate higher selection frequency, highlighting the most robust facility choices.

(d) Key Results: Cost Behavior Under Uncertainty

The Monte Carlo statistics show that total cost remains well-behaved under demand variability:

- Mean Total Cost: $92,614,252

- Std Dev of Total Cost: $6,006,979

- Min / Max Total Cost: $76,732,537 / $110,995,936

Even though demand is uncertain, the resulting total cost distribution remains within a manageable range. The relatively moderate standard deviation suggests that the network is not overly sensitive to random demand shocks.

In addition, the scatter plot reveals a near-linear relationship between Total Demand and Total Cost, meaning cost scales predictably as demand increases. Coloring by `Open_Brazil_Low` shows that Brazil Low opening is associated with demand-driven adjustments rather than unstable cost spikes.
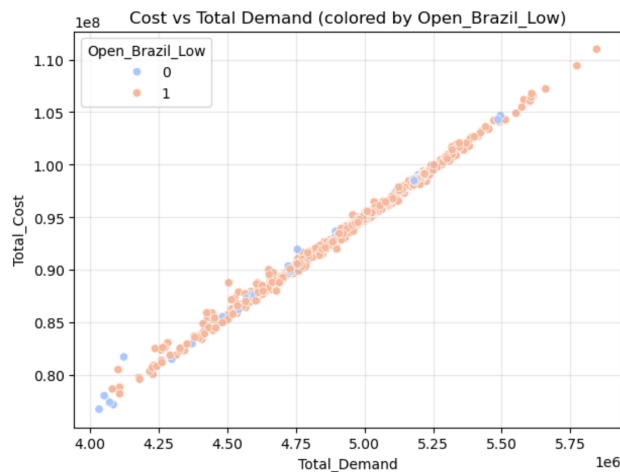
Figure 10: Total Cost vs. Total Demand Under Demand Uncertainty

This scatter plot (*Figure 10*) shows the relationship between total demand and total supply chain cost across Monte Carlo simulations. Points are colored by whether the Brazil low-capacity factory is opened, illustrating how cost scales predictably with demand.

(e) Key Drivers of the Brazil Low Decision

To understand why the model opens Brazil Low in most scenarios, we analyzed drivers of this binary decision. The feature importance results show:

- `Demand_Brazil` is the dominant driver (importance ≈ 0.866)

- Other markets (USA, Germany, Japan, India) contribute relatively little

This suggests Brazil Low is primarily a local responsiveness decision: when Brazil demand rises in a scenario, opening a small facility in Brazil becomes cost-effective to reduce transport cost and maintain fulfillment without overbuilding capacity.

```
--- Key Drivers for Open_Brazil_Low Decision ---
          Feature  Importance
3   Demand_Brazil    0.866112
0      Demand_USA    0.056009
1  Demand_Germany    0.032735
2    Demand_Japan    0.027250
4    Demand_India    0.017894
```

Figure 11: Key Demand Drivers for Opening the Brazil Low-Capacity Factory

(f) Managerial Implication

Overall, the Python simulation demonstrates that the proposed network is not only cost-efficient under average forecasts, but also robust under uncertainty. The repeated selection of USA/Japan/India high-capacity facilities provides a reliable backbone, while Brazil low-capacity acts as a flexible adjustment lever. This structure reduces operational risk and increases confidence that PrecisionLink's supply chain decisions will remain effective even when demand deviates from expectations.

Appendix

A1. Optimization Model Set-up GAP

To ensure a systematic approach to the PrecisionLink supply chain optimization, we follow the GAP framework to define and formulate our model.

1. Model Definition

- **Objectives**: To minimize the total cost of the global supply chain, which includes fixed facility costs, variable production costs, and international freight costs.

- **Decision Variables**:

  - *Binary decisions*: Whether to open a Low-capacity or High-capacity factory in each of the 5 candidate countries (USA, Germany, Japan, Brazil, India).
  - *Continuous decisions*: The quantity of units produced in each factory and shipped to each of the 5 regional markets.

- **Random Variables**: Market demand fluctuations in each region.

- **Constraints**:

  - *Capacity*: Total production at any site cannot exceed its established capacity (Low: 500k; High: 1.5M).
  - *Demand*: Total units delivered to each market must meet the forecasted regional demand.
  - *Exclusivity*: Each country can host at most one factory type (either Low or High, not both).

2. Formulate Model Mathematically

Decision Variables:

- $y_{i,s} \in \{0, 1\}$: 1 if factory size s ($s \in \{L, H\}$) is opened in country i, 0 otherwise.

- $x_{i,j}$: Units shipped from country i to market j.

  Objective Function:

$$\text{Minimize } Z = \sum_{i=1}^{5} \sum_{s \in \{L,H\}} f_{i,s} y_{i,s} + \sum_{i=1}^{5} \sum_{j=1}^{5} (v_i + t_{i,j}) x_{i,j}$$

Where f is fixed cost, v is variable cost, and t is transport cost.

  Constraints:

- **Capacity Constraint:** $\sum_{j=1}^{5} x_{i,j} \leq \sum_{s \in \{L,H\}} C_{i,s} y_{i,s} \quad \forall i$

- **Demand Fulfillment:** $\sum_{i=1}^{5} x_{i,j} \geq D_j \quad \forall j$

- **Single Site Selection:** $\sum_{s \in \{L,H\}} y_{i,s} \leq 1 \quad \forall i$

A2. Excel Simulation Model Set-up GAP

To evaluate the robustness of our optimized supply chain network under demand uncertainty, we perform a simulation using Excel. The model is structured following the Simulation GAP framework.

