Trajectory Generation

Thanks Ryan Brott: https://github.com/acmerobotics/road-runner/blob/master/doc/pdf/Quintic_Splines_for_FTC.pdf

For helping me understand this topic

As well as

http://www2.informatik.uni-freiburg.de/~lau/students/Sprunk2008.pdf

Quintic Spline Trajectories

Our splines will be expressed as a vector of single variable functions for the x and y component.

Quintic splines consist of a series of segments combined into a single peicewise curve. Each segment is a parametric curve with a quintic polynomial for each component.

For most of our segments in FTC, we will use one segment, which will also allow for some more brevity in our notation.

$r(t) = \begin{cases} x(t) = a_x t^5 + b_x t^4 + c_x t^3 + d_x t^2 + e_x t + f_x, \\ y(t) = a_y t^5 + b_y t^4 + c_y t^3 + d_y t^2 + e_y t + f_y \end{cases}$

where $0 \le t \le 1$

Reparametrization to Arc Length

We now need to reparametrize all the segments into a single variable of arclength ($$t \in [0, 1] \to s$$)

s is simply the displacement along the curve.

thus: $|r'(s)| = 1 = \sqrt{(\frac{dx}{ds})^2 + (\frac{dy}{ds})^2}$ as 1 unit

It is simple to represent $s(t)$ as arclength as a function of time

$$s(t) = \int_0^t |r'(\tau)|\, d\tau = \int_0^t \sqrt{\left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2}\, d\tau$$

We can numerically integrate this and match resulting arc lengths to times

We can now determine $r(s)$ by composition: $r(t(s))$

We can now differentiate to yield the parametrized derivative:

$$r'(s) = r'(t) \cdot t'(s) = \frac{r'(t)}{s'(t)} = \frac{r'(t)}{|r'(t)|}$$

We realize this makes sense as we can realize that $r'(s)$ = 1.

Now let us set $s(t)$ be the state of the motion profile at time t.

We now have

$$v(t) = \frac{d}{dt}[r(s(t))] = r'(s(t))s'(t)$$

Heading

In addition to the translational motion, we also need to interpolate some type of angular motion.

For nonholonomic drive trains, we can update the heading like so:

$\theta(t) = \arctan \frac{y'(t)}{x'(t)}$

For holonomic drive trains, we can consider the heading component to be a completely independent spline.

$$\theta'(t) = \frac{x'(t)y''(t) - x''(t)y'(t)}{x'(t)^2 + y'(t)^2}$$

We can now calculate this derivative at $t = 0$ and $t = 1$