Andrea Ferrero - Monetary Policy (NK Models)

NK Model Derivations Outline #flashcard

- Building blocks
  - Starting from Standard RBC
  -
    - Household Side: Add Money $\rightsquigarrow$ Real and Nominal Variables (An RBC Model with Money)
  -
    - Firm Side: Monopolistic Competition $\rightsquigarrow$ Inefficiency (Monopolistic Competition)
  -
    - Firm Side: Calvo-Style Price Rigidity $\implies$ Breaks classical dichotomy! (Sluggish Price Adjustment)
- $\dashrightarrow$ Log-linearisation around the ESS (RBC SS) (General Equilibrium and Gap Representation)
  - $\dashrightarrow$ Gap representation $\implies$ AD/AS(PC)
  - + Monetary Rule $\implies$ NK 3-Equation Model (AD/AS/MR)

The Real Effect of Monetary Policy: Theory

An RBC Model with Money

Solving the Money-in-Utility Household Optimisation in RBC Type Models

Example:

$$\max_{C_t,I_t,B_t,M_t,N_t,K_t} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\ln C_t+\chi\ln\frac{M_t}{P_t}-\frac{N_t^{1+\psi}}{1+\psi}\right)\right]$$

subject to:

- Budget Constraint:

$$P_tC_t+P_tI_t+B_t+M_t=(1+i_{t-1})B_{t-1}+M_{t-1}+R_t^kK_{t-1}+W_tN_t-T_t$$

- Capital Accounting Equation:

$$K_t=(1-\delta)K_{t-1}+I_t$$

#flashcard

- Set up Lagrangian:

$$L=\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left\{\left(\ln C_t+\chi\ln\frac{M_t}{P_t}-\frac{N_t^{1+\psi}}{1+\psi}\right)-\lambda_t\left[P_tC_t+P_tI_t+B_t+M_t-(1+i_{t-1})B_{t-1}-M_{t-1}-R_t^kK_{t-1}-W_tN_t+T_t\right]\right\}-\mu$$

1. Calculate the FOC wrt $C_t, I_t, B_t, M_t, N_t, K_t$
2. FOCs wrt flow variables (C_t, I_t) are used to obtain expressions for co-state variables λ_t, μ_t
   - FOC wrt C_t gives λ_t:

$$\lambda_t=\frac{1}{P_tC_t}$$

   - FOC wrt I_t gives μ_t:

$$\mu_t=\lambda_tP_t$$

3. Then substitute them into FOCs wrt stock variables B_t, M_t, K_t to get corresponding Euler Equations

- Euler Equation for Bond focuses on Nominal Interest Rate i_t:

$$\frac{1}{C_t} = \beta(1+i_t)\mathbb{E}_t\left[(\pi_{t+1})^{-1}\frac{1}{C_{t+1}}\right]$$

- Euler Equation for Capital focuses on Real Return on Capital $r^k_{t+1} \equiv \frac{R^k_{t+1}}{P_{t+1}}$:

$$\frac{1}{C_t} = \beta\mathbb{E}_t\left[\left(r^k_{t+1}+1-\delta\right)\frac{1}{C_{t+1}}\right]$$

- Euler Equation for Money focuses on the Real Money Demand / Demand for Real Balances $m \equiv \frac{M_t}{P_t}$:

$$\frac{1}{C_t} = \frac{\chi}{\frac{M_t}{P_t}} + \beta\mathbb{E}_t\left[(\pi_{t+1})^{-1}\frac{1}{C_{t+1}}\right]$$

- In all those equations, $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$

4. Similarly, FOC wrt N_t yields the Labour Supply Equation, which focuses on the Real Wage $\frac{W_t}{P_t}$:

$$\frac{W_t}{P_t} = N_t^{\psi}C_t$$

5. Combine Euler Equation for Money and Bonds to get link $m \equiv \frac{M_t}{P_t}$ and i_t, C_t and get the Real Money Demand / Demand for Real Balances:

$$m \equiv \frac{M_t}{P_t} = \chi\frac{1+i_t}{i_t}C_t$$

   - Real Money Demand is increasing in consumption C_t and decreasing in nominal interest rate i_t

- General intuition: *FOCs for flow variables are used to get expressions for Lagrangian Multipliers; FOCs for stock variables are used to get Euler Equations focusing on their returns*.
- Note: *the "asset pricing equation" of a variable comes from the FOC wrt it*

Consolidated Government Budget Constraint #flashcard

- Consolidated Government Budget Constraint:

$$\underbrace{(1+i_{t-1})B_{t-1}+G_t}_{\text{otal Go xpenditure}} = \underbrace{B_t+T_t+(M_t-M_{t-1})}_{\text{otal Go(+CB) Reenue}}$$

```
- Be aware that here $T_{t}$ is the tax -- sometimes it could be transfer/subsidy, and that will have the opposite sign.
```

Factor Demand for Cobb-Douglas Production Function

- Firm with Cobb-Douglas PF maximises real output in competitive markets:

$$\max_{K_t,L_t} Y_t - r_tK_t - w_tL_t$$

subject to CDPF:

$$Y_t = A_tK_t^{\alpha}L_t^{1-\alpha}$$

#flashcard

- Constant Cost Shares:

$$\begin{cases} \frac{r_tK_t}{TC} & = \alpha \\ \frac{w_tL_t}{TC} & = 1-\alpha \end{cases}$$

- Return on Inputs:

$$\begin{cases} r_t & = \frac{\alpha Y_t}{K_t} \\ w_t & = (1-\alpha)\frac{Y_t}{L_t} \end{cases}$$

- Use known result for Cobb-Douglas cost function:

$$\boxed{TC(Y_t) = \frac{Y_t}{A_t}\left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}}$$

- Plug the Cost Function into share-based input demands:

$$K^* = \frac{\alpha TC(Y_t)}{r_t}, \quad L^* = \frac{(1-\alpha)TC(Y_t)}{w_t}$$

- $\implies$ Conditional Factor Demand:$$\begin{align}
  K^* & = \frac{\alpha}{r} \cdot \frac{\bar{Y}}{A} \cdot \left(\frac{r}{\alpha} \right)^\alpha \left(\frac{w}{1 - \alpha} \right)^{1 - \alpha} \

L^* & = \frac{1 - \alpha}{w} \cdot \frac{\bar{Y}}{A} \cdot \left(\frac{r}{\alpha} \right)^\alpha \left(\frac{w}{1 - \alpha} \right)^{1 - \alpha}
\end{align}$$

Ricardian Equivalence #flashcard

- With lump-sum taxes and no financial friction, the way to finance government spending does not matter.
- Generally:
  - It holds if there's perfect foresight, infinite horizons, no liquidity constraints, lump-sum taxes
  - It breaks if agents are credit-constrained, have finite lives, face distortionary taxes, or expect future spending changes

Neutrality and Classical Dichotomy

Classical Dichotomy, Super Neutrality of Money, and Effects of Monetary Policy with Super Neutrality #flashcard

- In standard RBC models without nominal rigidities / non-separable MIU / CIA / financial frictions, we have the classical dichotomy and the super neutrality of money.
- **Classical Dichotomy**: theoretical separation between **real variables** and **nominal variables** in the economy.
- Classical Dichotomy $\implies$ **Super Neutrality of Money**: steady state real variables $\{C, K, N, I, Y, r, w, r^k\}$ are independent of the *level or the growth rate of money*.
  - $\implies$ **Neutrality of Money**: real variables are independent of the *level/amount of money*.
- Monetary policy affects:

$$\{m \text{ (real uantity of money)}, i \text{ (nominal interest rate)}, P \text{ (price leel)}, \text{(inflation rate)}\}$$

  - If MP is specified in terms of quantity of money:

$$M = \overline{M}$$

  then nominal price level will be fixed with constant inflation rate $= 1$
  - If MP is specified in terms of nominal interest rate:
  - Targeting constant inflation rate $= 1$ $\implies$ the price level and nominal money quantity will be indeterminate.
  - Targeting a nominal interest rate peg e.g. $i = 0$ $\implies$ inflation rate and price level indeterminate.
  - Using a feedback rule satisfying the Taylor Principle $\implies$ inflation will be determinate at the target.

