

OX Y1 Core Micro - Contract and Bargaining

L1: Hidden Information

Optimal Contracting, Signalling, and Market Screening #flashcard

3 key models with hidden information:

- **Optimal Contracting / Mechanism Design**
 - the uninformed party moves first, offering a menu of choices
 - then, the privately-informed party chooses the most attractive option from the menu
- **Signalling**
 - the privately-informed party chooses the signal
 - then, the uninformed party chooses to respond
- **Market Screening**
 - competing uninformed parties make offer
 - then, privately-informed parties choose between offers

	Monopolistic uninformed <i>(only 1 informed)</i>	Competition among uninformed <i>(many uninformed)</i>
Uninformed moves first	Optimal contracting	Market screening with exclusive contracts: U offers single option Market screening with non-exclusive contracts: U offers menu
Informed moves first	Signalling	Market signalling

L2: Optimal Contracting with a Single Privately-Informed Party

Direct Revelation Mechanism (DRM) and Truthful DRM #flashcard

- The principle commits to a **Direct Revelation Mechanism (DRM)** which specifies options contingent on agents' reported types:
 $DRM : \hat{s} \mapsto \text{option}$
- A **Truthful DRM** is a DRM for which truthful reporting is an optimal strategy for an agent regardless its type.

Directly Revelation Principle #flashcard

Given any contract and any DRM, there exists a truthful DRM which, for every type of agent, results in the same option being chosen as under the original contract or DRM.

- Implication: *when designing optimal contracts/mechanisms, we can consider truthful DRMs only. If an outcome is not achievable by a truthful DRM \implies it is not achievable at all.*

L3: Signalling Hidden Information

3 Types of PBE in the Signalling Model #flashcard

- **Separating Equilibrium**: different types of agents choose different signals
- **Pooling Equilibrium**: all agents, regardless of their types, choose the same signal
- **Hybrid Equilibrium**: some agents choose their signals deterministically (playing a pure strategy) while the others randomise between pooling and separating.

Result of a 2-type Signalling Model #flashcard

In the 2-type signalling model, the only PBE satisfying the intuitive criterion is the Pareto-dominant separating PBE:

- In the no-envy case: the Pareto-dominant separating PBE coincides with the full-information outcome

- In the envy case: the high-type will choose a smallest level of signal that can deter imitation from the low-type \rightsquigarrow distortion in allocation
 - In both the signalling and optimal contracting models, when \exists envy, there will be distortion in allocation for the envied type, and the direction of distortion for the envied type is the direction which allows self-selection (separation).
- Criticism of the signalling model:
 - Costly signal is the only information available to the uninformed party about the informed party's type.
 - The unique PBE surviving IC is independent of the prior probability that the informed party is the bad type. (even a super small probability can distort allocation)

2 × 2 Signalling Game General Method

- First mover (worker)
 - type (private information) $\in \{H, L\}$
 - decided by nature at the beginning, with $Pr(H) = \mu$
 - decides whether to signal by participating in a programme $\{Y, N\}$
- Second mover (firm)
 - prior belief on 1st mover's type $Pr(H) = \mu$
 - observes 1st mover's signal $\{Y, N\}$
 - decides whether to hire the 1st mover $\{Hire, Not\}$
- Solving this game #flashcard
- **Extensive form representation (crab diagram)**
 -
- **Looking for a separating PBE**
 1. Figure out 2nd mover's optimal response with updated posterior (signalling)
 - All information sets are reached, so there's no off-equilibrium-path belief
 2. Given 2nd mover's optimal response, figure out whether the 1st mover has an incentive to deviate
 - If so \implies not a separating PBE
- **Looking for a pooling PBE**
 - Note: if the question asks us just to construct a pooling PBE, we only need to find one example (typically pooling on taking the programme Y), no need to check for all possible situations
 1. Figure out 2nd mover's optimal on-equilibrium-path response with posterior = prior
 2. For each type of 1st mover, given 2nd mover's on-equilibrium-path response, check whether she will have a potential incentive to deviate
 3. If so, eliminate that incentive by choosing an off-equilibrium-path response for the 2nd mover
 4. Design the 2nd mover's off-equilibrium-path belief given which her off-equilibrium-path response is optimal
- **Intuitive Criterion**
 - Check whether the 2nd mover's off-equilibrium-path beliefs are consistent with 1st mover's strategy?
 - If the 1st mover indeed deviates, which type could it be?

