Khan Academy

Algebra 1

Unit 8: Functions

8.F.A.1, 8.F.B.5, HSF.BF.B.4, HSF.BF.B.4.a, HSF.BF.B.4.c, HSF.IF.A.1, HSF.IF.A.2, HSF.IF.B.4, HSF.IF.B.5, HSF.IF.B.6, HSF.IF.C.7

Functions can model real-world scenarios like a car's speed, market trends, population growth, and travel time. They can help students analyze data, solve problems, and prepare for advanced math and careers. They're also just a distinct way to represent some familiar equations.

START HERE

- ☐ **Evaluate functions**, using **function notation**, from equations and graphs
- ☐ **Determine the domain** of a function given an equation or situation and determine the domain and range given a graph
- ☐ Determine **whether a relation is a function** from tables, graphs, and word problems
- ☐ Describe a graph by locating **absolute maxima/minima** and **relative maxima/minima** and identifying intervals where the graph is **increasing**, **decreasing**, **positive**, and **negative**
- ☐ Calculate the **average rate of change** given an equation, table, or graph
- ☐ Determine the **inverse of linear functions** from graphs and equations

CCSS Full text of standards included at the end of this unit guide	**Common misconceptions**
8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	**Misunderstanding function notation |** Students often confuse the notation $f(x)$ with multiplication. They might think that $f(x)$ means f times x, rather than understanding it as the *function* f applied to the input x (said "f of x"). This confusion is of course very understandable, so be sure to acknowledge it and emphasize that context is key. **How to help:** Reiterate often that function notation is simply a shorthand, similar to using y. The beauty of function notation is that it tells us what value was substituted for x after it's been simplified. For example, if we simplify a function and get $y = 3$, we don't know what was plugged in to get that value. If we write $f(5) = 3$, then we know that when $x = 5$, and 5 is substituted into the function, we will get out 3. Function notation $f(x)$ is generally isolated by itself on one side of the equation. It is also useful in naming different functions. If you are working with two functions, they could be named $f(x)$ and $g(x)$ to note which is which.
8.F.B.5: Describe qualitatively the functional relationship between two quantities by	

analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**So much vocabulary! |** This unit has a lot of new vocabulary for students, particularly vocabulary that describes different aspects of graphs. It can be confusing for students to have so much new vocabulary at once, especially when so much of it is graph related. **How to help:** Have students use the Vocabulary and notation notetaker as they work through the unit. They should record all new vocabulary and notation so they have a reference. Review definitions as a class and ensure that all students have definitions and pictures or examples for all new words. See "Best practices" for a selection of definitions. **Mixing up domain/range or input/output |** *Domain* and *input* both describe x-values and *range* and *output* both describe $f(x)$-values, or y-values. However, it's important to note that domain and input are not synonymous, even though they both describe x-values. An input is a *specific number* that is substituted into a function for the x-value. The domain is the *set of all inputs* that a function can accept. Similarly for output and range, output is the specific resulting $f(x)$-value when an input is plugged in. The range is the set of all possible outputs. **How to help:** Associate the words domain, input, and x-value together, along with range, output, and y-value. Create a poster with the axes and words for easy reference. As students use these words regularly, they will become more automatic. **The domain is always all real numbers |** While many functions, especially those that students have regularly seen, have a domain of all real numbers, rational functions can have restrictions to the domain. Rational functions can have restrictions when the denominator is 0 and other functions, like those that include square roots, have different restrictions. **How to help:** It's important for students to be able to identify functions that have potential domain restrictions. Spend time going over the special cases included in this unit (rational and square root functions) and show examples of other types of functions that also have restrictions so that students see a variety of examples. Since they've mainly worked with linear functions in the past, which have no domain restrictions, this will be new for them and they'll need plenty of practice. **Confusing increasing/decreasing and positive/negative intervals on graphs |** Students might have difficulty correctly identifying where a function is increasing, decreasing, positive, and/or negative. **How to help:** Encourage students to reference their vocabulary on the vocabulary and notation notetaker to make sure they know what they're being asked to find. For increasing/decreasing intervals, students need to think about slope. They can use a piece of dry
HSF.BF.B.4: Find inverse functions.	
HSF.BF.B.4.a: Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.	
HSF.BF.B.4.c: Read values of an inverse function from a graph or a table, given that the function has an inverse.	
HSF.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.	
HSF.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
HSF.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in	

