0 E41 Econ of Migration and Job Search Index

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Week 1: Introduction, Motivation, and Migration, Return Migration

- 1 Introduction, Motivation, and Migration, Return Migration - A

Motivation

Motivation

- Definition of Migration: movement of individuals from one region to another
- *International Migration*: movements across borders
  - Migration is heterogeneous: the composition of the immigrant population differs across countries, with respect to ethnicity, origin, and educational composition
- *Internal Migration*: movements within a country
  - Many migrants move for economic reasons (e.g. for new jobs or look for new jobs)
  - Workers of different age and education levels move internally differently: younger and more educated people are more likely to migrate
  - Migration and job search decisions are tightly connected

More Motivation

- *Causes for Migration*:
  - Economic reasons
  - Persecution, displacement as a result of war or ethnic cleansing
  - Preference for the host country (family reason)
- *Consequences of migration*
  - For host countries: direct and immediate effects on economy and society along various dimensions (e.g. wage, consumption, cohesion, productivity, unemployment, etc.)
  - For source countries: immediate effects through withdrawing people (Brain Drain or Brain Gain), return, and remittances (migrants sent half a trillion dollars back home in 2015)
- Economically motivated migration generates a *surplus (gain)*, but has *distributional effects* at the same time
  - The main beneficiaries of migrations are the migrants themselves
  - Key question: *who else* gains/loses in source- and host-countries?

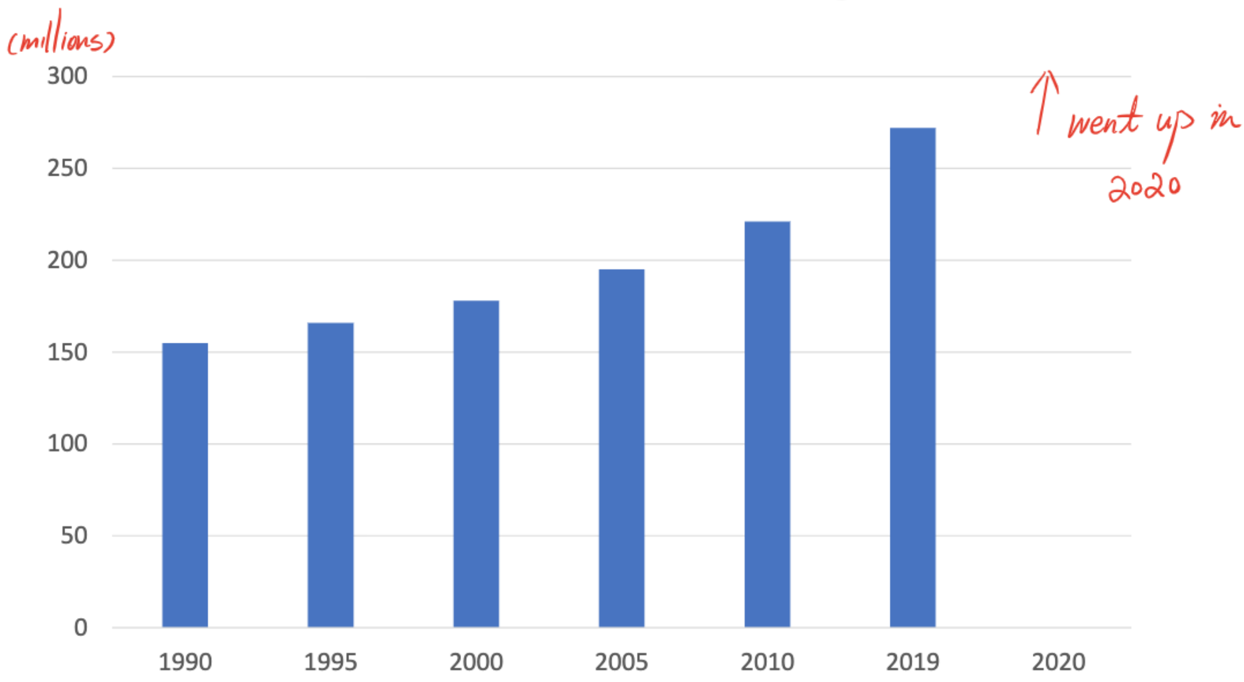
Economic Research Focuses

- *Migrants*
  - Migration and Re-migration decisions
  - Immigrant's performance in the receiving country
  - Selection of immigrants
  - Children of immigrants
- *Non-migrants* in host- and source country

    - Impact that immigration may have on receiving country: wages, employment, price, fiscal effects, innovations, crime, etc.
    - Impact that immigration may have on the sending country: employment, wages, income, children's education, etc.
    - Analysis of remittances
- *Interaction of immigrants and natives*
    - Social cohesion, attitudes to immigration, social integration

Empirical Facts

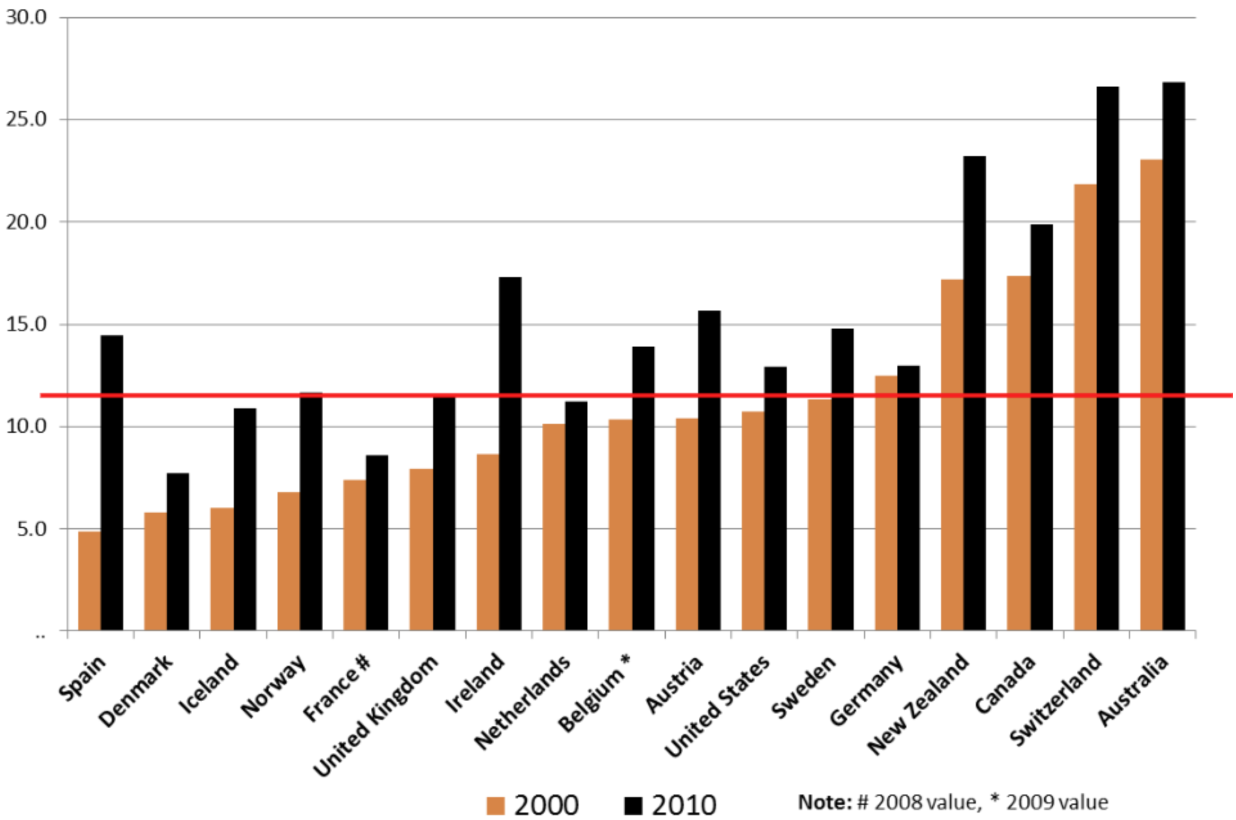
- Estimated *number of international migrants* in millions



Source: United nations

    - 270 million in 2019 for the world
    - the number went up in 2020
- Where did most international immigrants go in the past?
    - *Highly concentrated*: in 2015, 2/3 of international immigrants were living in only 20 countries
    - USA hosts 19% of all immigrants, followed by Germany, Russia, Saudi Arabia, the UK, and the UAE
- *Migration Crisis in Europe*
    - More than a million migrants and refugees crossed into Europe in recent years
    - Main nationalities: Syrian, Afghan, Nigerian, Pakistani, Iraqi, and Eritrean
    - Many fleeing wars
    - Some countries such as Hungary closed their borders

- Numbers crossing from Turkey to Greece fell sharply after the EU set a deal with Ankara in March to stem the flow
  - Many died when crossing the Mediterranean Sea when trying to cross to Europe
- *Increasing Share of foreign born in total population*



- *Percentage and Origin* of immigrants across Europe and in U.S.

Table 1: Immigrants as a percentage of total population, years 2007-2009

	% Immigrants in total population	*Composition of immigrant population by area of origin*							
		EU15	*NMS12*	*Other Europe*	*North Africa & Middle East*	*Other Africa*	*South and East Asia*	*North America and Oceania*	*Latin America*
Austria	15.68	17.55	18.7	51.18	3.58	1.2	5.44	1.07	1.29
Belgium	11.76	41.53	6.45	13.83	18.09	10.96	5.48	1.16	2.5
Germany	14.5	25.36	8.38	46.9	7.16	2.33	6.14	2.14	1.6
Denmark	7.98	20.05	5.39	26.27	16.12	4.76	16.75	8.04	2.63
Spain	13.09	13.83	13.76	3.89	15.13	2.86	3.28	0.65	**46.6**
Finland	2.71	29.86	10.51	33.75	7.16	5.08	8.89	2.73	2.02
France	10.66	27.57	2.99	6.11	40.23	12.08	6.79	1.56	2.67
Greece	7.79	5.85	12.89	61.34	11.98	1.02	4.36	2.21	0.35
Ireland	15.59	40.16	32.66	3.21	1.54	5.71	9.59	5.6	1.53
Italy	7.41	11.37	18.11	26.72	14.03	5.48	11.27	1.81	11.2
Netherlands	10.66	17.39	3.57	16.64	17.22	5.86	17.45	2.51	19.38
Norway	8.69	30.4	5.54	14.16	11.22	7.58	20.99	4.62	5.49
Portugal	6.48	18.51	3.06	8.31	0.23	45.04	1.73	2	21.12
Sweden	15.16	26.33	8.2	21.56	20.45	4.37	10.8	1.55	6.73
UK	**11.34**	**18.08**	**13.47**	**3.56**	**4.62**	**16.93**	**29.05**	**7.67**	**6.61**
Total	11.27	20.61	10.63	18.91	15.39	8.34	11.25	2.83	12.03
USA	**12.50**	**7.44**	**3.23**	**2.57**	**2.82**	**3.04**	**24.75**	**2.79**	**53.37**

