Ox Core Macro - Federica Romei - Deleveraging and Stagnation

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Deleveraging and Liquidity Traps (Eggertsen and Krugman 2013)

Deleveraging Shock: Eggertsson and Krugman 2013

- Closed economy
- **Key idea**: *when a deleveraging shock forces impatient households / borrowers to repay their debt $\rightsquigarrow$ lower consumption. In normal times, CB decrease the nominal IR and fully stabilise the economy by inducing more consumption from the savers. However, at ZLB with nominal rigidities, the real IR cannot fall enough to simulate demand from the savers, the AD will not be sufficient to support the full employment $\rightsquigarrow$ other margin of adjustment (if that margin is employment $\implies$ unemployment/recession).*
- **Heterogeneous (2 types of) households**:

$$i \in \{s \text{ (savers/patient)}, b \text{ borrowers/impatient}\}$$

  - There are n savers and $1-n$ borrowers.
  - Maximise utility:

$$\max_{C_{it}, B_{it}, L_{it}} \sum_{t=0}^{\infty} \beta_i^t \log C_{i,t},\ i \in \{s, b\},\ \beta_s > \beta_b$$

    subject to 2 constraints:
    - Budget Constraint:

$$B_{it} + P_t C_{it} = (1 + i_{t-1}) B_{t-1} + W_t L_{it}$$

    - Borrowing Constraint:

$$-\frac{1+i_t}{P_{t+1}} B_{it} \leq \overline{D}$$

  - Each household supplies a maximum of $\overline{L}$ units of labour in each period.
- **Perfectly Competitive Firms**:
  - Firms maximise profits:

$$\max_{L_t} P_t Y_t - W_t L_t$$

    subject to the production function:

$$Y_t = L_t$$

- **Solving the model** #flashcard
  - **Key steps**:
    1. **Normal time (steady SS, pre/post crisis)**: consumption/debt/price level unchanged $\rightsquigarrow$ nIR (calculate this first; remember to distinguish pre-post crisis consumption and debt)
    2. **Crisis above ZLB**: nominal rigidity + forced deleveraging $\rightsquigarrow$ borrowers' consumption from BC $\rightsquigarrow$ savers' consumption from symmetry (borrower's repayment = savers' extra income) $\rightsquigarrow$ nIR (or some other margin of adjustment) from savers' Euler Equation $t \to t+1$
    3. **Crisis at ZLB**: nominal rigidity + forced deleveraging $\rightsquigarrow$ borrowers' consumption from BC (same as above with flexible margin) $\rightsquigarrow$ savers' consumption from symmetry (borrower's repayment = savers' extra income, same as above but with flexible margin) $\rightsquigarrow$ savers' actual consumption from Euler Equation $t \to t+1$ and $i = 0$ $\rightsquigarrow$ equalise the two above to determine the level of unemployment (or some other margin of adjustment) $\rightsquigarrow$ substitute this in to determine borrow's consumption
- **Household Optimisation**
  - There's no disutility of working $\implies$ each household supplies $\overline{L}$ units of labour.
  - Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \log C_{it} - \lambda \left[B_{it} + P_t C_{it} - (1 + i_{t-1}) B_{i,t-1} + W_t L_t \right] - \mu_{it} \left[-\frac{1+i_t}{P_{t+1}} B_{it} - \overline{D} \right] \right\}$$

- Combine FOC wrt C_{it}, B_{it} to get the Euler Equation:

$$\frac{1}{C_{i,t}} = \beta_i \frac{1+i_t}{\pi_{t+1}} \frac{1}{C_{i,t+1}} + \mu_{it} \frac{1+i_t}{\pi_{t+1}}$$

- FOC wrt μ_{it} *w/CS*:

$$\mu_{it} \left[\frac{1+i_t}{P_{t+1}} B_{it} + \overline{D} \right] = 0$$

- We can already see that the extra term $\mu_{it} \frac{1+i_t}{\pi_{t+1}}$ in out Euler Equation, and a binding borrowing constraint suppresses C_t.