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<td>terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.</td>
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spaghetti, popsicle stick, or even their pencil to line it up with the section of the graph they are looking at to get an exaggerated view of the slope of the line. It's important for students to have lots of practice with these terms, especially writing the intervals for each. It may be helpful to give one graph as an example and have students find all four intervals to help them see the differences between them.

Misunderstanding the concept of inverses | It's important to clarify that the inverse function reverses the input-output pairs of the original function. What used to be the input is now the output, and vice versa. Sometimes this can make the inverse fail to be a function, and be merely a relation. Regardless, students might think that finding the inverse of a function is the same as just finding the reciprocal (it's not!).

How to help: Inverses are easiest to visualize in tables and with points on a graph, where the inputs and outputs are literally reversed. For example, if the point (a, b) is on the original function, the inverse function will have the point (b, a). It can also be helpful to share examples, like imagine you have a machine that turns apples into apple juice. The function is the process of making apple juice from apples. The inverse function would be a magical machine that turns apple juice back into apples. There are many more examples you could use that are more realistic, like converting back and forth from degrees fahrenheit to degrees celsius. Have students come up with their own examples!

Algebraic errors | There are so many places for potential algebraic mistakes! Evaluating functions, finding a domain with a restriction, and finding the inverse of an equation all require precision with arithmetic, order of operations, simplifying, and isolating a variable.

How to help: Encourage students to take their time with each algebraic step, show their work, and check their answers. Review common mistakes and show examples as needed. Have students help to find each other's mistakes as they arise, which helps build critical thinking, error analysis, and confidence.
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<td>HSF.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.</td>
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<td>HSF.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.</td>
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<td>HSF.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</td>
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Unit resources

- For the videos in this unit, use the Learning summary video notetaking guide.
- For the articles in this unit, use the Article notetaking guide.
- For the exercises in this unit, use the Blank workspace template.
- To record key terms and information, use the Vocabulary and notation notetaker.

Lesson overview

Lesson	Objective	Teaching tips
Lesson 1: Evaluating functions CCSS: 8.F.A.1, HSF.IF.A.1, HSF.IF.A.2 Video 5 · Article 0 · Exercise 3	Students will be able to evaluate functions from equations and graphs.	• **Warm up activity:** Give students problems where they use substitution and the order of operations to simplify. For example, Find the value of y when $x = 3$ in the equation $y = (2x + 1)^2 - 5$. • The first video provides a general overview of functions and how to solve them. Don't worry too much at this point about the definition of a function, focus on *understanding* and *using* function notation. We'll dive into the definition of a function in lesson 7. input $\underbrace{f\left(x\right)}_{\text{output}}$ • Evaluating a function is like solving a puzzle! For equations, you substitute the input, simplify, and get the output. For graphs, you look at the input on the x-axis and then find the corresponding y-value for the output. • Have students complete the Vocabulary and notation notetaker as they work through this unit—it's very vocabulary-heavy!
Lesson 2: Inputs and outputs of a function CCSS: 8.F.A.1, HSF.IF.A.1, HSF.IF.A.2	Students will be able to find the input for a given output of a function algebraically and graphically.	• In the previous lesson, students were given a value for the input and then found the corresponding output. Here, students are given a value for the output and are asked to find the corresponding input, both algebraically and graphically. When solving for the output algebraically, be sure students replace the output value given

