Radio Telescopes

Observing with a radio telescope can be very different from using a visible-wavelength telescope. The Sun does not light up the whole sky at radio wavelengths, as it does at visible wavelengths. The blue daytime sky you see is sunlight that is scattered by the atmosphere, which scatters blue light to a greater extent than the longer visible wavelengths. At radio wavelengths, there is no scattering by the atmosphere at all. Therefore, the daytime sky is dark at radio wavelengths, and so radio observations can be made during day or night. At long radio wavelengths, observations can occur even with a cloud-covered sky since clouds are transparent at these frequencies. At shorter radio wavelengths, though, the water in clouds is a significant source of light loss, and so shorter-wavelength observations require clear weather. Radio telescopes operating at these short-radio wavelengths are often placed in high and dry sites to minimize this loss.

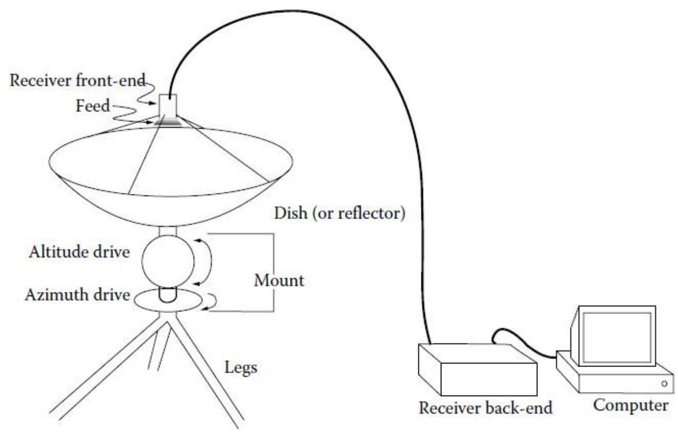

Figure 3.1: A Traditional Primary Reflector Radio Telescope

The component of a radio telescope responsible for collecting the radio signals is often referred to as an antenna or a reflector and these terms are often used interchangeably. There is a difference between the precise meanings of these two terms, however. An antenna is a device that couples electromagnetic (EM) waves in free space to confined waves in a transmission line, while reflectors, which are usually parabolic in shape, collect and concentrate the radiation. Most large radio telescopes employ a reflector as the first element, but they still need an antenna to couple the EM waves into a transmission line, which then carries these waves to the receiver. At long radio wavelengths, simple *dipole antennas* can be used as the first element. The famous Muchison Widefield Array (MWA) is one such example of a radio telescope, where first element is a dipole antenna. Two other words that are commonly used, and which help to avoid the misuse of the word antenna, are the dish, which is often used to refer to the reflector, and the feed, which is the device that couples the radiation concentrated by the reflector into a transmission line.

3.1 Primary Reflectors

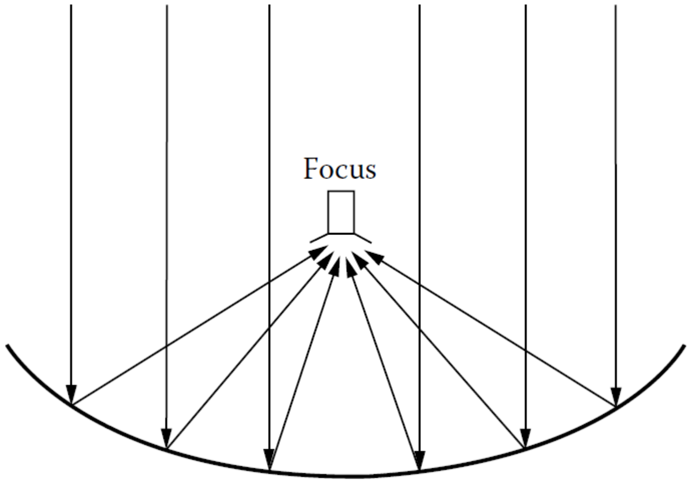

Figure 3.2: Primary Parabolic Reflector

The dishes of most radio telescopes are parabolic reflectors, similar to the primary components in telescopes used in the infrared, visible, and ultraviolet regimes. The parabolic shape causes all waves approaching the dish from the direction perpendicular to the entrance plane to come to a single point, known as the focus of the telescope. The EM waves emitted by an astronomical object, as they approach our telescope many light years away, are well approximated as plane waves and so enter the telescope in parallel paths. Figure 3.2 depicts the reflection of radio waves off a parabolic reflector and those arriving at the focus point

If the direction of the astronomical source is off the central axis of the reflector, the waves will still converge approximately to a point, but offset from that of an on-axis astronomical source. Therefore, in fact, the parabolic reflector has a focal plane and not just a single focus.