- **Firms Optimisation**
  - Perfectly competitive goods market $\implies$ Zero Profit:

$$P_t Y_t - W_t L_t = 0 \implies P_t = W_t$$

- **General Equilibrium (Market Clearing Conditions)**
  - Goods Market Clearing:

$$Y_t = n C_{s,t} + (1-n) C_{b,t}$$

  - Labour Market Clearing (if there is no nominal rigidities):

$$Y_t = \overline{L}$$

  - Credit Market Clearing:

$$n B_{s,t} = (1-n) B_{b,t}$$

  - Technically, we only need 2 out of 3 market clearing conditions and the Warlas Law will ensure the 3rd one.

- **Steady State**

- In SS, variables are constant:

$$\left\{ \overline{L}, Y, P, \pi, i \right\}$$

- Remember to distinguish between consumptions and debts for pre-crisis SS and post-crisis SS:

$$\{C_{s,\,pre}, C_{b,\,pre}, B_{s,\,pre}, B_{b,\,pre}\}$$
$$\{C_{s,\,post}, C_{b,\,post}, B_{s,\,post}, B_{b,\,post}\}$$

- But their analysis are exactly the same, so I omit the notation for pre or $post$ in this section.
- SS nIR:
- Apply the Euler Equation to both borrowers and savers, then take ratio to show that the borrower's borrowing constraint binds:

$$\begin{aligned} 1 &= \frac{\overset{0 \text{ savers' constraint slack}}{\cancel{\mu_s C_{s,\,pre}}} + \beta_s}{\mu_b C_b + \beta_b} \\ \mu_c C_b &= \beta_s - \beta_b \\ \implies \mu_c &> 0 \end{aligned}$$

- Apply the Savers' Euler Equation to get SS nIR:

$$\begin{aligned} \frac{1}{C_s} &= \beta_s (1+i) \frac{1}{C_s} \\ \implies i &= \frac{1}{\beta_s} - 1 \end{aligned}$$

- Use the budget constraint + borrowing constraint to derive C_s, C_b:

$$\begin{cases} C_b &= \overline{L} - (1-\beta_s)\overline{D} \\ C_s &= \overline{L} + \frac{1-n}{n}(1-\beta_s)\overline{D} \end{cases}$$

- **Deleveraging Shock**
  - Model the shock as an unexpected drop in $\overline{D}$ from D^h to D^l where $D^h > D^l$ at $t=0$, and the drop is permanent.

- **Effect of the Shock Above ZLB**

- Since there's no nominal rigidity, we can assume the CB pursuing a 0-inflation target and normalise $P=1$.
- SR
- Adjusting period $t=0$: *CB reduces nominal IR i_0 to fully stabilise the economy --- no change in real output/employment/price level*

1. *Start from figuring out borrowers' consumption from their BC*:

- Borrowers' consumption at $t=0$: borrowers will be forced to repay debt $\implies$ back out their consumption from budget constraint:

$$\begin{aligned}B_{b,0}+\cancel{P_0}^{1}C_{b,0}&=(1+\cancel{i_{-1}}^{=i})B_{b,-1}+\cancel{W_0}^{=P_0=1}L_{b,0}\\ C_{b,0}&=(1+i)\left(-\frac{P}{1+i}D^h\right)-\left(-\frac{P}{1+{\color{red}i_0}D^l}\right)+\overline{L}\\ C_{b,0}&=\overline{L}-D^h+\frac{1}{1+i_0}D^l\end{aligned}$$

