LOFO Cheat Sheet

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1 λ-calculus

Syntactic conventions:

$M ::= x \mid (\lambda x \cdot M) \mid (MM)$

- Omit outer parentheses $\qquad MN = (MN)$
- Application associates to the left $\qquad MNL = (MN)L$
- Multiple arguments as syntactic sugar $\lambda xy \cdot M = \lambda x \cdot \lambda y \cdot M$ (Currification)
- Abstraction associates to the right $\quad \lambda x \cdot MN = \lambda x \cdot (MN)$

2 Simply Typed λ-calculus

$$\frac{M : \sigma \to \tau \quad N : \sigma}{MN : \tau} \qquad \frac{\begin{array}{c}[x : \sigma]\\ \vdots \\ M : \tau\end{array}}{\lambda x \cdot M : \sigma \to \tau}$$

3 NJ — Intuitionistic Natural Deduction

$$\frac{\begin{array}{c}[A]\\ \vdots \\ B\end{array}}{A \Rightarrow B} \Rightarrow \mathcal{I} \qquad \frac{A \quad A \Rightarrow B}{B} \Rightarrow \mathcal{E} \qquad \frac{\bot}{A} \bot \mathcal{E} \qquad \neg A := A \Rightarrow \bot$$

$$\frac{A \quad B}{A \wedge B} \wedge \mathcal{I} \qquad \frac{A \wedge B}{A} \wedge l \mathcal{E} \qquad \frac{A \wedge B}{B} \wedge r \mathcal{E}$$

$$\frac{A}{A \vee B} \vee l \mathcal{I} \qquad \frac{B}{A \vee B} \vee r \mathcal{I} \qquad \frac{A \vee B \quad \begin{array}{c}[A]\\ \vdots \\ C\end{array} \quad \begin{array}{c}[B]\\ \vdots \\ C\end{array}}{C} \vee \mathcal{E}$$

4 LK — Classical Sequent Calculus

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \tau(\Delta)} \vdash X \quad \frac{\Gamma \vdash \Delta}{\sigma(\Gamma) \vdash \Delta} X \vdash \quad \frac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta} \vdash W \quad \frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta} W \vdash \quad \frac{\Gamma \vdash A, A, \Delta}{\Gamma \vdash A, \Delta} \vdash C \quad \frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta} C \vdash$$

$$\frac{}{F \vdash F} \text{Id} \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma', A \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'} \text{Cut}$$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta} \vdash \neg \qquad \frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta} \neg \vdash$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta} \vdash \wedge \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} l \wedge \vdash \qquad \frac{\Gamma, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} r \wedge \vdash$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma \vdash A \vee B, \Delta} \vdash l \vee \qquad \frac{\Gamma \vdash B, \Delta}{\Gamma \vdash A \vee B, \Delta} \vdash r \vee \qquad \frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta} \vee \vdash$$

$$\frac{\Gamma \vdash \Delta, A \quad \Gamma', B \vdash \Delta'}{\Gamma, \Gamma', A \Rightarrow B \vdash \Delta, \Delta'} \Rightarrow \vdash \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \Rightarrow B, \Delta} \vdash \Rightarrow$$

5 LJ — Intuitionistic Sequent Calculus

$$\frac{}{A \vdash A} \text{Id} \qquad \frac{\Gamma \vdash A \quad \Gamma', A \vdash B}{\Gamma, \Gamma' \vdash B} \text{Cut}$$

$$\frac{\Gamma \vdash B}{\sigma(\Gamma) \vdash B} X \vdash \qquad \frac{\Gamma \vdash B}{\Gamma, A \vdash B} W \vdash \qquad \frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B} C \vdash$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \vdash \wedge \qquad \frac{\Gamma, A \vdash C}{\Gamma, A \wedge B \vdash C} l \wedge \vdash \qquad \frac{\Gamma, B \vdash C}{\Gamma, A \wedge B \vdash C} r \wedge \vdash$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vdash l \vee \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vdash r \vee \qquad \frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \vee B \vdash C} \vee \vdash$$

$$\frac{\Gamma \vdash A \quad \Gamma', B \vdash C}{\Gamma, \Gamma', A \Rightarrow B \vdash C} \Rightarrow \vdash \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B} \vdash \Rightarrow$$