- For most countries, the share of immigrants in total population $> 10\%$
- *Distance and cultural ties* matter:
    - The US over 50% of the foreign born population comes from Latin America, this share is only 12% in Europe (except for Spain)

- *Undocumented/Illegal Immigrants*

Table 2: Estimates of undocumented immigrants, 2009

	As a % of total population		*As a % of immigrant population*	
	Min	*Max*	*Min*	*Max*
Austria*	0.22%	0.65%	2.2%	6.5%
Belgium*	0.82%	1.24%	9.4%	14.2%
Germany	0.24%	0.56%	2.7%	6.3%
Denmark	0.02%	0.09%	0.3%	1.7%
Spain*	0.62%	0.78%	6.1%	7.7%
Finland	0.15%	0.23%	6.6%	9.9%
France	0.28%	0.63%	4.9%	11.0%
Greece*	1.53%	1.86%	9.1%	19.2%
Ireland*	0.68%	1.41%	6.7%	13.8%
Italy*	0.47%	0.77%	9.5%	15.7%
Netherlands*	0.38%	0.80%	9.1%	19.2%
Norway	-	-	-	-
Portugal*	0.75%	0.94%	18.4%	23.0%
Sweden*	0.09%	0.13%	1.6%	2.4%
UK	0.68%	1.41%	11.4%	23.6%
EU 15	0.46%	0.83%	6.6%	11.9%
USA	3.50%		28.4%	

The table reports minimum and maximum estimates of the size of the undocumented immigrant population for each country in 2008, expressed a share of the total country population or as a share of the total immigrant population.

extrapolated from census / labour survey

** denotes low-quality estimates*

Source: Vogel and Kolacheva (2009) for European countries. Our calculations based on Hoefer et al. (2010) for the US.

  - extrapolated from census / labour survey

- *Education of Immigrants*

- by *Host Country*

Table 3: Immigration and education

	% with lower secondary education		% with tertiary education		Standard deviation of lower secondary education shares across origin groups
	Natives	*Immigrants*	*Natives*	*Immigrants*	
Austria	16.33	33.93	17.51	18.07	14.00
Belgium	29.03	42.72	32.8	28.4	15.92
Germany	**10.47**	**37.53**	**27.02**	**19.31**	**15.93**
Denmark	23.78	27.10	33.18	33.41	10.11
Spain	50.72	40.60	30.15	24.38	19.70
Finland	19.59	24.54	36.75	31.86	10.65
France	28.38	46.07	27.58	23.98	12.68
Greece	39.25	46.08	22.9	15.69	19.09
Ireland	33.04	18.51	31.32	46.34	10.43
Italy	48.36	45.32	13.62	12.85	13.19
Netherlands	27.18	37.91	31.14	25.91	12.71
Norway	19.90	27.02	34.01	38.51	12.34
Portugal	74.69	52.41	13.01	21.82	14.01
Sweden	15.31	25.18	30.9	31.94	9.19
UK	**30.00**	**24.28**	**30.57**	**33.96**	**6.79**
Total	31.74	38.05	25.83	23.51	15.4

  - High standard deviation → *heterogeneity* in education
  - The UK has a more educated immigration population (potential reason: strict policy)

- *by Origin*

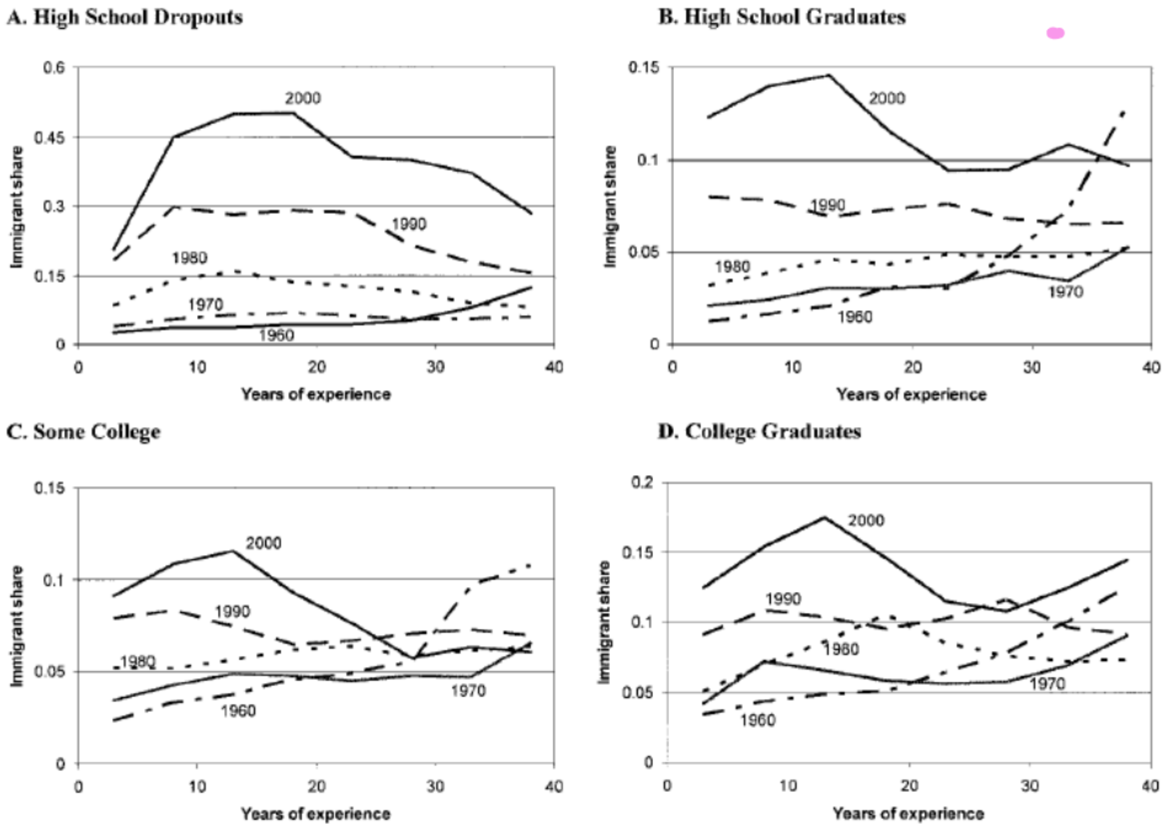
Table 4: Immigration and education, by area of origin

	% with lower secondary education	*% with tertiary education*
European Natives	31.74	25.83
EU15	35.08	**29.35**
NMS12	23.40	21.03
Other Europe	**49.01**	14.74
North Africa and near Middle East	**50.98**	20.52
Other Africa	39.01	27.84
South and East Asia	40.04	26.26
North America and Oceania	14.10	**49.55** Imm
Latin America	37.19	22.79
All immigrants	*38.05*	*23.51*

The table reports the percentage of natives and immigrants from each area of origin with low (column 1) and high (column 2) education, pooling all destination countries. The sample is restricted to working age population older than 25, not in full-time education and not in military service.
We define immigrants as "foreign born" in all countries, except for Germany where they are defined as foreign nationals.
Source: EU-LFS, years 2007, 2008 and 2009

- Immigrants from U.S. are generally more educated (potential reason: costs are higher for long-distance moving, so there have to be enough benefits)

- *in U.S.*

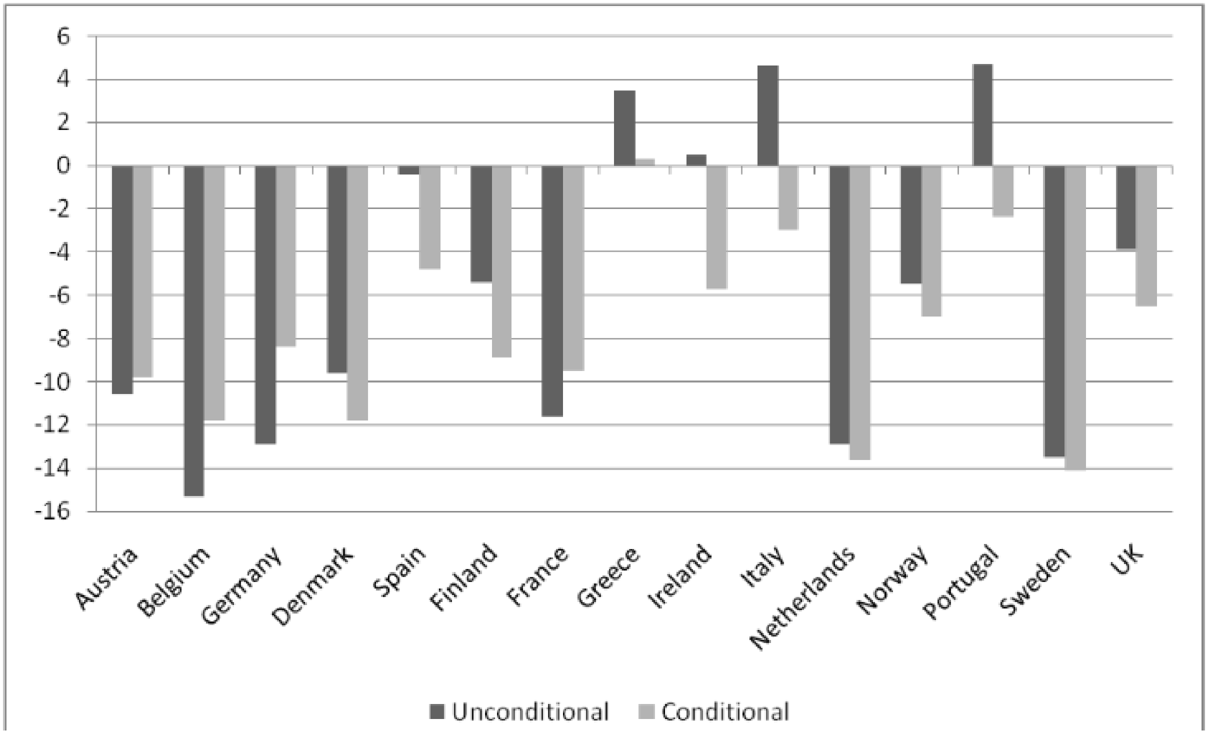


Source: Borjas (2003)

Comparison between Immigrants and Natives

- *Employment Differentials*

Figure 1: Immigrant-native employment differentials



- Unconditional: observed difference; Conditional: controlling for age, education, and areas of living
- In the *UK*:
  - Immigrants are 6.2% less likely to be employed than natives, conditional on age, education, and areas of living
  - 4% less likely unconditionally
- In *most countries*:
  - Immigrants are less likely to be employed than natives, except in Greece, Italy, and Portugal
  - Conditional on observable characteristics, immigrant are still worse off