2. *Then, back out savers' consumption from symmetry (borrower's repayment = savers' extra income)*:
- Savers' consumption at $t=0$: reduction in consumption from the borrower will be filled in by extra consumption from the savers, making the overall output the same $\implies$ back out saver' consumption:

$$C_{s,0}=\overline{L}+\frac{1-n}{n}\left(D^h-\frac{1}{1+i_0}D^l\right)$$

3. *Finally, back out the nominal IR through savers' Euler Equation (target is to restore SS next period. It's forward not backward!)*
- SR nominal IR dynamics: we focus on the *Euler Equation of the savers* since they price the debt:

$$\begin{aligned}\frac{1}{C_{s,0}}&=\beta_s(1+i_0)\frac{1}{C_{s,\,post}}\\ \implies i_0&=\frac{1}{\beta_s}\underbrace{\frac{C_{s,\,post}}{C_{s,0}}}_{<1}-1<\frac{1}{\beta_s}-1=i\end{aligned}$$

- CB has to cut nominal IR
- LR
- Borrowers spend more permanently while savers spend less permanently:

$$\begin{cases}C_{b,\,post} & =\overline{L}-(1-\beta_s)D^l & >C_{b,\,pre}\\ C_{s,\,post} & =\overline{L}+\frac{1-n}{n}(1-\beta_s)D^l & <C_{s,\,pre}\end{cases}$$

- **Effect of the Shock At ZLB**

- SR
- *With ZLB and nominal rigidities, CB cannot stabilise the economy by lowering nIR, nor does the price adjustment, so there will be recession and unemployment (here assumed to belong to savers $\overline{L}\to L_0$). Employment becomes the only margins of adjustment!*
- To generate a liquidity trap and a recession, we need an Additional Assumption: Temporary Perfect Wage Rigidity:

$$\begin{cases}P_0 & =W_0 & =\overline{W}\text{ is fully rigid at }t=0\\ P_t & =W_t & \text{is fully flexible when }t\neq 0\end{cases}$$

- without nominal rigidities, price level will change to fully stabilise the real economy.
- ZLB $\implies$ crisis nominal IR $i_0=0$
1. *Start from figuring out borrowers' consumption from their BC (will be the same as no ZLB case but with flexible margin L!)*:
- Borrower's consumption will be the same as the no ZLB case (forced deleveraging) but with flexible margin L:

$$C_{b,0}={\color{red}L_0}-D^h+D^l$$

2. *Then, back out savers' consumption from symmetry (borrower's repayment = savers' extra income; same as no ZLB case but with flexible margin L!)*:
- Savers' consumption at $t=0$: reduction in consumption from the borrower will be filled in by extra consumption from the savers, making the overall output the same $\implies$ back out saver' consumption:

$$C_{s,0}={\color{red}L_0}+\underbrace{\frac{1-n}{n}(D^h-D^l)}_{\text{extra transfer from borrower}}$$

3. *Then, back out savers' consumption from EULER EQUATION and nIR=0! not from goods market clearing!!*
- Savers' consumption: from savers' Euler Equation

$$\begin{aligned}\frac{1}{C_{s,0}}&=\beta_s(1+\cancel{i_0}^{0})\frac{1}{C_{s,\,post}}\\ \implies C_{s,0}&=\frac{1}{\beta_s}C_{s,\,post}\\ C_{s,0}&=\frac{1}{\beta_s}\left[\overline{L}+\frac{1-n}{n}(1-\beta_s)D^l\right]\end{aligned}$$

4. *Determine the size of unemployment by equalising the 2 calculations for C_s above*:

$$L_0 + \frac{1-n}{n}(D^h - D^l) = \frac{1}{\beta_s}\left[\overline{L} + \frac{1-n}{n}(1-\beta_s)D^l\right]$$
$$\implies L_0 = \frac{1}{\beta_s}\left\{\overline{L} - \frac{\beta_s(1-n)}{n}D^h - \frac{1-n}{n}D^l\right\}$$

- We can see $L_0 \downarrow$ as $D^l \downarrow$
5. *Finally, we can substitute in L_0 to get consumption for savers at the crisis*

