

$$|x_{k+1} - x_k| \leq L^k |x_1 - x_0| \leq 10^{-5}$$

第二章 作业

$$2. \quad k+1 \geq [\ln(b-a) - \ln \varepsilon] / \ln 2 \quad k = 16$$

$$= (\ln 1 - \ln 10^{-5}) / \ln 2 = 16.6096$$

$$3. (1) \quad g(x) = \sqrt{\frac{10}{4+x}} = \begin{cases} 1.414, & x = 1 \\ 1.291, & x = 2 \end{cases}$$

$$g'(x) = -\frac{\sqrt{10}}{2(4+x)^{3/2}} = \begin{cases} -0.1414, & x = 1 \\ -0.1076, & x = 2 \end{cases}$$

[1,2]区间满足定理1条件, 代入 $x_0=1.5$, $x_1=1.3484$, $L \approx 0.1414$, 得:
 $k=4.9215$, 至少迭代6次.

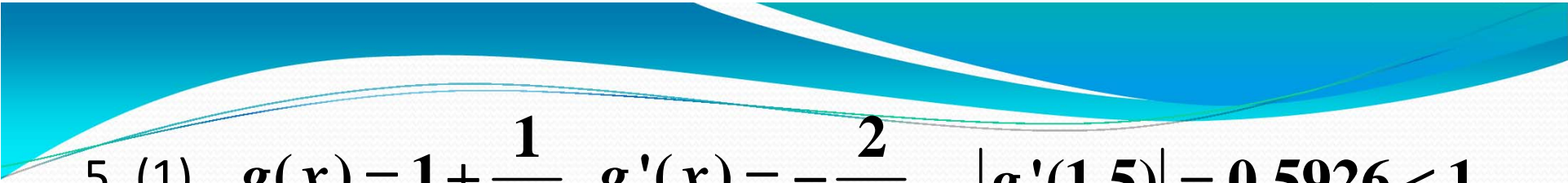
$$|x_{k+1} - x_k| \leq L^k |x_1 - x_0| \leq 10^{-5}$$

$$3. (2) \quad g(x) = \frac{1}{2} \sqrt{10 - x^3} = \begin{cases} 1.5, & x = 1 \\ 1.287, & x = 1.5 \end{cases}$$

$$g'(x) = -\frac{3x^2}{4\sqrt{10 - x^3}} = \begin{cases} -0.25, & x = 1 \\ -0.6556, & x = 1.5 \end{cases}$$

[1,2]区间满足定理1条件，代入 $x_0=1.5$, $x_1=1.2870$, $L \approx 0.6556$ ，得：

$k=23.6078$, 至少迭代25次.



5. (1) $g(x) = 1 + \frac{1}{x^2}$ $g'(x) = -\frac{2}{x^3}$ $|g'(1.5)| = 0.5926 < 1$

(2) $g(x) = \sqrt[3]{1+x^2}$ $g'(x) = \frac{2x}{3(1+x^2)^{2/3}}$

$$|g'(1.5)| = 0.4558 < 1$$

(3) $g(x) = \frac{1}{\sqrt{x-1}}$ $g'(x) = -\frac{1}{2(x-1)^{3/2}}$

$$|g'(1.5)| = 1.4142 > 1$$

6.

$$x_{k+1} = x_k - \frac{g(x_k)}{g'(x_k)} = x_k - \frac{x_k^3 + 4x_k^2 - 10 - x_k}{3x_k^2 + 8x_k}$$

1	1.37333	delta=0.12667
2	1.36526	delta=0.00807
3	1.36523	delta=0.00003
4	1.36523	delta=0.00000

8.

$$f(x) = x^3 - 3x^2 - x + 9 = 0$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f(x_k) - f(x_{k-1})} (x_k - x_{k-1})$$

$$x_0 = -1.6, x_1 = -1.4$$

1	-1.5203, delta=0.1203
2	-1.5254, delta=0.0051
3	-1.5251, delta=0.0003
4	-1.5251, delta=0.0000

9.

$$f(x) = x^3 + 4x^2 - 10 = 0$$

$$(1) y_k = g(x_k)$$

$$(2) z_k = g(y_k)$$

$$(3) x_{k+1} \approx x_k - \frac{(y_k - x_k)^2}{z_k - 2y_k + x_k}$$

$$g(x) = \sqrt{\frac{10}{4+x}}$$

1 1.36527 delta=0.13473

2 1.36523 delta=0.00004

$x \approx 1.3652$

基本要求

- 二分法计算及其迭代次数估计;
- 简单迭代法及其收敛性判断;
- 牛顿法计算;
- 简化牛顿法、弦割法、牛顿法下山法;
- 收敛阶的判断。