

## Ox Core Macro - Martin Ellison - Macrofinance

### Martin Ellison - Macrofianance

#### Asset Pricing

##### Consumption Capital Asset Pricing Model (CCAPM) #flashcard

- Fundamental CCAPM asset pricing equation:

$$\mathbb{E}_t [R_{risky,t+1}] = R_{f,t+1} - \frac{\text{Cov}_t [U'(C_{t+1}), R_{risky,t+1}]}{\mathbb{E}_t [U'(C_{t+1})]}$$

- Positive correlation between  $C_{t+1}, R_{risky,t+1} \iff$  Positive  $\text{Cov}_t[C_{t+1}, R_{risky,t+1}] \iff$  Negative  $\text{Cov}_t(U'(C_{t+1}, R_{risky,t+1})) \iff$  Positive excess returns  $\iff \mathbb{E}_t [R_{risky,t+1}] > R_{f,t+1}$ 
  - Intuition: this asset pays more when consumption is high and pays less when consumption is low, which means it's a risky asset. To justify its higher risk, it must have a higher return.

##### Discounted Dividend Model for Equity Prices #flashcard

- Strong Requirements:
  - Zero covariance term  $\iff$  Zero RP
  - Constant ratio of marginal utility
- Those 2 requirements  $\iff$  Linear Utility Function
- Expression:

$$p_t = \sum_{j=1}^{\infty} \beta^j \mathbb{E}_t [d_{t+j}]$$

##### Term Structure of Interest Rate #flashcard

- Decompose 2-period FAPE for the risk-free asset:

$$\frac{1}{R_{f,t \rightarrow t+2}} = \frac{1}{R_{f,t \rightarrow t+1}} \mathbb{E}_t \left[ \frac{1}{R_{f,t+1 \rightarrow t+2}} \right] + \underbrace{\text{Cov}_t \left[ \frac{\beta U'(C_{t+1})}{U'(C_t)}, \frac{1}{R_{f,t+1 \rightarrow t+2}} \right]}_{-TP_{t+1 \rightarrow t+2}}$$

- Suppose the Cov term is negative  $\iff \text{Cov}_t \left[ \frac{C_{t+1}}{C_t}, P_{f,t+1 \rightarrow t+2} \right] < 0 \iff \text{Cov}_t [C_t, P_{f,t+1 \rightarrow t+2}] > 0 \iff$  The capital gain at  $t+1$  from holding a 2-period bond increases when consumption in  $t+1$  decreases  $\iff$  2-period bond is a good hedge  $\iff$  Lower expected return needed  $\iff$  Positive TP

##### Special Case of Term Structure: Expectation Theory #flashcard

- If the utility function is linear, then the long-term return are product of short-term returns (up to a Jensen's Term):

$$U(C) = \delta C \iff \frac{1}{R_{f,t \rightarrow t+2}} = \frac{1}{R_{f,t \rightarrow t+1}} \mathbb{E}_t \left[ \frac{1}{R_{f,t+1 \rightarrow t+2}} \right]$$

##### Special Case of Term Structure: CRRA Utility #flashcard

- If:
  - The utility function is CRRA with RRA =  $\sigma$
  - Consumption growth is log normal with average growth rate  $\eta$
- Then: we can decompose average long-term IR into 3 components:

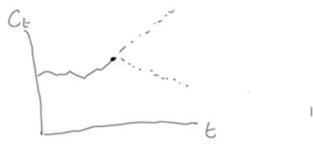
$$\frac{\ln R_{f,t \rightarrow t+j}}{j} = -\ln \beta + \sigma \eta - \frac{1}{2j} \text{Var} \left[ -\sigma \ln \left( \frac{C_{t+j}}{C_t} \right) \right]$$

Q5) Yield curve

$$\frac{\ln R_{t+j}}{j} = -\ln \beta + \bar{r}\eta - \frac{1}{2j} \text{Var}(-\sigma \ln \left( \frac{C_{t+j}}{C_t} \right))$$

$j \uparrow \rightarrow \frac{1}{j} \downarrow \rightarrow$  upward sloping

$\bar{r}\uparrow \rightarrow \text{var} \left( \frac{C_{t+j}}{C_t} \right) \uparrow \rightarrow$  downward sloping