Price Level and Inflation Determination

Some derivations have not been reviewed in this section!!! #notes/skipped

Friedman Rule #flashcard

- In short: *MB=MC argument: the opportunity cost of holding money faced by private agents should equal the social cost of creating additional fiat money. Assuming that Social MC of creating additional money is zero (or approximated by zero), then the MB --- nominal rates of interest should also be zero. With 0 nominal IR, it's optimal to have deflation in SS.*
- **Friedman (1969) Argument for Deflation**:
  - social marginal cost of producing money is approximately zero
  - $\Longrightarrow$ price of money should be zero
  - $\Longrightarrow$ the price of money is the opportunity cost of not earning the nominal interest rate
  - $\Longrightarrow$ nominal interest rate $i^* = 0$
  - $\Longrightarrow$ in steady state $i^* = 0 \iff {}^* = \beta < 1$ (optimal rate of inflation should be negative / deflation is optimal)
- **Limitation** of this deflationary argument:
  - If $i = 0$, then the money demand equation implies infinite real money balances $m \to \infty$.
  - Inflation can act as an optimal tax to finance government spending if other taxes are distortionary.
  - *If there is ZLB on nominal IR, optimal rate of inflation is typically positive to provide room for MP*.
  - See through the **Fisher Equation**:

$$i = r + \pi$$

  - In LR, we need $\pi \uparrow$ if r^* is low, in order to push up i and provide room for MP to stimulate the economy in the case of recessionary shocks.
  - *If there's nominal rigidity, changing price level is costly (inefficient), and only 0-inflation is optimal (e.g. 3-Equation NK Model)*.

When Will Money Have a Real Effect in RBC-type Models? (Breaking Super Neutrality of Money) #flashcard

Mechanism	Real Effect?	How?
Sticky prices/wages	✅ Yes	Prices adjust slowly; monetary shocks move real interest rates
Money in utility (MIU)	✅ *Sometimes*	Money provides liquidity or utility directly; *there's real effect only when utility is non-separable in money or introduce nominal rigidity / CIA etc.*
Cash-in-advance (CIA)	✅ Yes	Money directly limits consumption
Financial frictions	✅ Yes	Money affects credit and redistribution
Information frictions	✅ Temporary	Agents misinterpret nominal shocks
Open economy dynamics	✅ Sometimes	Via real exchange rate and trade channels
Standard RBC (flexible prices)	❌ No	Money affects only nominal variables

Generated by ChatGPT

The Real Effects of Monetary Policy: Evidence

The Real Effects of Monetary Policy: Evidence #flashcard

- In the **LR**, money is neutral:
  - Correlation between money growth rate and inflation is around 1

    - Correlation between money growth and real GDP growth is around 0
- In the **SR**:
    - Hard to identify the effect of money because it's endogenous $\implies$ Require MP shocks
    - Narrative evidence:
        - XVIII century France devaluation of "Livres" coin $\rightsquigarrow$ large contraction in production
        - US abandoned gold standard and adopted expansionary MP in 1933 (Roosevelt New Deal) $\rightsquigarrow$ real economy recovery
        - Volcker Disinflation (contractionary MP) $\rightsquigarrow$ unemployment
        - Romer and Romer Dates (FOMC minutes shocks) $\rightsquigarrow$ real effects
    - Econometrics evidence
        - High-frequency identification: MP surprises in a narrow time window and VAR --- contractionary MP decreases output and prices (mildly)
            - Potential problem: central bank information effect
- **Summary**: *money is neutral in the LR but has real effects in the SR*

The Three-Equation New Keynesian Model

Monopolistic Competition

Derivations skipped for now #notes/skipped

Monopolistic Competition in Wholesale Market #flashcard

- Structure
    - Inputs $\rightarrow$ Wholesale Producers $\rightarrow$ Retailers
    - Perfectly competitive input market $\rightarrow$ Monopolistic wholesale market $\rightarrow$ Perfectly competitive retail market
- Output is composed of a **continuum of differentiated goods**:

$$Y_t = \left[\int_0^1 [Y_t(i)]^{\frac{\theta-1}{\theta}}\, di\right]^{\frac{\theta}{\theta-1}}$$

  where $\theta > 1$ is the elasticity of substitution among differentiated goods.
- **Competitive retailers** takes of differentiated goods $P_t(i)$ as given and minimise costs for a given output level:

$$\min_{Y_t(i)} \int_0^1 P_t(i)Y_t(i)\, di\; s.t.\; Y_t = \left[\int_0^1 [Y_t(i)]^{\frac{\theta-1}{\theta}}\, di\right]^{\frac{\theta}{\theta-1}}$$

- **Wholesale producers** set price and choose inputs to maximise *real* profit given the demand function for each good from the retailer and the CDPF (here we simplify the CDPF to have only labour):

$$\max_{P_t(i),N_t(i)} (1-\tau_t)\frac{P_t(i)Y_t(i)}{P_t} - \frac{W_tN_t}{P_t}\; s.t.\; Y_t(i) = f(Y_t), Y_t(i) = A_tN_t(i)$$

#flashcard
- Use a **backward induction logic**: retailer's cost minimisation problem $\implies$ demand function for each good $\rightarrow$ wholesale producer's profit maximisation problem
- Solving the **cost minimisation problem for the retailer**, we have the retailer's demand for each good i:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t$$

where P_t is the Aggregate Price Index:

$$P_t = \left[\int_0^1 [P_t(i)]^{1-\theta}\, di\right]^{\frac{1}{1-\theta}}$$

- Given that, we can solve the **profit maximisation problem for the producer** and get the FOC:

$$\frac{P_t(i)}{P_t} = \frac{1+\mu}{1-\tau_t} MC_t$$

  i.e. the optimal relative price of an individual good is a **markup** $\mu = \frac{1}{\theta-1}$ over the real marginal cost, and the restriction $\theta > 1 \implies$ markup is positive $\mu > 0$
  - With perfect competition: $\lim_{\theta\to\infty} \mu = 0$ there is no markup
  - We can use subsidies $\tau_t = -\mu < 0$ to eliminate markup $\rightsquigarrow$ efficient output
- **Key results**:
  - *With flexible prices but NO nominal rigidities, classical dichotomy still holds! We need additional frictions to let MP have real effects.*
  - In a symmetric equilibrium:

$$P_t(i) = P_t \,\forall\, i \iff MC_t = \frac{1-\tau_t}{1+\mu}$$

  - Allocation is inefficient (output below the social optimum), but we can use subsidies $\tau_t = -\mu < 0$ to eliminate markup $\rightsquigarrow$ efficient output

Sluggish Price Adjustment

Derivations skipped for now #notes/skipped

Sticky Prices: Staggered Price Setting (Calvo-style Price Rigidity)