Figure 3.3 shows just one *feed* located at the focus of the telescope. However, multiple feeds are often used to collect the power from different directions simultaneously. The prime-focus configuration can be inconvenient, because the feed and receiver are in an awkward position, located high above the primary reflector and hence not easily accessible when the telescope is aimed at the sky. For this reason, most radio telescopes (and visible-light telescopes as well) are of *Cassegrain* design. In a *Cassegrain* telescope, a second reflector (or mirror) is placed before the focal plane of the primary reflector to redirect the waves to another focal point at or behind the vertex of the primary reflector. This arrangement is shown in Figure 3.3.

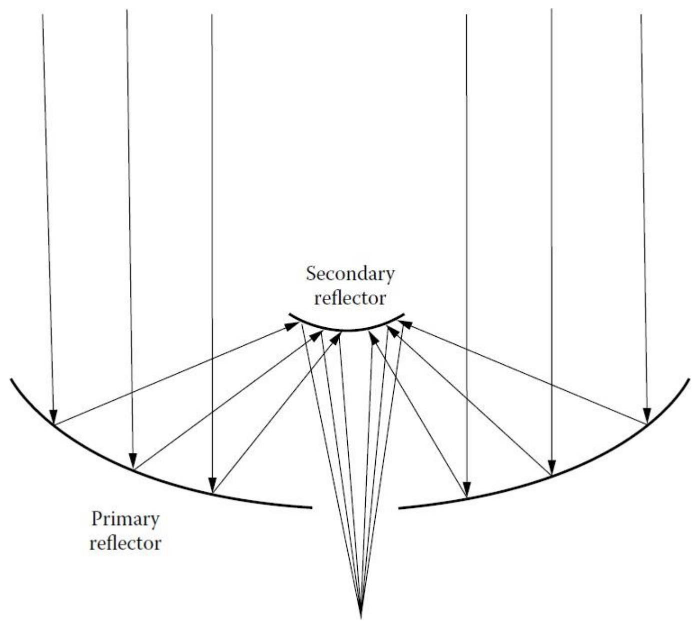

Figure 3.3: Optical Layout of Cassegrian Telescope

The primary reflector serves two important functions. First, it collects and focuses the radiation from astronomical sources, making faint sources more detectable. The amount of radiation collected depends on the telescope's effective area (A_{eff}), which is closely related to the physical area of the primary reflector. As given by Equation 2.2, the power, P, of radiation collected from an astronomical source of flux density F_{ν} is given by

$$P = F_{\nu} A_{eff} \Delta\nu$$

Where:
bandwidth, $\Delta\nu$ is the range of frequencies detected.

The bandwidth is determined by the receiver. The larger the physical area of the reflector, the more power is collected from an astronomical source and more readily faint sources can be detected. A number of factors affect the amount of radiation that enters the receiver and so the effective area of a radio telescope is always smaller than its geometrical area.
The second function of the primary reflector is to provide directivity, which is a telescope's ability to differentiate the emission from objects at different angular positions on the sky. When using a single radio telescope to make a map, the directivity determines the resolution in the map. The directivity of a telescope depends, largely, on the diameter of the primary reflector and is governed by the principle of diffraction. Diffraction, in fact, limits the directivity of all telescopes. The directivity of a radio telescope is commonly described as the telescope's beam pattern, which is the topic of the next section.

3.2 Beam Pattern

The beam pattern is a measure of the sensitivity of the telescope to incoming radio signals as a function of angle on the sky. This is similar to what optical astronomers often call the point-spread function. The term beam pattern derives from the idea of a beam of radio waves leaving a transmitting antenna. Because the sensitivity pattern is the same, whether the antenna receives or transmits—a principle known as the reciprocity theorem—we are free to describe the pattern either way. Ideally, we would like each feed in our telescope to collect radio signals from only one direction in the sky, so that when we point the telescope in a specific direction, the power detected through each feed corresponds to the radiation coming from only that spot in the sky. Unfortunately, this is not possible due to the diffraction of light. Consider a prime focus telescope