Uncertainty  $\uparrow$  with  $j$  more than proportionately  $\rightarrow$  downward sloping

Intuition: high uncertainty in future gives incentive for precautionary saving  
Price of long bond  $\uparrow$  yield on long bond  $\downarrow$

#notes/tbd

- The average  $j$ -period log return depends on:
- The discount factor  $\beta$  (common to all  $j$  -- hence the intercept)
- The expected average consumption growth over the next  $j$  periods  $\sigma\eta$  (assumed to be the same for all  $j$  here, but can change in reality)
- Variance of consumption growth:  $\frac{1}{j}$  decreases the effect of the variance term, but the variance of consumption growth typically increases faster  $\Rightarrow$  avg yield decreases with  $j \Rightarrow$  downward sloping yield curve
- Intuition: higher uncertainty in the future gives incentives for precautionary savings.
- not consistent with reality

Lucas Tree #notes/tbd

Equity Premium Puzzle #flashcard

- In short: *Risk-averse agents (concave utility functions) face a expected return - risk tradeoff, but given the amount of risk in equities and a reasonable RRA, equities have too high excess returns to be justified as compensation for risk. If we use a super high RRA to justify equity premium, it would imply a super high risk-free rate, which is also inconsistent with data.*
- A reasonable functional form of utility function is CRRA utility function:

$$U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$$

where  $\sigma$  is the RRA.

- In our asset pricing model, no arbitrage condition implies:

$$\mathbb{E}_t [R_{m,t \rightarrow t+1}] - R_{f,t \rightarrow t+1} = -\frac{\text{Cov}_t [U'(C_{t+1}), R_{m,t \rightarrow t+1}]}{\mathbb{E}_t [U'(C_{t+1})]} = -\frac{\text{Cov}_t [C_{t+1}^{-\sigma}, R_{m,t \rightarrow t+1}]}{\mathbb{E}_t [C_{t+1}^{-\sigma}]}$$

- This equation implies that, to generate the high equity premium as observed from real world data, we need a very high RRA. However, a high RRA will cause 2 further problems:

- High RRA is inconsistent with data
- High RRA implies a Risk-free Rate Puzzle, which is also inconsistent with data:

$$\frac{1}{R_{f,t \rightarrow t+1}} = \beta \mathbb{E}_t \left[ \left( \frac{C_t}{C_{t+1}} \right)^\sigma \right]$$

- Intuition:

- For CRRA utility functions, we have the inverse relationship between RRA ( $\sigma$ ) and the Intertemporal Elasticity of Substitution (IEoS):

$$\sigma = \frac{1}{IEoS}$$

- Therefore, a high RRA will have 2 effects:

- Consumers would like to consume similar amount of goods across good and bad states  $\Rightarrow$  Higher equity premium can be justified
- IEOS $\downarrow \Rightarrow$  Consumers are unwilling to move resources across periods  $\Rightarrow$  The risk-free rate has to be high ( $\Rightarrow$  Risk-free Rate Puzzle)
- How to reconcile?
  - Use some special forms of utility functions that separate RRA and IEoS.
  - Introduce habits so that  $U'(C_{t+1})$  becomes more volatile.
  - Disaster risk theory.

