Design and Optimization of Enhanced Magnitude Response IIR Full-Band Digital Differentiator Using the Atomic Orbital Search Algorithm

Abhishek Raj[1], Shikhar Pratap Singh[2] and Mridul Gupta[3]

[1] Department One, Institution One, City One, Country One
[2] Department Two, Institution Two, City Two, Country Two

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Abstract

This study utilizes the Atomic Orbital Search Algorithm (AOSA) to develop and optimize the magnitude response of an Infinite Impulse Response (IIR) full-band digital differentiator. The AOSA is employed to optimize the magnitude response of the differentiator, and the proposed design strategy exhibits superior performance compared to existing methods in terms of both magnitude response and Absolute Magnitude Error, as evidenced by simulation results. The AOSA is further demonstrated to effectively reduce the number of convergence iterations required for optimizing the magnitude response of the differentiator. The suggested design method holds potential applications in various fields, including voice processing, picture processing, and biological signal processing. Overall, the utilization of the AOSA in constructing and optimizing the magnitude response of the IIR full-band digital differentiator presents a promising approach to achieve improved performance in digital signals processing applications.

Keywords: Digital differentiator, linear phase, atomic orbital search algorithm, optimization.

1. Introduction

In recent years, the design of an integer order digital differentiator has become a prominent field of research for digital signal processing. The time-derivative of any applied signal is determined by the digital differentiator with integer order. Many technical, scientific, and technological applications including autonomous control [1], fluid dynamics [1], electrical theory [1], image processing [2], electromagnetic theory [3], probability and networks [4] have shown a lot of interest in the derivative idea in recent years.

A differentiator is a signal processing device that determines the time derivative of a given signal for the purpose of estimating velocity and acceleration in radars, biological studies, and picture processing [5-7]. It simplifies, a filter that produces the desired frequency response. An ideal digital differentiator is having the following frequency response:

Digital differentiators are defined as

$$H_{DD}(e^{j\omega}) = j\omega; \qquad |\omega| \leq \pi \qquad (1)$$

and are used to determine the time-derivative of an input signal [5-7]. These tools are used in almost all engineering disciplines, such as instrumentation, control systems, digital signal and image processing, bio-medical engineering, and other related subjects. The development of digital differentiators is therefore quite popular. For real-time dynamic applications, a differentiator should have a small group latency and a wide band frequency responsiveness. A low order is preferred for a straightforward practical implementation.

The two primary types of digital filters are finite impulse response (FIR) and infinite impulse response (IIR). These divisions pertain to the filter's impulse response, as suggested by the nomenclature [10].

1.1 IIR FIILTERS

They are known as "infinite impulse response filters" because their impulse response is endlessly lengthy. This is because they either use feedback or are recursive. [9]. It may be assessed and synthesized using more widely used conventional filter design techniques since they have more recognizable analogue counterparts (Butterworth, Chebyshev, Elliptic, and Bessel). [8]

1.2 FIR FIILTERS

A FIR filter may provide almost any frequency response characteristic by altering the coefficient weight and the number of filter taps. [9] These filters are capable of performance levels analogue filtering techniques are not (such as perfect linear phase response). Yet, since high performance FIR filters frequently need many multiply-accumulates, rapid and efficient DSPs are required. [8]

1.3 ADVANTAGE OF IIR OVER FIR

A filter's operation determines how accurate its computation is. The computational efficiency of FIR filters is much less than that of IIR filters.

1. IIR Filters need lesser processing power in comparison to FIR filters.
2. IIR filters function more rapidly and consume less memory than FIR filters.
3. Response time is slower when using a FIR filter than when using an IIR filter.
4. IIR filters function well when phase information is not required.

IIR filters are most efficient filters for digital differentiators , we are using nature inspired algorithms for designing and simulations purpose.

A differentiator's frequency response should be broad, and its group latency should be small for usage in real-time applications [4]. Like with any other digital filter, digital differentiators can be used in a FIR or IIR configuration. Linear phase is perfectly linear in digital differentiators with FIR-based designs. Several techniques have been used to construct digital differentiators of the FIR type with linear phase characteristics for use in low-, mid-, high-, and wideband frequency applications [11-13]

In signal processing, a filter's function is to either remove undesired signal components, such random noise, or to collect useful signal components, like components lying within a certain frequency range. A digital filter is a tool that uses mathematical operations on discrete temporal signals that have been sampled to enhance or reduce specific aspects of the signal as necessary. Since it is primarily used in signal processing, it differs from an analogue filter, an electrical circuit that processes continuous signals. [14].

