$$|x_{k+1}-x_k| \le L^k |x_1-x_0| \le 10^{-5}$$

第二章 作业

2.
$$k+1 \ge [\ln(b-a)-\ln\varepsilon]/\ln 2$$

$$=(\ln 1-\ln 10^{-5})/\ln 2 = 16.6096$$

$$k=16$$

3. (1)
$$g(x)=\sqrt{\frac{10}{4+x}}=\begin{cases}1.414, x=1\\ 1.291, x=2\end{cases}$$

$$g'(x)=-\frac{\sqrt{10}}{2(4+x)^{3/2}}=\begin{cases}-0.1414, x=1\\ -0.1076, x=2\end{cases}$$

[1,2]区间满足定理1条件，代入x_0=1.5, x_1=1.3484, L≈0.1414，得：

k=4.9215, 至少迭代6次.

$$|x_{k+1}-x_k| \le L^k |x_1-x_0| \le 10^{-5}$$

3. (2) $$g(x)=\frac{1}{2}\sqrt{10-x^3}=\begin{cases}1.5, x=1\\1.287, x=1.5\end{cases}$$

$$g'(x)=-\frac{3x^2}{4\sqrt{10-x^3}}=\begin{cases}-0.25, x=1\\-0.6556, x=1.5\end{cases}$$

[1,2]区间满足定理1条件，代入x_0=1.5, x_1=1.2870, L≈0.6556，得：

k=23.6078,至少迭代25次.

5. (1) $g(x) = 1 + \frac{1}{x^2}$ $g'(x) = -\frac{2}{x^3}$ $|g'(1.5)| = 0.5926 < 1$

(2) $g(x) = \sqrt[3]{1 + x^2}$ $g'(x) = \frac{2x}{3(1 + x^2)^{2/3}}$

$$|g'(1.5)| = 0.4558 < 1$$

(3) $g(x) = \frac{1}{\sqrt{x - 1}}$ $g'(x) = -\frac{1}{2(x - 1)^{3/2}}$

$$|g'(1.5)| = 1.4142 > 1$$

6.

$$x_{k+1} = x_k - \frac{g(x_k)}{g'(x_k)} = x_k - \frac{x_k^3 + 4x_k^2 - 10 - x_k}{3x_k^2 + 8x_k}$$

1	1.37333	delta=0.12667
2	1.36526	delta=0.00807
3	1.36523	delta=0.00003
4	1.36523	delta=0.00000

8.

$$f(x) = x^3 - 3x^2 - x + 9 = 0$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f(x_k) - f(x_{k-1})}(x_k - x_{k-1})$$

x0=-1.6, x1=-1.4

1	-1.5203, delta=0.1203
2	-1.5254, delta=0.0051
3	-1.5251, delta=0.0003
4	-1.5251, delta=0.0000

9. $$f(x) = x^3 + 4x^2 - 10 = 0$$

$$\boxed{\begin{aligned} &(1)\, y_k = g(x_k) \\ &(2)\, z_k = g(y_k) \\ &(3)\, x_{k+1} \approx x_k - \frac{(y_k - x_k)^2}{z_k - 2y_k + x_k} \end{aligned}}$$

$$g(x) = \sqrt{\frac{10}{4+x}}$$

1	1.36527	delta=0.13473
2	1.36523	delta=0.00004

$x \approx 1.3652$

基本要求

- 二分法计算及其迭代次数估计；
- 简单迭代法及其收敛性判断；
- 牛顿法计算；
- 简化牛顿法、弦割法、牛顿法下山法；
- 收敛阶的判断。