## Financial Frictions

### Ideas behind Financial Frictions/Crisis #flashcard

- **Financial Frictions**
  - **Key:** *net worth in economy*
    - Net worth  $\downarrow \Rightarrow$  Financial intermediation  $\downarrow \Rightarrow$  Spreads $\uparrow$
  - **Getler-Karadi-Kiyotaki Model of Abscond/Default:** the possibility to abscond adds an additional *no-default constraint* to bankers' optimisation. A binding constraint will induce a spread in interest rate, which causes less savings and distorts the intertemporal allocation. Putting more network ("skin in the game") will relax the constraint.
  - **Moral Hazard and Effort:** multiple equilibria: equilibrium in normal times contains a non-binding cash constraint  $\Rightarrow$  non-contingent payment from bankers to MF  $\Rightarrow$  bankers are the residual claimers  $\Rightarrow$  they choose the optimal level of effort; equilibrium in abnormal time where bankers have low networth  $\Rightarrow$  cash constraint binds  $\Rightarrow$  contingent payment to MF  $\Rightarrow$  MF are the residual claimers  $\Rightarrow$  bankers do not have enough incentive to exert the optimal level of effort.
  - **Mankiw Model of Adverse Selection:** The market fails under adverse selection because the price mechanism is unable to give sufficient incentives for bankers to activate their projects and drive up the return to deposits. In this model, revenues of MF is independent of the loan rate, so lowering IR cannot stimulate investment. Again, the net worth of bankers  $N$  is critical: we approach the socially optimal allocation as  $N \rightarrow 1$ .
  - **Bernanke, Gertler, and Gilchrist Model of Asymmetric Information and Monitoring Costs:** bankers rely on external finance (loans) to fund investment, but lenders (MFs) face asymmetric information -- if a bank declares bankruptcy, MF will have to incur a cost in order to verify its truthfulness (costly state verification). This leads to a financial accelerator and an important channel of propagation.
- **Financial Crisis**
  - **Key:** *multiple equilibria with a bad equilibrium due to liquidity crisis (bank run)*
  - **Diamond-Dybvig Model of Financial Crisis (Bank Run):** financial intermediation supports both a good equilibrium and a bad equilibrium. Both are NE. The bad equilibrium is caused by everyone withdrawing early  $\rightsquigarrow$  bank run. The bad equilibrium is worse than the competitive equilibrium and worse than the good equilibrium (socially optimal).

### Getler-Karadi-Kiyotaki: Moral Hazard in Financial Institutions - Abscond/Default

#### Getler-Karadi-Kiyotaki Model of Financial Frictions (Abscond/Default) #flashcard

- **Quick Summary**
  - **Main structure**
    - Bankers can abscond with funds (default on liabilities)  $\rightsquigarrow$  introduces a no-default constraint.
    - In normal times, the constraint does not bind  $\rightsquigarrow$  low spreads, efficient intermediation.
    - In bad times (e.g., after a shock), banker net worth is low  $\rightsquigarrow$  constraint binds  $\rightsquigarrow$  spreads rise, savings are discouraged, and investment falls.
  - **Policy Implications**
    - Capital requirements or encouraging "skin in the game" (high banker net worth) can relax the constraint and improve intertemporal efficiency.
    - Credit support policies (e.g., QE) can reduce spreads and restore intermediation during crises.
  - **Summary of structure**
    - Period 1:
      - Households earn income  $y$ : consume  $c_1$  and deposit  $d$  to bankers
      - Bankers combine households' deposit  $d$  with its own net worth  $N$  and invests  $s = N + d$  in firms
    - Period 2:
      - Firms pay back  $sR^k$  to bankers ( $R^k$  is exogenous)
      - Bankers can either default or not:

- If default:
  - Bankers retain  $\theta sR^k$  and remunerate households  $(1 - \theta)sR^k$
- If not default:
  - Bankers retain  $sR^k - Rd$  and pay back households  $Rd$

- **Setup**

- Bankers randomly matched to households, share equal amount of consumption; households receive all profits from bankers
- Households:

$$\max_{c_1, c_2} \{u(c_1) + \beta u(c_2)\}, u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

subject to:

$$\begin{cases} c_1 + d & \leq y \\ c_2 & \leq Rd + \pi \end{cases} \quad (\text{BC1}) \quad (\text{BC2})$$

where  $y, \pi, R$  are treated as given to households.

- Firms:
  - take in  $s$  in period 1 and pay back  $sR^k$  in period 2
- Bankers:
  - take in  $d$  from households in period 1, and pay back  $Rd$  in period 2
  - maximise profits:

$$\max_{d, s} (sR^k - Rd) \text{ s.t. } s \leq N + d$$

- **Benchmark: Social Planner (Optimal)**

- Social planner's optimisation:

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2) \text{ s.t. } c_2 \leq R^k(y + N - c_1)$$

- note that  $N$  is also included
- Substitute in / use Lagrangian to get the optimal allocation

- **Solving Basics**

- **Household Optimisation**
  - In 2 period model with simple 2 BCs and single asset, we can combine BCs:

$$c_1 + \frac{c_2}{R} \leq y + \frac{\pi}{R}$$

and solve the Lagrangian:

$$\mathcal{L} = u(c_1) + \beta u(c_2) + \lambda \left[ y + \frac{\pi}{R} - c_1 - \frac{c_2}{R} \right]$$