In 2013, first and higher order IIR digital differentiators based on Simulated Annealing, Genetic Algorithms, and Fletcher and Powell Optimization to optimize the Al-Alaoui differentiator [15]. In 2016 S. Mahata, S. K. Saha, R. Kar and D. Mandal, demonstrates an effective method for creating first, second, third, and fourth order stable, wideband, infinite impulse response of digital differentiators (DDs) using an evolutionary optimization algorithm named Harmony Search [16]. In 2017 T. K. Rawat and D. K. Upadhyay proposed to use the bat algorithm (BA) to optimize the L1-error fitness function in impulse response (IIR) digital differentiators of the second, third, and fourth orders. In comparison to designs created using other methods, such as particle swarm optimization and real-coded genetic algorithm, the outcomes for the solutions produced by the suggested L1-based BA (L1-BA) are superior [17]. In 2018 S. Mahata, S. K. Saha, R. Kar, and D. Mandal suggested a metaheuristic optimization approach, is used to construct wideband infinite impulse response digital differentiators (DDs) using the hybrid flower pollination algorithm (HFPA), RGA, jDE [18]. The particle swarm optimization (PSO) algorithm-based digital differentiators (DDs) of second, third, and fourth orders are given. The mean square error of the digital signal has been optimized using a modified particle swarm optimization (MPSO) method with decreasing maximum velocity [18].The CS optimization approach is used for the design and optimization of DDs and DIs due to its simplicity, efficiency, and resilience in handling generic multidimensional optimization issues [19].

2. Problem Formulation

In this section, the design of IIR DD is formulated as an optimization problem. Starting with the general frequency response of an Nth order IIR system given in (2), the design of DD and DI is completed by finding the coefficients (b_i, a_i , $0 \leq \mathrm{i} \leq \mathrm{N}$) in equation (2) to approximate the magnitude of the frequency response in equations (1) and (2), respectively. Therefore, the DD design problem can be typically formulated as an optimization problem with a suitable objective function based on the magnitude response of the ideal DD .

IIR DD design is presented as an optimization issue in this section. Using equation (2)'s general frequency response for a Nth order IIR system as a starting point, the design of DD is finished by finding the coefficients (b_i, a_i , $0 \leq \mathrm{i} \leq \mathrm{N}$) to roughly equal the size of the frequency response in equations (1).

As a result, the DD design problem may often be framed as an optimization problem with an appropriate objective function based on the magnitude response of the ideal DD.

$$H(e^{\mathrm{j}\omega}) = \frac{b_0 + b_1\, e^{-\mathrm{j}\omega} + b_2\, e^{-2\mathrm{j}\omega} + \cdots + b_N\, e^{-jN\omega}}{a_0 + a_1\, e^{-\mathrm{j}\omega} + a_2\, e^{-2\mathrm{j}\omega} + \cdots + a_N\, e^{-jN\omega}} \quad (2)$$

The absolute magnitude error (AME) mentioned in most the literature's published objective functions may be represented as a function of (3). The AME is the magnitude response difference between the desired and actual measurements.

$$AME = ||H_{DD}(e^{\mathrm{j}\omega})| - |H(e^{\mathrm{j}\omega})|| \quad (3)$$

The coefficient a and b of equation (2) can now be obtained by minimizing the AME values by applying the metaheuristic optimization algorithm for a digital differentiator of any desired order.

3. Optimization Algorithm

An optimization issue can be solved numerically in a variety of fields, including engineering, management, accounting, computer science, etc. Regarding the system parameters, the error surface in infinite impulse response (IIR) systems is often non-quadratic and multimodal. It is challenging to minimise such an error fitness function using a derivative-based search strategy. This is because it's possible for the derivative-based search method to become trapped in local minima instead of converging to the global minima. Additionally, IIR systems have stability problems since their poles could not be inside the unit circle. Such approaches are shown to be inadequate for resolving multi-objective, multimodal complicated issues, necessitating careful adjustment of algorithmic parameters. Many practitioners use metaheuristic algorithms, which are based on natural evolution, to get around these problems. The meta-heuristic algorithms are population-based search strategies inspired by nature that, by leveraging random search and selection principle, have the capacity to deliver a global optimal solution with quick convergence. Therefore, intelligent search paradigms and optimization techniques are used in this study to quickly and accurately compute the best differentiator design (a multimodal problem).