- With monopolistic competition, we add the additional assumption that: in each period, a firm has to keep the price unchanged with probability $\alpha \; (0,1)$ #flashcard
- Wholesale producer's profit optimisation becomes *dynamic*:
  - It has to set price at t taking into account the probability of not being able to readjust prices in the future
- **Markup Behaviour**:
  - the desired markup is constant:

$$\mu = \frac{1}{\theta - 1}$$

  - However, the actual markup fluctuates over time due to:
    - Price stickiness (not all firms can adjust)
    - Shocks to marginal cost or demand
  - Actual aggregate markup is countercyclical in many models: when output is high, marginal cost rises, but sticky prices lag behind $\rightsquigarrow$ lower actual markup.
- **Price Dispersion Behaviour**
  - Because not all firms adjust prices at the same time, the economy exhibits **price dispersion**:

$$\Delta_t = \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-} di$$

  - This affects aggregate productivity: more dispersion $\implies$ less efficient allocation of resources.
    - Even with identical technologies, some firms are too cheap or too expensive.
  - Dispersion disappears ($\Delta_t = 1$) when all firms adjust prices each period ($\alpha = 0$).

- **Summary**
  -

Concept	Description
Price rigidity	Only $1-\alpha$ fraction of firms can reset prices each period
Optimal price	Chosen to maximize expected discounted profits over future periods
Markup	$\mu = \frac{\varepsilon}{\varepsilon-1}$, but actual markup varies due to rigidity
Price dispersion	$\Delta_t > 1$ when not all prices adjust; reduces allocative efficiency
Implication	Sticky prices → aggregate output responds to shocks → **real effects of nominal frictions**

General Equilibrium and Gap Representation

Derivations skipped for now #notes/skipped

Efficient Steady State (ESS) Benchmark (RBC) #flashcard

- The **Efficient Steady State (ESS)** is the real allocation (output, labor, consumption, etc.) that would occur if prices were fully flexible and no monopoly distortions existed — i.e., a perfectly competitive RBC model.
  - It serves as a benchmark for evaluating welfare losses from price stickiness and market power
  - We log-linearise the model around this efficient steady state to get the 3-Equation NK Model
- **Summary of Real Variables**:

Feature	Standard Steady State	Efficient Steady State
Markup	$\mu = \varepsilon/(\varepsilon-1) > 1$	$\mu = 1$
Price stickiness	$\alpha > 0$	$\alpha = 0$
Price dispersion	$\Delta = 1$ (if inflation = 0)	$\Delta = 1$
Welfare	Suboptimal	Maximized
Output & labor	Too low	Efficient

- **Summary of Nominal Variables**:

Variable	Efficient Steady State Value
Inflation π	0
Price Level P_t	Constant ($P_t = \bar{P}$)
Nominal Wages	Grow in line with productivity (real wage constant × P_t)
Nominal Interest Rate i	Equals real interest rate r (or possibly 0 under Friedman Rule)
Money growth	Constant or zero (depending on utility specification)

  - Note that we typically define $= 1 + \pi$, so in ESS it will be 1
- **Log-Linearised Model around the ESS**:
  - Everything is represented in terms of log-gaps around the ESS $\implies$ all variables should be 0 in ESS after log-linearisation.

Gap Representation in NK Model (LL BME/LME/PC) $\implies$ AD/AS(PC)

- Log-Linearised Bond Market Equilibrium (LLBME):

$$y_t = -\sigma^{-1}(i_t - \mathbb{E}_t[\pi_{t+1}]) + \mathbb{E}_t[y_{t+1}]$$

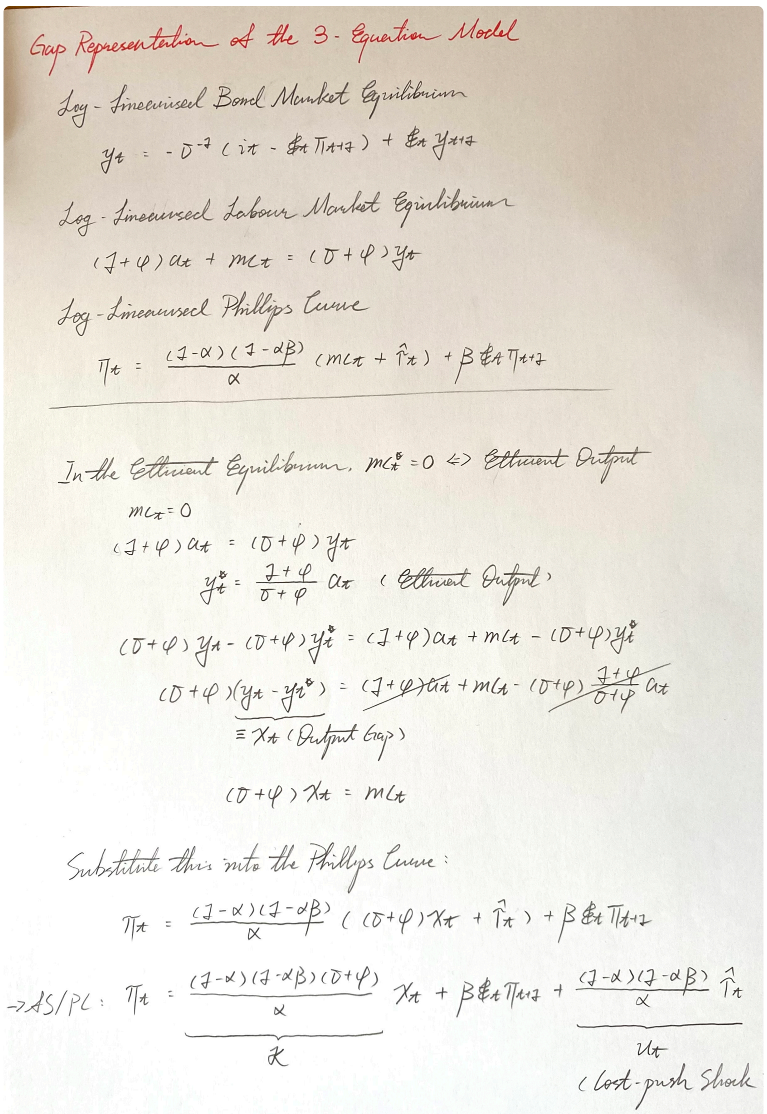
- Log-Linearised Labour Market Equilibrium (LLLME):

$$(1+\psi)a_t + mc_t = (\sigma+\psi)y_t$$

- Log-Linearised Philips Curve (LLPC):

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(mc_t + \tau_t) + \beta\mathbb{E}_t\left[\pi_{t+1}\right]$$

- Steps to obtain 3-Equations: #flashcard
  1. Use $mc^* = 0$ to get an expression of y^* in terms of the variable driving it (here it is a_t)
  2. Get AS (typically from Labour Market Equilibrium, Goods Market Clearing, and Production Function)
     1. Manipulate the LLLME to get an expression linking mc_t with x_t
     2. Replace mc_t in the Phillips Curve using that expression to get AS
  3. Get AD (typically from LLLBE/Euler Equation, and results above)
     - Manipulate c in LLLBE/Euler Equation to link y with the variable driving it
     - If only y, add and subtract y^*
- Example
  -