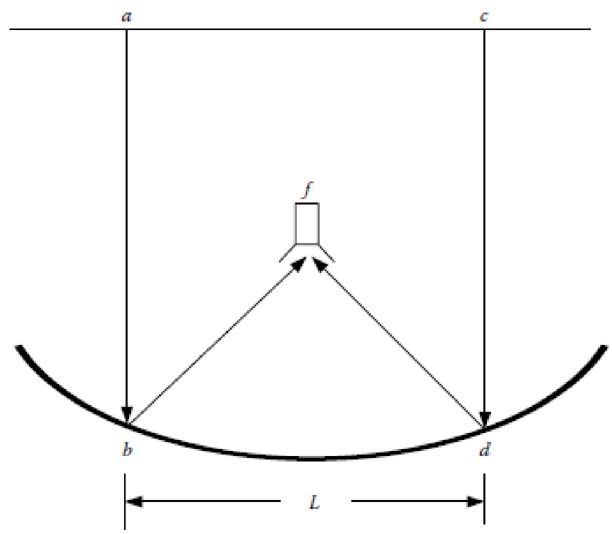

Figure 3.4: Path of two light rays, which reflect off two points on the primary reflector (b and d) and meet at the prime focus.

as depicted in Figure 3.4. Light from a distant astronomical source appears as a continuous chain of plane waves and the parabolic reflector brings these light rays to a single focus. Figure 3.6 shows the light rays from an astronomical source located along the optical axis of the telescope, that is, in the direction where the telescope is pointed. To simplify the calculations, we consider first only the contributions from two points on the reflector. The plane waves reflecting off the primary reflector at points b and d, which are separated by distance L, arrive at a common focus at point f. Note that for an on-axis source, the wave fronts are perpendicular to the optical axis and therefore the path length *abf* is equal to the path length *cdf*. Therefore, the phases of the waves from these two points are identical (see Figure 3.1, which shows the behavior of the wave fronts) so the waves add constructively at the focus. What happens to slightly off-axis plane waves, such as those coming from a source that is not along the optical axis? The ray tracing for off-axis waves is shown in Figure 3.5. The plane waves are now not perpendicular to the optical axis, but are tilted by an angle θ. In this case, the path length abf is slightly longer than the path length cdf. Since lines ac and ab are perpendicular, $\sin\theta = \Delta s/L$, and so the path length difference, Δs, is given by the distance from point a to the horizontal dashed line,

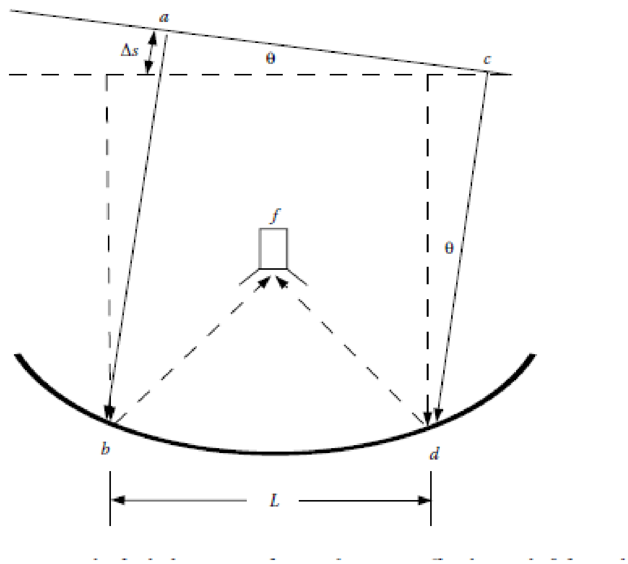

Figure 3.5: Path of two light rays, which reflect off two points on the primary reflector (b and d) and meet at the prime focus.

$$\Delta s = L\sin\theta$$

For a source located at a small angle, θ, from the telescope axis, we can use the small angle approximation for angles expressed in radians, that is,

$$\sin\theta \approx \theta$$

Therefore, the path difference is approximately,

$$\Delta s \approx L\theta$$

Because of this path difference, there is now a phase difference between these two rays of light when they arrive near the focus. Expressing the phase difference, $\Delta\phi$, in radians (remember that there are 2π radians in a complete cycle) gives

$$\Delta\phi = 2\pi\frac{\Delta s}{\lambda} = 2\pi\frac{L\theta}{\lambda}$$

If the path difference, Δs, is $\frac{\lambda}{2}$ then the phase difference will be π radians, and the waves will be exactly out of phase and will cancel at the focus. This destructive interference occurs when the off-axis angular distance is

$$\theta = \frac{\lambda}{2L}$$

We have only considered light from two opposing points on the reflector. When the rays reflect off all other points are also included, one finds that the cancellation is not complete, although the intensity is greatly reduced from when the source is located on-axis. Total destructive interference occurs for the sum of all the rays when the source is located at $\frac{\lambda}{D}$ radians from the central axis, where D is the distance across the aperture. For a uniformly illuminated circular aperture, like that of a typical optical telescope, the total collected power is zero when the source is $1.22(\frac{\lambda}{D})$ radians from the central axis. A sample plot of the sensitivity of a parabolic reflector as a function of angle is shown in Figure 3.6. The telescope's sensitivity pattern, like that shown in Figure 3.7,

