Andrea Chiavari - Growth Accounting and Misallocation

Growth Accounting

Within-Country Growth Accounting

Neoclassical Growth Accounting Framework (within Country Growth Decomposition)

- Start from the Aggregate Production Function:

$$Y_t = A_t F(K_t, L_t)$$

  - where A_t is the Total Factor Productivity / Solow Residual
- Bring the framework into data #flashcard
- **Manipulation**:
  1. Total Differencing:

$$dY_t = F(.)dA + \frac{\partial A_t F(.)}{\partial K_t} dK_t + \frac{\partial A_t F(.)}{\partial L_t} dL_t$$

  2. Converted into Percentage Changes g and Elasticities ϵ:

$$\underbrace{\frac{dY_t}{Y_t}}_{\equiv g_Y} = \frac{A_t \cancelto{1}{F(.)}}{\cancel{Y_t}} \underbrace{\frac{dA_t}{A_t}}_{\equiv g_A} + \underbrace{\frac{\partial A_t F(.)}{\partial K_t} \frac{K_t}{Y_t}}_{\equiv \epsilon_K} \underbrace{\frac{dK_t}{K_t}}_{\equiv g_K} + \underbrace{\frac{\partial A_t F(.)}{\partial L_t} \frac{L_t}{Y_t}}_{\equiv \epsilon_L} \underbrace{\frac{dL_t}{L_t}}_{\equiv g_L}$$
$$g_Y = g_A + \epsilon_K g_K + \epsilon_L g_L B$$

  3. Rearrange to get the Residual Expression:

$$\boxed{g_A = g_Y - \epsilon_K g_K - \epsilon_L g_L}$$

- **Estimation**:
  - Measuring Input Growth Rates:
  - Measuring Aggregate Capital using the Perpetual Inventory Method:

$$K_t = \underbrace{(1-\delta)K_{t-1}}_{\text{Capital after Depreciation}} + \underbrace{\frac{_{t-1}}{P_{t-1}}}_{\text{Real Investment}}$$

  Iterate backward for a given K_0 (which vanishes in the long term):

$$K_t = \sum_{i=1}^{T-1} (1-\delta)^i \frac{_{t-i}}{P_{t-i}} + (1-\delta)^{T-1} K_0$$

  - From this, we can calculate $g_K \equiv \frac{dK_t}{K_t}$
  - Measuring Aggregate Labour is easy from national statistics
  - Measuring Output Elasticity of Inputs:
  - Output Elasticity of Labour: from firm's FOC for labour:

$$\frac{\partial A_t F(K_t, L_t)}{\partial L_t} = W_t$$
$$\underbrace{\frac{\partial A_t F(K_t, L_t)}{\partial L_t} \frac{L_t}{Y_t}}_{\equiv \epsilon_L} = \underbrace{\frac{W_t L_t}{Y_t}}_{\text{abour Share}}$$
$$\epsilon_L = \frac{W_t L_t}{Y_t}$$

  which is easy to estimate from data

- Output Elasticity of Capital: same as above:

$$\epsilon_K = \frac{R_t K_t}{Y_t}$$

but this is hard to measure, so we choose to back it up:
- from CRS $\implies$ zero profits:

$$\begin{aligned} \Pi_t &= 0 \\ \implies Y_t &= R_t K_t + W_t L_t \\ 1 &= \underbrace{\frac{R_t K_t}{Y_t}}_{\epsilon_K} + \underbrace{\frac{W_t L_t}{Y_t}}_{\epsilon_L} \\ \implies \epsilon_K &= 1 - \epsilon_L \end{aligned}$$

- **Limitations**:
  - This framework has poor measurement of the quality of inputs
    - Ignore improvement in capital/labour quality
- **Empirical Evidence**:
  - Baseline framework: TFP accounts for 70% of growth
  - Adjusted for input qualities: TFP still accounts for 1/3 to 1/2 of growth.

