Info-Spé

Corrigé du contrôle 1

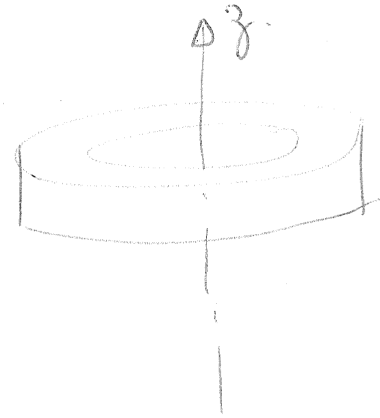
(2010/2011)

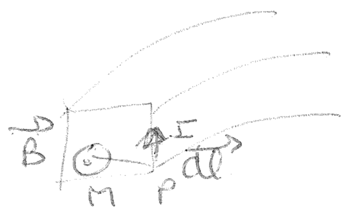
1.

coupe

Ex1 :

1)

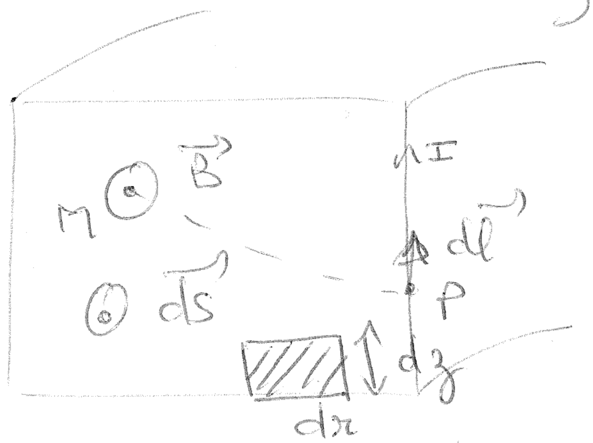




$\vec{B} \perp (\vec{dl}, \vec{PM})$

$\rightarrow \vec{B} \perp$ plan des spires.

d'où $\vec{B}$ est tangentiel.



b) les surfaces traversées par les lignes de $\vec{B}$ sont les plans des spires tel que.

$$\boxed{dS = dr \cdot dz}$$

c) $\Phi(\vec{B}) = N \iint_S \vec{B} \cdot \vec{dS} = +N \int_0^h \int_{R_1}^{R_2} \frac{\mu_0 N I}{2\pi r} dr dz$

$$\Phi(\vec{B}) = \frac{\mu_0 N^2 I}{2\pi} \int_{R_1}^{R_2} \frac{dr}{r} \cdot \int_0^h dz$$

$\longrightarrow$

$$\boxed{\Phi(\vec{B}) = \frac{\mu_0 N^2 I}{2\pi} \ln\left(\frac{R_2}{R_1}\right) \cdot h}$$

2) $I(t) = At^2$

a) I varie en fct du temps, donc B varie aussi " " car B est proportionnel à I.

le flux sera donc variable en fct du temps. car le flux est proportionnel à B, on a donc un phénomène d'autoinduction.

b) $$e = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\frac{\mu_0 N^2 A . t^2}{2\pi} \ln() h\right)$$

$$\boxed{e(t) = -\frac{\cancel{2}At}{\cancel{2}\pi} \mu_0 N^2 \ln\left(\frac{R_2}{R_1}\right) \cdot h}$$

<u>Ex2</u> : $\vec{\nabla} \cdot (\vec{\nabla} \wedge \vec{A}) = 0$?

$\mathrm{div}(\vec{\mathrm{rot}}\,\vec{A}) = 0$.

$$\vec{\mathrm{rot}}\,\vec{A} = \begin{pmatrix} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \\ \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \\ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{pmatrix}$$

$\text{div}(\overrightarrow{\text{rot}}\,\vec{A}) = 0$ (2)

$$\Rightarrow \quad \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)$$

$$+ \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right).$$

$$\text{div}(\overrightarrow{\text{rot}}\,\vec{A}) = \cancel{\frac{\partial^2 A_z}{\partial x \partial y}} - \cancel{\frac{\partial^2 A_y}{\partial x \partial z}} + \cancel{\frac{\partial^2 A_x}{\partial y \partial z}} - \cancel{\frac{\partial^2 A_z}{\partial y \partial x}}.$$

$$+ \cancel{\frac{\partial^2 A_y}{\partial z \partial x}} - \cancel{\frac{\partial^2 A_x}{\partial z \partial y}}.$$