- *Index of Dissimilarity: Occupational Segregation*

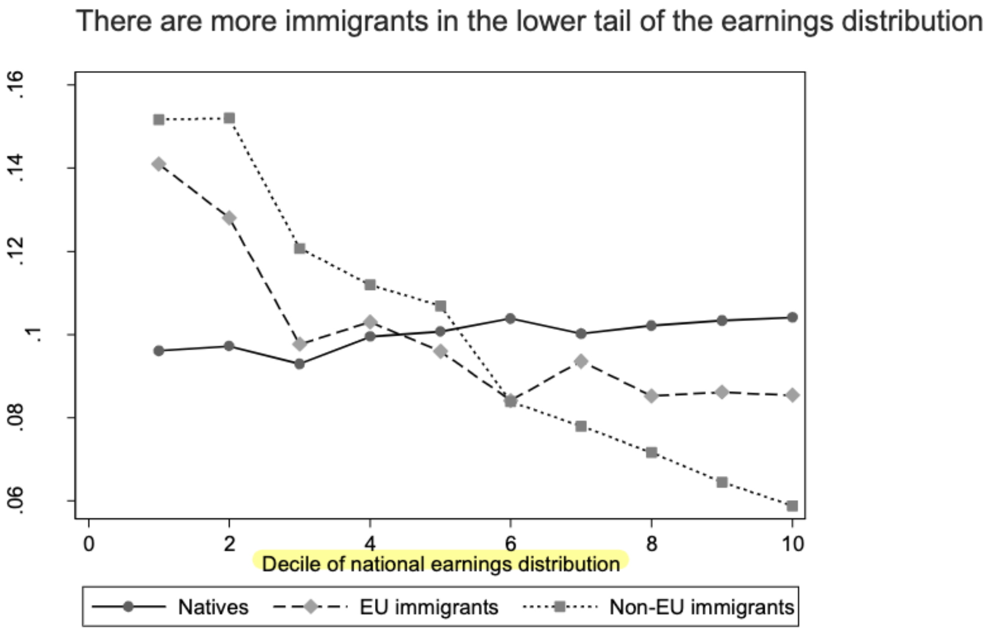
Table 6: Dissimilarity in occupational distribution

	Overall index of dissimilarity		*Index of dissimilarity by educational level* Low		Medium		High		*Weighted average across education*	
	EU (1)	Non-EU (2)	EU (3)	Non-EU (4)	EU (5)	Non-EU (6)	EU (7)	Non-EU (8)	EU (9)	Non-EU (10)
Austria	11.4	34.4	10.2	30.5	11.7	32.9	9.6	19.4	11.1	30.0
Belgium	9.7	18.4	9.5	12.7	7.7	12.8	10.1	16.5	9.0	14.2
Germany	12.2	26.1	12.1	16.1	11.6	19.5	0.1	13.4	8.6	17.4
Denmark	4.1	18.3	1.8	14.5	7.6	18.1	4.1	12.8	5.0	15.5
Spain	17.1	31.4	12.6	21.4	31.9	31.0	20.0	29.7	19.8	26.6
Finland	1.2	13.4	2.1	14.8	6.7	12.0	8.4	19.5	6.6	15.3
France	17.5	12.2	25.0	14.6	8.8	6.7	6.7	13.2	12.0	10.6
Greece	31.9	50.0	32.3	43.9	33.7	45.5	18.5	58.8	29.3	48.4
Ireland	12.1	19.4	9.0	12.7	19.1	20.4	14.8	12.8	15.2	15.8
Italy	27.5	36.2	19.7	19.8	41.7	42.9	19.8	44.5	29.8	34.5
Netherlands	5.4	14.8	8.1	12.9	4.7	9.6	2.0	11.2	4.7	11.0
Norway	10.4	17.2	11.3	9.4	9.0	19.2	9.7	19.6	9.7	17.3
Portugal	8.3	12.2	5.0	15.3	15.2	26.6	6.9	15.6	7.0	17.2
Sweden	4.7	20.8	6.0	19.6	1.8	21.0	8.3	25.8	4.5	22.3
UK	12.5	9.9	18.7	15.3	18.1	12.2	2.8	4.4	13.1	10.3

The table reports the Duncan dissimilarity index for the distribution of EU (odd columns) and non-EU (even columns) immigrants and natives across 1-digit ISCO occupations. Columns 1-2 reports the overall index. Columns 3-8 report the index by education group. Columns 9 and 10 report the average of the index by education group weighted by the share of total population in each education group.
Source: EULFS, 2007-2009

  - *Index of Dissimilarity*: percentage of immigrants that would be required for immigrants and natives to have the same occupational distribution
    - Higher index → more dissimilar is the occupation distribution of immigrants and natives
  - The index tends to be lower for EU immigrant than non-EU immigrants and natives for most countries
  - Stronger *downgrading* (skilled labour doing low-skill works) among non-EU immigrants
- *Earnings*

- Immigrant and native *Earnings Distribution*



The figure reports the share of natives (circles), EU immigrants (rhomb) and non-EU immigrants (squares) in each decile of the national earnings distribution in Belgium, Germany, Finland, France and Italy pooled. Source: EULFS 2009.

- Position in Earnings Distribution by Origin

Table 8: Position in national earnings distribution

	Decile of national earnings distribution									
	1	2	3	4	5	6	7	8	9	10
Natives	9.6	9.7	9.3	10.0	10.1	10.4	10.0	10.2	10.4	10.5
EU15	11.8	10.7	8.2	9.0	8.9	9.2	10.3	10.0	10.8	11.3
NMS12	18.9	17.3	13.0	13.0	11.1	6.9	7.5	5.5	4.2	2.8
Other Europe	16.1	15.2	10.9	10.7	9.7	9.2	9.4	8.3	6.7	3.8
N.Africa & Middle East	12.8	12.7	12.7	11.4	11.9	8.2	7.5	7.2	7.1	8.6
Other Africa	13.7	15.2	15.0	11.4	13.2	8.0	6.6	6.3	5.5	5.1
South and East Asia	**17.0**	**19.7**	12.0	13.7	9.0	7.6	6.5	5.3	4.8	4.5
N.America & Oceania	7.9	6.9	11.6	10.3	10.6	9.2	6.0	9.9	8.6	**19.0**
Latin America	**20.8**	**19.8**	11.8	9.4	10.4	6.2	5.4	4.7	6.1	5.4

The table reports the percentage of natives and immigrants in each decile of the national earnings distribution in Belgium, Germany, Finland, France and Italy pooled. We define immigrants as "foreign born" in all countries except for Germany, where they are defined as foreign nationals. Source: EULFS, 2009

Second Generation

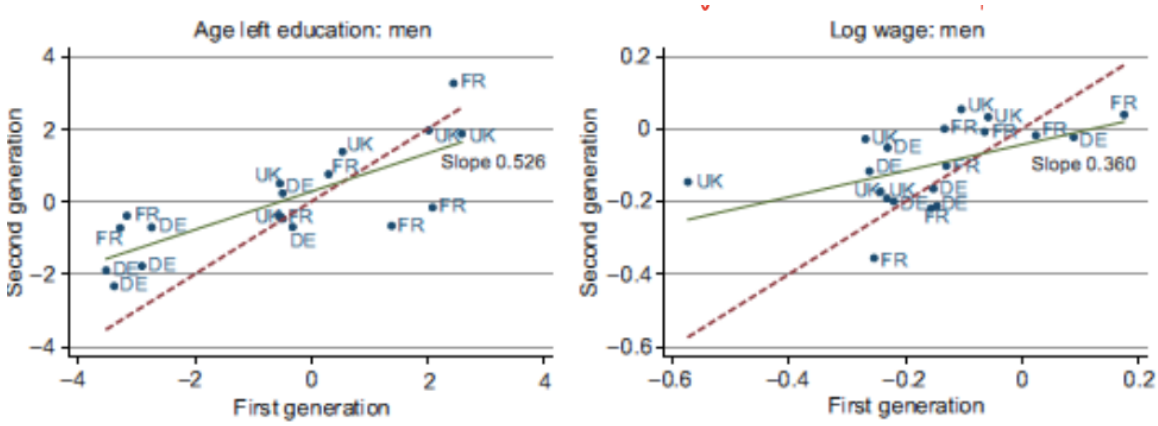
- Percent of children in immigrant households

Table 10: Children in Immigrant households

	Percentage of children (<15) who live in an immigrant household <not trivial>					Percentage of immigrants in adult population	
	EU	*Non-EU*	EU/Non-EU	*Mixed* EU/ Native	Non-EU/ Native	*EU*	*Non-EU*
Austria	3.16	**17.47**	0.66	4.47	4.32	5.21	8.36
Belgium	4.09	10.69	0.69	3.78	5.11	5.08	5.49
Germany	1.68	7.97	0.38	2.89	6.05	2.11	3.8
Spain	1.8	8.04	0.21	2.92	3.43	3.39	8.51
France	1.68	10.08	0.28	2.94	6.52	2.89	6.99
Greece	0.93	9.68	0.08	2.16	2.4	1.18	5.4
Ireland	7.73	4.94	0.61	9.86	2.41	8.96	3.3
Italy	1.66	7.81	0.17	2.94	3.91	1.72	4.6
Netherlands	0.84	12.9	0.35	3.11	6.18	1.5	8.14
Portugal	0.68	5.89	0.32	3.24	6.59	0.54	4.02
UK	2.12	**11.03**	0.48	2.37	**5.06**	3.03	7.44
Total	**1.86**	**9.43**	0.34	2.95	5.16	2.58	5.96

The left panel of the table reports the share of children under the age of 15 who live in an immigrant or a mixed household. The right panel reports the share of immigrants in the total population above the age of 15.
EU (Non-EU) households are defined as households where the reference person and her or his spouse – if there is a spouse - is an EU(Non-EU) immigrant. Mixed households are households where the reference person and her or his partner have a different immigrant status. We define immigrants as "foreign born" in all countries except for Germany, where they are defined as foreign nationals.
Source: EULFS, 2007-2009.

- First and Second Generation: *Education and Wage*



Each dot represents an ethnic group of immigrants in each country: UK, Germany, France. The dotted line is a 45 degree line. We can see that for most groups, the second generation performs better and obtains higher education.

