EEE401

ELECTROMAGNETIC FIELDS & WAVES III

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Course Content

Basic Concepts

- Definition of basic terms
- Motion of charged particles in uniform electric field
- Electric Field as an accelerating field
- Electric Field as a deflecting Field
- Motion of charged particles in a uniform magnetic field
- Motion of charged particles in transverse Magnetic Field
- Motion of charged particles projected at an angle
- Deflection in Magnetic Field

Course Content

Applications

- Velocity Selector
- Cathode Rays
- Canal Rays
- Thomson’s Parabola Method
- Bethe’s Law
- Electrostatic Lens
- Cathode Ray Tubes
- Cathode Ray Oscilloscope
- Lissajous Figures
- Magnetic Field Focusing
- Magnetic lens
- Bainbridge mass spectrograph
- Linear Accelerator
- Cyclotron

Assignment 1

Derive the Lorentz force for crossed Electric and Magnetic Fields

1.0 Basic Definitions

Electric Field

- ✓ An electric field is said to exist in a region of space if an electric charge, at rest, experiences a force of electric origin.
- ✓ The electric field strength or intensity (**E**) at a point in space is defined as the amount of force (**F**) experienced by a unit positive charge (**q**) placed at that point. $E = \frac{F}{q}$
- ✓ Electric field between two parallel plates having a potential difference of V volts and having separation d metres between them is given by $E = V/d$
- ✓ The SI unit of E is Newtons/Couloumb (N/C)

Quiz 1

1.0 Basic Definitions

Electric Potential

- ✓ Electric potential (**V**) at a point is defined as the work done on a positive charge to bring it from infinity to that point.
- ✓ The SI unit for Electric potential is Volts (V) or Joules/Coulomb (J/C).

1.0 Basic Definitions

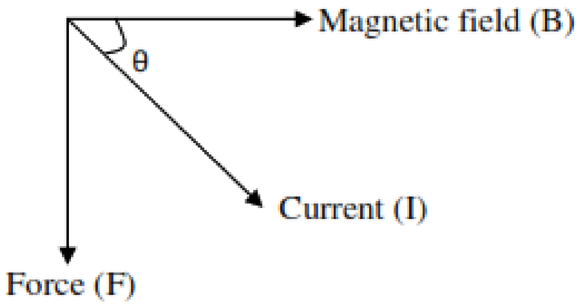
Magnetic Field

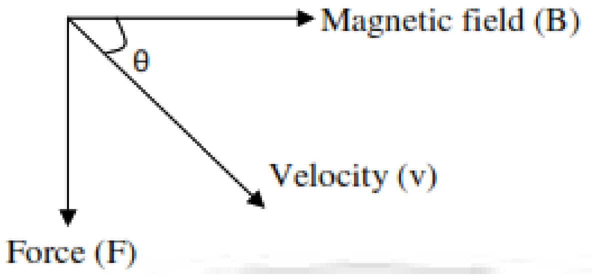
- ✓ Moving charges experience an additional interaction in addition to the Electrostatic interaction.
- ✓ magnetic field and is characterized by the magnetic induction vector **B**.
- ✓ The SI unit for **B** is Tesla (T) or Webers/metre2 (Wb/m^2).
- ✓ magnetic field strength **H** is defined as the ratio of magnetic induction in vacuum to the permeability μ_0
- ✓ $H = B/\mu_0$
- ✓ SI unit for magnetic field strength is Ampere-turn/metre (A/m).

1.0 Basic Definitions

Force in a Magnetic Field

- ✓ Consider a conductor of length l, carrying current I and is placed in a magnetic field of flux density **B** then the magnitude of the force is given as, $F = BIlsin\theta$
- ✓ Consider a charge q moving with velocity in a magnetic field B, the force acting on the charge is given by $F = Bqvsin\theta$