- Euler Equation:

$$c_2 = (\beta R)^{\frac{1}{\gamma}} c_1$$

- We can explicitly solve for  $c_1^*, c_2^*, d^*$ .
- Assume  $0 < \gamma < 1$  to have  $R \uparrow \implies d \uparrow \implies \frac{\partial c_1}{\partial R} < 0$  (substitution effect dominates income effect)

- **Bankers Optimisation**

- As long as  $R > R^k$ , bankers will invest all available funds ( $s = d + N$ )

- **Equilibrium Conditions**

- Households optimisation
- Bankers optimisation
- Markets clear
- Consumption is non-negative in each period

- **Solving the Model with Moral Hazard (Possibility to Abscond)**

- Introduce the **possibility to abscond**: in the period 2, bankers have the option to declare default and keep  $\theta$  of the assets
- Now, we need to modify the bankers' optimisation, satisfying the **No Default Condition**:

$$\underbrace{(N + d)R^k - Rd}_{\text{Profits} \mid \text{Not Default}} \geq \underbrace{\theta(N + d)R^k}_{\text{Profits} \mid \text{Default}}$$

- **Equilibrium Conditions:**

- Households maximise utility
- Bankers maximise profits + *no default condition holds*
- Markets clear
- Consumption is non-negative in each period

- **Bankers Optimisation and Spreads:**

$$\max_d \{(N+d)R^k - Rd\} \text{ s.t. } (N+d)R^k - Rd \geq \theta(N+d)R^k$$

- Lagrangian:

$$\mathcal{L} = (N+d)R^k - Rd + \lambda[(N+d)R^k - Rd - \theta(N+d)R^k]$$

- FOC wrt  $d$  with CS:

$$R^k - R + \lambda(R^k - R - \theta R^k) \geq 0 \text{ and } \lambda \geq 0 \text{ w/CS}$$

- *If the no default condition does not bind*, then:

$$\lambda = 0 \iff R = R^k$$

i.e. the socially optimal allocation obtained.

- *If the no default condition binds*, then:

$$\lambda > 0 \iff R > R^k \implies d < d^*$$

i.e. there exists a spread between  $R^k$  and  $R$ ; savings are less than optimal.

- $\implies$  There is a distortion to the socially optimal equilibrium.
- Specifically:

$$R - R^k = \frac{\lambda}{1 + \lambda} \theta R^k$$

- *As the net worth increases, the no default constraint becomes less binding  $\lambda \downarrow$ , compressing the spreads.*
- In a dynamic model with financial accelerator, we will introduce propagation by falling net-worth  $N$ . (see flashcard below)

- **Summary:** *the possibility to abscond adds an additional no-default condition to bankers' optimisation. A binding constraint will induce a spread in interest rate, which causes less savings and distorts the intertemporal allocation. Putting more network ("skin in the game") will relax the constraint.*

## Amplification and Propagation of Shocks in Getler-Karadi-Kiyutaki Model #flashcard

- **NOT LIKELY TO BE IN THE EXAM!!!**

- Effects of a **Negative Productivity Shock**:

- **Amplification (immediate):** Shock

- $\rightsquigarrow$  Net Worth  $N \downarrow$
- $\rightsquigarrow$  No-default Constraint becomes more binding  $\lambda \uparrow$
- $\rightsquigarrow$  Spread  $(R - R^k) \uparrow$
- $\rightsquigarrow$  Capital Demand  $\downarrow$
- $\rightsquigarrow$  Asset Price  $\downarrow$
- $\rightsquigarrow$  Net Worth further  $N \downarrow\downarrow$

- **Propagation (dynamic):** Capital Today  $\downarrow$

- $\rightsquigarrow$  Profits  $\downarrow$
- $\rightsquigarrow$  Investment  $\downarrow$
- $\rightsquigarrow$  Productivity  $\downarrow$
- $\rightsquigarrow$  Net Worth Tomorrow  $\downarrow$
- $\rightsquigarrow$  Capital Tomorrow  $\downarrow$
- ... (persistent effect due to time needed to rebuild net worth  $N$ )

- **Propagation to Other Sectors:** if firms use assets as collaterals, productivity shock