Gap Representation of the 3-Equation Model

Log-Linearised Bond Market Equilibrium

$$y_t = -\sigma^{-1}(i_t - \mathbb{E}_t\pi_{t+1}) + \mathbb{E}_t y_{t+1}$$

Log-Linearised Labour Market Equilibrium

$$(1+\varphi)a_t + mc_t = (\sigma+\varphi)y_t$$

Log-Linearised Phillips Curve

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(mc_t + \hat{\tau}_t) + \beta\mathbb{E}_t\pi_{t+1}$$

In the Efficient Equilibrium, $mc_t^* = 0 \Leftrightarrow$ Efficient Output

$$mc_t = 0$$

$$(1+\varphi)a_t = (\sigma+\varphi)y_t$$

$$y_t^* = \frac{1+\varphi}{\sigma+\varphi}a_t \quad \text{(Efficient Output)}$$

$$(\sigma+\varphi)y_t - (\sigma+\varphi)y_t^* = (1+\varphi)a_t + mc_t - (\sigma+\varphi)y_t^*$$

$$(\sigma+\varphi)\underbrace{(y_t - y_t^*)}_{\equiv x_t \text{ (Output Gap)}} = \cancel{(1+\varphi)a_t} + mc_t - \cancel{(\sigma+\varphi)\frac{1+\varphi}{\sigma+\varphi}a_t}$$

$$(\sigma+\varphi)x_t = mc_t$$

Substitute this into the Phillips Curve:

$$\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}((\sigma+\varphi)x_t + \hat{\tau}_t) + \beta\mathbb{E}_t\pi_{t+1}$$

$$\to AS/PC: \quad \pi_t = \underbrace{\frac{(1-\alpha)(1-\alpha\beta)(\sigma+\varphi)}{\alpha}}_{\kappa}x_t + \beta\mathbb{E}_t\pi_{t+1} + \underbrace{\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\hat{\tau}_t}_{u_t \text{ (Cost-push Shock)}}$$

  -

Use the similar method in the Bond-Market Equilibrium / Euler Equation + Market Clearing

$$y_t = -\sigma^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t y_{t+1}$$

$$\underbrace{y_t - y_t^*}_{\equiv x_t} + y_t^* = -\sigma^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t [\underbrace{y_{t+1} - y_{t+1}^*}_{\equiv x_{t+1}^*} + y_{t+1}^*]$$

$$\downarrow$$

$$x_t + \frac{1+\varphi}{\sigma+\varphi} a_t = -\sigma^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t \left[x_{t+1}^* + \frac{1+\varphi}{\sigma+\varphi} a_{t+1}\right]$$

$$x_t = -\sigma^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) + \frac{1+\varphi}{\sigma+\varphi}(\mathbb{E}_t a_{t+1} - a_t)$$

$\rightarrow$ AD/IS: $$x_t = -\sigma^{-1}\Big(i_t - \mathbb{E}_t \pi_{t+1} - \underbrace{\frac{\sigma(1+\varphi)}{\sigma+\varphi}(\mathbb{E}_t a_{t+1} - a_t)}_{r_t^*}\Big)$$

(Efficient Real Interest Rate)

† We can also choose use the formula $r_t^* = \sigma(\mathbb{E}_t y_{t+1}^* - y_t^*)$ without substituting for $y^*_{t \text{ or } t+1}$.

Together with the Monetary Rule:

$$i_t = \phi_\pi \pi_t + \phi_x x_t + \varepsilon_t$$

We got our 3-equation model.

AD/AS(PC) + MR $\Longrightarrow$ 3-Equation NK Model #flashcard

NOT YET FINISHED

SR/LR Money Neutrality in NK Models #flashcard

📋 **Summary Table**

Time Horizon	Price Flexibility	Monetary Policy Impact on Real Variables	Monetary Neutrality
Short Run	Incomplete (Calvo)	✅ Yes — affects output, employment	❌ Not neutral
Long Run	Full (all firms have reset)	❌ No — only affects nominal prices	✅ Neutral

---

🔁 **Contrast With RBC**

- **RBC models**: money is always neutral (no nominal frictions)
- **NK models**: money is **non-neutral in short run**, but **neutral in long run**
- **Only models with persistent nominal frictions or hysteresis** (e.g., some Post-Keynesian models) break long-run neutrality

Properties of the Three-Equation New Keynesian Model

The Transmission of Exogenous Shocks

Derivations skipped for now #notes/skipped

Transmission of 3-Types of Shocks in 3-Equation NK Model

- **3 Equations and 3 Shocks**
  - Aggregate Supply (AS) / Phillip's Curve (PC):

$$\pi_t = x_t + \beta \mathbb{E}_t [\pi_{t+1}] + u_t$$

  and Cost-Push Shocks:

  $$u_t = \rho_u u_{t-1} + \epsilon_{mt}$$

- Aggregate Demand (AD):

  $$x_t = -\sigma^{-1} i_t - \mathbb{E}_t[\pi_{t+1}] - \underbrace{\sigma(\mathbb{E}_t[y^*_{t+1}] - y^*_t)}_{r^*_t} + \mathbb{E}_t[x_{t+1}]$$

  where

  $$y^*_t = \frac{(1+\psi)a_t}{\sigma+\psi}$$

  and Productivity Shock:

  $$a_t = \rho_a a_{t-1} + \epsilon_{at}$$

- Monetary Rule:

  $$i_t = \phi_\pi \pi_t + \phi_x x_t + \epsilon_t$$

  and Monetary Policy Shock:

  $$\epsilon_t = \rho_m \epsilon_{t-1} + \epsilon_{mt}$$

- Shocks follow:

  $$\rho_j \quad [0,1), \epsilon_{jt} {}^{iid} N(0,1) \text{ for } j \quad \{m,a,u\}$$

- **SR Dynamics: Transmission of Shocks** #flashcard
  - **Transmission of a Positive Cost-Push/Markup/AS(PC) Shocks**
    - Assume the economy starts from a zero-inflation SS (all variable = 0)
    - A positive cost-push shock can be caused by temporary input price or markup pike
    - A positive cost-push shock means $u_t > 0$
    - Immediate effect: higher inflation:

      $$u_t > 0 \rightsquigarrow \pi_t \uparrow > 0$$

    - Effect on nominal IR through MR: CB fights higher inflation by increasing nominal IR:

      $$\pi_t \uparrow \rightsquigarrow i_t \uparrow$$

    - Effect on output gap through AD: higher nominal IR depresses real output, causing a negative output gap:

      $$i_t \uparrow \rightsquigarrow x_t \downarrow < 0$$

      - 2nd effect on inflation: negative output gap partially mitigates inflation
    - Effect on actual output: since the efficient level of real output y^*_t does not change, the actual real output falls with the output gap
    - Real IR increases less than nominal IR due to higher expected inflation
    - Summary: *in SR, inflation increases while real output decreases*.
  - **Transmission of a Positive Technology (Productivity) Shock**
    - Assume the economy starts from a zero-inflation SS (all variable = 0)
    - A positive technology shock means an increase in productivity $\epsilon_{at} > 0$
    - Immediate effect: higher efficient level of real output:

      $$\epsilon_{at} \uparrow \rightsquigarrow y^*_t \uparrow = \frac{(1+\psi)a_t \uparrow}{\sigma+\psi}$$