3.1 ATOMIC ORBITAL SEARCH ALGORITHM

Based on the idea of atomic orbitals in quantum physics, the Atomic Orbital Search Algorithm (AOSA) is an optimization technique inspired by nature. Wang et al. originally put up the idea in 2015.The atomic orbitals that electrons occupy around an atom's nucleus are the source of inspiration for the algorithm. Candidate solutions are represented in the AOSA as a collection of electrons, where each electron corresponds to a certain energy level and geographical location within the search space. The way the AOSA operates is by segmenting the search space into several atomic orbital zones, each of which is connected to a certain energy level. Next, using a set of quantum operators that mimic the behaviour of electrons in an atom, the electrons are positioned inside these areas and their locations are modified.

According to the conventional atomic models, the atomic orbital is the actual space or area surrounding an atom's nucleus where electrons can be present with a particular probability. It is found that the electrons appear as a cloud of charge that rapidly changes their location over time when an atom with moving electrons surrounding the nucleus is photographed using a time-exposure technique. The location of the electrons around the nucleus is unknown at any given time, but the electron probability density may be used to estimate the likelihood that they reside there. The volume around the nucleus is hypothetically split into thin, spherical, concentric layers with radius of r in order to calculate the overall probability of finding any electron at any distance from the nucleus. The radial probability distribution map is shown in Figure. Since each layer's volume grows faster than its probability density, the second hypothetical layer has a larger likelihood overall of containing any electron than the first.

It has been demonstrated that the AOSA is successful in resolving a variety of optimization issues, including feature selection, function optimization, and data clustering. Other optimization techniques may struggle with issues involving highly nonlinear and multimodal functions, but this one excels in these scenarios. The fact that the AOSA is highly parallelizable and can operate well on parallel computing architectures like graphics processing units is one of its main benefits (GPUs).

Overall, the Atomic Orbital Search Algorithm is a creative and interesting optimization technique with a lot of promise for a variety of applications. The efficiency of this optimization process, however, will be influenced by the issue being resolved and the selection of algorithmic parameters.

The AOS algorithm is used to search for the system coefficient (b_i, a_i , $0 \leq i \leq N$) of the IIR system that result in the best value of the objective function.

The Pseudo code implementation of the AOS algorithm are summarized as follows:

Input:

- *Population size N*
- *Maximum number of iterations T*
- *Lower and upper bounds of the search space*

Output:

- *The best solution found.*

1. *Initialize the population P of size N with random solutions in the search space.*
2. *Evaluate the fitness of each solution in P.*
3. *Find the best solution in P, Pbest.*
4. *For t = 1 to T:*
    1. *Generate a new solution Q by applying a mutation operator to Pbest.*
    2. *Evaluate the fitness of Q.*
    3. *If Q is better than Pbest, replace Pbest with Q.*
5. *Return Pbest.*

Mutation operator:

1. *Generate a random number Ψ in the range (0, 1).*
2. *If Ψ ≥ Photon rate, then:*
    1. *Generate random numbers α, β and γ in the range (0, 1).*
    2. *Calculate the new position X_{i+1}^{k} of the i^{th} solution of the k^{th} layer using the following equation:*

$$X_{i+1}^{k} = \frac{X_i^k + \alpha_i + (\beta_i \times LE - \gamma_i \times BS)}{K}$$

where LE is the solution candidate with lowest energy level in the atom, BS is Binding State and BE is Binding Energy for a random layer.

3. *If Ψ < Photon rate, then:*
    1. *Generate a random number r_i in the range (0, 1).*
    2. *Calculate the new position X_{i+1}^{k} of the i^{th} solution of the k^{th} layer using the following equation:*

$$X_{i+1}^{k} = X_i^k + r_i$$

4. *Return X_{i+1}^{k} .*

The following are the key steps in implementation of the AOS algorithm for IIR DD Design:

The Designed parameters are simulated with the upper bound of 1 and lower bound of -1 and iteration of total 2000 with the photon rate compared as error rate of value of 0.10 and 80 number of imaginary layers around the nucleus.

Step 1: Create a Population of n=25(n = 1…i) initial candidate and then set maximum number of iteration(iter=2000).

Step 2: Compute the initial fitness (E_i) the initial energy state of candidate solution. Determine their binding state (BS), binding energy and candidate with lowest energy level, where the fitness value is defined by E_i, Step 3: Create 'N=80' number of imaginary layer around nucleus. In an imagined layer, distribute solution candidates in ascending or descending order using pdf.

Step 4: Again, we find binding state (BS) and binding energy (BE) for a random layer and also determine candidate have lowest Energy Level in random layer.

Step 5: We generate random parameter Ψ, α, β, γ and find Photon rate.

