Measurement of Subpixel Structure

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Foreword

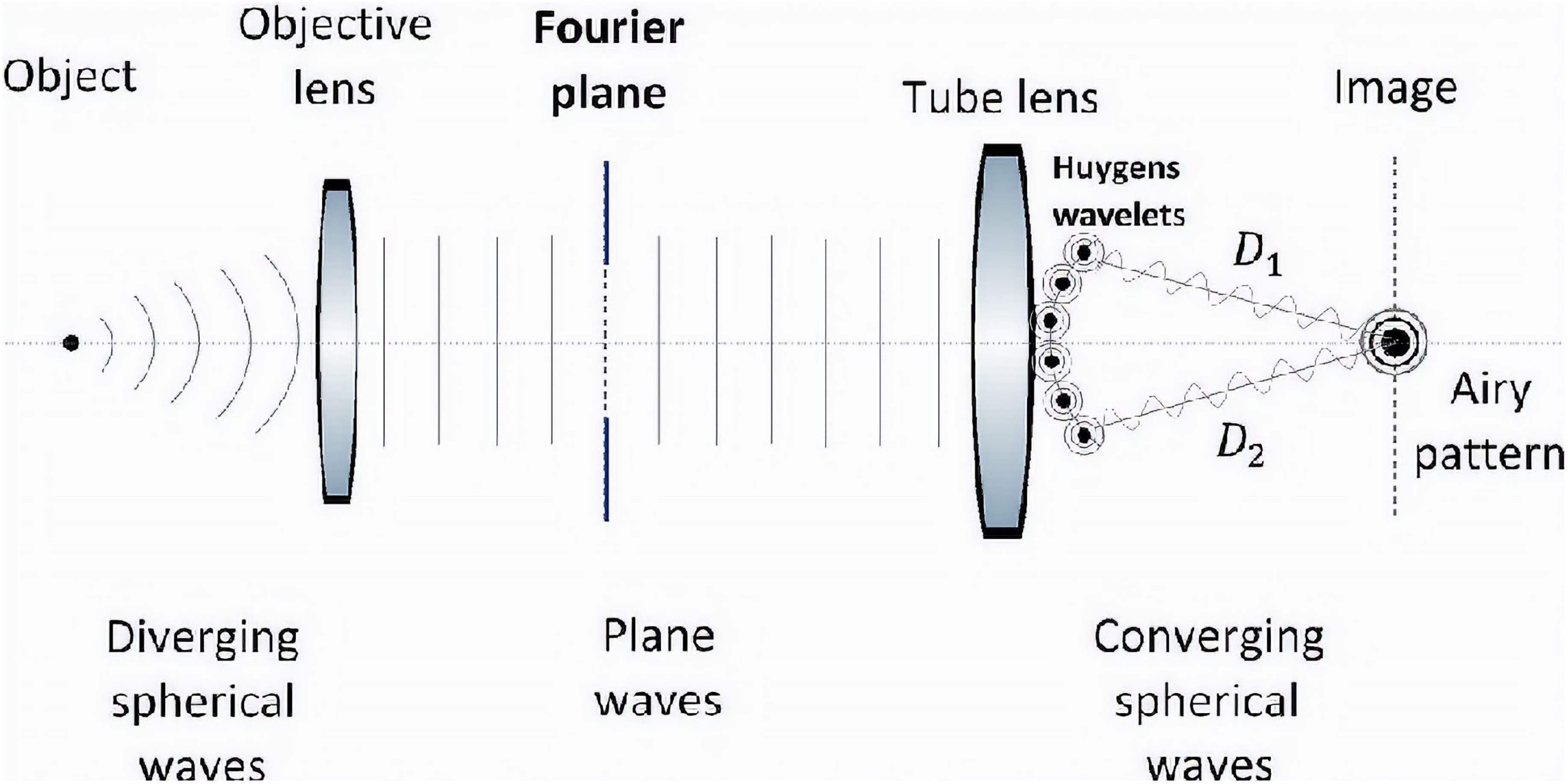
I would like to express my thanks to Dr. Kaloyan Penev who graciously set aside time every day to help me with this project and explain new concepts to me.