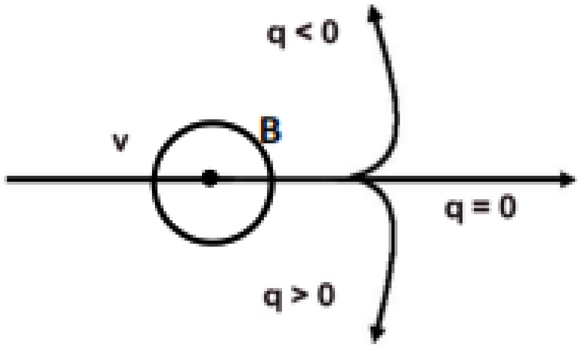




1.0 Basic Definitions

Force in a Magnetic Field

- ✓ The trajectories for positively and negatively charged particles in a magnetic field can be shown as



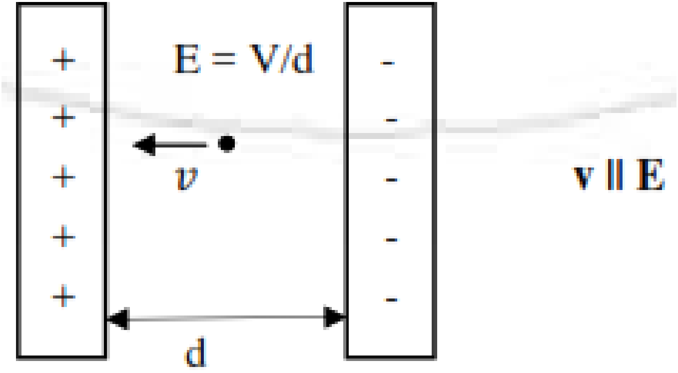
- ✓ The Lorentz Force on a point charge due to electromagnetic fields is: $F = q(E + v \times B)$

2. Motion of Charged Particles in Uniform Electric Field

Motion Parallel to Electric Field

- ✓ Assume two parallel plates in vacuum separated by a distance d metres as shown in the Figure
- ✓ The Electric field E, developed across the plates due to the applied voltage V is:

$$E = V/d$$



- ✓ A moving electron of mass m and charge q experience a force F due to the electric field

$$F = -qE$$

- ✓ According to newton's second law of motion

$$F = ma$$

2. Motion of Charged Particles in Uniform Electric Field

Motion Parallel to Electric Field

- ✓ Acceleration of the electron in Electric Field is given as

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md} \qquad (2.1)$$

- ✓ Applying the laws of kinematics and assuming the particle starts from rest, the velocity traveled by the particles at any time t is:

$$v = u + at = \frac{qEt}{m} = \frac{qVt}{md} \qquad (2.2)$$

2. Motion of Charged Particles in Uniform Electric Field

Motion Parallel to Electric Field

- ✓ The distance travelled by the particle during time t is:

$$S = ut + \frac{1}{2}at^2 = \frac{qEt^2}{2m} = \frac{qVt^2}{2md} \qquad (2.3)$$

Also

$$v^2 = u^2 + 2as = \frac{2qES}{m} = \frac{2qVS}{md}$$

$$\Rightarrow v = \sqrt{\frac{2qES}{m}} = \sqrt{\frac{2qVS}{md}} \qquad (2.4)$$

2. Motion of Charged Particles in Uniform Electric Field

Motion Parallel to Electric Field

- ✓ The time required by the particle to travel a distance S is given by

$$t = \sqrt{\frac{2mS}{qE}} = \sqrt{\frac{2mdS}{qV}} \qquad (2.5)$$

- ✓ The kinetic energy acquired by the electron after travelling a distance S is:

$$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}m\right)\left(\frac{2qES}{m}\right) = qES = \frac{qVS}{d} \qquad (2.6)$$

2. Motion of Charged Particles in Uniform Electric Field

Motion Parallel to Electric Field

- ✓ The electron gains energy from the electric field which it converts to kinetic energy. Hence when S = d, the kinetic energy is $\frac{1}{2}mv^2 = qV$