Comme les fct = (composantes des vecteurs) sont des D.T.E.

alors $\dfrac{\partial^2 A_z}{\partial x \partial y} = \dfrac{\partial^2 A_z}{\partial y \partial x}$

— — — —

$$\Rightarrow \quad \text{div}(\overrightarrow{\text{rot}}\,\vec{A}) = 0.$$

2) $\vec{\nabla} \cdot (\vec{A} \wedge \vec{B}) = \vec{B} \cdot (\vec{\nabla} \wedge \vec{A}) - \vec{A} \cdot (\vec{\nabla} \wedge \vec{B})$

$$\text{div}(\vec{A} \wedge \vec{B}) = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \wedge \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \frac{\partial}{\partial x}\left(A_y B_z - A_z B_y\right) +$$

$$\frac{\partial}{\partial y}\left(A_z B_x - A_x B_z\right) + \frac{\partial}{\partial z}\left(A_x B_y - A_y B_x\right)$$

③

$$\operatorname{div}(\vec{A}\wedge\vec{B}) = \frac{\partial A_y}{\partial x} B_z + A_y \frac{\partial B_z}{\partial x} - \frac{\partial A_z}{\partial x} B_y -$$

$$A_z \frac{\partial B_y}{\partial x} + \frac{\partial A_z}{\partial y} B_x + A_z \frac{\partial B_x}{\partial y} - \frac{\partial A_x}{\partial y} B_z.$$

$$- A_x \frac{\partial B_z}{\partial y} + \frac{\partial A_x}{\partial z} B_y + A_x \frac{\partial B_y}{\partial z} - \frac{\partial A_y}{\partial z} B_x - A_y \frac{\partial B_x}{\partial z}$$

(en appliquant la dérivé d'un produit).

et en factorisant correctement on obtient.

$$\operatorname{div}(\vec{A}\wedge\vec{B}) = B_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + B_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right).$$

$$+ B_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) - A_x\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) -$$

$$A_y\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) - A_z\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)$$

$$= \vec{B}\cdot(\vec{\nabla}\wedge\vec{A}) - \vec{A}\cdot(\vec{\nabla}\wedge\vec{B}). \quad (C.Q.F.D)$$

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2) $\vec{rot}(\vec{grad}(f)) = \vec{0}$

$\Rightarrow \iint_S \vec{rot}(\vec{grad}(f)) = \vec{0}$

$\oint_C \vec{grad}(f)) = 0$ or $\vec{grad}(V) = -E$

$$\Rightarrow \oint_C \vec{E} \cdot \vec{dl} = 0$$

circulation de vecteur nulle.

Conl : un vecteur dont le rotationnel est nul, sa circulation sera également nulle.

EX3

$$V(r) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} e^{-r/a}$$

1) $\vec{E} = -\vec{grad}(V)$.

V ne depend que de r $\rightarrow$ $\vec{E}$ est radial.

$$\begin{cases} E_r = -\frac{\partial V}{\partial r} . \\ E_\theta = -\frac{1}{r} \underbrace{\frac{\partial V}{\partial \theta}}_{=0} \quad V = V(r) \\ E_\varphi = -\frac{1}{r\sin\theta} \cdot \underbrace{\frac{\partial V}{\partial \varphi}}_{} = 0 \end{cases}$$

$$\text{ó } V = V(r)$$

$$E = E_r = -\frac{q}{4\pi\varepsilon_0} \cdot \frac{\partial}{\partial r}\left(\frac{e^{-r/a}}{r}\right)$$ 4.

$$= \frac{-q}{4\pi\varepsilon_0} \cdot \left(-\frac{1}{r^2} e^{-r/a} + \frac{1}{r}\left(-\frac{1}{a}\right)e^{-r/a}\right)$$

$$\boxed{E_r(r) = \frac{q}{4\pi\varepsilon_0 r^2} e^{-r/a}\left(1 + \frac{r}{a}\right)}$$

2) $\Phi(\vec{E}) = \frac{Q}{\varepsilon_0}$ (Gauss)

$\quad \shortparallel$

$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$

$\Rightarrow \quad Q = E \cdot 4\pi r^2 \varepsilon_0$

$$Q = \frac{q}{4\pi\varepsilon_0 r^2} e^{-r/a}\left(1 + \frac{r}{a}\right) 4\pi r^2 \varepsilon_0$$

$$\Rightarrow \boxed{Q = q \cdot e^{-r/a}\left(1 + \frac{r}{a}\right)}$$

Fin