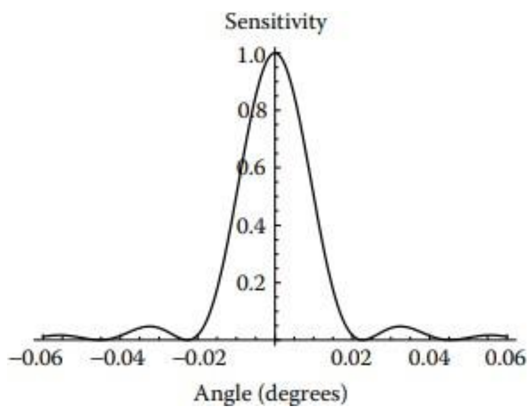

Figure 3.6: Sensitivity Pattern of a Typical Telescope

is called the Airy pattern. The central peak of the sensitivity pattern is called the *main beam*. A telescope, then, can detect power from a source a good ways off-axis, although with much less sensitivity. As the off-axis angle increases further, the response of the telescope goes through a series of peaks and valleys in which there is partial constructive and destructive interference. These off-axis responses are called sidelobes and are undesirable as they can add confusion to observations. The width of the central peak of the Airy pattern is used to define the angular resolution of a single-dish telescope. By convention, the angular width of this peak is taken to be equal to Full Width at Half Maxima(FWHM) of the peak. FWHM is equal to $\frac{1.02\lambda}{D}$, (where D is the distance across the aperture) which is smaller than the radius of the Airy Disc, which is $\frac{1.22\lambda}{D}$. In Radio Astronomy, FWHM of the central Airy Peak is used to define the resolution angle of the telescope. This means, that if two sources lie within this angle, the telescopes sees them as one single source in the sky. Since the FWHM of the main lobe is inversely proportional to the diameter of the reflector, we have that large diameter telescopes not only collect more power from an astronomical source, but also provide better angular resolution. The sensitivity of a Radio Telescope can also be increased by having longer integration times of observation, which can be calculated using Radiometer Equation, which we will come back to later.

3.3 Feeds

At the focus of the telescope, we need antennas to couple EM Waves form space to confined waves in transmission lines for signal to be transmitted to the receiver. These feeds are generally in the shape of horns. They are often flared, with the radiation entering the larger end and tapering down to the proper size for a type of transmission line called the *waveguide*. The larger end of the flare horn should be of at least the size of one wavelength of the radiation being received. The minimum size of a feed horn opening at long wavelengths, then, can be quite large, thus limiting the number of feeds that can fit in the focal plane of a radio telescope. The radiation reflected off the dish enters the feed horn through a finite-sized opening, approaching from many different angles, and is then combined inside the feed. Diffraction, therefore, again determines the amount of power the feed collects and passes onto the receiver. The beam pattern of the feed determines the *illumination pattern* of the primary reflector.

A quantity that describes how the feed horn's beam is distributed on the primary reflector is called the edge taper, which is defined as the ratio of the sensitivity at the center of the reflector to that at the edge. The shape of the illumination pattern on the primary reflector affects (1) the angular

Figure 3.8: Example of a Circular Feed Horn

resolution of the telescope, (2) the sensitivity level in the sidelobes, and (3) the effective collecting area of the telescope.

. On one hand, with a large edge taper much of the power coming from the outer regions of the reflector is not detected, and so the effective collecting area of the telescope is reduced. The ratio of the effective collecting area considering this effect to the physical area is called the illumination efficiency. On the other hand, with a small edge taper, since the illumination is still fairly large at the edge of the reflector, some of the feed's beam pattern misses the reflector, and so the signal entering the feed is diluted by its sensitivity to area beyond the physical reflector. The illumination of the feed beyond the reflector is called spillover. Thus, a large edge taper optimizes the spillover efficiency, while a small edge taper optimizes the illumination efficiency. The maximum collecting area results from a compromise between spillover and illumination efficiency. The edge taper that maximizes the collecting area of the telescope is one in which the power per unit area transmitted to the center of the reflector is 10 times larger than that at the edge; this is called a 10-dB edge taper.