    - Effect on output gap through AD --- actual output will increase, but not as much as the increase in efficient level because some firms will not be able to adjust price, leading to a negative output gap:

$$y_t^* \uparrow + \text{Nominal Riidity} \rightsquigarrow x_t \downarrow < 0$$

        - Effect on inflation through AS(PC) --- a negative output depresses inflation:

$$x_t \downarrow < 0 \rightsquigarrow \pi_t \downarrow < 0$$

        - Effect on nominal IR through MR --- the central bank cut nominal IR to fight deflation and negative output gap:

$$x_t \downarrow < 0 \text{ and } \pi_t \downarrow < 0 \rightsquigarrow i_t \downarrow$$

          but not enough to mitigate the negative output gap completely
        - Summary: *in SR, real output increases while inflation decreases*.
    - **Transmission of a Contractionary Monetary Policy (MP) Shock $\epsilon_t > 0$**
        - Assume the economy starts from a zero-inflation SS (all variable = 0)
        - A contractionary monetary policy shock means an increase in the interest rate: $\epsilon_t > 0$
        - Immediate effect on nominal interest rate through MR curve --- higher nominal IR:

$$\epsilon_t > 0 \rightsquigarrow i_t \uparrow > 0$$

        - Effect on output gap through AD curve --- for given $\mathbb{E}_t[\pi_{t+1}]$, the real IR falls, pushing down the output gap.:

$$i_t \uparrow \rightsquigarrow \text{rR} = (i_t - \mathbb{E}_t[\pi_{t+1}]) \uparrow \rightsquigarrow x_t \downarrow < 0$$

        - Effect on inflation through AS curve --- negative output gap lowers current inflation:

$$x_t \downarrow \rightsquigarrow \pi_t \downarrow < 0$$

        - Note:
            - Since the efficient level of output y^* is unaffected, output falls exactly as the output gap.
            - MR rule will respond to the negative output gap by decreasing nominal IR, but since this is less than the shock itself, so overall still $i_t \uparrow$.
            - Real IR will increase more than nominal IR due to lower inflation expectation $\mathbb{E}_t[\pi_{t+1}] \downarrow$
        - Summary: *in SR, output decreases and inflation decreases*.
    - Note on expectation formation: here we assume *Rational Expectations*. With RE and AR(1) shocks, the expected next period variables will have the same sign as current period variables (shocks are persistent).
- **Summary**:

Summary of Transmission Mechanisms

Shock Type	Immediate Effect on x_t (Output Gap)	Effect on Inflation π_t	Summary
Contractionary MP Shock	Decreases x_t (output gap)	Decreases π_t	Higher interest rates reduce demand and output, leading to lower inflation.
Positive Technology Shock	Increases x_t (output gap)	Decreases π_t	Higher productivity increases output and reduces inflation.
Positive Cost-Push Shock	Decreases x_t (output gap)	Increases π_t	Higher costs raise inflation and reduce output.

- **LR Dynamics: Anchored Expectation is Important**
  - *If expected inflation remains anchored, in the following periods, the economy adjusts and all variables $\rightarrow 0$ in the LR.*

Interpreting AS(PC): Modelling Supply Side Changes/Shocks in 3-Equation NK Models

- Aggregate Supply (AS) / Phillip's Curve (PC):

$$\pi_t = x_t + \beta \mathbb{E}_t[\pi_{t+1}] + u_t$$

  and Cost-Push Shocks:

$$u_t = \rho_u u_{t-1} + \epsilon_{mt}$$

- When to model supply-side changes/shocks as changes in slope or shocks u_t?
- Whether more flexible prices $\iff$ a steeper PC is desirable? #flashcard
- Modelling as changes in slope :

- is the slope of the Phillips curve — it governs how responsive inflation is to the output gap. In canonical NK models:

$$= \frac{(1-\alpha)(1-\alpha\beta)(\sigma+\psi)}{\alpha}$$

  where:
    - α: Calvo price stickiness (probability of not being able to change price) $-$ slope of AS (decrease responsiveness)
    - β: discount factor $-$ slope of AS (decrease responsiveness)
    - σ: RRA slope of AS (increase responsiveness)
    - ψ: inverse Frisch elasticity of labor slope of AS (increase responsiveness)
- Modelling as shock u_t:
  - Use this when the supply shock is temporary or unexpected, such as: oil price spikes, one-time import cost shocks, wage increases due to strikes or regulation, fluctuations in pricing power (markup shocks), weather disruptions
  - These are modelled as shocks to u_t because they affect inflation directly, without changing the structural inflation-output trade-off.
- Summary:

Scenario	Shock to κ	Shock to u_t
Long-run structural reform	✅ Yes	❌ No
Technology makes pricing easier	✅ Yes	❌ No
Temporary oil price spike	❌ No	✅ Yes
Wage hike due to union negotiations	❌ No	✅ Yes
Global competition changes markup behavior	✅ Maybe (if permanent)	✅ Maybe (if short-term)

- Whether more flexible prices $\iff$ a steeper PC is desirable?
  - A steeper PC means that inflation is more responsive to output gap.
  - **Paradox of flexibility**:
  - In normal times, this makes it easier for CB to control inflation.
  - However, at ZLB, this make the economy more vulnerable to negative demand shocks, yet it also makes it easier for a positive cost-push shock to pull out the economy out of ZLB.

Interpreting AD in 3-Equation NK Models

- Aggregate Demand (AD):

$$x_t = -\sigma^{-1}\underbrace{\underbrace{i_t - \mathbb{E}_t\left[\pi_{t+1}\right]}_{\text{ctual RR}} - \underbrace{\sigma(\mathbb{E}_t\left[y^*_{t+1}\right] - y^*_t)}_{\text{fficient RR } r^*_t}\qquad}_{\text{RR Gap}} + \mathbb{E}_t\left[x_{t+1}\right]$$

  - The output gap moves in the same direction as *variables in red*, and in the opposite direction as **variables in blue**.
  - where y^*_t is the Efficient Level of Real Output:

$$y^*_t = \frac{(1+\psi)a_t}{\sigma+\psi}$$

  and Productivity Shock:

$$a_t = \rho_a a_{t-1} + \epsilon_{at}$$

- What drives the dynamics of this equation? #flashcard
  - **Intertemporal Elasticity of Substitution Parameter / Inverse of RRA** $\frac{1}{\sigma}$:
  - Role: Controls how responsive the output gap x_t is to deviations of the real interest rate from its efficient (natural) level.
  - Interpretation: A higher RRA $\sigma \iff$ lower IES $\implies$ less responsive consumption (and hence x_t) to real interest rate changes.

- Slope: IES / Inverse of RRA $\frac{1}{\sigma}$ is the slope coefficient in front of the real interest rate gap:

$$\text{slope} = \frac{\partial x_t}{\partial \text{ RR Gap}} = -\frac{1}{\sigma}$$

- A larger RRA σ flattens the AD curve (less sensitive output to interest rate gap), and vice versa.
- The **Real Interest Rate Gap** ($-x_t$):

$$\underbrace{i_t - \mathbb{E}_t[\pi_{t+1}]}_{\text{ctural RR}} - \underbrace{r_t^*}_{\text{fficient RR}}$$

- This is the key driver of short-run deviations in demand --- *output moves oppositely with RIR Gap*.
- If the Actual RIR is above the natural/efficient rate r_t^*, output falls ($x_t < 0$).
- If monetary policy is too tight relative to fundamentals, it contracts the economy.
- **Efficient Level of Real Output** y_t^* , **Efficient RIR** r_t^*, and their Dynamics:
- **Efficient RIR** r_t^* (x_t):

$$r_t^* = \sigma\left(\mathbb{E}_t[y_{t+1}^*] - y_t^*\right)$$

- This part comes from the gap-represented Phillip's Curve
- ERIR r_t^* is model-specific (even state-specific, e.g. at ZLB)
- **Efficient Level of Real Output** y_t^* ($-x_t$):

$$y_t^* = \frac{1+\psi}{\sigma+\psi} a_t, \quad \text{with} \quad a_t = \rho_a a_{t-1} + \epsilon_{at}$$

- The natural output is driven by productivity a_t with shocks, IES σ, and inverse Frisch elasticity of labor ψ.
- A positive productivity shock raises $a_t \uparrow$, and the resulting growth of $y_t^* \uparrow$ (immediate effect: real output expansion) decreases $r_t^* \downarrow$, increasing the RIR gap and decreasing the output gap $x_t \downarrow$ and inflation $\pi_t \downarrow$ (inflation decreases).
- **Expectations of Future Output Gap** $\mathbb{E}_t[x_{t+1}]$:
- The AD equation is forward-looking — expectations of the future output gap influence today's output gap.
- This leads to dynamics with inertia and amplification: agents smooth consumption across time.
- The output gap becomes persistent, even if current shocks are temporary.