Problem

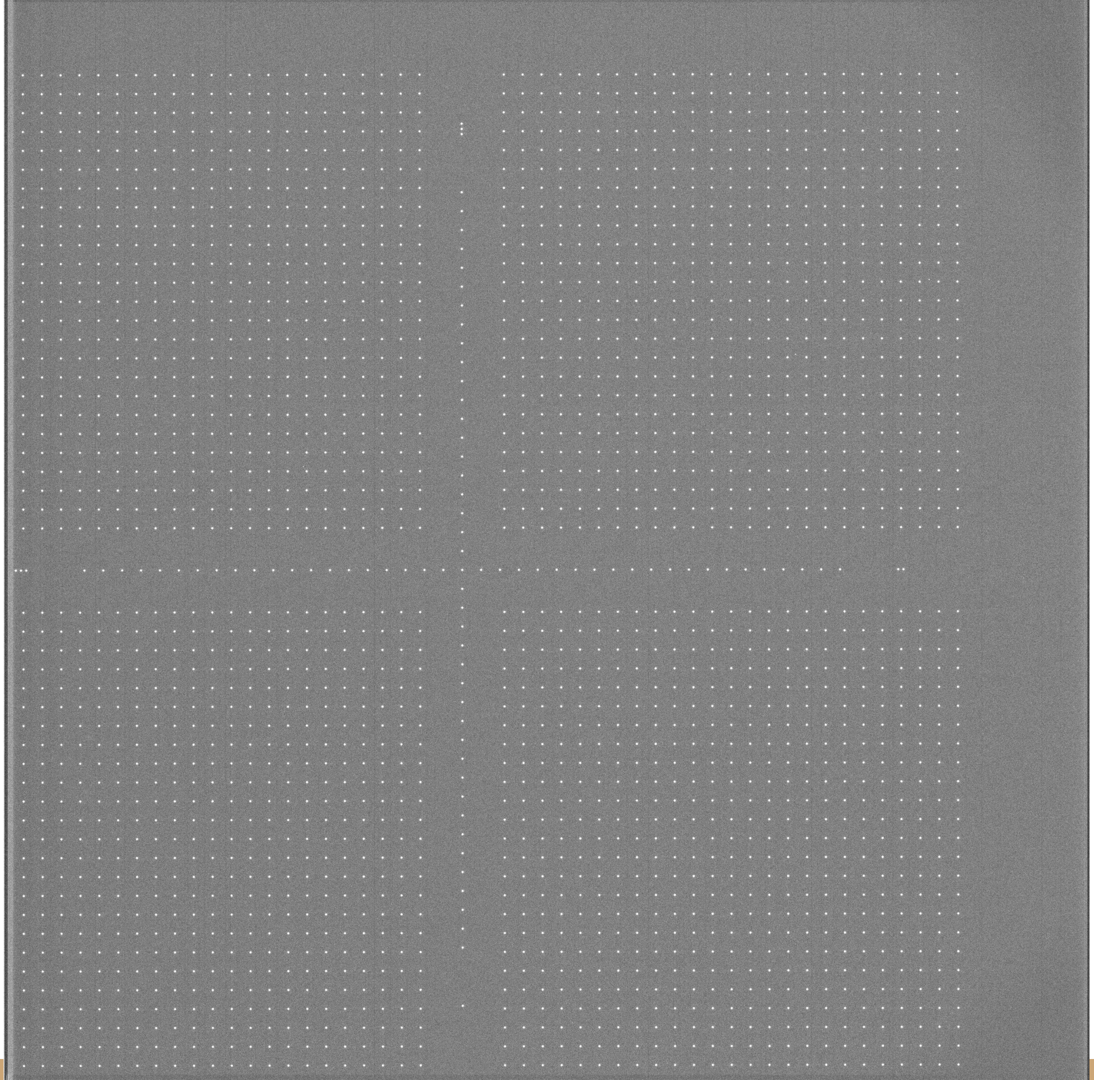
- Detection of exoplanets yields lots of noise
- Inconsistencies on outer edges of pixels on detector plane
- If we were able to deduce how sensitivity changes at some point on the pixel, we could mathematically correct for this phenomenon

Experiment

- We emit spherical waves through a mask which are reflected off mirrors and converge to the detector plane
- The mask allows for the light to form a certain pattern on the detector plane
- We do this for a grid of spots to create more data points to interpolate on.