$$thus\ v = \sqrt{\frac{2qV}{m}} \qquad (2.7)$$

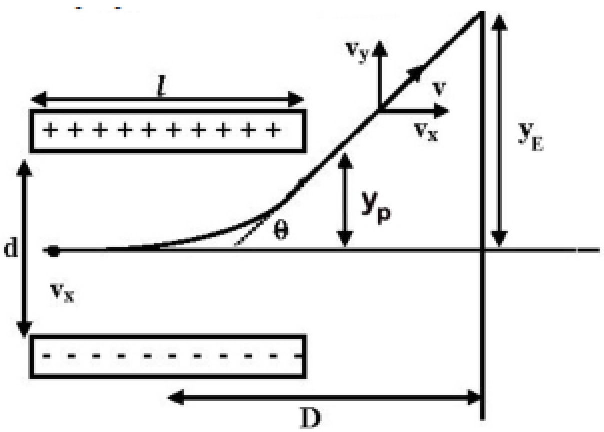
- ✓ The velocity of eqn. (2.7) is known as the impact or terminal velocity
- ✓ An electron volt (eV) is the energy acquired by an electron when accelerated through a potential of 1V

$$1eV = 1.6 \times 10^{-19} J$$

2. Motion of Charged Particles in Uniform Electric Field

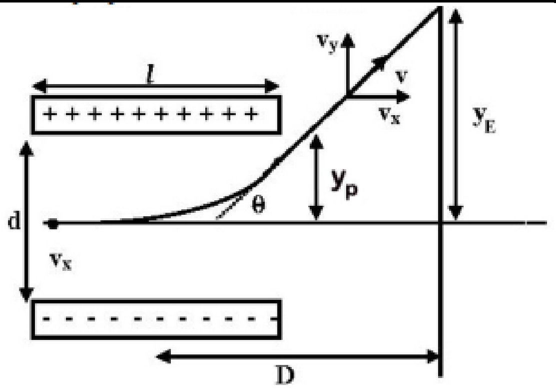
Motion Perpendicular to Electric Field

- ✓ Consider two parallel plates of length l having a potential difference $\boldsymbol{V}$ in between them. The plates are at a distance d from each other. The electric field (E) is directed from the positive to the negative plate. A Particle having charge q, mass m and initial velocity in the x-direction (v_x) enters the space between the plates having direction perpendicular to that of the electric field.



2. Motion of Charged Particles in Uniform Electric Field

Motion Perpendicular to Electric Field



- ✓ The vertical acceleration a_y is

$$a_y = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md} \qquad (2.8)$$

- ✓ The displacement in the x-direction is:

$$x = v_x t \qquad (2.9)$$

- ✓ t in eqn. 2.9 is the time taken for the particle to transverse the electric field
- ✓ The constant vertical velocity of the particle while leaving the field is:

$$v_y = u_y + a_y t = \frac{qEt}{m} = \frac{qVt}{md} \qquad (2.10)$$

2. Motion of Charged Particles in Uniform Electric Field

Motion Perpendicular to Electric Field

- ✓ The displacement in the y-direction is given as

$$S_y = \frac{1}{2} a_y t^2 = \frac{qE}{2m} t^2 = \frac{qV}{2md} t^2 \qquad (2.11)$$

- ✓ Substituting for t in (2.11), we get

$$S_y == \frac{qE}{2mv_x^2} x^2 = \frac{qV}{2mdv_x^2} x^2 = kx^2 \qquad (2.12)$$

- ✓ Eqn. (2.12) implies that a charged particle describes a parabolic path in a transverse electric field

2. Motion of Charged Particles in Uniform Electric Field

Motion Perpendicular to Electric Field

- ✓ When the charged particle leaves E, the deflecting force ceases to act on the particle. The particle now moves in a straight line with a resultant velocity:

$$v = \sqrt{v_x^2 + v_y^2} \qquad (2.13)$$

- ✓ The particle is deflected towards one of the plates by an angle, θ:

$$\theta = tan^{-1}\left(\frac{v_y}{v_x}\right) \qquad (2.14)$$

2. Motion of Charged Particles in Uniform Electric Field

Motion Perpendicular to Electric Field

- ✓ The vertical deflection is given by

$$y = D\tan\theta = D\frac{v_y}{v_x} \qquad (2.15)$$

- ✓ Where D is the distance from the point the projection of the resultant velocity cuts the x-axis to where the electron hits the screen (stops)
- ✓ Using $x = l$; we have

$$v_y = \frac{qEl}{mv_x} = \frac{qVl}{mdv_x} \qquad (2.16)$$

$$=> y = \frac{DqEl}{mv_x^2} = \frac{DqVl}{mdv_x^2} \qquad (2.17)$$