Video: 3 Article: 0 Exercise: 2		with the $f(x)$ and then solve for x (or whichever variable is used). When solving graphically, the output given is the y-value, so make sure students locate that on the y-axis and find the corresponding x-values. • It may be helpful to remind students that the notation $f(x)$ represents the output, just as y does. To warm up, they can replace $f(x)$ with y if that makes it easier for them to make sense of the problem.
Lesson 3: Functions and equations CCSS: 8.F.A.1, HSF.IF.A.1 Video: 2 Article: 0 Exercise: 1	Students will be able to write equations in function notation.	• This lesson focuses on understanding the difference between equations and functions. In the first video, Sal makes a Venn diagram, giving examples of equations, functions, and both. Make a Venn diagram together as a class and add examples from students to help clarify the differences and similarities. If you create the Venn diagram on poster paper, hang it up for future reference. • This lesson encourages students to begin to think about functions as an idea, not tied to any specific letters. For example $h(b)$ is the function h acting on the input b. • Spend time reviewing the wording of problems to help students understand what they are being asked to do. For example, For a given input value b, the function f outputs a value a to satisfy the equation $a - 5 = 2(b + 3)$. Write a formula for $f(b)$ in terms of b. Since students are familiar with x as the input and y as the output, it may be helpful to relate (or rename) the variables given. In the example above, b is like x (the input) and a is like y (the output), so we need to solve the equation for a (the output) before replacing a with $f(b)$.

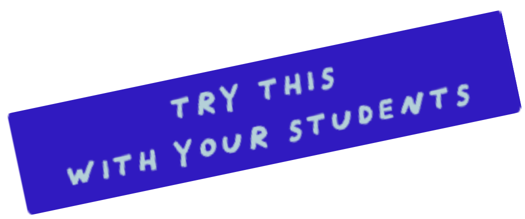
		$$\begin{aligned} a-5&=2(b+3)\\ a&=2(b+3)+5\\ a&=2b+6+5\\ a&=2b+11\\ f(b)&=2b+11 \end{aligned}$$
Lesson 4: Interpreting function notation CCSS: 8.F.A.1, HSF.IF.A.2 Video 2 · Article 0 · Exercise 1	Students will be able to interpret function notation in a context.	• Encourage students to read each problem slowly and carefully to interpret the meaning of the variables in the situation. Have them write the equation on their paper and label each part with the meaning and value, if it's given. It can be very confusing for students to see so many letters together! • Spend time reviewing how to translate function notation in different contexts. For example, if they are given a problem like "$V(d)$ models the vertical distance of a bug walking up a wall (in cm) after d seconds. What does the statement $V(5) = 3$ mean?," have students translate that $V(5)$ means the "vertical distance after 5 seconds." input (time) $\underbrace{V(5)}_{\text{output (vertical distance)}}$
Lesson 5: Introduction to the domain and range of a function CCSS: 8.F.A.1, HSF.IF.A.1, HSF.IF.B.5 Video 4 · Article 0 · Exercise 1	Students will be able to write the domain and range of a function given a graph.	• **Warm up activity:** Give problems where students practice graphing one-variable inequalities on a number line. You can use this [number line template](#). Graph $x \geq -3$  • The first video shows multiple ways to indicate an interval, including whether the endpoints are included or not included. Many different notations are shown, which are good for students to be familiar with, but they will only need to know how to write intervals with inequalities. • Domain and range may seem very abstract! However, it is concrete to show on a graph. Have students use colored pencils or mark-up tools to show the domain and range on a graph before trying to describe it numerically. • The videos introduce different notations for representing the domain and range of functions.