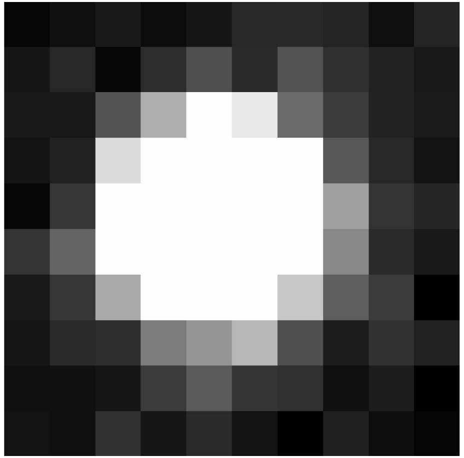
Source: [Wikipedia]

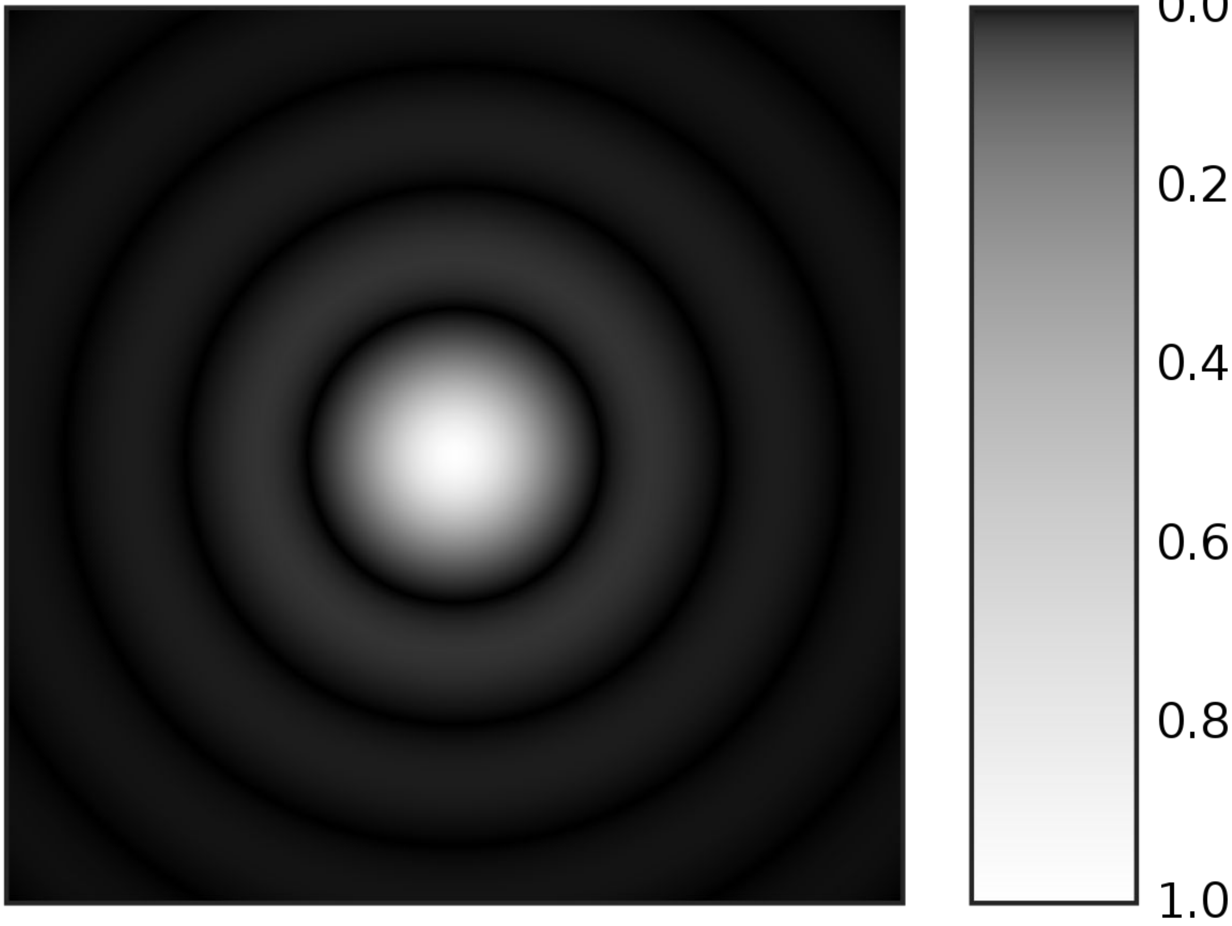


- Sample of spots on a grid and their respective point spread functions

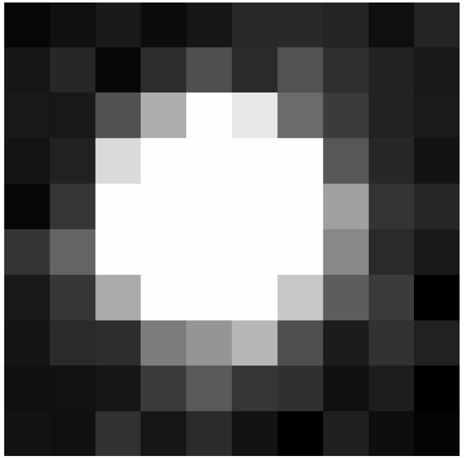
Point Spread Function

- The waves converge to a spot, and the distribution of intensity is modeled by the point spread function.
- The point spread function is defined as the convolution between an Airy Disk Function and a 2D gaussian function.





Example of airy rings produced by the airy disk function, and its similarity to the response of the pixels of a spot



Point Spread Function (Fourier Space)