- $\rightsquigarrow$  devaluation of assets
- $\rightsquigarrow$  borrowing constraint becomes more binding

- **Key Mechanism:** how much you can borrow depends on your net worth  $N$

- Negative shock  $\rightsquigarrow N \downarrow$

- ↘ Borrowing capacity ↓
- ↗ IR ↑
- ↘ Output ↓

## Moral Hazard and Effort

### Financial Friction: Moral Hazard and Effort #flashcard

- **Main Structure**

- Adverse selection in loan markets: only banks with bad projects accept low loan rates.
- Bankers won't activate their projects unless the return exceeds their private opportunity cost.
- The model assumes mutual funds' revenue is independent of loan rates, so cutting rates doesn't help.
- Indeed, lowering the interest rate worsens the pool of borrowers ↘ can cause market collapse (aka Akerlof lemons problem).

- **Policy Implications**

- Interest rate policy is ineffective during crises dominated by adverse selection.
- The key variable is banker net worth  $N$ : the higher it is, the better the market functions.
- Policy should aim to recapitalise banks or provide guarantees to improve selection quality and restore credit flows.
- Transparency or certification mechanisms can help mitigate adverse selection.

• **Summary:** *multiple equilibria: equilibrium in normal times contains a non-binding cash constraint ⇒ non-contingent payment from bankers to MF ⇒ bankers are the residual claimers ⇒ they choose the optimal level of effort; equilibrium in abnormal time where bankers have low networth ⇒ cash constraint binds ⇒ contingent payment to MF ⇒ MF are the residual claimers ⇒ bankers do not have enough incentive to exert the optimal level of effort*

## Mankiw: Adverse Selection

### Financial Friction: Mankiw Model of Adverse Selection #flashcard

- Model

- Information asymmetry: bankers know the riskiness of her projects, but MF does not and hence unable to distinguish the "good"/"bad" bankers, so they charge all bankers with the same interest rate on borrowing. Consequently, only high-risk projects will be initiated, leading to inefficiency.

- Main Structure:

- Information asymmetry: bankers know the riskiness of her projects, but MF does not and hence unable to distinguish the "good"/"bad" bankers
- Thus, MF charges all bankers with the same interest rate on borrowing.
- Consequently, only high-risk projects will be initiated, leading to inefficiency.

- Policy Implications:

- Justifies intervention: Markets may fail without help.
- Loan subsidies/guarantees: Encourage safe borrowers to participate.
- Better credit info: Reduces info gaps, limits adverse selection.
- Limits of monetary policy: Lower rates may not boost lending (may actually worsens the problem) --- non-monetary tools needed.

• **Summary:** *The market fails under adverse selection because the price mechanism is unable to give sufficient incentives for bankers to activate their projects and drive up the return to deposits. In this model, revenues of MF is independent of the loan rate, so lowering IR cannot stimulate investment. Again, the net worth of bankers  $N$  is critical: we approach the socially optimal allocation as  $N \rightarrow 1$ .*

## Bernanke, Gertler, and Gilchrist: Asymmetric Information and Monitoring Costs

### Financial Friction: Bernanke, Gertler, and Gilchrist's Model of Asymmetric Information and Monitoring Costs #flashcard

- **Main Structure**

- Bankers borrow from MF and invest
- Asymmetric information and Costly State Verification: bankers, with limited liabilities, can fake bankruptcy, and MF has to incur a cost to determine whether it's true or not
- This leads to insufficient financial intermediation and incentivises more risky investments and higher leverages, leading to more bankruptcies in equilibrium

- **Summary:** bankers rely on external finance (loans) to fund investment, but lenders (MFs) face asymmetric information -- if a bank declares bankruptcy, MF will have to incur a cost in order to verify its truthfulness (costly state verification). This leads to a financial accelerator and an important channel of propagation.