Step 6: If Ψ ≥Photon rate=0.10 and also $E^k = BE^k$ then best solution candidate:

$$X_{i+1}^{k} = \frac{X_i^k + \alpha_i + (\beta_i \times LE - \gamma_i \times BS)}{K}$$

other wise –

$$X_{i+1}^{k} = \frac{X_i^k + \alpha_i \times (\beta_i \times LE^k - \gamma_i \times BS^K)}{K}$$

where X_{i+1}^k and X_{i+1}^k are the current and upcoming position for the i^{th} solution of the k^{th} layer. LE is the solution candidate with lowest energy level in the atom. α_i, β_i and γ_i are vectors containing randomly generated numbers which are distributed uniformly in range of (0,1) for determining the amount of energy that is emitted.

Step 7: If Ψ < Photon rate then best solution candidate:

$$X_{I+1}^{k} = X_i^k + r_i$$

Step 8: Repeat step 2-7 until best candidate solution X_{i+1}^k provide the problem ideal solution and the maximum number of iterations has been achieved**.**

Step 9: After the evaluation of step, we come into the conclusion that the best candidate solution having their lowest energy which are the coefficient of the desired equation. (b_i, a_i , 0 ≤ i ≤ N).

4. RESULT ANALYSIS AND DISCUSSION

Numerous simulations are done on MATLAB software to reduce the AME value defined in equation (3). To obtain an optimal first order digital differentiator. Full band IIR design for digital differentiator of order N=1,2,3 is obtained in the present work and the coefficient are listed in **Table 1**.

Table 1: Coefficient of proposed IIR Digital Differentiator (N=1,2,3) using AOSA.

Order(N)	NUMERATOR $(a_0, a_1 \dots \dots a_n)$	DENOMINATOR $(b_0, b_1 \dots \dots b_n)$
1	1.8051 -1.6681	-1.5315 -0.2901
2	-0.9484 0.4489 0.5340	0.8312 0.6072 0.0599
3	0.2402 1.9970 1.9957 0.5157	0.5594 1.6059 - 0.0647 -1.9586

Magnitude Comparison:

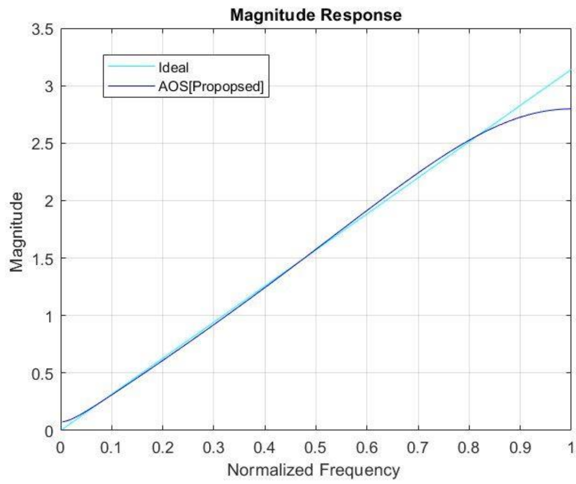

Figure 1. Magnitude response comparison of the proposed IIR digital differentiator with ideal one (N=1)

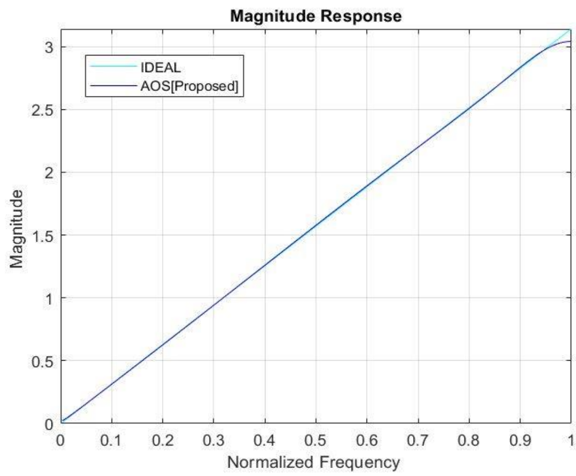

Figure 2. Magnitude response comparison of the proposed IIR digital differentiator with ideal one (N=2)