Source: Algan, Dustmann, Glitz, Manning (2010)

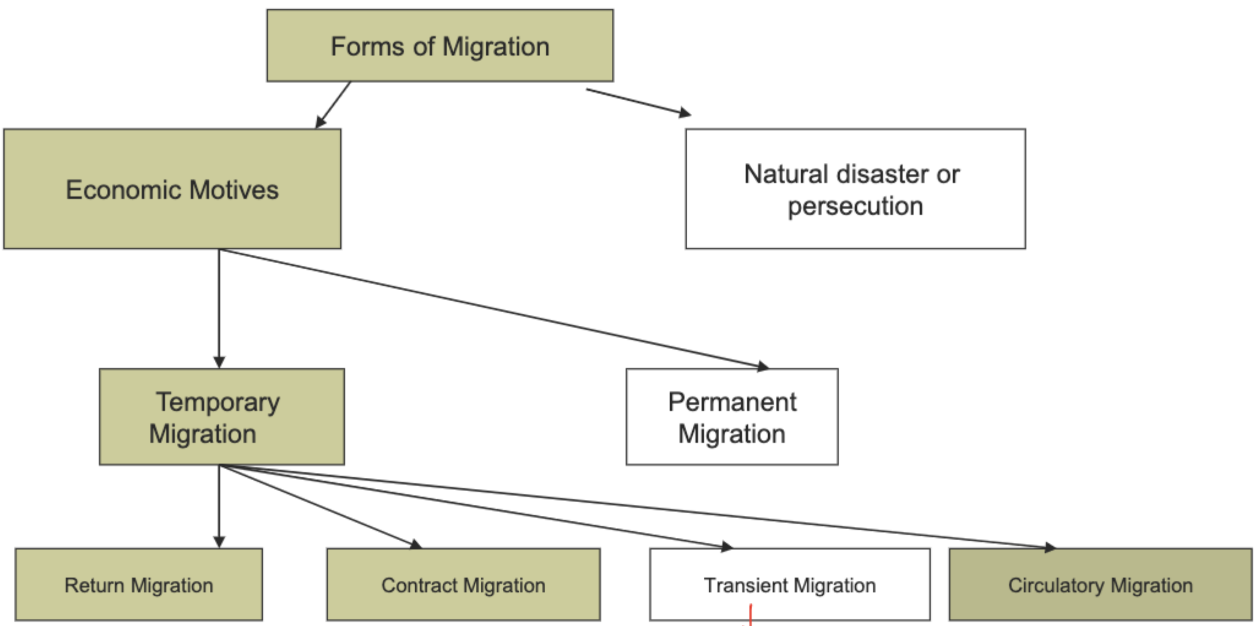
  - Above 45 degree line: the 2nd generation is doing better than the 1st generation

- *Poverty*

Table 11: Households with both spouses in bottom decile of earnings distribution

	Percentage of households with both spouses in bottom decile of earnings distribution	**Percentage of children (<15) in households with both parents in bottom decile of earnings distribution**	**Percentage of children (<15) in immigrant household out of all children in households with both parents in bottom decile of the earnings distribution**				
					Mixed		
			EU	*Non-EU*	EU/Non-EU	EU/ Native	Non-EU/ Native
Belgium	4.88	4.60	6.50	**23.0**	0.22	2.86	3.39
Germany	1.15	0.80	0	**19.2**	0	0	5.25
France	4.22	3.35	2.54	**19.1**	0.08	0.69	5.70
Italy	4.05	3.30	5.55	**20.0**	0.11	2.01	3.03
Total	**2.98**	**2.53**	3.62	**19.8**	0.10	1.26	4.57

Classification of Migrations



did not arrive the destination planned

-
    - *Circulatory migration*: migrant workers move frequently between the host and the source country
    - *Transient migration*: the migrant moves across different host countries before possibly reaching a final destination
    - *Contract migration*: a temporary migration where the migrant lives in the host country for a limited number of years, and the length of the migration is exogenously determined (e.g. by a residence permit or a working contract)

- *Return migration*: the migrant return to their country of origin *by their own choice*, often after a significant period abroad.
    - Many migrations to Europe over the last decades fall into this category

A Simple Model of Migration Decisions

Two Main Reasons for Migration:

- *Non-economic* reasons: natural disaster or persecution
- Better *economic* prospects in other areas: higher wages, cheaper housing, etc.
    - We focus on this category: migration due to individual decisions based on optimisation

Wages

- An individual has *skills*:
    - manual skill: S_m
    - cognitive skill: S_c
- *Prices for skills*: $j \in \{0, 1\} : b_{jm}, b_{jc}$
    - In the home country:
        - price for manual skill: b_{0m}
        - price for cognitive skill: b_{0c}
    - In the host country:
        - price for manual skill: b_{1m}
        - price for cognitive skill: b_{1c}
- *Productive capacity / Human capital* of an individual, in efficiency unit: $j \in \{0, 1\} : b_j S = b_{jm} S_m + b_{jc} S_c$
    - In the home country: $b_0 S = b_{0m} S_m + b_{0c} S_c$
    - In the host country: $b_1 S = b_{1m} S_m + b_{1c} S_c$
- *Return to human capital:* $j \in \{0, 1\} : R_{jt}$
    - In the home country: R_{0t}
    - In the host country: R_{1t}
- *Wages*: $j \in \{0, 1\} : w_{jt} = R_{jt} \exp(b_j S)$
    - In the home country: $w_{0t} = R_{0t} \exp(b_0 S)$
    - In the host country: $w_{1t} = R_{1t} \exp(b_1 S)$
- *Log wages*: $j \in \{0, 1\} : \ln w_{jt} = r_{jt} + b_j S$ where $r_{jt} = \ln R_{jt}$
    - In the home country: $\ln w_{0t} = r_{0t} + b_0 S$
    - In the host country: $\ln w_{1t} = r_{1t} + b_1 S$
- Thus, the wages of a given individual could differ in different countries due to:
    1. Different returns to human capital
    2. Different skill prices

One-Shot Migration Decisions

Setup

- Suppose an individual lives for T periods (*life span*)
- C denotes the *moving costs*
- The Net Gains from Migration (net present value of monetary gains from migration):

$$\Delta = \sum_{t=0}^{T} \frac{w_{1it}}{(1+r)^t} - \sum_{t=0}^{T} \frac{w_{0it}}{(1+r)^t} - C$$

  - this depends on the individual's expectation of future earning streams in the two countries

Decision

- Migrate to the host country if $\Delta > 0$
- Tendency to migrate $\uparrow$ if:
  - skill prices in the host country increases $b_{1m}, b_{1c} \uparrow$
  - return to human capital in the host country increases: $R_{1t} \uparrow$
- Suppose $\Delta > 0$, there are *reasons* why an individual *still does not migrate*:
  - *Credit constraints*: the individual cannot borrow enough to pay for C
  - *Preference for amenities*: the individual also values non-pecuniary factors such as weather, quality of goods and services, etc.

Possible Extensions

- Allow for investment in skills
- Allow for changes in the skill prices
- Allow for return migration in a dynamic framework
- Consider non-wage factors such as housing prices and amenities
- Uncertainty about future earnings can also matter

Return Migration Decisions

Setup

- Suppose an individual decides:
  1. whether to migrate
  2. how long to remain in the host country
  3. how much to spend in the host country
- The *fraction of time spending in the host country* as t
- Individual's *utility* in each country: $j \in \{0, 1\} : v(\xi_j, c_j)$
  - ξ_j is a parameter reflecting preferences for consumption in country j

- c_j is the individual's consumption in country j
- Typically we assume the individual has *home-biased preferences*: $\xi_0 > \xi_1$

Decision

- The individual *maximises life-time utility* by choosing c_0, c_1 and t subject to the *budget constraint* where $j \in \{0, 1\} : p_j$ denotes the price for goods in each country
- We can set up a *Lagrangian* to find t^*, c_0^*, c_1^*
- Some *Results*:
  1. If $\xi_0 > \xi_1$ and $p_0 < p_1$, then for migration to happen, it must be that $w_0 < w_1$
  2. What happens to t^* if *home wage increases* $w_0 \uparrow$?
     - $t^* \downarrow$ (unambiguous as income/substitution effects are in the same direction)
  3. What happens to t^* if *host wage increases* $w_1 \uparrow$?
     - ambiguous due to 2 conflicting effects:
       1. *Substitution Effects* (about relative wages): host country's wage is relatively higher than home country's wage, so the individual would like to stay in the host country longer ($t \uparrow$)
       2. *Income Effects* (about purchasing power): for a given unit of time worked in the host country, the individual makes more money, so he/she needs to work less to earn an equal amount of money $\rightarrow$ stay in the host country shorter ($t \downarrow$)

Week 2: Selection of Migration - The Roy Model

- 2 The Roy Model and Selection of Immigrants Completed - A

Introduction

- Dates back to a paper by A.D. Roy in the Oxford Economic Papers in 1951
- Roy developed the implications of *multi-dimensional abilities* for occupational choice and earnings distribution
- We will develop the key arguments of the Roy Model in the context of migration

Model Setup

Basics

$$\boxed{S \underbrace{\rightarrow}_{=bS} u \underbrace{\rightarrow}_{=R\ln u} y \underbrace{\rightarrow}_{\log y} Y}$$

- 2 Countries (E $\rightarrow$ I)
  - an Emigration/home (E) and Immigration/host (I) country

- Immigrants move from Emigration/home (E) $\rightarrow$ Immigration/host (I)

- Individual's Skills (S): each individual i is characterised by a pair of skills S_{1i}, S_{2i}
  - e.g. S_{1i}: cognitive, S_{2i}: manual
  - *Distributional assumption*: assume further skills S_{1i}, S_{2i} have standard normal distribution $\mathcal{N}(0,1)$ and covariance zero:

$$\begin{pmatrix} S_{1i} \\ S_{2i} \end{pmatrix} \sim \mathcal{N} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- Individual Production Capacity (u_{ji}) in corresponding countries (b_{I1}, b_{I2} are rewards to skills in the immigration/host country; b_{E1}, b_{E2} are rewards to skills in the emigration/home country):

$$u_{ji} = \begin{cases} u_{Ii} = b_{I1}S_{1i} + b_{I2}S_{2i} \\ u_{Ei} = b_{E1}S_{1i} + b_{E2}S_{2i} \end{cases}$$

Labour Income

- Labour Income (y_{ji}) of individual i in country j is the return to human capital

$$j \in \{I, E\} : y_{ji} = R_j \exp u_i = R_j \exp(b_{j1}S_{1i} + b_{j2}S_{2i})$$

  where R_j is the *rent rates of human capital*

- Log Labour Income (Y_{ji})

$$j \in \{I, E\} : Y_{ji} = \ln y_{ji} = \underbrace{\ln R_j}_{\mu_j} + \underbrace{b_{j1}S_{1i} + b_{j2}S_{2i}}_{u_{ji}} = \mu_j + u_{ji}$$

- Distribution of Log Income
  - With assumptions above, we can drive the distribution of Y_{Ei}, Y_{Ii}:

$$\begin{pmatrix} Y_{Ei} \\ Y_{Ii} \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} \mu_E \\ \mu_I \end{pmatrix}, \begin{pmatrix} \sigma_I^2 & \sigma_{IE} \\ \sigma_{IE} & \sigma_E^2 \end{pmatrix} \right)$$

  - where:
  - $\mu_E = \ln R_E, \mu_I = \ln R_I$
  - $\sigma_I^2 = Var(Y_{Ii}) = Var(u_{Ii}) = b_{I1}^2 + b_{I2}^2$
  - $\sigma_E^2 = Var(Y_{Ei}) = Var(u_{Ei}) = b_{E1}^2 + b_{E2}^2$
  - $\sigma_{IE} = Cov(Y_I, Y_E) = Cov(u_I, u_E) = b_{E1}b_{I1} + b_{E2}b_{I2}$