Airy Disk

Represents the intensity when given an x,y coordinate on the aperture

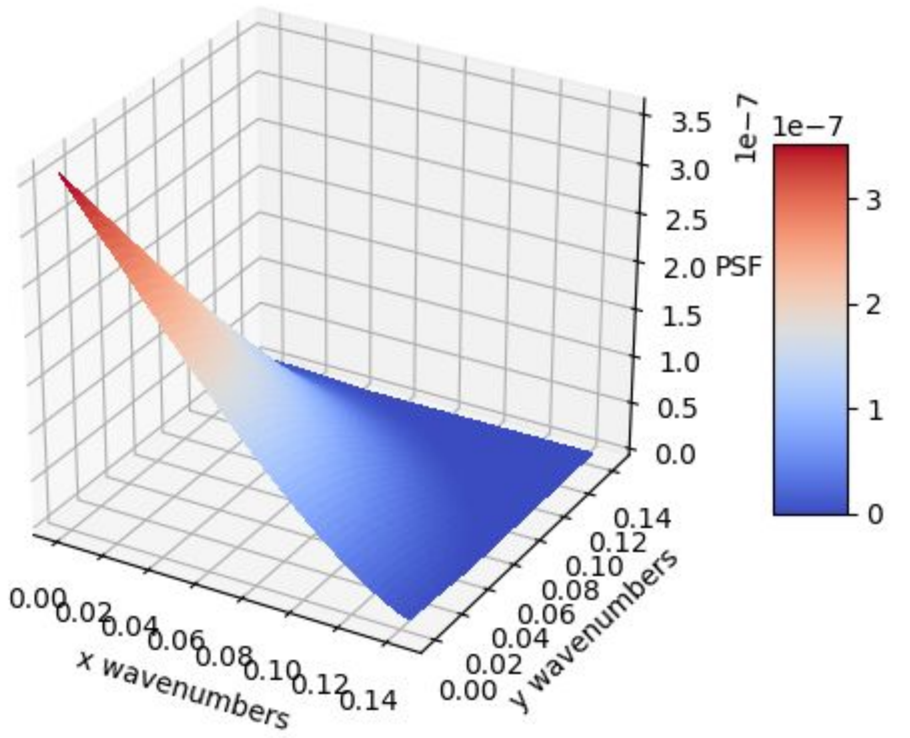
Gaussian

Used to account for mechanical jitter

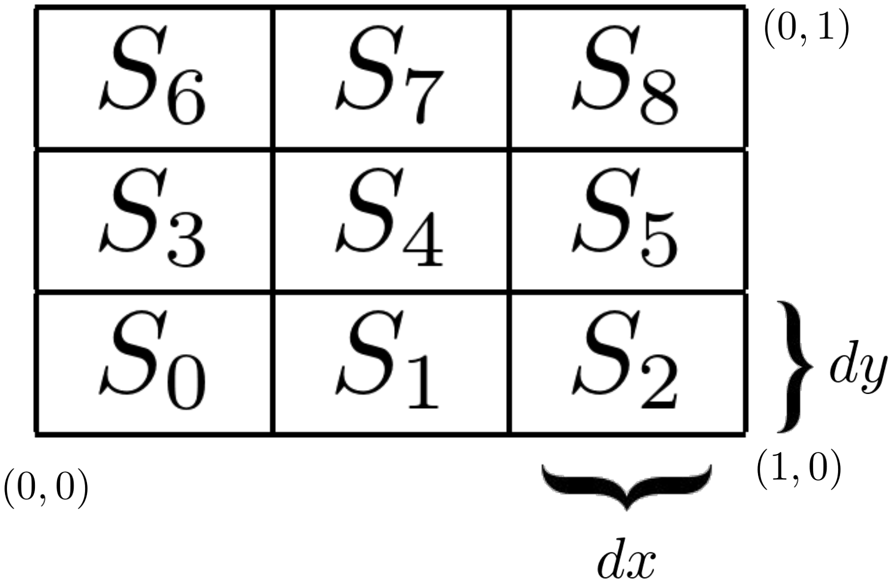
Fourier Point Spread Function (PSF)

$$PSF(\lambda, N, k_x, k_y, \sigma_x, \sigma_y) = G(\sigma_x) \cdot G(\sigma_y) \cdot \mathrm{OTF}$$

PSF Values Graph (In the fourier space)