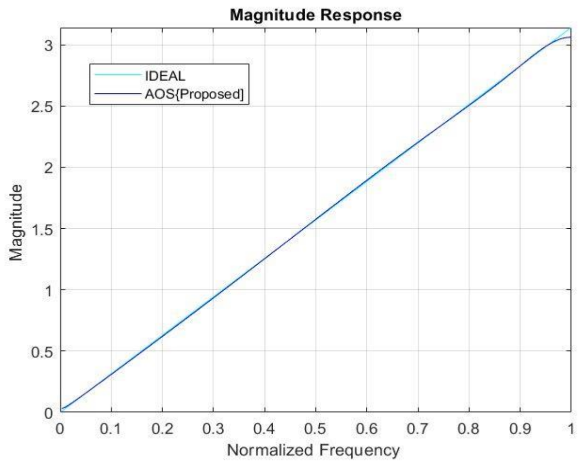

Figure 3. Magnitude response comparison of the proposed IIR digital differentiator with ideal one (N=3)

Now, the performance of the proposed design is directly identified on the basis of AME response plot given in figure 4,5 & 6 for the proposed IIR digital Differentiator of order N=1,2 & 3 respectively.

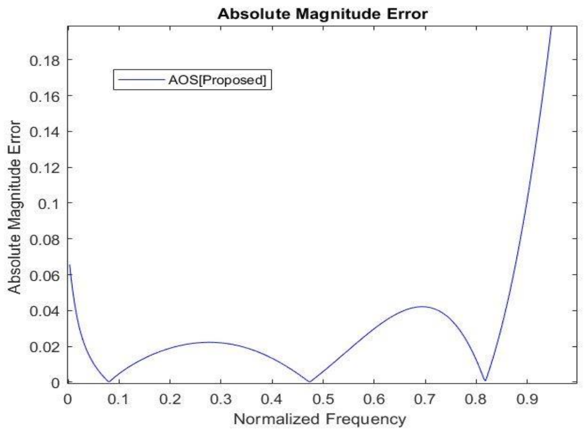

Figure 4. Absolute Magnitude Response of the proposed IIR digital differentiator (N=1)

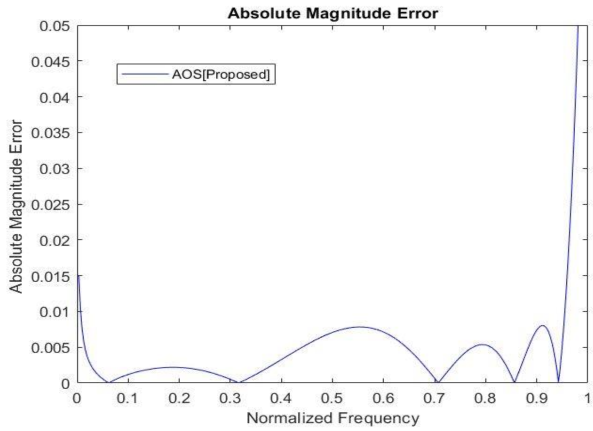

Figure 5. Absolute Magnitude Response of the proposed IIR digital differentiator (N=2)

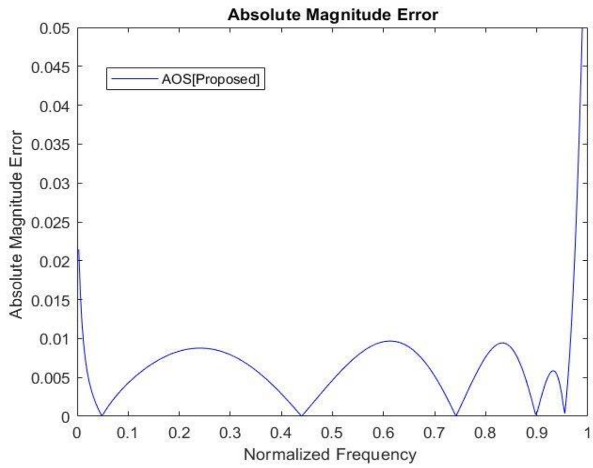

Figure 6. Absolute Magnitude Response of the proposed IIR digital differentiator (N=3)

Now, the performance of the proposed differentiator is compared with some state of the art exisiting designs.For this purpose,the coefficients of the exisiting digital differentiator design for different order are listed in **Table 2**.

Table 2 : Coefficient of existing IIR Digital Differentiator (N=1,2,3) using different optimization algorithm.