Cross-Country Development Accounting

Neoclassical Development Accounting Framework (Cross-Country Comparison)

- Start from a Parameterised Aggregate Production Function (Labour Augmenting Cobb-Douglas PF):

$$Y_{it} = K_{it}^{\alpha} \left(A_{it} \times \underbrace{e^{\phi(E_{it})} L_{it}}_{H_{it}} \right)^{1-\alpha}$$

  where:
  - K_{it} is the stock of capital
  - H_{it} is human-adjusted labour factor:

$$H_{it} = e^{\phi(E_{it})} L_{it}$$

  - A_{it} is TFP
- Bring this framework into data #flashcard
- **Manipulation**:
  - Write the PF in terms of output per worker

$$y_{it} \equiv \frac{Y_{it}}{L_{it}}$$

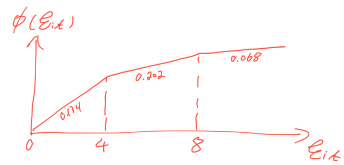
  - Rearrange to get:

$$\boxed{A_{it} = \frac{y_{it}}{\left(\frac{K_{it}}{Y_{it}}\right)^{\frac{\alpha}{1-\alpha}} e^{\phi(E_{it})}}}$$

- **Estimation**:
  - Capital-output Ratio $\frac{K_{it}}{Y_{it}}$ and Output per Worker y_{it} can be directly calculated from Penn World Table
  - Human capital can be mapped from year of schooling:

$\mathcal{E} \mapsto w$

- To construct $e^{\phi(E_{it})}$ we use Barro and Lee data plus information on returns of education
- To construct the returns we do
  Avg. Human Capital
  - For the first 4 year: 13.4%
  - For years from 4 to 8: 10.1%
  - For year from 9 onward: 6.8%



- Thus, our human capital function becomes
  - if $E_{it} \leq 4$
    $$\phi(E_{it}) = 0.134 \times E_{it}$$
  - if $4 < E_{it} \leq 8$
    $$\phi(E_{it}) = 0.134 \times 4 + 0.101 \times (E_{it} - 4)$$
  - if $E_{it} > 8$
    $$\phi(E_{it}) = 0.134 \times 4 + 0.101 \times 4 + 0.068 \times (E_{it} - 8)$$

- α is assumed to be around 1/3

- **Empirical Evidence**: difference in TFP A_{it} is the main driver of difference in output per worker

Misallocation

≈### International Capital Misallocation

Prediction of an Efficient International Capital Market on MRPK #flashcard

- Gross return on capital in country i at time t:

$$R_{it}^{k} = \underbrace{MRPK_{it}}_{\text{arginal Revenue Product of Capital}} + \underbrace{(1-\delta)\frac{P_{it+1}^{k}}{P_{it}^{k}}}_{\text{Capital Gain after Depreciation}}$$

where the Marginal Revenue Product of Capital is

$$MRPK_{it} = \frac{P_{it}^{y} MPK_{it}}{P_{it}^{k}}$$

- A well-functioning international capital market would imply:

$$R_{it}^{k} > R_{jt}^{k} \implies K_{jt} \to K_{it}$$

- Capital will flow to countries with higher returns to arbitrage.
- This will lead to the equalisation of returns across countries:

$$R_{it}^{k} = R_{jt}^{k} \; \forall \, i, j, t$$

- Assuming constant price level $\frac{P_{it+1}^{k}}{P_{it}^{k}} \approx 1$, we have:

$$MRPK_{it} = MRPK_{jt} \; \forall \, i, j, t$$

Further Refinement on Capital Return Accounting from Caselli and Feyrer: MPKN, MPKL, PMPKN, PMPKL #flashcard

- Measure from Caselli and Feyrer:
- **MPKN (Naive MPK)**:

$$\underbrace{(1-\alpha_L)}_{\text{Capital Share}} \underbrace{\frac{Y}{K}}_{\text{Real utput to Capital Ratio}}$$

- where α_L is the labour share
- **MPKL (MPK Land and natural resources corrected)**:

$$\kappa(1-\alpha_L)\frac{Y}{K}$$

where κ is the percentage of reproducable capital (i.e. capital excluding land and natural resources) among all capital
- **PMPKN (Price corrected MPKN)**:

$$(1-\alpha_L)\frac{P^yY}{P^kK} = \frac{P^y}{P_K} \times MPKN$$

which is MPKN adjust for the relative price of capital relative to labour $\frac{P^y}{P^k}$
- **PMPKL (Price corrected MPKL)**:

$$\kappa(1-\alpha_L)\frac{P^yY}{P^kK}$$

which is MPKL adjusted for the relative price of capital relative to labour $\frac{P^y}{P^k}$
- Result: *the most refined version of capital return (PMPKL) is roughly the same across all countries, suggesting a reasonable efficient international capital market*

Misallocation within a Country

Here, we try to study the mechanism in which allocation of capital drives TFP.