- **Debt Deflation**
  - If we allow price to decrease during the recession $P_0 \downarrow$, the recession will be amplified since deflation increase the real value of nominal debts, further tightening the borrowing constraint.
  - The employment equation will become:

$$L_0 = \frac{C_s}{\beta_s} - \frac{1-n}{n}\left(\frac{D^h}{P_0} - D^l\right)$$

    - We can see $P_0 \downarrow \rightsquigarrow L_0 \downarrow$
- **Mitigating Policies**
  - Raise inflation target (forward guidance)
  - Fisher Equation:

$$1 + r_t = \frac{1 + i_t}{\pi_{t+1}}$$

  - CB can relax ZLB by promising higher π_{t+1}
  - In the simple setup, there is no cost of inflation (no nominal rigidities/MIU/CIA), so there will be no efficiency loss.
  - Government spending
  - G_t can be used to boost Y_t
  - Effect depends on the method of financing:
  - G_t financed by taxing borrowers will only worse the recession; G_t financed by future taxes (gov debt) or taxing on savers will work
  - Transfer to debtors
  - Transferring from savers $\rightarrow$ borrowers will boost AD due to borrowers' higher propensity to consume
  - Example: debt relief policy (cancelling debt)
  - Quantitative Easing
  - Channel 1: seignorage and rebate everyone
  - Channel 2: direct lending to borrowers to relax constraint
  - Channel 3: sustain collateral prices to relax borrowing constraint;o

Solving Alternating Endowment Models #flashcard

- Use periodicity! If the endowment has a k-period cycle, then the stationary solution should have that too.

Secular Stagnation

Secular Stagnation: protracted period of weak demand and high unemployment. There is no self-correcting force towards full employment.

- Two complementary approaches to model:
  - Stagnation traps: multiple equilibria --- liquidity trap driven by fulfilling pessimistic expectations about future growth
  - Secular Fundamental Factors: liquidity traps as the outcome of secular fundamental factors (e.g. demography, technological progress, inequality)

Modelling Stagnation Traps in a Keynesian Growth Framework

- Model: *standard Keynesian model with nominal wage rigidities and MR with ZLB*
- **Household**:
  - Representative household maximising expected lifetime utility:

$$\max_{C_t, L_t, b_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma}\right)$$

    - Labour endowment = 1 with no labour disutility, but unemployment possible due to nominal wage rigidities

- Owns a firm and has access to a nominal bond with IR $i_t \implies$ Budget constraint:

$$P_t C_t + \frac{b_{t+1}}{1+i_t} = W_t L_t + b_t + d_t$$

- **Firms**:
  - Perfect competition $\implies \pi_t = 0$
  - Optimisation:

$$\max_{L_t} P_t Y_t - W_t L_t \text{ s.t. } Y_t = A_t L_t$$

  - Define the Productivity Growth Rate:

$$g_{t+1} \equiv \frac{A_{t+1}}{A_t}$$

- **Monetary Policy**:
  - Monetary Rule with ZLB:

$$1 + i_t = \max\left\{\left(1+\bar{i}\right) L_t^{\phi}, 1\right\}$$

- **Solving this model** #flashcard
- **AD Curve in SS**
  - We have close-form solution for this model $\implies$ no need for log-linearisation
  - Household Optimisation
  - Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} \right) - \lambda_t \left(P_t C_t + \frac{b_{t+1}}{1+i_t} - W_t L_t - b_t - d_t \right) \right\}$$

  - Euler Equation for Bond:

$$C_t^{-\sigma} = \beta(1+i_t)\frac{1}{\pi_{t+1}} C_t^{-\sigma}$$

  where $\pi_{t+1} = \frac{P_{t+1}}{P_t}$
  - Firm Optimisation
  - Perfect competition $\implies$

$$\begin{aligned} d_t &= 0 \\ \implies P_t Y_t &= P_t A_t L_t = W_t L_t \\ \implies P_t A_t &= W_t \\ P_t &= \frac{W_t}{A_t} \end{aligned}$$