Equilibrium Properties (Determinacy)

Check Determinacy of 3-Equation NK Models and Generalised Taylor Principle

- Start from 3 equations: AD/AS/MR #flashcard

1. Substitute MR into AD to get rid of nominal IR i_t:

$$\sigma\mathbb{E}_t\left[x_{t+1}\right] + \mathbb{E}_t\left[\pi_{t+1}\right] = (\sigma+\phi_x)x_t + \phi_\pi \pi_t - r_t^* + \epsilon_t$$

2. Together with AS, write the equations into a system:

$$\begin{bmatrix}\sigma & 1\\ 0 & \beta\end{bmatrix}\begin{bmatrix}\mathbb{E}_t\left[x_{t+1}\right]\\ \mathbb{E}_t\left[\pi_{t+1}\right]\end{bmatrix} = \begin{bmatrix}\sigma+\phi_x & \phi_\pi\\ - & 1\end{bmatrix}\begin{bmatrix}x_t\\ \pi_t\end{bmatrix} + \begin{bmatrix}1 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}\begin{matrix}\epsilon_t\\ r_t^*\\ u_t\end{matrix}$$

- Rearrange into the form of $\mathbb{E}_t\left[_{t+1}\right] = A_t + BV_t$:

$$\underbrace{\begin{bmatrix}\mathbb{E}_t\left[x_{t+1}\right]\\ \mathbb{E}_t\left[\pi_{t+1}\right]\end{bmatrix}}_{\mathbb{E}_t[_{t+1}]} = \underbrace{\frac{1}{\beta\sigma}\begin{bmatrix}\beta\sigma+\beta\phi_x+ & \beta\phi_x - 1\\ -\sigma & \sigma\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}x_t\\ \pi_t\end{bmatrix}}_{t} + \frac{1}{\beta\sigma}\underbrace{\begin{bmatrix}\beta & -\beta & 0\\ 0 & 0 & -\sigma\end{bmatrix}}_{B}\underbrace{\begin{matrix}\epsilon_t\\ r_t^*\\ u_t\end{matrix}}_{V_t}$$

3. Check **Determinacy (Blanchard Kahn) Condition**:
   - For the equilibrium to be unique and determinate:

$$\text{non-stable roots (einalue outside the unit circle) of } = \text{ non-predetermined ariables}$$

- **Equivalent Criterion by Woodford**:

$$\text{B Condition} \iff \begin{cases} \det(A) & > 1 \\ \det(A) - tr(A) & > -1 \\ \det(A) + tr(A) & > -1 \end{cases} \text{or} \begin{cases} \det(A) & \leq 1 \\ \det(A) - tr(A) & < -1 \\ \det(A) + tr(A) & < -1 \end{cases}$$

- $\implies$ In our standard model, determinacy is equivalent to **Generalised Taylor Principle**:

$$\phi_\pi > 1 - \frac{(1-\beta)\phi_x}{}$$

  - Intuition: the responsiveness of nominal IR to inflation gap has to be large enough to induce a output gap (through AD) in the counter direction, stabilising the economy.
  - *Taylor principle is model-specific, but ϕ_π is always crucial*.
  - for Rational Expectations at least; other expectation formation processes could lead to different results
  - Particular implication: *inflation expectation shocks will have no effect*!
  - a positive inflation expectation shock will induce a large increase in nIR, creating a large negative output gap and offsetting the inflationary pressure through expectation.
  - fundamental shocks cannot be stabilised still
  - On the other side, if ϕ_π is small, economy may not be stabilised from an inflation expectation shock, leading to self-fulfilling fluctuations.

The Lower Bound of the Nominal Interest Rate

Demand Driven Recessions and the Lower Bound

Modelling Demand Shocks in 3-Equation NK Models

- Aggregate Demand (AD):

$$x_t = -\sigma^{-1} \underbrace{\underbrace{i_t - \mathbb{E}_t\left[\pi_{t+1}\right]}_{\text{ctual RR}} - \underbrace{\sigma(\mathbb{E}_t\left[y^*_{t+1}\right] - y^*_t)}_{\text{fficient RR } r^*_t} \qquad}_{\text{RR Gap}} + \mathbb{E}_t\left[x_{t+1}\right]$$

  where

$$y^*_t = \frac{(1+\psi)a_t}{\sigma+\psi}$$

  and Productivity Shock:

$$a_t = \rho_a a_{t-1} + \epsilon_{at}$$

- We would like to introduce a large demand shock #flashcard
- In our 3 shocks frameworks, large demand decrease can only be introduced by a large positive productivity shock ϵ_{at} $\uparrow\uparrow$, which is not very realistic.
- **More attractive way to (explicitly) model demand shocks**
  - Microfoundation: tightening the borrowing constraint [Eggertsen and Krugman 2013]
  - Reduced form: **discount factor shock** that makes the representative household more patient:
  - Modify the agent's utility function:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta_t \left(\ln C_t - \frac{N^{1+\psi}}{1+\psi}\right)\right]$$

  where $_t$ is a discount factor shock

- Euler Equation for Bonds becomes:

$$1 = \beta \mathbb{E}_t \left[\frac{{}_{t+1}}{{}_{t}} \frac{C_t}{C_{t+1}} \frac{1+i_t}{1+i_{t+1}} \right]$$

and there will be no change on Labour Supply Equation
- After log-linearisation, the Euler Equation becomes:

$$y_t = -\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + \mathbb{E}_t\left[y_{t+1}\right] - {}_t$$

where ${}_t = \mathbb{E}_t\left[\ln\left(\frac{{}_{t+1}}{{}_{t}}\right)\right]$
- After transforming into gap representation, our **New Formula for Efficient RIR** becomes:

$$r_t^* = \mathbb{E}_t\left[y_{t+1}^* - y_t^* - \boxed{{}_t}\right]$$

and the rest of AS(PC) stays the same.
- Transmission: *A positive discount factor shock ${}_t \uparrow > 0$ will induce a lower ERIR $r_t^* \downarrow$, increasing the RIR gap and creating a negative output gap $x_t \downarrow < 0$.*