Modeling Pixels



- To the left is the way we chose to model a pixel, which is a grid of subpixels
- In our actual model, each pixel was split into a 10x10 grid of subpixels

Response (PRF)

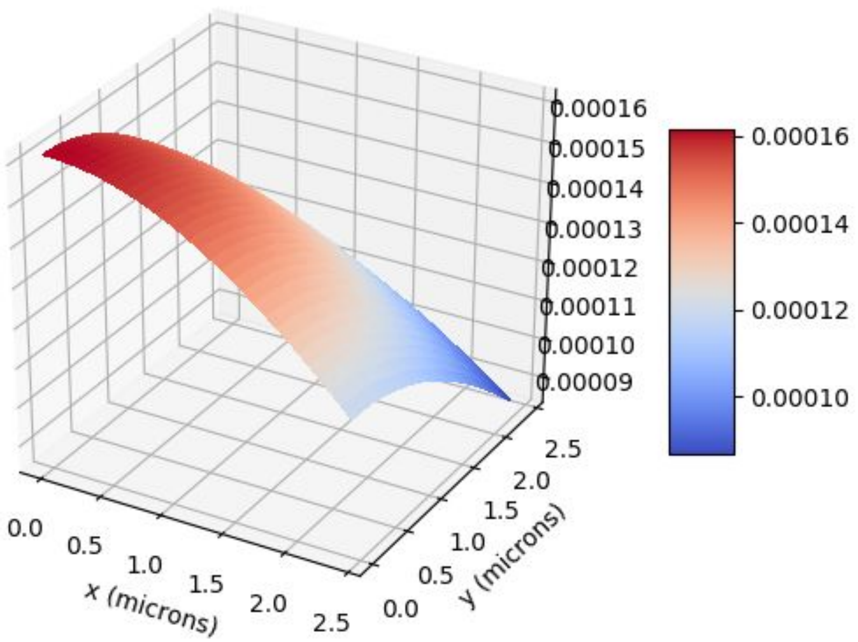
- The PRF can be evaluated as the point spread function integrated over each subpixel, and then multiplied by its corresponding subpixel sensitivity map coefficient.