		Students should be familiar with these notations, but they don't appear in the exercise. They'll need to use inequalities, which are only shown in the first video. See "Best practices" for more on domain and range with inequalities. • The domains and ranges of continuous graphs are written with inequality notation while the same for discrete graphs are written as a list. See "Best practices," below, for more.
Lesson 6: Determining the domain of a function CCSS: 8.F.A.1, HSF.IF.A.1, HSF.IF.B.5 Video 5 · Article 0 · Exercise 3	Students will be able to determine the domain of a function given its equation. Students will be able to determine the domain of a function in a word problem.	• **Warm up activity:** Have students substitute and simplify equations with square roots, exponents, parentheses, and rational expressions with a variable in the denominator. For example, Find the output of each function when the input is 6: $f(x) = \sqrt{x+3}$ $g(x) = \frac{2x+4}{x-2}$ $h(x) = (x+5)^2$ • When determining the domain of a function given its equation, it's important to check for undefined input values. Review types of functions that have restrictions on their domains, like square roots (the value under the square root must be positive) and division (the denominator must not be equal to 0) and *why* these constraints must be met. This is also a good opportunity to review what real numbers are if necessary. Challenge students to identify other functions where the domain is restricted. • Students must read and interpret word problems before they are able to determine the domain. Encourage them to read carefully to understand each situation, and draw a picture if it's helpful. • The videos use bracket notation for the domain but the exercise uses inequality notation (as in previous exercises).
Lesson 7: Recognizing functions CCSS: 8.F.A.1, HSF.IF.A.1	Students will be able to determine whether a relation is a function from tables, graphs, and words.	• Determining whether a relation is a function can be confusing as it is quite abstract. Provide a concrete example for students to reference as they work through the lesson. For example, a soda machine (or snack machine) is a function because

Video 5 · Article 0 · Exercise 2		when you press a button, you know what you're getting out. If you press the grape soda button, you get out a grape soda, not an orange soda. A soda machine would NOT be a function if you could get different types of soda when pushing the same button—the grape soda button could give you either a grape soda or an orange soda. See "Best practices" for more on functions. • This lesson shows examples of how functions can be presented in tables, graphs, and words. Show connections between each presentation and how one function can be represented in multiple ways. When graphing, be sure to discuss the vertical line test! Even when no graph is provided, drawing or visualizing the idea of the vertical line test can be very helpful for students when determining if a relation is a function. • When students get to the word problems, encourage them to work slowly and read carefully!
Lesson 8: Maximum and minimum points CCSS: 8.F.B.5, HSF.IF.C.7 Video 2 · Article 0 · Exercise 2	Students will be able to identify absolute and relative maxima and minima on a graph.	• The first two videos present the definition of absolute and relative maxima and minima. The notation in the mathematical definition may be confusing for students, and that's okay at this point. For now, they only need to identify the key points on graphs. See "Best practices" for definitions.
Lesson 9: Intervals where a function is positive, negative, increasing, or decreasing CCSS: HSF.IF.C.7 Video 2 · Article 0 · Exercise 2	Students will be able to identify intervals on a graph where the function is positive, negative, increasing, and decreasing.	• Students continue to use inequalities to denote intervals on the axes of graphs. • Encourage students to use the markup tool in exercises to highlight where the function is positive, negative, increasing, or decreasing so they can better visualize what is happening. • The first exercise focuses on positive and negative intervals while the second exercise focuses on increasing and decreasing intervals. See "Best practices" for definitions.