METHOD	ORDER	NUMERATOR $(a_{0,}a_1 \dots \dots a_n)$	DENOMINATOR $(b_{0,}b_1 \dots \dots b_n)$
SA [15]		1.1369 -1.1369	1.0000 0.1955
RGA [18]	1	2.0835 -1.7262	2.1368 0.8488
CS [19]		1.8780 -1.7367	1.6022 0.3646
SA [15]		1.1538 -0.5408 -0.6130	1.0000 0.7121 0.0670
jDE [18]	2	1.3206 -0.5033 -0.7729	1.1330 0.8847 0.0873
RGA [18]		1.2145 -0.5137 -0.5695	1.0234 0.7298 0.0879
HFPA [18]		1.1454 -0.4541 -0.7294	0.9945 0.7911 0.0766
CS [19]		1.1454 -0.4542 -0.7286	0.9945 0.7911 0.0766
SA [15]		1.0000 0.8662 0.1612 0.0028	1.1555 -0.3582 -0.714 -0.0833
jDE [18]		1.0330 0.8066 0.1056 0.0195	1.1904 -0.4880 -0.6820 0.0191
RGA [18]	3	0.9988 1.2478 0.3964 0.0083	1.1432 0.0638 -0.9520 -0.2959
HFPA [18]		0.9235 1.3408 0.5129 0.0413	1.0681 0.2861 -0.9853 -0.3742
CS [19]		0.9317 1.3448 0.5121 0.0389	1.0759 0.2798 -0.9792 -0.3712

The magnitude response comparison of the proposed IIR digital differentiator with existing differentiator is shown in Figure 7,8 & 9.

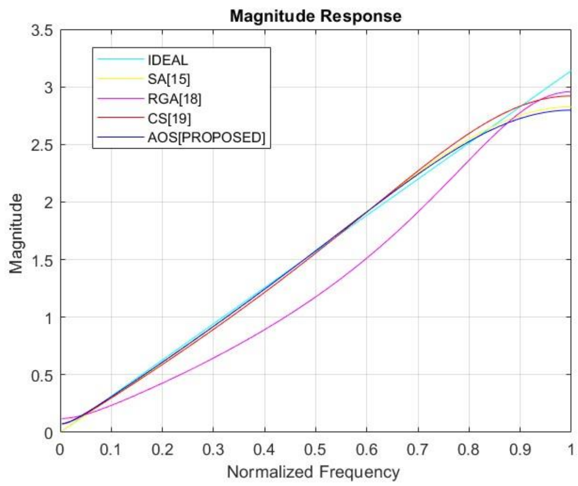

Figure 7 : Magnitude response comparison of the proposed IIR digital differentiator with existing differentiators (N=1)

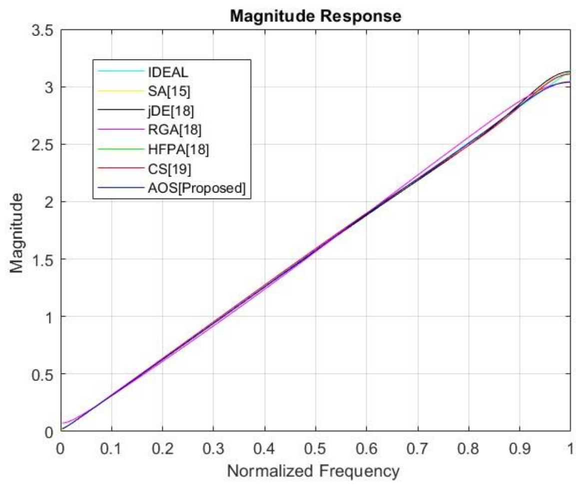

Figure 8 : Magnitude response comparison of the proposed IIR digital differentiator with existing differentiators (N=2)

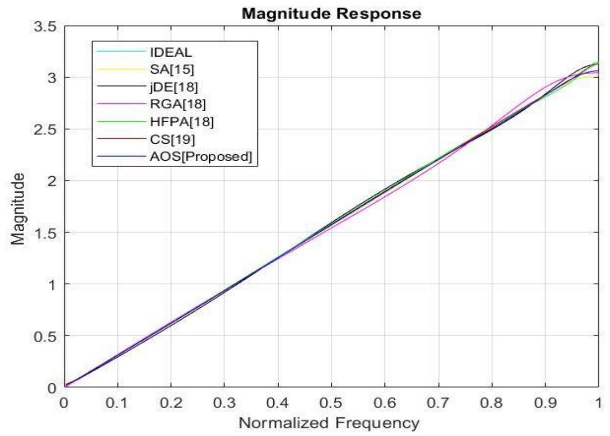

Figure 9 : Magnitude response comparison of the proposed IIR digital differentiator with existing differentiators (N=3)