L4: Market Screening

Results of Screening in Competitive Markets with Exclusive Contracts #flashcard

- Setup
 - Privately informed workers and large number of identical firms
 - Timing:
 - Workers have private information about their types; firms know the probability
 - Each firm offers a contract
 - Workers choose the most-preferred contract
 - Workers acquire the education level e and receive the wage w specified in the contract
 - Equilibrium concept: SPNE
- Result:
 - In **no envy** case, the unique SPNE is the same as the full-information case
 - In **envy** case, whether or not a SPNE exists depends on the prior probability of being a high type q . There exists a cutoff \bar{q} such that:
 - $q \leq \bar{q} \implies$ there is a unique SPE where L-type choosing $w^*(L), e^*(L)$ and H-type choosing the minimal amount of education that prevents L from mimicking, and get the wage making 0-profit for the firm

L5: Hidden Action I

Linear-Exponential-Normal (LEN) Principal-Agent Model #flashcard

- Setup:
 - Agent's CARA utility function:

$$U(w, e) = -\exp\{-rm\}, \quad m = (w - C(e))$$

$$= -\exp\left\{-r\underbrace{(w - C(e))}_m\right\}$$

with reservation utility \bar{U} and:

- r is the Coefficient of Absolute Risk Aversion of this CARA utility function
- Principle's risk-neutral utility function:

$$V(w, e) = \mathbb{E}[\pi(e)] - w$$

- Performance measurement:

$$z = e + x, \quad x \sim N(0, \sigma_x^2)$$

- Additional contractable signal:

$$y \sim N(0, \sigma_y^2)$$

- Linear contract:

$$w = \alpha + \beta(z + \gamma y) = \alpha + \beta(e + x + \gamma y)$$

- Solving the LEN model (backward induction):
 - Starting from the Agent's optimisation:
 - 1. Calculate the Agent's CE use the following fact:

$$\begin{cases} u(m) &= -\exp\{-rm\} \\ m &\sim N(\bar{m}, \sigma_m^2) \end{cases} \implies CE(m) = \bar{m} - \frac{1}{2}r\sigma_m^2$$

- Thus:

$$CE_{Agent}(w = \alpha + \beta(e + x + \gamma y), e) = \alpha + \beta e - C(e) - \frac{1}{2}r\beta^2 Var(x + \gamma y)$$

- 2. Agent maximising utility \iff maximising CE \implies FOC wrt $e \implies e^*$
- 3. Calculate the $CE_{Agent}|_{e^*}$ and Principal set α^* to make the Agent's participation constraint binding:

$$CE_{Agent}|_{e^*}, \alpha^* = 0$$

- 4. Calculate the Profit of the Principal $\pi^P|_{e^*}, \alpha^*$ and let γ^* maximise it for a given effort level $\bar{e} \implies$ FOC wrt $\gamma \implies \gamma^*$

- Conclusion: risk-averseness + uncertainty in payoff \rightsquigarrow weak strength of incentive from the principal $\rightsquigarrow e^* <$ full-information benchmark

L6: Hidden Action II

Equal Compensation Principle (ECP) #flashcard

- Suppose efforts on 2 activities are perfect substitutes in the Agent's cost function with MRS = 1. If the Agent's allocation of effort between these activities cannot be observed by the Principle, then the Agent will devote positive effort to **both** activities **only if** the marginal returns for the two are the same.
- Otherwise, the Agent will devote NO effort to the activity with a lower marginal return.