The Lower Bound in the 3-Equation NK Model

AD/AS at ZLB in Demand Crisis in 3-Equation NK Models

- We need a explicit formula for ZLB and a well-behaved process for the crisis
- **Monetary Rule (MR) with ZLB**:

$$i_t = \max\left\{-i, \phi_\pi \pi_t + \phi_x x_t\right\}$$

where $i = r = \beta^{-1} - 1$ is the nominal IR in 0-inflation ESS.
- **Two-State Process for ERIR r_t^***:
- ERIR can take 2 values

$$\left\{\begin{array}{lll} \tilde{r}_c & -i < 0 & \text{in crisis} \\ 0 & & \text{in normal times} \end{array}\right.$$

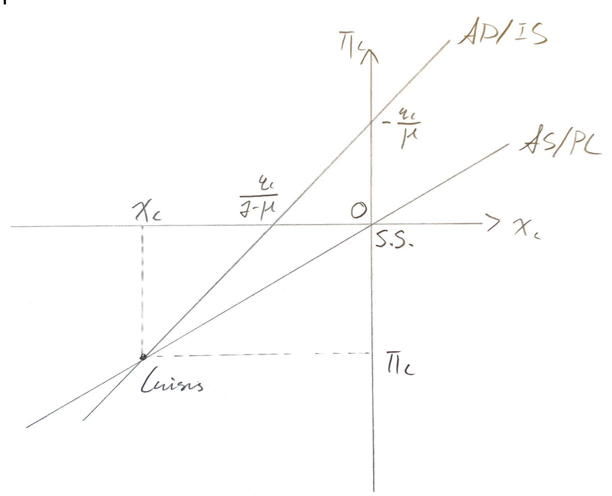
- ERIR process:
1. At t_0, the crisis hits and $r_{t_0}^* = \tilde{r}_c$
2. In each period after t_0, there are 2 possibilities:
- r_t^* jumps back to normal (0) with probability μ
- *All variables immediately jump back to normal (0)*
- r_t^* stays at the crisis level ($\tilde{r}_c$) with probability $1-\mu$
- All variables will be at their crisis value
3. In the LR, ERIR will eventually return back to normal.
- **Simplification Assumption**: assume $\sigma = 1$ for simplicity
- **Economic Dynamics at Crisis + ZLB**
    - When at ZLB, the AD/AS become: #flashcard
    - **Crisis AD(PC):**

$$\begin{aligned} x_c &= - -i - \underbrace{\left[\mu \times \pi_c + (1-\mu) \times 0\right]}_{\mathbb{E}_t[\pi_{t+1}]} - \tilde{r}_c + \underbrace{\left[\mu \times x_c + (1-\mu) \times 0\right]}_{\mathbb{E}_t[x_{t+1}]} \\ &= \frac{1}{1-\mu}\left(\tilde{r}_c + i + \mu \pi_c\right) \\ \Longrightarrow \pi_c &= -\frac{1}{\mu}\left(\tilde{r}_c + i\right) + \frac{1-\mu}{\mu} x_c \end{aligned}$$

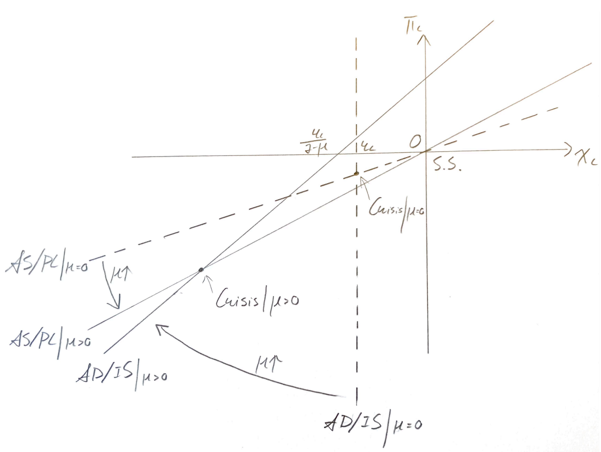
    - Crisis AD(PC) becomes upward sloping!

- **Crisis AS**:

$$\pi_t = x_c + \beta\underbrace{[\mu \times \pi_c + (1-\mu) \times 0]}_{\mathbb{E}_t[\pi_{t+1}]}$$
$$= \frac{}{1-\beta\mu}x_c$$

- We can explicitly solve for $\{x_c, \pi_c\}$
- The condition for $\{x_c, \pi_c\} < 0$ also ensures AD to be steeper than AS.
  - If this condition fails (AS steeper than AD), then it means $\mathbb{E}_t[\pi_{t+1}] \downarrow$ so that the economy become trapped in a **deflationary spiral**. We enter indeterminacy and $x, \pi \to -\infty$
- Graph:
  - 

- **Effect of the Shock Persistence**:
  - No persistence $\mu = 0$ and higher persistence
  -



Effects of Shocks at ZLB

- What will be the effects of 3 shocks at ZLB? #flashcard
- *Effects will be different from the normal times!*
- PS3 Q1:
  - A **positive cost-push shock / markup shock** $u_t \uparrow > 0$ will increase inflation one-to-one, helping counter the deflationary pressure (shrinking RIR Gap through Fisher Equation) and suppressing the output gap $\pi_c \uparrow, x_c \uparrow$ (Desirable!)
  - Graphically, shifting up AS by exactly the amount of u_t
  - A **positive productivity shock** $\epsilon_{at} \uparrow > 0$ will expand the efficient level of real output y_t^*, further inflating the RIR Gap and hence lowering inflation and output gap $\pi_c \downarrow, x_c \downarrow$ (Bad!)
  - Monetary policy is at ZLB, so no MP shock.

Applications: the Great Depression and the Great Recession

The Great Depression and the Great Recession can be modelled as two AD crisis with the former one having a higher persistence μ.

Unconventional Monetary Policy

Forward Guidance

Forward Guidance in 3-Equation NK Models #flashcard

- Solve forward the AD/AS equation:

$$\begin{cases} x_t & = -\frac{1}{\sigma}\mathbb{E}_t\left[\sum_{j=0}^{\infty} i_{t+j} - \pi_{t+j+1} - r^*_{t+j}\right] \quad \text{(D)} \\ \pi_t & = \mathbb{E}_t\left[\sum_{j=0}^{\infty} \beta^j (x_{t+j} + u_{t+j})\right] \quad \text{(S(PC))} \end{cases}$$

  - This is like to exaggerate the effectiveness of FG since there's no discounting and there's full credibility.
  - Recent literatures introduced discounting:

$$\begin{aligned} x_t &= -\frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - r^*_t) + \delta\mathbb{E}_t[x_{t+1}] \\ &= -\frac{1}{\sigma}\mathbb{E}_t\left[\sum_{j=0}^{\infty} \delta^j (i_{t+j} - \pi_{t+j+1} - r^*_{t+j})\right] \end{aligned}$$

- In 3ENK models, forward guidance works by CB announcing to keep nIR i_t at 0 for more periods than implied by MR.
- Direct Effect: expected future nominal IRs decreases $\mathbb{E}_t\left[\sum_{j=0}^{\infty} i_{t+j}\right] \downarrow$, which stimulates current real economy and pushing up the output gap $x_t \uparrow$, relieving the crisis.
- Indirect (GE) Effect: with smaller negative output gap $x_t \uparrow$, the current inflation $\pi_t \uparrow$, adding additional stimulus to demand.