Migration Decision

- Migration Cost of an individual i from $E \rightarrow I$ is k
- An individual i *migrates ($E \rightarrow I$) if*:

$$Y_{Ii} - k > Y_{Ei}$$

equivalently:

$$\mu_I - \mu_E + (b_{I1} - b_{E1})S_{1i} + (b_{I2} - b_{E2})S_{2i} > k$$
$$\underbrace{(b_{I1}S_{1i} + b_{I2}S_{2i})}_{u_I} - \underbrace{(b_{E1}S_{1i} + b_{E2}S_{2i})}_{u_E} > \mu_E - \mu_I + k$$
$$\iff \underbrace{u_I - u_E}_{u} > \underbrace{\mu_E - \mu_I + k}_{z}$$

Result: Migration Selection

Results

- Caveat: *Y is log wage here -- it can be negative!!!!*
- Mean Income of 4 Types of Individuals:
  1. *Migrants* in *Host(I)* Country, with mean log-income:

$$E[Y_{Ii}|Y_{Ii} - k > Y_E] = \mu_I + \sigma_{IU} \times \frac{\phi(z)}{1 - \Phi(z)}$$

  2. *Non-migrants* in *Home(E)* Country, with mean log-income:

$$E[Y_{Ei}|Y_{Ei} > Y_{Ii} - k] = \mu_E - \sigma_{EU} \times \frac{\phi(z)}{\Phi(z)}$$

  3. *Migrants* in *Home(E)* Country (counterfactual), with mean log-income:

$$E[Y_{Ei}|Y_{Ii} - k > Y_E] = \mu_E + \sigma_{EU} \times \frac{\phi(z)}{1 - \Phi(z)}$$

  4. *Non-migrants* in *Host(I)* Country (counterfactual), with mean log-income:

$$E[Y_{Ii}|Y_{Ei} > Y_{Ii} - k] = \mu_I - \sigma_{IU} \times \frac{\phi(z)}{\Phi(z)}$$

  - where
    - $z = \mu_E + k - \mu_I$
    - $u = u_i - u_E$
    - $\mu_E = \ln R_E,\ \mu_I = \ln R_I$
    - $\sigma_I^2 = Var(Y_{Ii}) = Var(u_{Ii}) = b_{I1}^2 + b_{I2}^2$
    - $\sigma_E^2 = Var(Y_{Ei}) = Var(u_{Ei}) = b_{E1}^2 + b_{E2}^2$
    - $\sigma_{IE} = Cov(Y_I, Y_E) = Cov(u_I, u_E) = b_{E1}b_{I1} + b_{E2}b_{I2}$
    - $\sigma^2 = Var(u_I - u_E) = Var(u) = b_{I1}^2 + b_{I2}^2 + b_{E1}^2 + b_{E2}^2 - 2b_{E1}b_{I1} - 2b_{E2}b_{I2}$
    - $\sigma_{IU} = \frac{Cov(u_I,u)}{\sigma} = \frac{\sigma_I^2 - \sigma_{IE}}{\sigma} = \frac{b_{I1}(b_{I1}-b_{E1})+b_{I2}(b_{I2}-b_{E2})}{\sigma}$
    - $\sigma_{EU} = \frac{Cov(u_E,u)}{\sigma} = \frac{\sigma_{IE} - \sigma_E^2}{\sigma} = \frac{b_{E1}(b_{I1}-b_{E1})+b_{E2}(b_{I2}-b_{E2})}{\sigma}$
    - $\rho = Corr(u_E, u_I) = \frac{\sigma_{IE}}{\sigma_I \sigma_E} = \frac{b_{I1}b_{E1}+b_{I2}b_{E2}}{\sqrt{(b_{I1}^2+b_{I2}^2)(b_{E1}^2+b_{E2}^2)}}$

- $\frac{\phi(z)}{1-\Phi(z)}$ and $-\frac{\phi(z)}{\Phi(z)}$ are *Inverse Mills Ratios* -- the means/expectations of upper and lower tails of a truncated normal distribution
  - suppose $x \sim \mathcal{N}(\mu, \sigma^2)$ and z is a constant, then:

$$E[X|X > \alpha] = \mu + \sigma \times \frac{\phi\left(\frac{\alpha-\mu}{\sigma}\right)}{1 - \Phi\left(\frac{\alpha-\mu}{\sigma}\right)}$$

where $\phi(.), \Phi(.)$ are pdf and cdf of the standard normal distribution with mean μ and variance σ^2

Derivations (not in exam)

- Example: mean income of migrants in the host country (type 1):

$$\begin{aligned} E[Y_{Ii}|Y_{Ii} - k > Y_E] &= E[\mu_{Ii} + u_{Ii}|u > z] \\ &= \mu_I + E[u_{Ii}|u > z] \\ &= \mu_I + \frac{\sigma_{IU}}{\sigma} E[u|u > z] && \text{(use Linear Reg.)} \\ &= \mu_I + \sigma_{IU} \frac{\phi\left(\frac{z}{\sigma}\right)}{1 - \Phi\left(\frac{z}{\sigma}\right)} && \text{(use Inv. M. Ratio)} \end{aligned}$$

where the second last line uses a linear regression / OLS estimator: $u_I = \frac{\sigma_{IU}}{\sigma} u + \epsilon$ and we drop $\frac{1}{\sigma}$ in $\phi(.), \Phi(.)$ for ease of exposition

Different Cases of Selection

- **画图要点**
  - 表示combinations of earnings的斜线永远过(μ_I, μ_E)，斜率为$\frac{c}{b}$
  - 45度线($Y_E = Y_I$或$Y_E = Y_I - k$)永远过$(0, -k)$，斜率为1
  - 斜线在45度线下方时migrate
  - 只有在$k = 0, \mu_E = \mu_I$时，斜线和45度线才会交于(μ_E, μ_I)

Case 1: Positive Selection

Basics

- Rich individuals migrate and remain rich; higher variance in host is desirable for them
- *Condition*:
  - $\sigma_{IU} > 0, \sigma_{EU} > 0 \iff \sigma_I^2 > \sigma_{IE} > \sigma_E^2 \iff \frac{\sigma_I}{\sigma_E} > \rho > \frac{\sigma_E}{\sigma_I}$ where $\rho = \frac{\sigma_{IE}}{\sigma_I \sigma_E}$
  - i.e. variance of log income in host country σ_I^2 is large than the covariance, and variance of log income in home country σ_E^2 is smaller than the covariance
- *Results*:

  - Migrants have higher than average earnings in both host and home countries:

$$\begin{cases} E[Y_{Ii}|Y_{Ii}-k>Y_E]=\mu_I+\underbrace{\sigma_{IU}\times\frac{\phi(z)}{1-\Phi(z)}}_{>0} & >\mu_I \text{ (in host/immigration country)} \\ E[Y_{Ei}|Y_{Ii}-k>Y_E]=\mu_E+\underbrace{\sigma_{EU}\times\frac{\phi(z)}{1-\Phi(z)}}_{>0} & >\mu_E \text{ (in home/emigration country)} \end{cases}$$

- Those who don't migrate have lower (potential) earnings in both countries:

$$\begin{cases} E[Y_{Ii}|Y_{Ei}>Y_{Ii}-k]=\mu_I-\underbrace{\sigma_{IU}\times\frac{\phi(z)}{\Phi(z)}}_{>0}<\mu_I \text{ (in host/immigration country)} \\ E[Y_{Ei}|Y_{Ei}>Y_{Ii}-k]=\mu_E-\underbrace{\sigma_{EU}\times\frac{\phi(z)}{\Phi(z)}}_{>0}<\mu_E \text{ (in home/emigration country)} \end{cases}$$

- *Intuition*:
  - Higher variance in log income in host country is desirable for migrants here: only individuals in the upper distribution of both countries will migrate, so a fatter upper tail means they have bigger chance to earn a very high income. Higher variance typically means the host country does not have a highly progressive income tax, etc.

1.1 Benchmark Case (Same average, No migration cost)

- Further assumptions:

  - *no immigration cost* $(k=0)$
  - and:

$$\begin{cases} b_{E1}=b_{E2}=c \\ b_{I1}=b_{I2}=b \\ c<b \implies \sigma_I^2>\sigma_{IE}>\sigma_E^2 \\ \mu_E=\mu_I \end{cases}$$

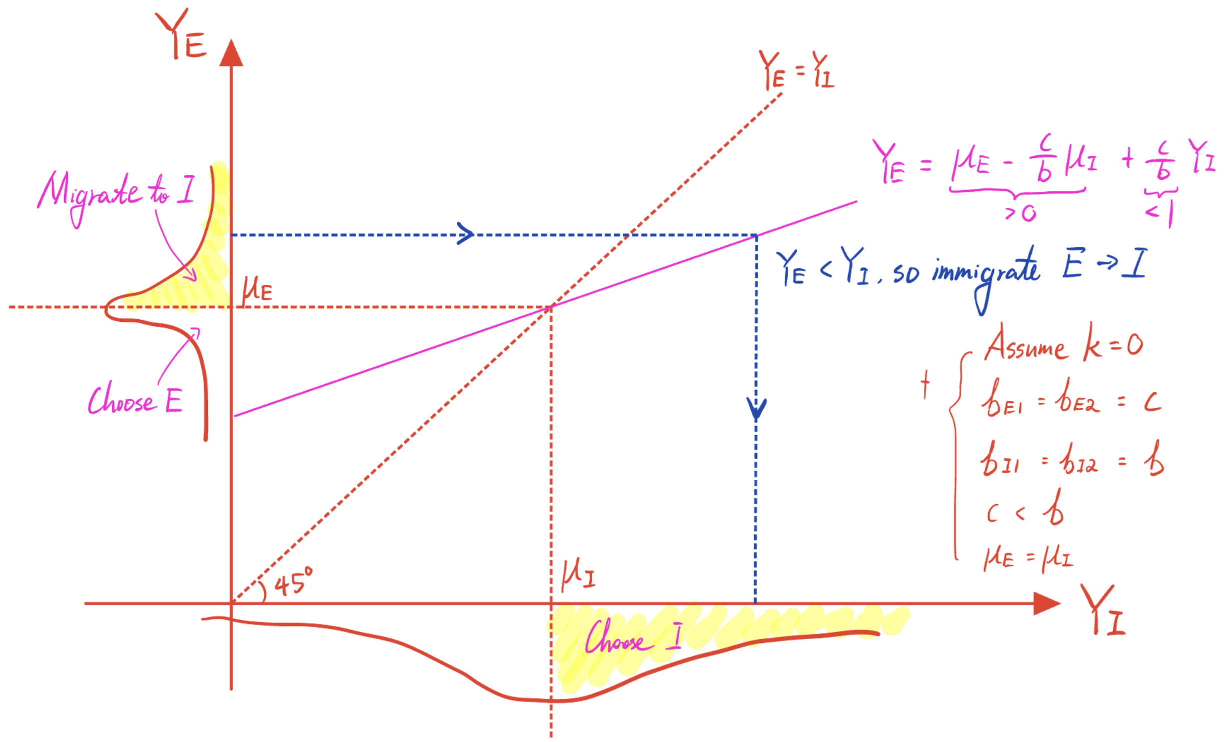
  - Since $b_{I1}=b_{I2}=b; b_{E1}=b_{E2}=c$:

$$\begin{cases} Y_I & =\underbrace{\ln R_I}_{\mu_I}+b(S_{1i}+S_{2i}) \\ Y_E & =\underbrace{\ln R_E}_{\mu_E}+c(S_{1i}+S_{2i}) \end{cases} \implies Y_E=\mu_E-\frac{c}{b}\mu_I+\frac{c}{b}Y_I$$