  - Define productivity and nominal wage growth rate:

$$\begin{cases} g_t & \equiv \frac{A_t}{A_{t-1}} \\ \pi_t^w & \equiv \frac{W_t}{W_{t-1}} \end{cases}$$

  - Then, we can decompose inflation rate

$$\begin{aligned} \pi_t &= \frac{P_t}{P_{t-1}} \\ &= \frac{\frac{W_t}{A_t}}{\frac{W_{t-1}}{A_{t-1}}} \\ &= \frac{W_t}{W_{t-1}} \cdot \frac{A_{t-1}}{A_t} \\ &= \frac{\pi_t^w}{g_t} \end{aligned}$$

  - Natural/Efficient Level of Output
  - In absence of nominal rigidities, LS should be 1 since there's no disutility of working, hence the Natural/Efficient Level of Output should be:

$$Y_t^* = A_t \times 1 = A_t$$

  - Output Gap:

$$\frac{Y_t}{Y_t^*} = \frac{A_t L_t}{A_t} = L_t$$

output gap is the same as employment!

- $L_t = 1 \iff Y_t = Y_t^*$ efficient full employment and no output gap
- $L_t < 1 \iff Y_t < Y_t^*$ involuntary unemployment and negative output gap
- IS Equation (Market Clearing + Euler Equation + PF)
- Combine the Market Clearing Condition $Y_t = C_t$ with the Euler Equation and Production Function to get the IS Equation:

$$\begin{aligned}
Y_t^{-\sigma} &= \beta(1+i_t)\frac{1}{\pi_{t+1}}Y_t^{-\sigma} \\
(A_tL_t)^{-\sigma} &= \beta(1+i_t)\frac{1}{\pi_{t+1}}(A_tL_t)^{-\sigma} \\
L_t^{\sigma} &= \frac{\pi_{t+1}g_{t+1}^{\sigma}}{\beta(1+i_t)}L_{t+1}^{\sigma} \\
L_t^{\sigma} &= \frac{\frac{\pi_{t+1}^w}{g_{t+1}}g_{t+1}^{\sigma}}{\beta(1+i_t)}L_{t+1}^{\sigma} \\
&\boxed{L_t^{\sigma} = \frac{\pi_{t+1}^w g_{t+1}^{\sigma-1}}{\beta(1+i_t)}L_{t+1}^{\sigma}}
\end{aligned}$$

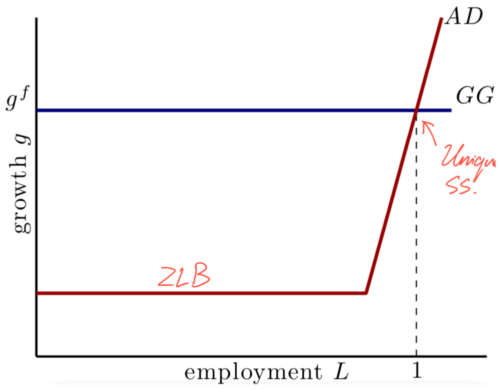
- AD Equation (IS + MR)
- Substitute MR into IS:

$$\begin{aligned}
L_t^{\sigma} &= \frac{\pi_{t+1}^w g_{t+1}^{\sigma-1}}{\beta(1+i_t)}L_{t+1}^{\sigma} \\
&= \frac{\pi_{t+1}^w g_{t+1}^{\sigma-1}}{\beta\max\left\{\left(1+\bar{i}\right)L_t^{\phi},1\right\}}L_{t+1}^{\sigma} \\
\implies &\boxed{L_t^{\sigma} = \frac{\pi_{t+1}^w L_{t+1}^{\sigma}}{\beta\max\left\{\left(1+\bar{i}\right)L_t^{\phi},1\right\}}g_{t+1}^{\sigma-1}}
\end{aligned}$$