Empirical Findings on Firm Sizes across Countries #flashcard

- In richer countries, firms
  - have larger sizes
  - grow faster
- This might be a result of:
  - cross-country difference in firm-level productivity
  - cross-country difference in input allocations

Toy Model of Misallocation

- 2 types of intermediate output:

$$Y_s, Y_c$$

- Aggregate output:

$$y = Y_s^{0.} Y_c^{0.}$$

- 2 firms with PF:

$$\begin{cases} Y_s & = A_s L_s \\ Y_c & = A_c L_c \end{cases}$$

  - Assume same productivity across firms:

$$A_s = A_c = \overline{A}$$

- Resource constraint:

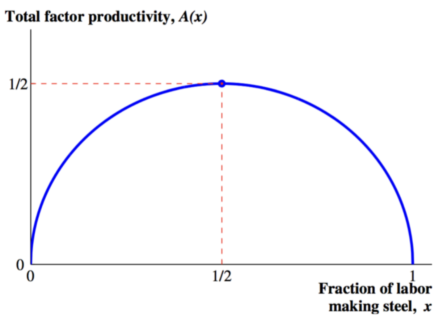
$$L_s + L_c = \overline{L}$$

- Key insights: #flashcard
  - Define:

$$x \equiv \frac{L_s}{\overline{L}}$$

- Solve for aggregate output:

$$
\begin{aligned}
Y &= \overline{A}\left(\frac{L_s}{\overline{L}}\right)^{0.}\left(\frac{L_c}{\overline{L}}\right)^{0.}\overline{L} \\
&= \underbrace{\overline{A}x(1-x)\ }_{TFP:\ A(x)}\overline{L}
\end{aligned}
$$

- $\implies$ Allocation affects TFP!
- 

The Indirect Approach to Misallocation

- Monopolistic competitive goods market:
  - Final output is a CES aggregate of M differentiated products:

$$
Y = \left(\sum_{i=1}^{M} Y_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \sigma > 1
$$

  - Each individual product is produced by a firm with CDPF:

$$
Y_i = A_i K_i^{\alpha} L_i^{1-\alpha}, 0 < \alpha < 1
$$

    - Note that the productivity A_i is allowed to be different across firms.
    - Each firm acts as a monopolist in its market.
  - There is a tax τ_i imposed on each firm $\implies$ firm's profit is:

$$
\pi_i = (1-\tau_i)P_i Y_i - wL_i - rK_i
$$

- Results: #flashcard
- Derivation skipped for now, similar version in PS #notes/tbd
  - A general result:

$$
TFPR_i = \left(\frac{MRPK_i}{\alpha}\right)^{\alpha}\left(\frac{MRPL_i}{1-\alpha}\right)^{1-\alpha}
$$

  - No distortion/misallocation benchmark ($\tau_i = 0$)
  - Result: constant TFPR across firms

$$
\begin{aligned}
TFPR_i &= \left(\frac{MRPK_i}{\alpha}\right)^{\alpha}\left(\frac{MRPL_i}{1-\alpha}\right)^{1-\alpha} \\
&= \overline{TFPR}\ \forall\, i
\end{aligned}
$$

  - Distortion/misallocation case ($\tau_i \ \ 0$, varies by firms):
  - Result:
  - Positive τ_i (tax) makes the firm size too small; negative τ_i (subsidy) makes the firm size too large.
  - Distorted MRPK, MRPL:

$$
\begin{cases}
MRPL_i &= w \cdot \frac{\sigma}{(1-\tau_i)(\sigma-1)} \\
MRPK_i &= r \cdot \frac{\sigma}{(1-\tau_i)(\sigma-1)}
\end{cases}
$$