1. Model Definition

- **Decision Variables**:

  - *Network Configuration*: The fixed factory locations and capacities determined in the Optimization phase (e.g., High-capacity factories in USA and Japan).

  - *Production Policy*: The allocation logic used to fulfill regional demand from active factories.

- **Random Variables**:

  - *Market Demand (D_j)*: Monthly demand for each of the 5 markets, modeled as a continuous random variable (e.g., Normal distribution) with a ±5% fluctuation range.

- **Objective and Logic**:

  - *Total Cost Calculation*: Sum of Fixed Costs + (Actual Production × Unit Variable Costs) + Penalty Costs for unmet demand.

  - *Fulfillment Logic*: If *Total Capacity < Total Demand*, the model calculates the "Unmet Demand" and applies a penalty cost per unit.

- **Statistics/Performance Measures**:

  - Average (Mean) Total Cost across 1,000 iterations.

  - Standard Deviation and 95% Confidence Interval of the total cost.

  - Service Level: The probability that all market demands are fully met without stockouts.

2. Formulate Simulation Logic Mathematically

For each iteration $k \in \{1, \ldots, 1000\}$:

    Step 1: Generate Random Demand

$$D_j^{(k)} \sim \text{Distribution}(\mu_j, \sigma_j) \quad \forall \text{ market } j$$

    Step 2: Calculate Production and Shortage

$$\text{Actual Production}^{(k)} = \min\left(\sum D_j^{(k)}, \text{Total Network Capacity}\right)$$

$$\text{Unmet Demand}^{(k)} = \max\left(0, \sum D_j^{(k)} - \text{Total Network Capacity}\right)$$

    Step 3: Calculate Total Cost per Iteration

$$\text{Total Cost}^{(k)} = \text{Fixed Costs} + (\text{Var. Cost} \times \text{Actual Prod.}^{(k)}) + (\text{Penalty Cost} \times \text{Unmet Demand}^{(k)})$$

    Step 4: Statistical Aggregate

$$\text{Expected Total Cost} = E[\text{Total Cost}] = \frac{1}{1000} \sum_{k=1}^{1000} \text{Total Cost}^{(k)}$$

A3. Python Simulation Model Set-up GAP

While the Excel simulation evaluates the performance of a *single fixed* network configuration, it cannot systematically assess the structural stability of the supply chain. To overcome this, we developed a Python-based simulation using the **Optimization-in-the-Loop** approach. This algorithm re-optimizes the entire network structure for 500 randomized demand scenarios to quantify the "Robustness" of each facility location.

1. Model Definition

- **Decision Variables (Dynamic per Iteration)**:

    - Unlike the Excel model where factories are fixed inputs, the Python model treats Factory Decisions ($y_{i,s}^{(k)}$) and Flow Decisions ($x_{i,j}^{(k)}$) as variables to be solved *from scratch* in each iteration k.

- **Random Variables**:

    - *Stochastic Demand (D_j)*: Modeled using a Normal Distribution $\mathcal{N}(\mu_j, \sigma_j)$ for each market, capturing volatility risks.

- **Objective and Logic**:

  - *Algorithm*: The script runs a loop $k = 1, \ldots, 500$. In each pass, it instantiates a Mixed-Integer Linear Programming (MILP) solver (using the `PuLP` library).

  - *Goal*: Minimize Total Cost for the specific random demand profile of that iteration.

- **Statistics/Performance Measures**:

  - **Robustness Score**: The probability $P(\text{Open}_{i,s})$ that a specific factory configuration is chosen by the optimizer across all scenarios.

  - **Cost Sensitivity**: Analysis of how the optimal Total Cost correlates with aggregated Global Demand.

2. Formulate Python Simulation Logic Mathematically

For each simulation iteration $k \in \{1, \ldots, 500\}$:

Step 1: Generate Random Environment

$$D_j^{(k)} \leftarrow \mathcal{N}(\text{Forecast}_j, \text{StdDev}_j) \quad \forall j \in \{\text{USA, DE, JP, BR, IN}\}$$

Step 2: Solve the MILP Instance

Given $D_j^{(k)}$, find $y_{i,s}^{(k)}$ and $x_{i,j}^{(k)}$ to:

$$\text{Minimize } Z^{(k)} = \sum_{i,s} f_{i,s} y_{i,s}^{(k)} + \sum_{i,j} (v_i + t_{i,j}) x_{i,j}^{(k)}$$

Subject to:

$$\sum_i x_{i,j}^{(k)} \geq D_j^{(k)} \qquad \qquad \text{(Demand Satisfaction)}$$

$$\sum_j x_{i,j}^{(k)} \leq \sum_s C_{i,s} y_{i,s}^{(k)} \qquad \qquad \text{(Capacity Constraint)}$$

$$\sum_s y_{i,s}^{(k)} \leq 1 \qquad \qquad \text{(Single Site Constraint)}$$

Step 3: Aggregate Robustness Metrics

After N iterations, calculate the opening frequency for each facility candidate:

$$\text{Robustness}_{i,s} = \frac{1}{N} \sum_{k=1}^{N} y_{i,s}^{(k)}$$