- Steady State
- In SS, employment and growth rates are constant: $L_t = L, g_t = g, \pi_t^w = \pi^w$:

$$\begin{aligned}
L^{\sigma} &= \frac{\pi^w L^{\sigma}}{\beta\max\left\{\left(1+\bar{i}\right)L^{\phi},1\right\}}g^{\sigma-1} \\
\implies &\boxed{g^{\sigma-1} = \frac{\beta\max\left\{\left(1+\bar{i}\right)L^{\phi},1\right\}}{\pi^w}}
\end{aligned}$$

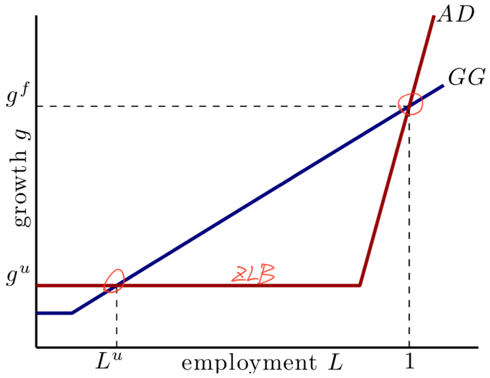
- For reasonable RRA ($\sigma > 1$), there is a positive relationship between g and L, except at ZLB.

- **Growth Rate Curve: Exogenous/Endogenous Productivity Growth**
    - Exogenous Productivity Growth ($g = g^f$) $\implies$ Unique SS
        - 

    - Endogenous Productivity Growth
        - Productivity growth rate $\propto$ Output/Employment:

$$g = f(L), f'(L) > 0$$

            - Possible rationale: demand↓ profits↓ investment↓ productivity growth↓
        - Result: **dual steady states**:

- 

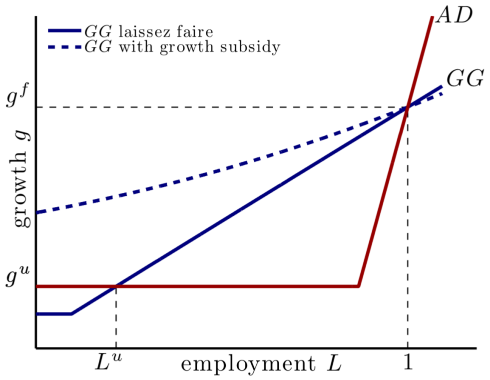
- The bad SS (i.e. Stagnation Trap) has unemployment $L^u < 1$, weak growth $g^u < g^f$, and nIR trapped at ZLB $i^u = 0$
  .

- **Importance of Expectations**: the equilibrium is determined by expectation and shocks(sunspots): low expectation of future growth $\implies$ low future income $\rightsquigarrow$ low AD $\rightsquigarrow$ CB cannot help due to ZLB $\rightsquigarrow$ firms have low profit $\rightsquigarrow$ low investment $\rightsquigarrow$ low growth (**self-fulfilling expectations**)

- **Policy Implications**:
  - Key: *lifting up the GG curve to eliminate the bad SS*
  - FG may not help since the trap could last forever
  - Desirable policy: **countercyclical subsidy to innovations**:

$$g = f(L) + s(g), s'(g) < 0$$

  - Increase g when unemployment is high:
  -



Stagnation Trap: Key Model Insights #flashcard

- Model: simplified Keynesian model with AL production function (real output $\iff$ employment)
  - Solving: Euler Equation + 0-Profit Condition + Inflation decomposition $\rightsquigarrow$ IS curve + MR $\rightsquigarrow$ AD (g, L relationship) + different assumptions on g (GG curve)
- Pessimistic expectation with ZLB can generate stagnation traps
- Inverse of Say's Law: lack of demand creates lack of supply
- Aggressive supply-side policies can stimulate demand and drive the economy out of stagnation.