- This results in different TFPR across firms:

$$TFPR_i = \left(\frac{MRPK_i}{\alpha}\right)^{\alpha} \left(\frac{MRPL_i}{1-\alpha}\right)^{1-\alpha} \quad TFPR_j$$

- Key insight: *with no tax/subsidy distortion i.e. allocation, TFPR will be the same for all firms; with firm-specific tax/subsidy distortion, TFPR varies across firms* $\implies$ *Dispersion of TFPR is a measure of misallocation in an economy.*
- Empirical findings:
  - Measure TFPR by:

$$TFPR_i = P_i A_i = \frac{P_i Y_i}{K_i^{\alpha} L_i^{1-\alpha}}$$

    assuming $\alpha = 0.33$, the rest variables are available from data.
  - Misallocation accounts for around 1/3 of TFP difference between China and the US, and around 1/2 for India.
- Problems and limitations:
  - Measurement error
  - Adjustment costs
  - Unobserved investments (e.g. RnD)
  - Within-industry variations in technology (e.g. capital intensities)

Misallocation Causal Identification (Alternative Methods to Measure Misallocation)

- Bau and Matray 2003 study a policy that liberalised FDI in some sectors
- With imperfect domestic capital market, smaller sizes of firms can be the result of limited capital access. FDI may be a relief.
- Method and findings #flashcard
- **Conceptual Framework**:
  - Firm i has profit:

$$\pi_i = P_i F(K_i, L_i, M_i) - \sum_{x \in \{K,L,M\}} (1+\tau_i^x) P^x x_i$$

  - FOC:

$$x \in \{K, L, M\} : MRPx_i \equiv \underbrace{P_i \frac{\partial F(K_i, L_i, M_i)}{\partial x_i}}_{MR} = \underbrace{(1+\tau_i^x) P^x}_{MC}$$

  - This implies, if a firm has less than optimal capital $\iff$ it is taxed:

$$K_i < K^* \iff MRPK_i > P^K \iff \tau_i^K > 0$$

  - *Policies providing additional capital to such firms $K_i \uparrow$ and hence reduce $MRPK_i \downarrow$ will help reduce misallocation*.
  - Aggregate Productivity Change:

$$\Delta \log TFP_{It} = \sum_{i \in I} \lambda_i \Delta \log A_i + \sum_{i \in I, \in \{M,L,K\}} \lambda_i \alpha \left(\frac{\tau_i}{1+\tau_i}\right) \Delta \log{}_i$$

    where:
    - i is a firm in an industry I
    - λ_i is the share of firm i's output in the industry
    - α is the relative importance of input
    - τ_i is the tax imposed on input
  - we can see: for $\tau_i > 0$, $\Delta \log{}_i > 0 \rightsquigarrow \Delta \log TFP_{It} > 0$
- **Empirical Strategy**:
  - Regression:

$$y_{ijt} = \beta_1 \text{Reform}_{jt} + \beta_2 \text{Reform}_{jt} \times \text{igh RPK}_i + {}_{it} + \theta_i + \delta_t + \epsilon_{ijt}$$

where:

- Reform_{jt} is 1 if the FDI liberalisation affected industry j
- igh RPK_i is 1 if the firm has high MRPK before the reform, indicating capital deficiency
- ${}_{it}$ are controls
- θ_i, δ_t are FEs
- *Key variable of interest: β_2 which captures the additional effect of the reform on previously high MRPK firms compared with other affected firms*
- We found β_2 significantly >0, which indicates that the liberalisation of FDI provided additional capital to firms with high MRPK $\iff$ insufficient capital, reducing the misallocation.
- Meanwhile, β_1 is not significantly different from 0, which means the reform did not affect other firms.
- Also, there is no significant effect on each individual firm's TFP A_i. Similarly, $\Delta \log A_i = 0, \Delta \log M_i = 0, \Delta \log L_i = 0,$ which implies:

$$\Delta \log TFP_{It} = \sum_{i \in I} \lambda_i \alpha^K \left(\frac{\tau_i^K}{1 + \tau_i^K} \right) \Delta \log K_i$$

and $\Delta \log TFP_{It}$ is indeed >0 as we expected. *Policies that improve allocation of resources can generate substantial aggregate TFP gains!*