

2005-2006 第二学期概率论数理统计

试题 A 标准答案及评分标准

一. 选择题($3 \times 5 = 15$ 分)

BDABD

二. 填空题($3 \times 5 = 15$ 分)

- (1): 1; (2): $DX = npq = np(1-p)$; (3): $\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$;
(4) 0.6 (5): 0.2

三. 简答题($8 \times 7 = 56$ 分)

1.A: 3 件中恰有 1 件次品

$$P(A) = \frac{C_3^1 C_7^2}{C_{10}^3} = \frac{3 \times 7 \times 6 \times 3!}{2 \times 10 \times 9 \times 8} = \frac{21}{40} = 0.525 \quad 3 \text{ 分}$$

B: 3 件全是次品

$$P(B) = \frac{C_3^3}{C_{10}^3} = \frac{1 \times 3!}{10 \times 9 \times 8} = \frac{1}{120} = 0.0083 \quad 3 \text{ 分}$$

C: 3 件中至少有 1 件次品

$$P(C) = 1 - \frac{C_7^3}{C_{10}^3} = 1 - \frac{7 \times 6 \times 5 \times 3!}{10 \times 9 \times 8 \times 3!} = \frac{17}{24} = 0.708 \quad 2 \text{ 分}$$

$$2.(1) \quad 1 = \int_0^2 k(4x - 2x^2)dx = k \left(2x^2 - \frac{2}{3}x^3 \right) \Big|_0^2 = k \frac{8}{3} \quad 3 \text{ 分}$$

$$k = \frac{3}{8} \quad 1 \text{ 分}$$

$$(2) \quad P\{1 < X < 3\} = \int_1^3 f(x)dx = \int_1^2 \frac{3}{8}(4x - 2x^2)dx \quad 2 \text{ 分}$$

$$= \frac{3}{8} \left(2x^2 - \frac{2}{3}x^3 \right) \Big|_1^2 = 0.5 \quad 2 \text{ 分}$$

$$3.(1) \quad f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{其它} \end{cases} \quad 1 \text{ 分}$$

$$x < a, F(x) = 0$$

$$a \leq x < b, F(x) = \int_a^x \frac{1}{b-a} dt = \frac{x-a}{b-a}$$

$$x \geq b, F(x) = 1$$

$$F(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b \\ 1, & x \geq b \end{cases} \quad 3 \text{ 分}$$

$$(2) P\{a < X < 2\} = \int_a^2 \frac{1}{b-a} dx = \frac{2-a}{b-a} \quad 4 \text{ 分}$$

$$\text{或 } P\{a < X < 2\} = F(2) - F(a) = \frac{2-a}{b-a}$$

4. A: 加工零件 A 的时间, B: 加工零件 B 的时间, C: 停机

$$(1) P(C) = P(A)P(C|A) + P(B)P(C|B) \quad 2 \text{ 分}$$

$$= \frac{1}{3} \times 0.3 + \frac{2}{3} \times 0 = \frac{1}{3} \quad 2 \text{ 分}$$

$$(2) P(B|C) = \frac{P(B)P(C|B)}{P(C)} \quad 2 \text{ 分}$$

$$= \frac{\frac{2}{3} \times 0.4}{\frac{11}{30}} = \frac{8}{11} \quad 2 \text{ 分}$$

$$5. (1) EX = 0 \times 0.7 + 1 \times 0.1 + 2 \times 0.1 + 3 \times 0.1 = 0.6 \quad 3 \text{ 分}$$

$$EY = 0 \times 0.5 + 1 \times 0.3 + 2 \times 0.2 + 3 \times 0 = 0.7 \quad 3 \text{ 分}$$

$$(2) EX^2 = 0^2 \times 0.7 + 1^2 \times 0.1 + 2^2 \times 0.1 + 3^2 \times 0.1 = 1.4$$

$$DX = EX^2 - (EX)^2 = 1.4 - 0.36 = 1.04 \quad 1 \text{ 分}$$

$$EY^2 = 0^2 \times 0.5 + 1^2 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0 = 1.1$$

$$DY = EY^2 - (EY)^2 = 1.1 - 0.49 = 0.61 \quad 1 \text{ 分}$$

$$6. (1) P\{2 < X \leq 5\} = P\left\{\frac{2-3}{2} < \frac{X-3}{2} \leq \frac{5-3}{2}\right\} = P\left\{-0.5 < \frac{X-3}{2} \leq 1\right\} \quad 2 \text{ 分}$$

$$= \Phi(1) - \Phi(-0.5) = 0.53 \quad 2 \text{ 分}$$

$$(2) P\{X > C\} = 1 - P\{X \leq C\} = P\{X \leq C\}$$

$$P\{X \leq C\} = \frac{1}{2} \quad 2 \text{ 分}$$

$$P\{X \leq C\} = P\left\{\frac{X-3}{2} \leq \frac{C-3}{2}\right\} = \Phi\left(\frac{C-3}{2}\right) = \frac{1}{2} \quad 1 \text{ 分}$$

$$C = 3 \quad 1 \text{ 分}$$

7. $P\{A\} = 0.6$, $P\{B|\bar{A}\} = 0.5$, 求母亲孩子都得病的概率.

$$P\{A\} = 0.6, P\{\bar{A}\} = 0.4, \quad 4 \text{ 分}$$

$$P\{\bar{A}B\} = P(\bar{A})P(B|\bar{A}) = 0.4 \times 0.5 = 0.2 \quad 4 \text{ 分}$$

四.综合题(7×2=14 分)

1.A:元件需要更换

$$P(A) = P\{X < 150\} = \int_{-\infty}^{150} \frac{100}{x^2} dx = \int_{100}^{150} \frac{100}{x^2} dx \quad 2 \text{ 分}$$

$$= -\frac{1}{x} \Big|_{100}^{150} = \frac{1}{3} \quad 2 \text{ 分}$$

B: 5 个同类型的元件在使用的前 150 小时内恰有 2 个需要更换.

$$P(B) = C_5^2 P(A)^2 P(\bar{A})^3 = 10 \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^3 = \frac{80}{243} \quad 3 \text{ 分}$$

2.设车门的高度为 h cm,由题意得

$$P\{X \geq h\} \leq 0.01$$

$$\text{或者 } P\{X < h\} \geq 0.99 \quad 2 \text{ 分}$$

$$X \sim N(168, 7^2), P\{X < h\} = \Phi\left(\frac{h-168}{7}\right) \geq 0.99 \quad 1 \text{ 分}$$

$$\text{查表得 } \Phi(2.33) \approx 0.99, \text{ 因此有 } 2.33 = \frac{h-168}{7} \quad 2 \text{ 分}$$

$$\text{于是 } h = 168 + 7 \times 2.33 = 184.31 \quad 2 \text{ 分}$$

故车门的高度为 184.31cm 时,男子与车门碰头得机会不超过 0.01.