## Financial Crisis

### Diamond-Dybvig: Liquidity and Bank Runs

#### Diamond-Dybvig Model of Financial Crisis

- 3 Settings:
  - Competitive equilibrium
  - Socially optimal allocation
  - Financial intermediaries
    - Good equilibrium
    - Bad equilibrium (bank run/crisis) #flashcard

- **Summary of Structure**

- Key idea of modelling financial crisis: *multiple equilibria*
- Switching between equilibria can be happen quickly.
- There are 2 types of consumers: impatient (type 1) / patient (type 2):

$$U = \begin{cases} U(c_1^1) & \text{if impatient} \\ \rho U(c_1^2 + c_2^2) & \text{if patient} \end{cases} \quad \begin{matrix} Pr = t \\ Pr = 1 - t \end{matrix}$$

- There are 3 periods:  $T = 0, 1, 2$
- In period 0:
  - All consumers get 1 endowment
  - Consumers do not know their own types
  - Consumers can:
    - invest in technology which pays  $R \geq \rho^{-1} > 1$  if held until period 2 and pays 1 if liquidated in period 1
    - store the endowment without depreciation
    - Consumers decide the amount of endowment to invest in period 0
    - Consumers can trade and make contracts, but they cannot be contingent on the types revealed in the future.
  - In period 1:
    - Consumers' type revealed
    - Consumers decide the amount to liquidate
  - In period 2:
    - All investment pays off
    - Notation:  $c_{\text{period}}^{\text{type}}$  i.e.

$$\begin{cases} c_1^1 & \text{consumption of impatient consumers in period 1} \\ c_2^1 & \text{consumption of impatient consumers in period 2} \\ c_1^2 & \text{consumption of patient consumers in period 1} \\ c_2^2 & \text{consumption of patient consumers in period 2} \end{cases}$$

- **Competitive Equilibrium with No Financial Intermediation**

- In period 0:
  - All consumers invest 1 in the technology (since storing is dominated)
  - There will be no trade or contract because everyone is identical
- In period 1:
  - If a consumer turns out to be impatient, then she liquidates all investments and consume ( $c_1^1 = 1$ )
  - If a consumer turns out to be patient, she keep all the investments ( $c_2^2 = R$ )
- In period 2:
  - Patient consumers consume the return to investment
- Result:

$$\begin{cases} c_1^1 = 1, & c_2^1 = 0 \\ c_1^2 = 0, & c_2^2 = R \end{cases} \iff \begin{cases} U^1 = U(1) \\ U^2 = \rho U(R) \end{cases}$$

- This is *same as autarky* since there is no trade, insurance, or contract -- the market serves no useful purpose.

- **Efficient Case with a Social Planner**

- The optimal allocation should have  $c_1^2 = c_2^1 = 0$  because:
  - the impatient type only benefits from  $c_1^1$

- the impatient type always prefer holding the investment to maturity
- Thus, Social planner's problem:

$$\max_{c_1^1, c_2^2} \{tu(c_1^1) + (1-t)\rho u(c_2^2)\} \text{ s.t. } (1-t)c_2^2 = R(1-tc_1^1)$$

- We can just rewrite the constraint and substitute it in
- FOC:

$$u'(c_1^{1*}) = \rho R c'(c_2^{2*})$$

- Since  $\rho R > 1$ , we have:

$$1 < c_1^{1*} < c_2^{2*} < R$$

which implies:

- Impatient consumers consume more at  $T = 1$  than in the competitive equilibrium.
- Patient consumers consume less at  $T = 2$  than in the competitive equilibrium.
- Interpretation: the impatient need to liquidate some of their investments at  $T = 1$  as an "insurance" for the impatient.

#### • Financial Intermediation and Crisis

- Try to retrieve the optimal allocation through financial intermediation
- Banks Setup:
  - on its liability side:
  - Takes in deposits in period 0
  - Promise a return:

$$\begin{cases} r_1 & \text{if withdraw at } T = 1 \\ r_2 & \text{if withdraw at } T = 2 \end{cases}$$

- on its asset side:
- Invests all deposits in period 0
- Liquidate some investment to meet the demand in period 1
- Distribute the return to remaining investment in period 2