Quantitative Easing

Quantitative Easing: Practice, Mechanisms, and Empirical Evidence #flashcard

- **Asset Purchasing in QE**:
  - Long-term government bonds (our focus)
  - Private assets
- **Potential Mechanisms**:
  - **Portfolio channel**: change the composition of private sector portfolio
  - **Interest rate channel**: price of long-term bonds $\uparrow$, long-term nominal IR $\downarrow$, which stimulates the economy when short-term nominal IR is already at ZLB
  - **Signalling / FG credibility channel**: provide signal that CB is indeed committed to keep future nominal IR low by putting in CB's own stakes. (*This can work even Wallace Irrelevance Theorem holds*)
- **Empirical Evidence**:
  - Krishnamurthy and Vissing-Jorgensen 2011:
  - Event study by looking at high-frequency data around announcements
  - Evidence mainly supports the signalling channel

Wallace Irrelevance Theorem #flashcard

- **Wallace Irrelevance Theorem**: in standard general equilibrium models (e.g. baseline RBC models), asset purchases are irrelevant. $\implies$ QE has no real effects.
  - e.g. If CB purchases risky assets with safe assets (bonds), then there will be no effect on real variables because the quantity of safe asset and the ownership of capital is irrelevant to the equilibrium.
- Wallace Irrelevance holds in baseline RBC with prices or even with *money-in-cashless* models, but does not holds once we add some financial frictions, MIU, or nominal rigidities.
  - However, at ZLB, Wallace Irrelevance Theorem will hold even with MIU!

Modelling QE as Long Bond Purchase in 3-Equation NK Model #flashcard

- Derivation skipped for now #notes/skipped
- Summary:
  - For QE to have real effect in 3-equation NK model, we introduce **preferred habitat** in utility function:

$$U\left(C_t, N_t, \frac{B_t}{B_{Lt}}\right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\tilde{v}}{2}\left(\frac{B_t}{B_{lt}} - \delta\right)^2 - \frac{N_t^{1+\psi}}{1+\psi} \right\}$$

  - Household prefer a particular mix of long-term and short-term bonds.
  - How QE works?
  - The central bank buys long-term bonds, reducing the supply available to households.
  - Since households want to maintain their preferred mix (e.g., ratio of long-term to short-term bonds near δ), they bid up prices of long-term bonds.
  - This lowers long-term yields, independent of short-term rates, which stimulate consumption, boosts investment, and raise inflation.

Fiscal-Monetary Policy Interaction

Government Debt Sustainability in NK 3-Equation Model #flashcard

- **(log-linearised) Government Budget Constraint**:

$$_t = \frac{1}{\beta}(i_{t-1} - \pi_t + _{t-1}) + \eta_g g_t - \eta_\tau \tau_t$$

- Debt is *inherently unstable*: holding everything else equal, government debt $_t$ has an unstable process since $\frac{1}{\beta} > 1$
- Deriving the sustainability condition:
  - Assume the government uses **tax rule**:

$$\tau_t = \phi_{t-1}$$

    - Substitute in to the gov budget constraint:

$$_t = \left(\frac{1}{\beta} - \eta_\tau \phi\right)_{t-1} + \frac{1}{\beta}(i_{t-1} - \pi_t) + \eta_g g_t$$

    - $\implies$ **Sustainability condition**:

$$\frac{1}{\beta} - \eta_\tau \phi < 1 \iff \boxed{\phi > \frac{1-\beta}{\beta \eta_\tau}}$$

      - We can also generalise this to responding to higher-order lags of debts
      - Quantitatively, this is not very stringent since $1-\beta$ is typically small
      - Intuition: the government has to collect enough tax to decrease the level of debt
- Additional complication: increasing government debt is typically with an increasing interest rate (risk premium) $\rightsquigarrow$ potentially strong non-linearities (default crisis)

Leeper's Matrix: Active/Passive Monetary/Fiscal Policy and Determinancy #flashcard

- Simplification: $\sigma = 1, \phi_x = 0$
- Derivation of determinancy conditions:
  - If we can ensure the government debt is sustainable $\forall$ MP ($\phi > \frac{1-\beta}{\beta\eta_\pi}$), then we can effectively ignore government BC and use our old analysis of determinancy
  - Specific derivation #notes/skipped
- **Leeper's Matrix**
  -

		Fiscal policy	
		Active $\left(\phi_b < \frac{1-\beta}{\beta\eta_\tau}\right)$	Passive $\left(\phi_b > \frac{1-\beta}{\beta\eta_\tau}\right)$
Monetary policy	Active $(\phi_\pi > 1)$	Non-existence (Explode / $\nexists$ stable eq.)	Determinacy
	Passive $(\phi_\pi < 1)$	Determinacy	Indeterminacy (Many equilibria driven by sunspots)

symmetric

  - "Active" means responds well to economic shocks (not including government debt)
    - "Active MP": stabilise inflation
    - "Active FP": respond to things other than debt
- **Active MP + Passive FP $\implies$ Separation Principle (Determinate)**
  - CB stabilises inflation (active):

$$\phi_\pi > 1$$

  - Fiscal authority guarantees sustainability of government debt (passive):

$$\phi > \frac{1-\beta}{\beta\eta_\pi}$$

  - This reflects well institutional arrangements in many advanced economies.
- **Passive MP + Active FP $\implies$ Fiscal Dominance (Determinate)**
  - CB gives up inflation stabilisation (passive):

$$\phi_\pi < 1$$

    - Intuition: if the government does not respond enough to debt, we need to inflate out the debt to have determinancy
  - Fiscal authority does not necessarily ensures sustainability of government debt (active):

$$\phi < \frac{1-\beta}{\beta\eta_\pi}$$

  - Cochrane argues in his "fiscal theory of price level" that this is a good description of recent years.
  - Possibility of hyperinflation due to irresponsible fiscal authority: if treasury runs perpetual deficits financed by the central bank, debt and inflation will move in locksteps and there could be hyperinflation.
- **Passive MP + Passive MP $\implies$ Indeterminate Equilibria**
  - CB gives up inflation stabilisation (passive):

$$\phi_\pi < 1$$

  - Fiscal authority guarantees sustainability of government debt (passive):

$$\phi > \frac{1-\beta}{\beta\eta_\pi}$$

  - We have indeterminacy of inflation rate, which allows for hyperinflaiton.
  - Clarida, Gali and Gertler argue this is consistent with Fed policy in 1970s (great inflation)
- **Active MP + Active FP $\implies$ Explosive Equilibria (No Stable Equilibrium)**
  - CB stabilises inflation (active):

$$\phi_\pi > 1$$

- Fiscal authority does not necessarily ensures sustainability of government debt (active):

$$\phi < \frac{1-\beta}{\beta\eta_\pi}$$

- There's no stable equilibrium with active MP + active FP. It could generate hyperinflation.
- **Extra**:
  - Temporary switches between active/passive MP/FP separation make Leeper's matrix less clear-cut (we can have hyper-inflation periods due to passive MP + active FP)

Which combinations of FP and MP can lead to hyperinflation? #flashcard

- **Active MP + Active FP**:
  - Non-stable equilibrium with exploding inflation.
- **Passive MP + Active FP**:
  - If treasury runs perpetual deficits financed by the central bank, debt and inflation will move in locksteps and there could be hyperinflation.
- **Passive MP + Passive FP**:
  - Indeterminancy where everything can happen.
- Therefore:
  - It is NOT true that "inflation is always and everywhere a monetary phenomenon" (Friedman, 1963)
  - Hyperinflation may also be caused by irresponsible behaviour of fiscal authority (e.g. Weimar Republic, Zimbabwe, and Venezuela)
  - Rather: "Persistent inflation is always and everywhere a fiscal phenomenon" (Sargent, 2013)