NOT FINISHED #notes/tbd

L7: Bargaining

Nash Bargaining Solution #flashcard

- Nash's Axioms:** invariance to equivalent utility representations, symmetry, Pareto efficiency, independent of irrelevant alternatives

- **Nash Bargaining Solution:** (in a 2 player bargaining game) there is a unique bargaining solution $f^N : \mathbb{B} \rightarrow \mathbb{R}^2$ satisfying Nash's 4 axioms and f^N satisfies:

$$f^N(S, d) = \arg \max_{d \leq s \in S} (s_1 - d_1)(s_2 - d_2)$$

- i.e. maximise the product of the difference in payoff between the bargaining solution and the disagreement (outside) option

Rubinstein's (1982) Model of Alternating-Offer Bargaining

- Setup:
 - Rules:
 - 2 player dividing a pie of size 1
 - make offer alternatively (one in each round)
 - acceptance ends the bargaining
 - no limit on # rounds (infinite game)
 - Outcome:
 - either agreement (x, t) or perpetual disagreement D
 - Preference:

$$U_i(x_i, t) = \delta_i^t x_i; U_i(D) = 0$$

- Information: complete information (structure, preference, history)
- Notations:

$$\begin{cases} \text{P1's Proposed Division : } \{x_1 \text{ for P1, } x_2 \text{ for P2}\} \\ \text{P2's Proposed Division : } \{y_1 \text{ for P1, } y_2 \text{ for P2}\} \end{cases}$$

#flashcard

- Results:
 - For NE \implies no unique solution:
 - Proposing any feasible split in period 0 and the counterparty accepting is a NE.
 - The proposer can raise any proposal and make threats for future periods
 - There are NEs with delayed agreements.
 - Just rejecting every offer at period 0, and play the same strategy as above in later periods.
 - For SPNE (no non-credible threats) \implies unique solution:
 - **Rubinstein's Theorem:**
 - the *unique SPNE* involves:

$$\begin{cases} \text{Player 1 always propose } x^* = \{x_1^*, x_2^*\} \text{ and accepts } y \iff y_1 \geq y_1^* \\ \text{Player 2 always propose } y^* = \{y_1^*, y_2^*\} \text{ and accepts } x \iff x_2 \geq x_2^* \end{cases}$$

where x^*, y^* are the unique solution to the **indifference conditions**:

Payoff at Accepting This period = Discounted Payoff Accepting Next Period

$$\iff \begin{cases} y_1^* = \delta_1 x_1^* & \text{for P1} \\ x_2^* = \delta_2 y_2^* & \text{for P2} \end{cases}$$

or more generally:

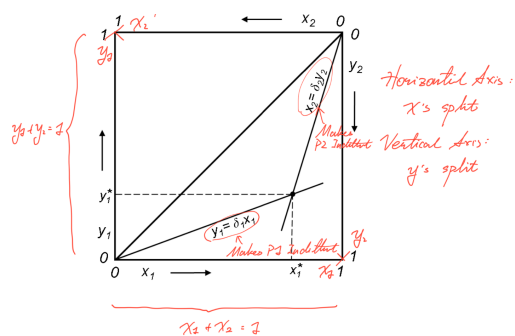
$$\begin{cases} u_1(y_1^*) = \delta \mathbb{E}[u_1(x_1^*)] & \text{for P1} \\ u_2(x_2^*) = \delta \mathbb{E}[u_2(y_2^*)] & \text{for P2} \end{cases}$$

and the SPNE outcome is that the first mover player 1 proposes x^* at $t = 0$ and player 2 accepts. Since there's no delay, the SPNE will be Pareto Efficient.

