

DRIVE KINEMATICS

Drive update

- get encoder values of all wheel positions
- do (change in wheel positions)/(change in time) for each wheel to determine the wheel velocity

- **Tank Kinematics**

x pointing forward, y pointing left

We have

S_L, S_R = left distance and right distance

r = turn radius from inner wheel

b = track width

θ is turn in radians

$$S_L = r\theta$$

$$S_R = (r + b)\theta$$

$$S_M = (r + b/2)\theta$$

We are now interested in how the x and y coordinates and heading change with respect to time.

$$\frac{dx}{dt} = m(t) \cos(\theta(t))$$

$$\frac{dy}{dt} = m(t) \sin(\theta(t))$$

Derivation

Frame of reference: center point of left wheel

We have the following equation:

$$\frac{d\theta}{dt} = \frac{(V_R - V_L)}{b}$$

Integrating and setting $\theta(0) = \theta_0$ we find a function for calculating the robot's orientation as a function of wheel velocity and time:

$$\theta(t) = \frac{(V_R - V_L)t}{b} + \theta_0$$

The velocity of the robot is the average of the two wheels: $\frac{(V_R + V_L)}{2}$

We now have the following differential equations:

$$\frac{dx}{dt} = \frac{(V_R + V_L)}{2} \cos(\theta(t))$$

$$\frac{dy}{dt} = \frac{(V_R + V_L)}{2} \sin(\theta(t))$$

Mecanum Kinematics

Each wheel has an additional x and y velocity component due to the vehicles rotational velocity ω_v given by

$$V_x r = -Y_n \cdot \omega_v$$

$$V_y r = X_n \cdot \omega_v$$

The total linear velocity given by the vector componenets is

$$V_{x_n} = V_x + V_{x_{r_n}} = V_x - Y_n \cdot \omega_v$$

$$V_{y_n} = V_y + V_{y_{r_n}} = V_y + X_n \cdot \omega_v$$

$$\omega = \frac{V_x + V_y \cdot \tan \theta}{r} = \text{wheel velocity}$$

Odometry

We have our target velocity as

$$v(t) = (V_x \cos(\theta_0 + \omega\tau) - v_y \sin(\theta_0 + \omega\tau), V_x \sin(\theta_0 + \omega\tau) + v_y \cos(\theta_0 + \omega\tau))$$

Integrating results in

$$x(t) = \int_0^{\Delta t} v(\tau) d\tau = \left(\frac{v_x}{\omega} [\sin \theta_1 - \sin \theta_0] + \frac{v_y}{\omega} [\cos \theta_1 - \cos \theta_0] \right), \left(-\frac{v_x}{\omega} [\cos \theta_1 - \cos \theta_0] + \frac{v_y}{\omega} [\sin \theta_1 - \sin \theta_0] \right)$$

our updated robot pose (x and y component)