With regard to angular resolution and sidelobe levels, A large edge taper minimizes the sidelobe level, but produces poorer angular resolution, whereas a small edge taper increases resolution of the telescope but also increases the sidelobe levels. The edge taper that maximizes the effective collecting area, the 10-dB taper, fortunately, is also a good compromise between good resolution and low sidelobe level. The FWHM of the central Airy Peak with 10-dB taper is given by:

$$\theta_{FWHM} = \frac{1.15\lambda}{D}$$

For this optimum edge taper, the first sidelobe level is approximately 0.4 percent of the peak, and the maximum collecting area of the telescope is about 82 percent of the reflector's physical area.

3.4 Surface Errors

The primary reflector of a radio telescope is never a perfect parabola. There are always manufacturing imperfections that limit its surface accuracy. We can characterize an imperfect reflector by the root mean square (rms) deviations, δz, of the real surface from that of an ideal parabola measured parallel to the optical axis. Such deviations will cause the path length to the focus to be slightly different for various parts of the reflector. This effect is sketched in Figure 3.9. As we saw in earlier path differences cause phase differences that produce less than full constructive interference; therefore, these deviations reduce the power collected by the telescope. Because the light is reflected off the surface, the total path difference is twice the deviation, $2\delta z$; therefore, these deviations produce rms phase errors of $4\pi\frac{\delta z}{\lambda}$.

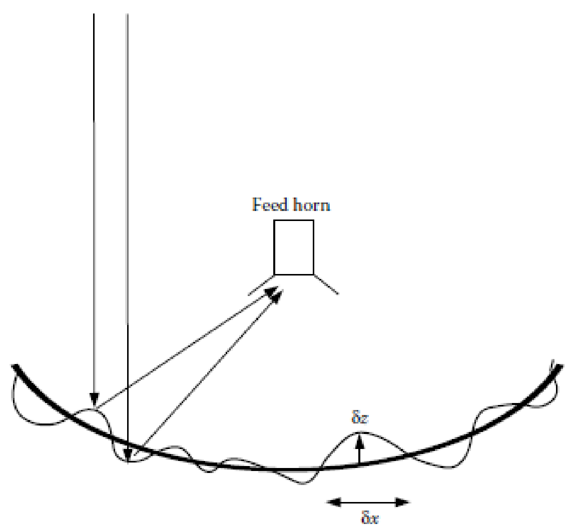

Figure 3.9: Surface irregularities on the reflector cause deviations from a perfect parabola with rms values δz and δx in the directions parallel and perpendicular, respectively, to the optical axis

The presence of surface errors therefore reduces the on-axis sensitivity of the telescope, and this can be viewed as a loss in the collecting area. The effect of surface errors on the collecting area is described by the *Ruze* equation, which is given by

$$A_\delta = A_0 e^{-(4\pi\delta z)^2}$$

3.5 Noise, Temperature and Antenna Temperature

Characterizing noise signals generated in electrical circuits has, of course, always been of great interest in electronics. Nyquist in 1928, for example, found that a resistor in the circuit will add electrical noise with a power per Hz that depends solely on the resistor's temperature. For this reason, the electronic power in a circuit, in general, can be described in terms of an equivalent temperature, T_{equiv}, which is equal to the temperature of a resistor that would produce the same amount of power as the resistor. Following this convention, radio astronomers also describe the power traveling in the transmission lines and receiver in terms of an equivalent temperature given by

$$T_{equiv} = \frac{P}{k\Delta\nu}$$

Where,
k is the Boltzmann Constant and
$\Delta\nu$ is the Bandwidth of the Radiation with Power P. Some of the detected power is due to the astronomical source, which was converted by the antenna to electronic power in the transmission line. We call the equivalent temperature of the power that the antenna delivers to the transmission line, the antenna temperature, T_A. The far majority of the detected power, though, is due to noise from the receiver components. We describe the total noise power by the noise temperature, T_N, and each component in the receiver is characterized by its own noise temperature. Both the source signals and the noise are affected by the amplification and losses that occur along the path through the receiver. The equivalent temperature of the final power output, then, is not simply the sum of the equivalent temperatures of all the sources in the path.
Let us first focus on the signal from the astronomical source and see how its power is affected by the processes in the receiver. At each stage, the source signal is either amplified (when passing through an amplifier) or reduced by a loss (such as in a transmission line). We can use the gain, G for each step; when passing through an amplifier, $G > 1$, and when there is a loss, $G < 1$. For example, if we assign a gain of G_1 to the first element, which is the RF amplifier, then the power in the source signal after this stage is