- Being the first-mover or the more impatient person will be advantageous.
- Together with the constraints $x_1^* + x_2^* = y_1^* + y_2^* = 1$, we can solve for explicit solutions:

$$\begin{cases} x_1^* = \frac{1-\delta_2}{1-\delta_1\delta_2} \\ x_2^* = \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2} \end{cases} \text{ and } \begin{cases} y_1^* = \frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2} \\ y_2^* = \frac{1-\delta_1}{1-\delta_1\delta_2} \end{cases}$$

- Draw in an Edgeworth Box:

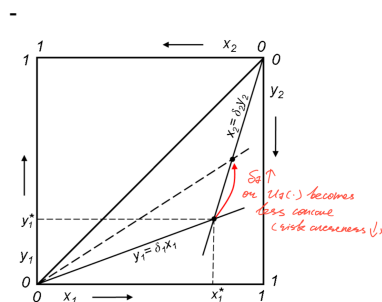


- Importance of complete information:

- A player who is uncertain about the opponent's preference might find it optimal to use some strategies to get information about the opponent.

How Patience and Risk-Averseness Affect the Outcome of Bargaining? #flashcard

- In Rubinstein's model: *being more patient / less risk-averse will offer an advantage:*



- Since NBS is the limit of Rubinstein SPE, the same result applies to NBS.

Finite Version of Rubinstein's Model of Bargaining

- Suppose bargaining can last for at most T rounds. If there is no agreement at T th round, both get 0. #flashcard
- This can be solved by backward induction: $\forall t$, the unique SPE is to accept any proposed offer:
 - In the T th (last) round, SPE: proposer offers responder 0, and responder accepts.
 - In the $(T - 1)$ th round, SPE: proposer offers s.t. responder is indifferent between accepts and rejects, and the responder accepts (indifference conditions hold)
 -
- We can show that: if $\delta_1 = \delta_2 = \delta$, then for any T , the SPE payoff of the 1st proposer is:

$$\sum_{t=0}^{T-1} (-\delta)^t \rightarrow \underbrace{\frac{1}{1+\delta}}_{\text{Infinite Ver Result}} \quad \text{as } T \rightarrow \infty$$

Relationship between Rubinstein SPE and NBS #flashcard

- Equivalence 1: Infinite Horizon, No Discounting**
- Keep the settings of the Rubinstein's model, but set:

$$\delta_1 = \delta_2 = \delta \rightarrow 1 \text{ (approaching no discounting)}$$

- Let:

$$U_i(x_1, t) = \delta^t u_i(x_i) \text{ (strictly increasing and weakly concave)}$$

with:

$$u_i(0) = 0, U_i(D) = 0$$

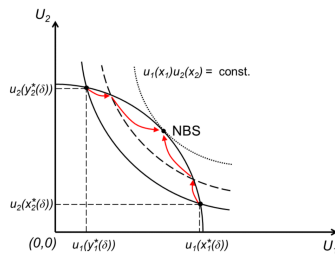
- The unique SPNE is Pareto efficient and satisfies the following Indifference Conditions:

$$\text{Payoff at Accepting This period} = \text{Discounted Payoff Accepting Next Period} \iff \begin{cases} u_1(y_1^*(\delta)) &= \delta u_1(x_1^*(\delta)) \\ u_2(x_2^*(\delta)) &= \delta u_2(y_2^*(\delta)) \end{cases}$$

- As $\delta \rightarrow 1$, both $x^*(\delta), y^*(\delta) \rightarrow$ Nash Bargaining Solution corresponding to $d = (0, 0)$

- Intuition: Rubinstein's model offers the first-mover an advantage, but that diminishes as $\delta \rightarrow 0$, leading to a symmetric solution as NBS.

- Graph:



• Equivalence 2: Risk of Breakdown $\rightarrow 0$

- Introduce an exogenous risk of breakdown (bargaining stops and everyone gets 0) with probability q in each round
- Indifference Conditions:

$$\begin{cases} x_2^* &= (1-q)y_2^* + qd_2 \\ y_1^* &= (1-q)x_1^* + qd_1 \end{cases}$$

- *As $q \rightarrow 0$, both $x^*(\delta), y^*(\delta) \rightarrow$ Nash Bargaining Solution corresponding to $d = (0, 0)$
- Breakdown risk plays the role of discounting in this setup.