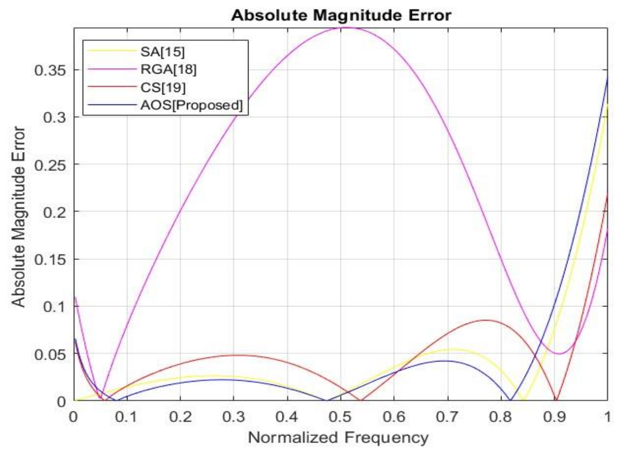

Figure 10: AME comparison of the proposed IIR digital differentiator with exsiting differentiator (N=1)

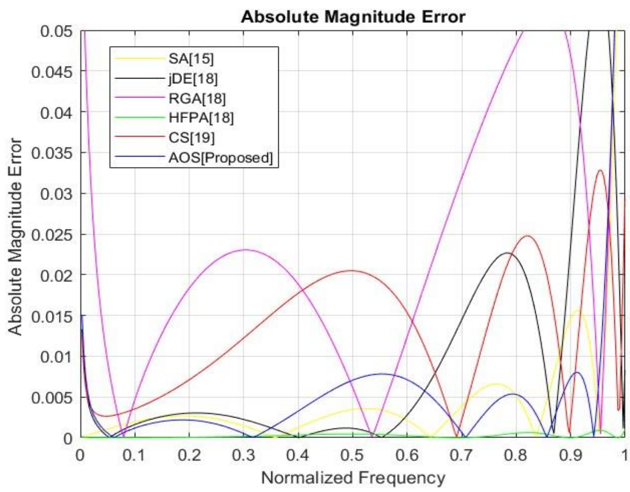

Figure 11: AME comparison of the proposed IIR digital differentiator with exsiting differentiator (N=2)

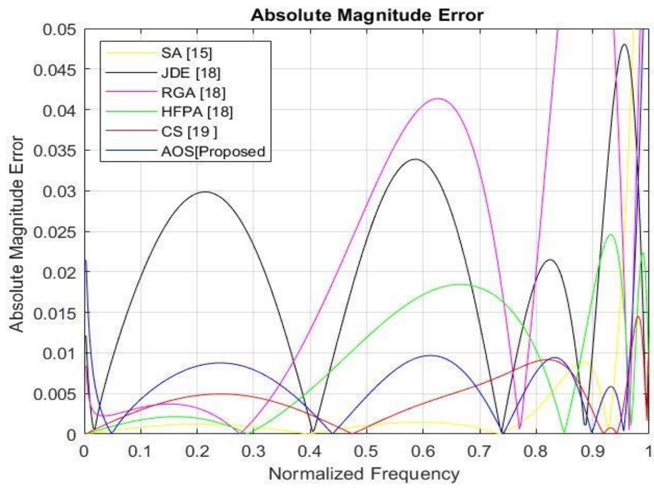

Figure 12: AME comparison of the proposed IIR digital differentiator with exsiting differentiator (N=3)

Furthermore the performance of the proposed design is evaluated on the basis of MAME,Mame and RMSME value given by :

$$i)\ MAME(dB) = 20\log_{10}\{\max(||H_D(jw)| - |H_I(jw)||)\}$$

$$\boldsymbol{ii)\ mAME(dB) = 20\log_{10}\left\{\mathrm{sum}\left(\frac{||H_D(jw)| - |H_I(jw)||}{N}\right)\right\}}$$

$$where\ N = No.of\ points$$

$$iii)\ MAME(dB) = 20\log_{10}\sqrt{1}/\mathrm{N} \times (||H_D(jw)| - |H_I(jw)||)^2$$

$$where\ N = No.of\ points$$

These values are calculated for the proposed IIR digital differentiator and are calculated with that of the existing design in the **Table 3**.