Lesson 10: Interpreting features of graphs CCSS: HSF.IF.B.4 Video 3 · Article 0 · Exercise 1	Students will be able to interpret features of graphs in context.	• In this lesson, students interpret key features of a graph (relative max or min, positive or negative interval, increasing or decreasing interval, y-intercept, etc.) in a context. Encourage students to read each scenario carefully to determine the meaning of the key features.
Lesson 11: Average rate of change CCSS: HSF.IF.B.6 Video 3 · Article 0 · Exercise 1	Students will be able to calculate the average rate of change given a graph or a table. Students will be able to determine the interval on a graph that has a given rate of change.	• **Warm up activity.** Find the slope of a line given two points. Remind students of the formula to find slope, $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta x}{\Delta y}$. For example, Find the slope of the line between the points (-1, 6) and (3, -2). • This lesson is all about slope! The average rate of change (ARC) is the slope for linear functions. For nonlinear functions, the ARC is the slope of the line that connects the endpoints of the given interval. Watch out for typical mistakes when finding slope, like errors with negative numbers and not plugging into the formula correctly.
Lesson 12: Average rate of change word problems CCSS: HSF.IF.B.6 Video 2 · Article 1 · Exercise 1	Students will be able to calculate the average rate of change given a situation and graph or table.	• Students will calculate and compare the ARC over different intervals of a graph. Now the graphs are given in context, where students need to interpret the meaning of the ARC in a specific situation. See “Best practices” for a formal definition.
Lesson 13: Intro to inverse functions CCSS: HSF.BF.B.4, HSF.BF.B.4.a, HSF.BF.B.4.c Video 4 · Article 2 · Exercise 2	Students will be able to find the inverse of linear functions from graphs and equations.	• **Warm up activity:** Have students work on composite function notation problems. They haven't seen this before, so let them try to figure it out with a partner. For example, Given $f(x) = 4x - 6$, find: $f(2) + f(-1)$ $\quad\quad$ $f(f(3))$ • The first video introduces inverse functions with equations and graphs. The first article is an excellent resource for students who want a

		detailed explanation and additional opportunities for practice. • The graph of the inverse of a function is the original function reflected over the line $y = x$. On a graph, individual points can be reflected simply by swapping the x- and y-values. For example, if the point (-3, 2) is on the function, then the point (2, -3) will be its corresponding point on the inverse of the function. This is true for all points on the function. • When working with equations, find the inverse of the function by swapping the position of x and y in the equation to get the inverse relationship. Then solve for y to get the equation in a form where we can write it in function notation.



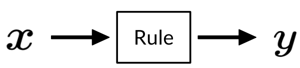
Best practices

CHECK THIS OUT

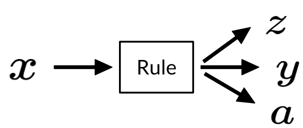
Functions, explained

A **relation** is a relationship between a set of values. All functions are relations but not all relations are functions—a function is a special kind of relation. A relation is a **function** if and only if *every* domain (x) value is mapped to *exactly one* range (y or $f(x)$) value. This definition is quite abstract and may sound more complicated than it is, so let's dig into it further. And when in doubt, we can always use the vertical line test by making a quick graph!

A "function machine" is typically used to demonstrate the difference between a function and a general relation.



This is a **function** because when you put a number into the rule, it always gives one answer out.

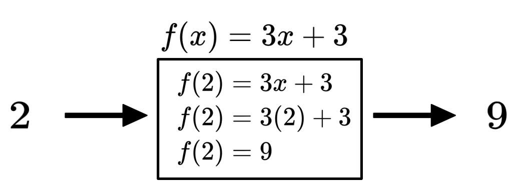


This is **NOT a function** because when you put a number into the rule, you could get multiple answers out.

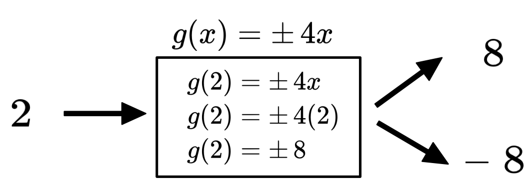
The soda machine example, mentioned above in the lesson overview, is a great example to share with students. Another example is height because every person only has one height. If the input is a student's name, the output would be their height and there is only one possible outcome. Some students may ask, what if two students have the same height? And this is a very important nuance. It is still a function if two inputs have the same output, so it is still a function if two students have the same height. What would make this example not a function is if one student had two heights! When in doubt, students can use the vertical line test here as well, once they have learned this tool.

An example of a relation that is not a function would be the sister function: the input is the student's name, and the output is their sister's name. It is okay for multiple students to have "none," because multiple inputs can have the same output, but if a single student has two sisters this is not a function! Because then one input would map onto multiple outputs, the two sisters' names. Have students come up with their own examples of functions and non-function relations!