- Lines:

$$\begin{cases} Y_E=\mu_E-\frac{c}{b}\mu_I+\frac{c}{b}Y_I \\ Y_E=Y_I \end{cases} \implies Y_I^*=\left(\mu_E-\frac{c}{b}\mu_I\right)\times\underbrace{\frac{b}{b-c}}_{>0}=\mu_I$$

- Graph



1.2 Positive Migration Costs, but Same Averages

- Setup:
  - Allows *positive immigration cost* ($k > 0$)
  - Notations:

$$\begin{cases} k > 0 \\ b_{E1} = b_{E2} = c \\ b_{I1} = b_{I2} = b \\ c < b \implies \sigma_I^2 > \sigma_{IE} > \sigma_E^2 \\ \mu_E = \mu_I \end{cases}$$

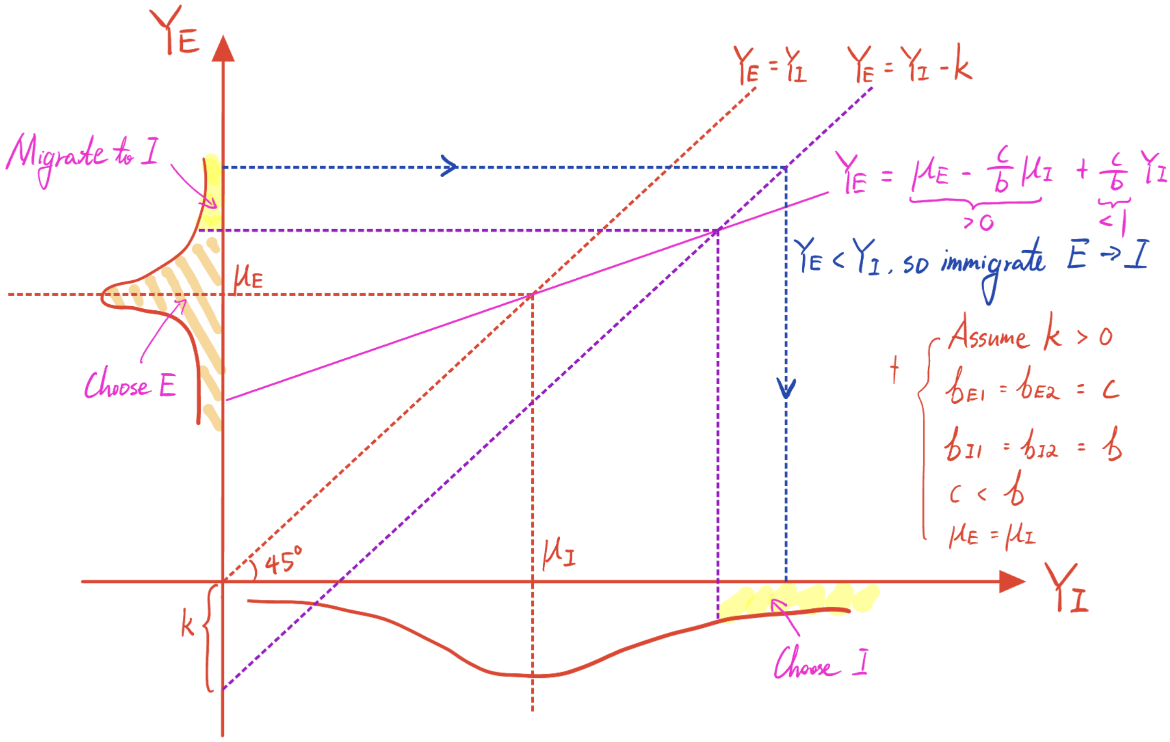
  - Since $b_{I1} = b_{I2} = b; b_{E1} = b_{E2} = c$:

$$\begin{cases} Y_I & = \underbrace{\ln R_I}_{\mu_I} + b(S_{1i} + S_{2i}) \\ Y_E & = \underbrace{\ln R_E}_{\mu_E} + c(S_{1i} + S_{2i}) \end{cases} \implies Y_E = \mu_E - \frac{c}{b}\mu_I + \frac{c}{b}Y_I$$

- Lines:

$$\begin{cases} Y_E = \mu_E - \frac{c}{b}\mu_I + \frac{c}{b}Y_I \\ Y_E = Y_I - k \end{cases}$$

- Graph



Case 2: Negative Selection

Basics

- Poor individuals migrate and remain poor; lower variance in host is desirable
- *Condition*:
  - $\sigma_{IU} < 0, \sigma_{EU} < 0 \iff \sigma_I^2 < \sigma_{IE} < \sigma_E^2 \iff \frac{\sigma_I}{\sigma_E} < \rho < \frac{\sigma_E}{\sigma_I}$ where $\rho = \frac{\sigma_{IE}}{\sigma_I \sigma_E}$
  - i.e. variance of log income in host country σ_I^2 is lower than the covariance, and variance of log income in home country σ_E^2 is lower than the covariance
- *Results*:
  - Migrants have lower than average earnings in both host and home countries:

$$\begin{cases} E[Y_{Ii}|Y_{Ii} - k > Y_E] = \mu_I + \underbrace{\sigma_{IU} \times \frac{\phi(z)}{1-\Phi(z)}}_{<0} & < \mu_I \text{ (in host/immigration country)} \\ E[Y_{Ei}|Y_{Ii} - k > Y_E] = \mu_E + \underbrace{\sigma_{EU} \times \frac{\phi(z)}{1-\Phi(z)}}_{<0} & < \mu_E \text{ (in home/emigration country)} \end{cases}$$

  - Those who don't migrate have higher (potential) earnings in both countries:

$$\begin{cases} E[Y_{Ii}|Y_{Ei} > Y_{Ii} - k] = \mu_I - \underbrace{\sigma_{IU} \times \frac{\phi(z)}{\Phi(z)}}_{<0} > \mu_I \text{ (in host/immigration country)} \\ E[Y_{Ei}|Y_{Ei} > Y_{Ii} - k] = \mu_E - \underbrace{\sigma_{EU} \times \frac{\phi(z)}{\Phi(z)}}_{<0} > \mu_E \text{ (in home/emigration country)} \end{cases}$$

- *Intuition*:
  - Lower variance in log income in host country is desirable for migrants here: only individuals in the lower distribution of both countries will migrate, so a thinner

lower tail means they have smaller chance to earn a very low income. Lower variance typically means the host country insures low-income worker agains poor labour market outcomes while taxing high-income workers

- If there is cost of migration k, then average quality of migrants increases as cost increases
  - because in negative selection, marginal migrants are more productive than other migrants
- Cases and Graphs:

2.1 Benchmark Case (Same average, No migration cost)

- Setup:

$$\begin{cases} k = 0 \\ b_{I1} = b_{I2} = b \\ b_{E1} = b_{E2} = c \\ c > b \\ \mu_E = \mu_I \end{cases}$$

- Show this satisfies conditions:

$$c > b \implies 2b^2 < 2bc < 2c^2 \implies \sigma_I^2 < \sigma_{IE} < \sigma_E^2 \implies \sigma_{IU} < 0, \sigma_{EU} < 0$$

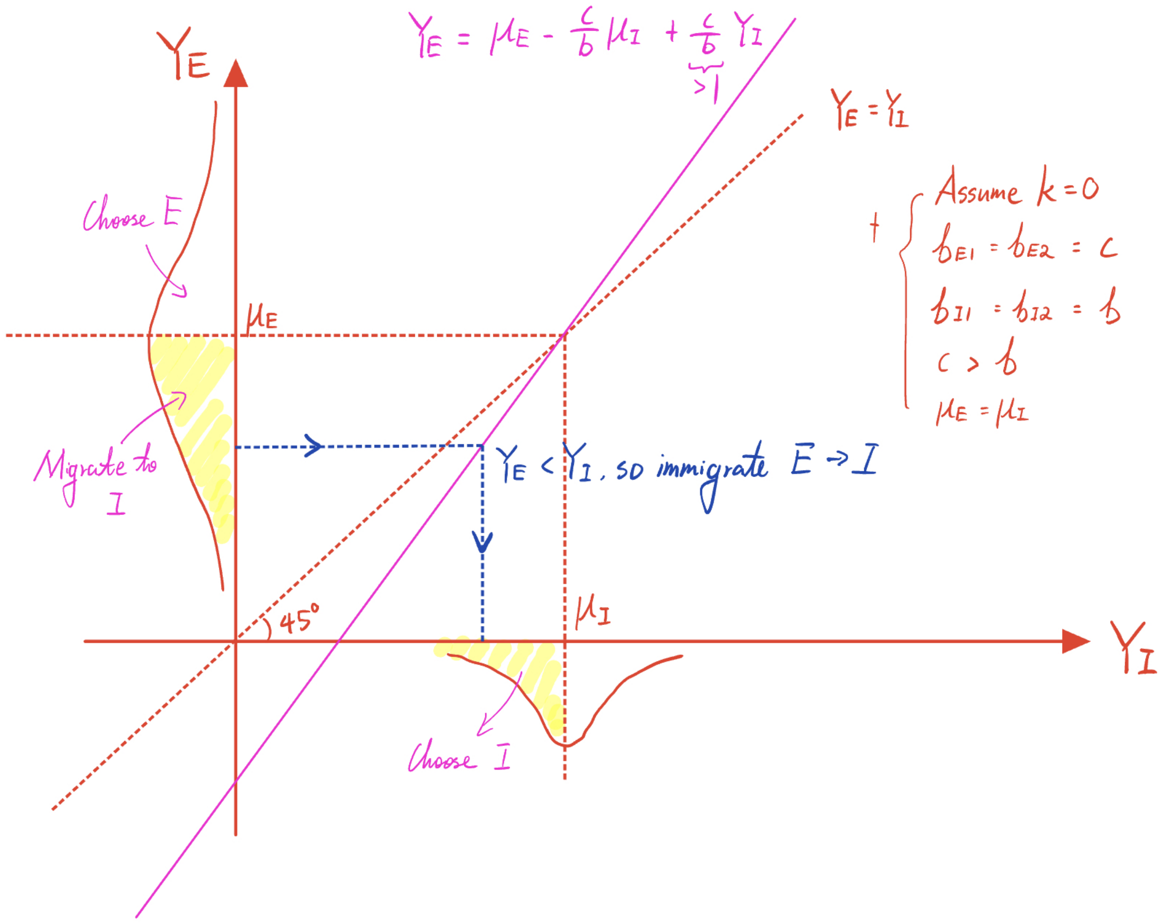
- Since $b_{I1} = b_{I2} = b; b_{E1} = b_{E2} = c$:

$$\begin{cases} Y_I &= \underbrace{\ln R_I}_{\mu_I} + b(S_{1i} + S_{2i}) \\ Y_E &= \underbrace{\ln R_E}_{\mu_E} + c(S_{1i} + S_{2i}) \end{cases} \implies Y_E = \mu_E - \frac{c}{b}\mu_I + \frac{c}{b}Y_I$$