#### - Socially Optimal Contract & Good Nash Equilibrium

- Banks offer the Optimal Contract where

$$r_1 = c_1^{1*}$$

for withdrawals at  $T = 1$

- If only the impatient consumers withdraw at  $T = 1$ , then:

$$c_2^2 = \frac{R(1 - c_1^{1*}t)}{1 - t} = c_2^{2*}$$

- This is a Nash Equilibrium:
- The impatient will only withdraw in period 1 due to the nature of their utility function
- The patient will only withdraw in period 2 since an early withdraw yields lower utility
- Consumers have incentive to deposit in period 0 because  $(c_1^{1*}, c_2^{2*})$  yields a higher expected utility than  $(1, R)$
- **Bad Nash Equilibrium (Crisis / Bank Run)**
- If all consumers (patient + impatient) withdraw in period 1, the bank will only be able to pay  $c_1^{1*}$  to  $\frac{1}{c_1^{1*}}$  consumers. Thus, a consumer will get:

$$\begin{cases} c_1^{1*} & \text{with probability } \frac{1}{c_1^{1*}} \\ 0 & \text{with probability } 1 - \frac{1}{c_1^{1*}} \end{cases}$$

- This is also a Nash Equilibrium:
- The impatient always choose to withdraw in period 1 due to the nature of their utility function
- The patient also choose to withdraw in period 1 because they still have a chance to get something if they do so, but they will get nothing if wait to period 2.
- Households still have an incentive to deposit in the bank in period 0 if the probability of a bank run is low.
- **Summary:** financial intermediation supports both a good equilibrium and a bad equilibrium. Both are NE. The bad equilibrium is caused by everyone withdrawing early  $\rightsquigarrow$  bank run. The bad equilibrium is worse than the competitive equilibrium and worse than the good equilibrium (socially optimal).

Use Deposit Insurance to Eliminate the Bad Equilibrium in Diamond-Dybvig Model #flashcard

- **Policy: Deposit Insurance:** insure the deposits by:
  - Imposing a conditional tax  $\tau$  on withdraws in period 1:

$$\begin{cases} \tau = 0 & \text{if only the patient withdraw at } T = 1 \\ \tau = 1 - \frac{1}{c_1^{1-\rho}} & \text{if some patient also withdraw at } T = 1 \end{cases}$$

- Everyone will get  $(1 - \tau)c_1^* = 1$  if some patient consumers try to withdraw early.
- Ensuring a repayment of  $c_2^{2*}$  in period 2
- With this policy, the **bad equilibrium is eliminated**:
  - If only the impatient consumer withdraw, then this becomes the socially optimal case (good equilibrium)
  - If some patient consumers also withdraw, for other patient consumers, there will be no incentive to join:

$$\rho u(c_2^{2*}) > \rho u(1)$$

- $\Rightarrow$  All patient consumers will wait to period 2.

- **Cons of this policy (Karenken and Wallace):** there may be moral hazard problem for the banks if they know the government will help, so we may need Basel-II kind regulations to prevent banks to have too risky portfolios.

### Simplified Karenken-Wallece Model: Moral Hazard of Deposit Insurance #flashcard

- **Setup:** a household is trying to move 1 endowment from period 1 to period 2
  - Quadratic utility:

$$u(c) = -(c - \bar{c})^2$$

- 2 Assets:
  - Safe asset: pays a return  $R > 1$  for sure in period 2
  - Risky asset: pays  $\theta + \epsilon$  or  $\theta - \epsilon$  with equal probabilities
    - $\theta > R$
  - Denote  $\mu$  as the fraction invested in the risky asset

- **Benchmark Case: No Deposit Insurance**

- Household Optimisation:

$$\max_{\mu} \left\{ -\frac{1}{2} \left( (1 - \mu)R + \mu(\theta - \epsilon) - \bar{c} \right)^2 - \frac{1}{2} \left( (1 - \mu)R + \mu(\theta + \epsilon) - \bar{c} \right)^2 \right\}$$

- Take FOC wrt  $\mu$  and the optimal holding of risky asset is:

$$\mu^* = \frac{(\bar{c} - R)(\theta - R)}{\epsilon^2 + (\theta - R)^2} < 1$$

- **With Deposit Insurance**

- Household Optimisation:

$$\max_{\mu} \left\{ -\frac{1}{2} \underbrace{\left( R - \bar{c} \right)^2}_{\text{Get R for sure in the bad case}} - \frac{1}{2} \left( (1 - \mu)R + \mu(\theta + \epsilon) - \bar{c} \right)^2 \right\}$$

- With the deposit insurance, holding the safe asset is dominated by holding the risky asset, hence:

$$\mu^* = 1$$

- **Summary:** deposit insurance encourages over-aggressive position on risky assets.