- This represents the intensity of the subpixel. Summing these integrals results in the PRF value of the pixel

Discrete Cosine Transform (DCT-II)

- Function for intensity is in the fourier space, so an inverse fourier transform is required to get back into the real space
- Results in a DCT-II (Discrete Cosine Transform type 2) due to the imaginary portions of our integral cancelling out
- We use FFTW to calculate this transform
- FFTW is written purely in C and results in excellent performance

Intensities



Solving for the sensitivity map

- After executing the DCT, we now have the values of the intensities for all subpixels of each pixel
- We can now use least squares to solve for the coefficients

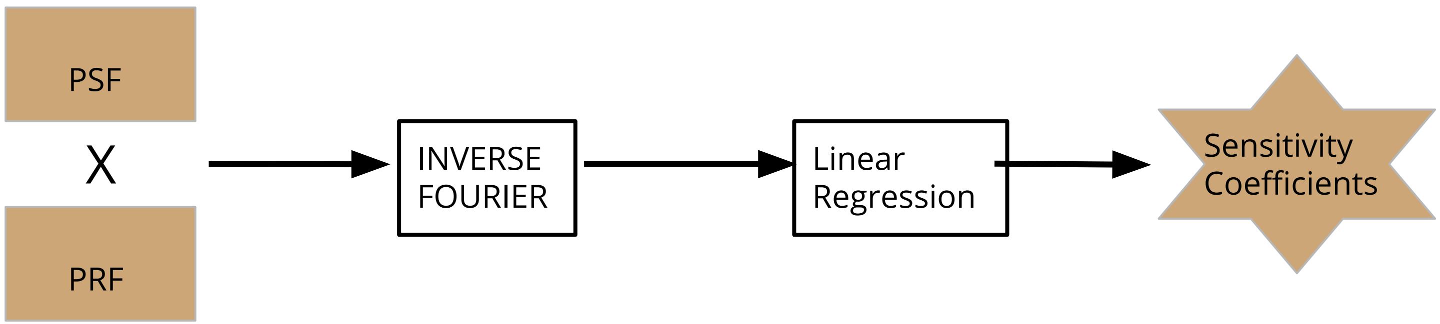
$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ \vdots \\ r_{p_y+p_xP_y} \end{bmatrix} = \begin{bmatrix} I_{0,0} & I_{0,1} & I_{0,2} & \cdots & I_{0,n_y+n_xN_y} \\ I_{1,0} & I_{1,1} & I_{1,2} & \cdots & I_{1,n_y+n_xN_y} \\ I_{2,0} & I_{2,1} & I_{2,2} & \cdots & I_{2,n_y+n_xN_y} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I_{p_y+p_xP_y,0} & I_{p_y+p_xP_y,1} & I_{p_y+p_xP_y,0,2} & \cdots & I_{p_y+p_xP_y,n_y+n_xN_y} \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ \vdots \\ S_{n_y+n_xN_y} \end{bmatrix}$$

Optimizations

- PSF and PRF models were written in C++

- Convolutions were performed using fast fourier transforms rather than numeric integration

Outcomes


PSF
X
PRF
INVERSE
FOURIER
Linear
Regression
Sensitivity
Coefficients

References

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- “Point Spread Function.” *Wikipedia*, Wikimedia Foundation, 27 July 2019, en.wikipedia.org/wiki/Point_spread_function.

Software Libraries Used

C/C++:

- FFTW (Fastest Fourier Transform in the West) - used to compute fast fourier transforms
- boost - used to compute the bessel integral

Python:

- numpy - used for various numerical methods involving matrices
- scipy - used for spline generation and interpolation
- matplotlib - used for the generation of graphs