- Math:

$$\begin{cases} Y_E = \mu_E - \frac{c}{b}\mu_I + \frac{c}{b}Y_I \\ Y_E = Y_I \end{cases}$$

- Graph:



2.2 Increased Home Avg. Earnings

- *Average earning in home increases* $\mu_E \uparrow \Longrightarrow$ *Quality of migrants decreases*
- Setup:

$$\begin{cases} k = 0 \\ b_{I1} = b_{I2} = b \\ b_{E1} = b_{E2} = c \\ c > b \\ \mu_E = \mu_I > \mu_{E1} > \mu_{E0} \end{cases}$$

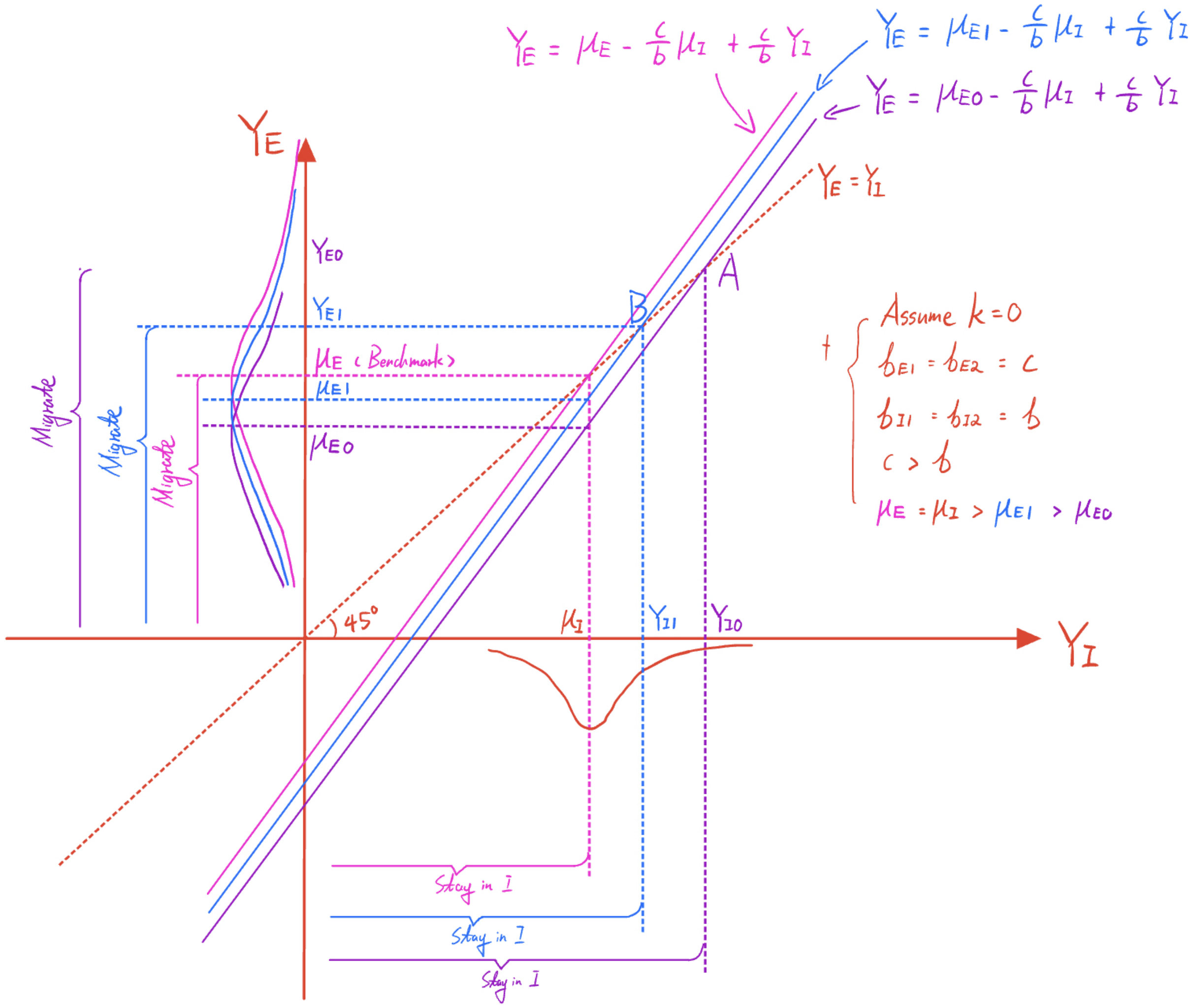
- Since $b_{I1} = b_{I2} = b; b_{E1} = b_{E2} = c$:

$$\begin{cases} Y_I &= \underbrace{\ln R_I}_{\mu_I} + b(S_{1i} + S_{2i}) \\ Y_E &= \underbrace{\ln R_E}_{\mu_E} + c(S_{1i} + S_{2i}) \end{cases} \Longrightarrow Y_E = \mu_E - \frac{c}{b}\mu_I + \frac{c}{b} Y_I$$

- Math:

$$\begin{cases} Y_E = \mu_E - \frac{c}{b}\mu_I + \frac{c}{b} Y_I \\ Y_E = Y_I \end{cases} \Longrightarrow Y_I^* \downarrow = \left(\frac{c}{b}\mu_I - \mu_E \uparrow\right) \times \underbrace{\frac{-b}{b-c}}_{>0}$$

- Graph

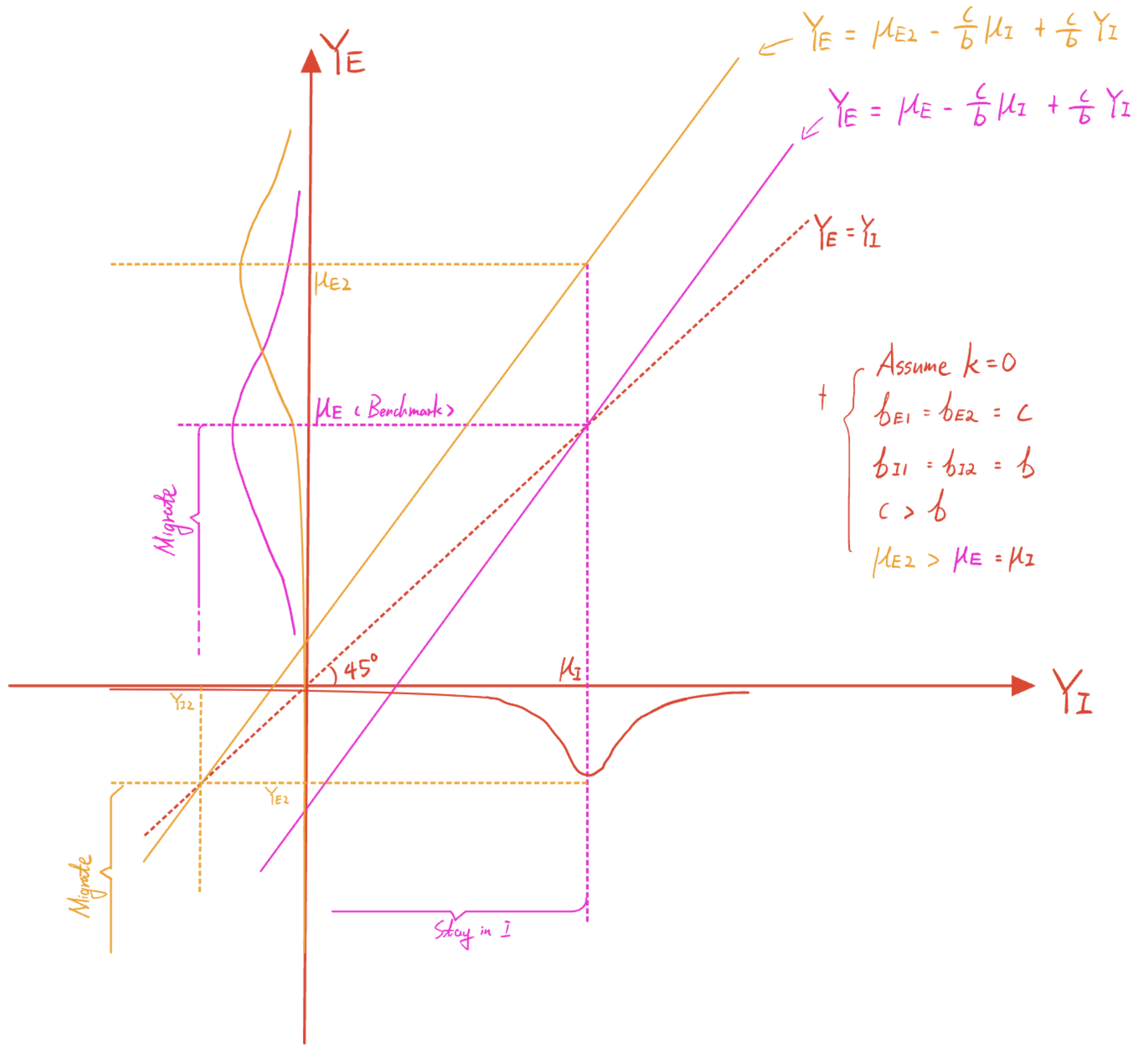


2.3 Higher Avg. Earning at Home

- *As long as* $c > b$*, there will ALWAYS be migration; Extreme Case: average earnings at home is higher than host* $(\mu_E > \mu_I)$
- Math: threshold log-income $Y_I = Y_E < 0$, only super low-income workers migrate

$$\begin{cases} Y_E = \mu_E - \frac{c}{b}\mu_I + \frac{c}{b}Y_I \\ Y_E = Y_I \end{cases} \implies Y_I^* = \underbrace{\left(\frac{c}{b}\mu_I - \mu_E\right)}_{<0} \times \underbrace{\frac{-b}{b-c}}_{>0}$$