Here is an *algebraic* example to show the difference between a relation and a function:

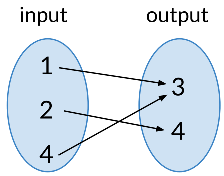
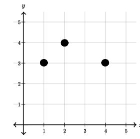
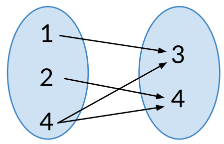
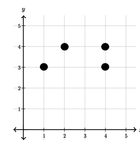


This **is** a function because every *x*-value will have exactly one *y*-value. When $x = 2$, $f(x) = 9$ for example.



This is **NOT** a function because when $x = 2$, $g(x) = 8$ ***and*** -8.

Let's look at the same data set in a picture, table, and graph to see how they compare.

	Pictures	*Tables*	*Graphs*
These **are functions** because every input has exactly one output.		see table below	
These are **not functions** because when we input 4, we can get either 3 or 4 as the output.		see table below	

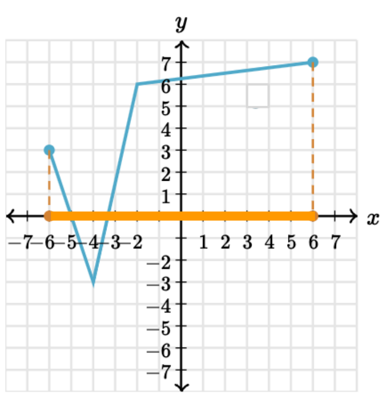
x	y
1	3
2	4
4	3

x	y
1	3
2	4
4	3
4	4

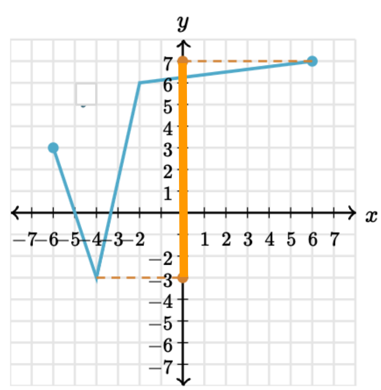
Domain and Range

The domain is all possible x-values of a function and the range is all possible y-values of a function. The domain and range tell us about the possible inputs (x) and outputs (y) of the function. Graphs help us make this concept visual. When we work with a continuous graph (an unbroken line), we can use inequality notation to make sure that we include all values between the endpoints. When we work with a discrete graph (points), we can simply list all of the values.

Continuous graph

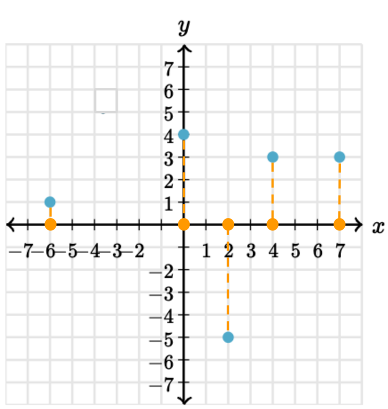


Domain:
$-6 \leq x \leq 6$

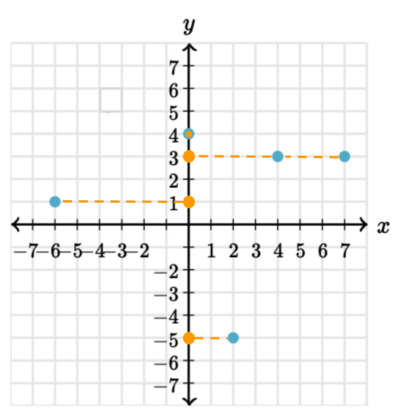


Range:
$-3 \leq y \leq 7$

Discrete graph



Domain:
$x = -6,\ 0,\ 2,\ 4,\ 7$



Range:
$y = -5,\ 1,\ 3,\ 4$

Vocabulary

This unit is riddled with new vocabulary, and some of it is quite complicated! Here is a *selection*:

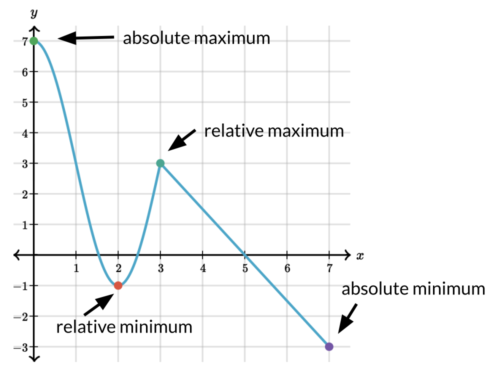
Lesson 8: Maximum and minimum points

A ***relative maximum*** point is a point that is higher than all other graph points around it. It's like the highest point on a "peak."

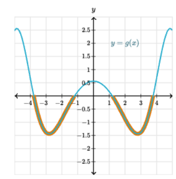
A ***relative minimum*** point is a point that is lower than all other graph points around it. It's like the lowest point in a "valley."

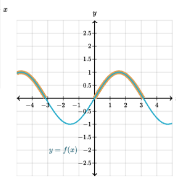
The ***absolute maximum*** point is the point that is higher than *all* other points on a graph.

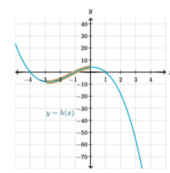
The ***absolute minimum*** point is the point that is lower than *all* other points on a graph.

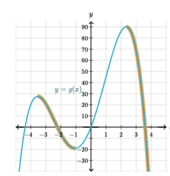


Lesson 9: Intervals where a function is positive, negative, increasing, or decreasing









A function, g, is ***negative*** ($g(x) < 0$) whenever its graph is *below* the x-axis.

A function, h, is ***positive*** ($h(x) > 0$) whenever its graph is *above* the x-axis.

A function, p, is ***increasing*** in an interval where as x increases, $p(x)$ also *increases*. Visually, this is where the graph goes *upward* as we go from left to right.

A function, q, is ***decreasing*** in an interval where as x increases, $q(x)$ *decreases*. Visually, this is where the graph goes *downward* as we go from left to right.

Lesson 12: Average rate of change word problems

To find the ***average rate of change*** (ARC) of a function over an interval, take the *total change in the function value of the interval* and divide it by the *length of the interval*. The ARC of function f over the interval $a \leq x \leq b$ is given by this expression: $\dfrac{f(b) - f(a)}{b - a}$

It's a measure of how much the function changed per unit, on average, over that interval.

Notice that it's the formula for slope written with function notation! For points $(a, f(a))$ and $(b, f(b))$ and (x_1, y_1) and (x_2, y_2):

$$\frac{f(b) - f(a)}{b - a} = \frac{y_2 - y_1}{x_2 - x_1}$$

Lesson 13: Intro to inverse functions

An ***inverse*** is the reverse of something else. For a function, $f(x)$, it's inverse is written as $f^{-1}(x)$.

If f(input) = output, then f^{-1}(output) = input. On a graph, the inverse function is a reflection, or mirror image, of the original function across the line $y = x$.

GENERAL CLASSROOM IMPLEMENTATION RESOURCES:

- Weekly Khan Academy quick planning guide: Use this template to plan your week using Khan Academy.
- Using Khan Academy in the classroom: Learn teaching techniques and strategies to support your students and save time with Khan Academy.
- Differentiation strategies for the classroom: Discover strategies to support the learning of all students.

Common Core State Standards

8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

HSF.BF.B.4: Find inverse functions.

HSF.BF.B.4.a: Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.

HSF.BF.B.4.c: Read values of an inverse function from a graph or a table, given that the function has an inverse.

HSF.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

HSF.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

HSF.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.

HSF.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

HSF.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

HSF.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.