$$P = G_1 k\Delta\nu T_A$$

Note that the antenna temperature describes the power in the input radiation before any amplification. Even though the amount of power increases when the signal is passed through an amplifier, the radiation is still described by the same equivalent temperature. Therefore, regardless of the amount of amplification in the system, the input radiation power will still be described by the same antenna temperature. An amplifier's noise temperature is defined by the equivalent temperature of the noise power as if it was introduced at the input to the amplifier, and hence it is amplified along with the astronomical signal. Then, the Power Output P would be

$$P = G_1 k\Delta\nu (T_A + T_N)$$

If we have two amplifiers in succession, the first characterized by G_1 and T_{N1}, and the second by G_2 and T_{N2}, then the noise power due to the first is amplified by a factor of G_2 along with the noise produced by the second amplifier. So the total noise power coming out of the second amplifier is

$$P_N = G_1 G_2 k\Delta\nu T_{N1} + G_2 k\Delta\nu T_{N2}$$

The total gain in succession of devices is therefore just a product of individual gains. Now, if for a succession of devices, we define the *total noise temperature* T_N by

$$P_N = Gk\Delta\nu T_N$$

Where, G is the total gain as described above.
Now it remains a trivial calculation to arrive at the formula for *total noise temperature* (just compare the expressions for total noise power) which is given by

$$T_N = T_{N1} + \frac{T_{N2}}{G_1} + \frac{T_{N3}}{G_1 G_2} + \ldots.$$

With total gain and total noise temperature defined, we can define total detected power as :

$$P = Gk\Delta\nu(T_A + T_N)$$

Therefore, even if we observe blank sky, the power out of the receiver is not zero because of the noise power. For these reasons, we cannot readily make total power measurements, but instead, we must make switched power measurements in which we measure the difference in voltage between when the telescope is aimed at the astronomical source (called an on-source observation) and when it is aimed at blank sky (called an off-source observation).It is tempting to believe that the switched observations completely subtract off the noise power, and hence that noise is not really a concern. However, this is not the case. The switched observations remove the offset in the measured power caused by the noise, but the fluctuations in the noise power still affect our measurement and dominate our uncertainty in the antenna temperature. It is these fluctuations that limit the sensitivity of a radio telescope to detect a faint astronomical source. Therefore, we will take a slight digression into statistics of noise power.

3.5.1 A Brief Statistical Analysis of Noise Power

First, recall that the variance is the mean square deviation from the average. Following common convention, we will use σ to indicate the standard deviation, and thus σ^2 as the variance. Now, when measuring the amount of power in EM radiation, the variance in the measure of that power depends on two effects. Since the amount of power in the radiation is proportional to the number of photons arriving per second, the variance in power must relate to fluctuations in the arrival rate of the photons. The power in the radiation is also proportional to E^2 in the waves, and so the variance also depends on the fluctuations in the waves. The former effect is a consequence of the particle aspect of light while the latter is often called the wave noise and is a purely classical effect. When both effects are included in the statistics, the variance is found to have two terms; the variance due to fluctuations in the photon arrival rate depends on the number of photons per mode (with units of photons per second per Hz) and the wave noise depends on the number squared. More specifically,

$$\sigma^2 \propto n^2 + n$$

Where, n is

$$n = \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Therefore, at higher frequencies, the first term of the equation dominates and σ is directly proportional to the square root of number of photons per mode, whereas, at low frequencies, the second

term of the equation dominates, and σ is proportional to number of photons per mode.
Now, with the necessary background, we can talk about the *Radiometer Equation* mentioned in the section 3.2. We know that with any measurement involving random errors, the uncertainty in the measure decreases by averaging more values, by a factor equal to the square root of the number of measurements. Similarly, by making the measurement for more seconds or by increasing the bandwidth, we make many independent measurements of the power. In general, the number of independent measurements (or modes) made over a time period of Δt and bandwidth $\Delta\nu$ is given by $\Delta t\Delta\nu$. Therefore, in an observation with a bandwidth $\Delta\nu$ and integration time of Δt, the uncertainty in the power measured, σ_P, is given by

$$\sigma_P = \sqrt{\frac{P_N}{\Delta t\Delta\nu}}$$

This is the Radiometer Equation and is essential for planning of any radio observation.