Table 3: Comparison of mAME,MAME and RMSME values of proposed IIR digital differentiator.(N=1)

ALGORITHM	ORDER	RMSME(dB)	AME	
			$MAME_D(dB)$	$mAME_D(dB)$
AOS [Proposed]	1	-3.666005e+00	-9.314578e+00	-2.968661e+01
SA [15]		-3.763423e+00	-1.007109e+01	-2.978402e+01
RGA [18]		1.107612e+01	-8.083251e+00	-1.494448e+01
CS [19]		-2.866408e+00	-1.318047e+01	-2.888701e+01

Table 4: Comparison of mAME,MAME and RMSME values of proposed IIR digital differentiator.(N=2)

ALGORITHM	ORDER	RMSME(dB)	AME	
			$MAME_D(dB)$	$mAME_D(dB)$
AOS [Proposed]	2	-2.098532e+01	-2.007437e+01	-4.700592e+01
SA [15]		-2.195636e+01	-2.068772e+01	-4.797696e+01
jDE [18]		-1.679885e+01	-2.496066e+01	-4.281945e+01
RGA [18]		-8.443511e+00	-1.976154e+01	-3.446411e+01
HFPA [18]		-1.355366e+01	-5.951886e+01	-3.957426e+01
CS [19]		-1.365845e+01	-2.966278e+01	-3.967905e+01

Table 5: Comparison of mAME,MAME and RMSME values of proposed IIR digital differentiator.(N=3)

ALGORITHM	ORDER	RMSME(dB)	AME	
			$MAME_D(dB)$	$mAME_D(dB)$
AOS [Proposed]	3	-2.243352e+01	-2.209240e+01	-4.515327e+01
SA [15]		-1.913267e+01	-1.855476e+01	-4.845412e+01
jDE [18]		-1.000096e+01	-2.636402e+01	-3.602156e+01
RGA [18]		-8.807505e+00	-2.027639e+01	-3.482810e+01
HFPA [18]		-1.716013e+01	-3.218089e+01	-4.318073e+01
CS [19]		-2.353792e+01	-3.673381e+01	-4.955852e+01

5. CONCLUSION

The magnitude of IIR DDs are to be optimised using a unique method. Finding the parameters for the ideal design of IIR full-band DDs using the potent AOS algorithm is effective. It is demonstrated that the developed DDs utilising the suggested approach have a superior magnitude response than previous techniques. The mean of the squared absolute magnitude error showed that the AOS-designed DDs outperformed many of the developed DDs utilising other approaches in the literature. The AOS method once more demonstrated its ability to tackle challenging design issues while being a strong optimisation tool. Finally, the suggested method can be quickly expanded to design different kinds of digital filters.

One of the key metrics used to evaluate the performance of the AOS-designed digital filters is the mean of the squared absolute magnitude error. This metric quantifies the deviation between the desired magnitude response and the actual response of the filters. The results demonstrate that the AOS-designed digital filters outperform numerous other filters developed using alternative approaches. This showcases the effectiveness of the AOS algorithm in optimizing the magnitude response of IIR digital filters.

The AOS algorithm once again proves its capability to tackle challenging design issues and serves as a robust optimization tool in the realm of digital filter design. Its potency is evident in achieving superior results compared to previous techniques, highlighting its efficacy and reliability.

Furthermore, the suggested method possesses the advantage of versatility. It can be quickly expanded to design different kinds of digital filters, catering to a wide range of applications and requirements. This flexibility is crucial in today's dynamic and evolving technological landscape, where various types of digital filters are needed to address diverse signal processing tasks.

By leveraging the AOS algorithm, the optimization process becomes more efficient and effective. The algorithm's ability to explore and exploit the solution space enables it to find optimal parameter values for IIR full-band digital filters. This, in turn, leads to improved magnitude responses, which have a direct impact on the overall performance and quality of the filters.

The significance of the AOS-designed digital filters extends beyond their superior magnitude response. They have the potential to enhance various applications that rely on digital filters, such as audio processing, image processing, telecommunications, and biomedical signal analysis. The improved magnitude response translates into better signal fidelity, noise reduction, and overall system performance.

It is worth noting that the success of the AOS algorithm in optimizing IIR digital filters' magnitude response can be attributed to its unique characteristics. The algorithm's ability to handle complex design problems and its adaptability to different scenarios make it a valuable tool for researchers and engineers in the field of digital filter design.

In summary, the use of the AOS algorithm for optimizing the magnitude response of IIR full-band digital filters has been demonstrated to be effective and superior to previous techniques. The mean of the squared absolute magnitude error serves as a quantitative measure to validate the improved performance of the AOS-designed filters. With its strong optimization capabilities, the AOS algorithm continues to prove its efficacy in tackling challenging design issues. Furthermore, the suggested method's versatility allows for the quick expansion to design various types of digital filters, thereby catering to diverse application requirements. The AOS-designed digital filters hold promise for enhancing a wide range of signal processing applications, leading to improved signal fidelity and system performance.

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