第四章 作业

1. 对**n**次多项式**f(x)**:

$$f^{n+1}(x) \equiv 0$$

设**p_n(x)**为**n**次插值多项式，余项为零：

$$R_n(x) = \frac{f^{n+1}(\xi)}{(n+1)!}\omega_{n+1}(x) \equiv 0$$

因此，**p_n(x)≡f(x)**

2. (1)线性插值：

$$L_1(x)=\sum_{j=0}^{1} f(x_j)l_j(x)= f(x_0)\frac{(x-x_1)}{(x_0-x_1)}+ f(x_1)\frac{(x-x_0)}{(x_1-x_0)}$$

$$=2.3979\frac{-0.25}{-1}+2.4849\frac{0.75}{1}$$

$$\approx 2.4632$$

误差： $$R_1(x)=\frac{f^{2}(\xi)}{2!}\omega_2(x)$$

$$\left|R_1(x)\right|=\frac{1}{2}\xi^{-2}0.75\times 0.25\le 7.7479\times 10^{-4}$$

2. (2)二次插值，取**11**、**12**、**13**三点：

$$L_2(x)=\sum_{j=0}^{2} f(x_j)l_j(x)= f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$+ f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}+ f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$=2.3979\frac{-0.25\times-1.25}{-1\times-2}+2.4849\frac{0.75\times-1.25}{1\times-1}+2.5649\frac{0.75\times-0.25}{2\times1}$$

$$\approx 2.4638$$

误差： $$R_2(x)=\frac{f^{3}(\xi)}{3!}\omega_3(x)$$

$$\left|R_2(x)\right|=\frac{1}{3}\xi^{-3}0.75\times0.25\times1.25\le 5.8696\times10^{-5}$$

3. 二次插值多项式：

$$L_2(x)=\sum_{j=0}^{2} f(x_j)l_j(x)= f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$+ f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}+ f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$=0\frac{(x-3)(x-4)}{-1\times-2}+0\frac{(x-2)(x-4)}{1\times-1}+2\frac{(x-2)(x-3)}{2\times1}$$

$$=x^2-5x+6$$

极值： $L'_2(x)=0 \Rightarrow x=2.5 \qquad L_2(2.5)=-0.25$

5. 差商表

$$f[x_0, x_1, \ldots, x_k] = \frac{f[x_1, \ldots, x_k] - f[x_0, \ldots, x_{k-1}]}{x_k - x_0}$$

x_i	$f(x_i)$	一阶差商	二阶差商	三阶差商
0.7	0.6442			
0.9	0.7833	0.6955		
1.1	0.8912	0.5395	-0.3900	
1.3	0.9636	0.3620	-0.4438	-0.0897
1.5	0.9975	0.1695	-0.4813	-0.0625
1.7	0.9917	-0.0290	-0.4962	-0.0248

$$N_n(x) = f(x_0) + \sum_{k=1}^{n} f[x_0, x_1, \ldots, x_k] \prod_{j=0}^{k-1} (x - x_j)$$

5. 两点、三点

$$N_1(1.0) = f(0.9) + f[x_0, x_1](1.0 - 0.9)$$

$$= 0.7833 + 0.5395 \times 0.1 \approx 0.8373$$

$$N_2(1.0) = f(0.9) + \sum_{k=1}^{2} f[x_0, x_1, ..., x_k] \prod_{j=0}^{k-1} (x - x_j)$$

=0.7833+0.5395*0.1+0.4438*0.01≈0.8417

$$N_2(1.0) = f(0.7) + \sum_{k=1}^{2} f[x_0, x_1, ..., x_k] \prod_{j=0}^{k-1} (x - x_j)$$

=0.6442+0.6955*0.3-0.3900*0.03≈0.8411

真实值: sin(1.0)≈0.8414710

5. 四点　　真实值: **sin(1.0)≈0.8414710**

$$N_3(x) = f(0.7) + \sum_{k=1}^{3} f[x_0, x_1, \ldots, x_k] \prod_{j=0}^{k-1} (x - x_j)$$

=0.6442+0.6955*0.3-0.3900*0.03+0.0897*0.003

≈0.8414(191)

$$N_3(x) = f(0.9) + \sum_{k=2}^{4} f[x_1, x_2, \ldots, x_k] \prod_{j=1}^{k-1} (x - x_j)$$

=0.7833+0.5395*0.1+0.4438*0.01-0.0625*0.003

≈0.8415(005)

12.

$$(\varphi_0,\varphi_0)=\sum_{i=0}^{4}\omega_i=5 \qquad (\varphi_0,\varphi_1)=(\varphi_1,\varphi_0)=\sum_{i=0}^{4}\omega_i x_i^2=5327$$

$$(\varphi_1,\varphi_1)=\sum_{i=0}^{4}\omega_i x_i^4=7277699$$

$$(f,\varphi_0)=\sum_{i=0}^{4}\omega_i y_i=271.4 \qquad (f,\varphi_1)=\sum_{i=0}^{4}\omega_i x_i^2 y_i=369321.5$$

$$\begin{pmatrix}(\phi_0,\phi_0) & (\phi_0,\phi_1)\\ (\phi_1,\phi_0) & (\phi_1,\phi_1)\end{pmatrix}\begin{pmatrix}a_0\\ a_1\end{pmatrix}=\begin{pmatrix}(f,\phi_0)\\ (f,\phi_1)\end{pmatrix}$$

$y=0.972579+0.050035*x^2$

14. $$H(x)=N_2(x_1)+(Ax+B)x(x-1)(x-2)$$

或 $$H(x)=L_2(x_1)+(Ax+B)x(x-1)(x-2)$$

$$N_2(x_1)=0+x-\frac{1}{2}x(x-1)=-\frac{1}{2}x(x-3)$$

$$H'(x)=\frac{3}{2}-x+Ax(x-1)(x-2)+(Ax+B)(x-1)(x-2)$$

$$+(Ax+B)x(x-2)+(Ax+B)x(x-1)$$

$$H'(0)=\frac{3}{2}+2B=0\Rightarrow B=-\frac{3}{4}$$

$$H'(1)=\frac{1}{2}-A-B=1\Rightarrow A=\frac{1}{4}$$

$$H(x)=\frac{1}{4}x^2(x-3)^2$$

$$=\frac{1}{4}x^4-\frac{3}{2}x^3+\frac{9}{4}x^2$$

基本要求

- 拉格朗日插值法计算；
- 一次和二次分段拉格朗日插值法计算；
- 差商的计算、牛顿插值法计算；
- 埃尔米特插值法计算；
- 最小二乘法数据拟合的法方程组方法。