- Graph



2.4 Migration Costs

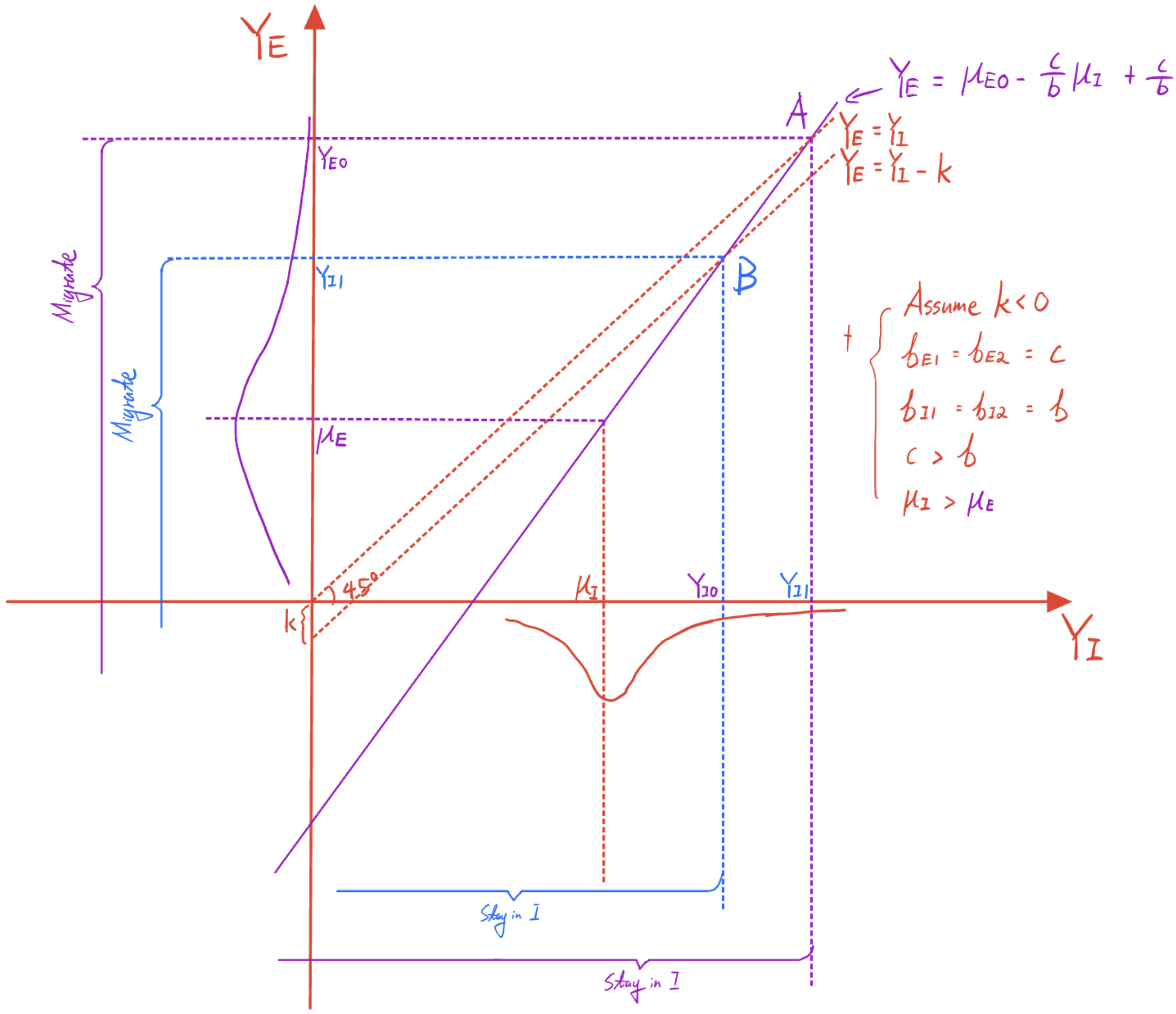
- *Migration costs increase $k \uparrow \implies$ Quality of migrants decreases
- Setup:

$$\begin{cases} k > 0 \\ b_{I1} = b_{I2} = b \\ b_{E1} = b_{E2} = c \\ c > b \\ \mu_E < \mu_I \end{cases}$$

- Math:

$$\begin{cases} Y_E = \mu_E - \frac{c}{b}\mu_I + \frac{c}{b}Y_I \\ Y_E = Y_I - k \end{cases} \implies Y_I \downarrow = \left(\frac{c}{b}\mu_I - \mu_E - k \uparrow\right) \times \underbrace{\frac{-b}{b-c}}_{>0}$$

- Graph:



Case 3: Non-Hierarchical Sorting (Refugee Sorting)

- Earnings of those who migrate are higher than the average earnings in the host country (I), but are lower than the average earnings in the home country (E)
- *Condition*:
  - $\sigma_{IU} > 0, \sigma_{EU} < 0 \iff \sigma_I^2 > \sigma_{IE}, \sigma_E^2 > \sigma_{IE} \iff \frac{\sigma_I}{\sigma_E} > \rho, \frac{\sigma_E}{\sigma_I} > \rho$ where $\rho = \frac{\sigma_{IE}}{\sigma_I \sigma_E}$
  - i.e. both variance of log income in host country σ_I^2 and in home country σ_E^2 are large than the covariance (*low correlation*)
- *Results*

  - Migrants have lower than average earnings in home country, but higher than average earnings in the host country:

$$\begin{cases} E[Y_{Ii} | Y_{Ii} - k > Y_E] = \mu_I + \underbrace{\sigma_{IU} \times \frac{\phi(z)}{1-\Phi(z)}}_{>0} & > \mu_I \text{ (in host/immigration country)} \\ E[Y_{Ei} | Y_{Ii} - k > Y_E] = \mu_E + \underbrace{\sigma_{EU} \times \frac{\phi(z)}{1-\Phi(z)}}_{<0} & < \mu_E \text{ (in home/emigration country)} \end{cases}$$

  - Those who don't migrate have higher than average earnings in home country, but would earn lower than average earnings in host country:

$$\begin{cases} E[Y_{Ii}|Y_{Ei} > Y_{Ii} - k] = \mu_I - \underbrace{\sigma_{IU} \times \frac{\phi(z)}{\Phi(z)}}_{>0} > \mu_I \text{ (in host/immigration country)} \\ E[Y_{Ei}|Y_{Ei} > Y_{Ii} - k] = \mu_E - \underbrace{\sigma_{EU} \times \frac{\phi(z)}{\Phi(z)}}_{<0} > \mu_E \text{ (in home/emigration country)} \end{cases}$$

- no statement about selection can be made (cannot say it's positive/negative)
- *Brain Gain for Both Countries*: this non-hierarchical sorting may increase the average productivity level in the home county (since individuals below average productivity leave the country), as well as in the host country.
- Referred by Borjas (1987) as Refugee Sorting: highly skilled individuals are discriminated against in dictatorial systems, receiving a return for their skills that is below average, while being rewarded according to market prices in countries that accommodate refugees.
- However, this case seems to be more important for modern migrations where *countries differ in their industry structure and, thus, skill requirements*
- Special/Extreme case: $b_{I1} = b_{E2} = 0$, which means each skill is only valuable in one country. There's no correlation ($\rho = 0$)

Case 4: Not Possible

- $\sigma_{IU} < 0, \sigma_{EU} > 0 \iff \sigma_I^2 < \sigma_{IE}, \sigma_E^2 < \sigma_{IE}$
- This is not possible

Selection Patterns in Reality

Overview

- Large empirical literature attempts to assess the direction of migration selection.
- Most papers use the Borjas 1987 framework and consider the special case of proportional skill price ($u_E = cu_I$ where c is a constant)
- The evidence is mixed

Chicquiar and Hanson 2005: Mexico → US Immigration

- Chicquiar and Hanson 2005 investigated migration from Mexico to the U.S.
- To determine selection we need to know wages that immigrants would have received, had they not migrated. But we don't usually observe such wages.
- They constructed *counterfactual wages* had immigrants stayed in Mexico based on *observed characteristics*
  - Specifically, they compare actual wage densities for residents of Mexico with counterfactual wage densities that would obtain were Mexican immigrants paid according to skill prices in Mexico based on observed characteristics.

    - Drawback: there could be selection on unobservables: there could be unobserved factors that determine wages
- *Finding*:
    - Mexican immigrants are *from the upper income distribution of Mexico* (were Mexican immigrants in the United States paid according to Mexican skill prices, they would fall disproportionately in the middle and upper portions of Mexico's wage distribution)
    - This rejects negative selection
        - support intermediate/positive selection
        - *Inconsistent with simple Roy model prediction*: Variance of income distribution in Mexico is larger than variance of income distribution in the US. ($\sigma_I^2 < \sigma_E^2$) So the simple Roy model would imply negative selection.
- Reason 1: selection on unobservables: wrong counterfactual $\rightarrow$ indeed negative selection
- Reason 2: problem with the model. We can improve it to reconcile this (*next section*)

Selection with Non-Linear Migration Costs

Setup

- *Proportional Skill Price* (only one skill S, assumed to be schooling here):

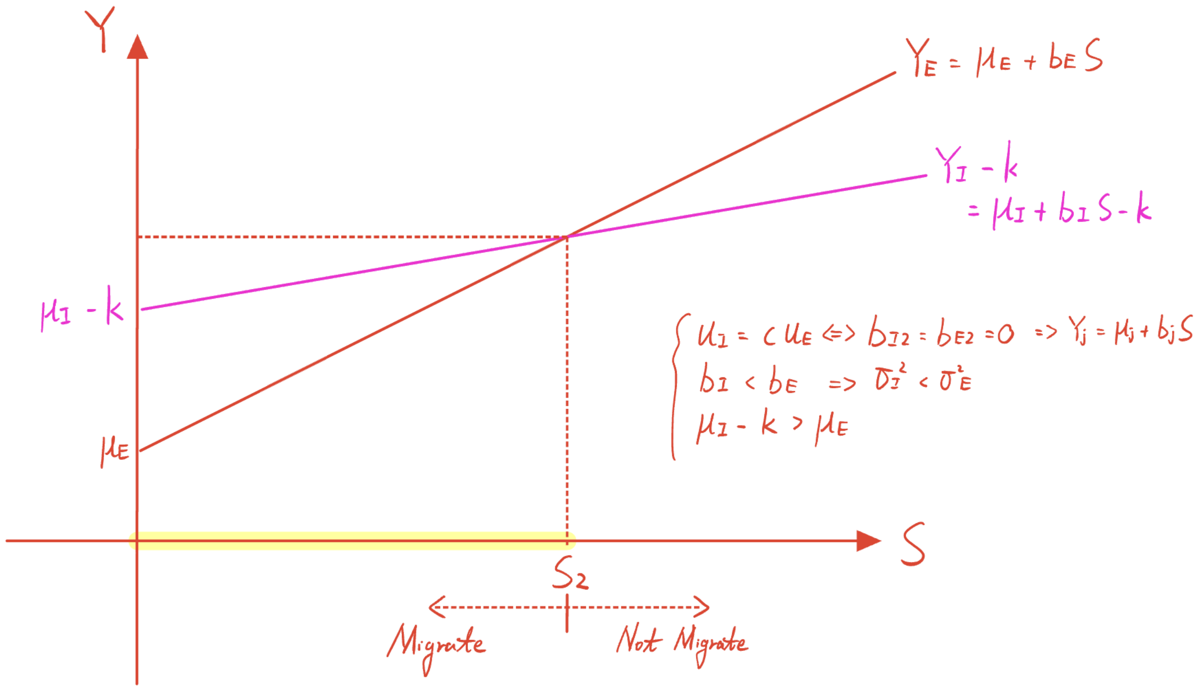
$$u_I = cu_E \iff b_{I2} = b_{E2} = 0 \implies j \in \{I, E\} : Y_j = \mu_j + b_j S$$

- *Higher Variance in Home Country*:

$$b_I < b_E \implies \sigma_I^2 < \sigma_E^2$$

Constant Migration Cost

- When the migration cost is constant k, workers with $S < S_2$ migrate

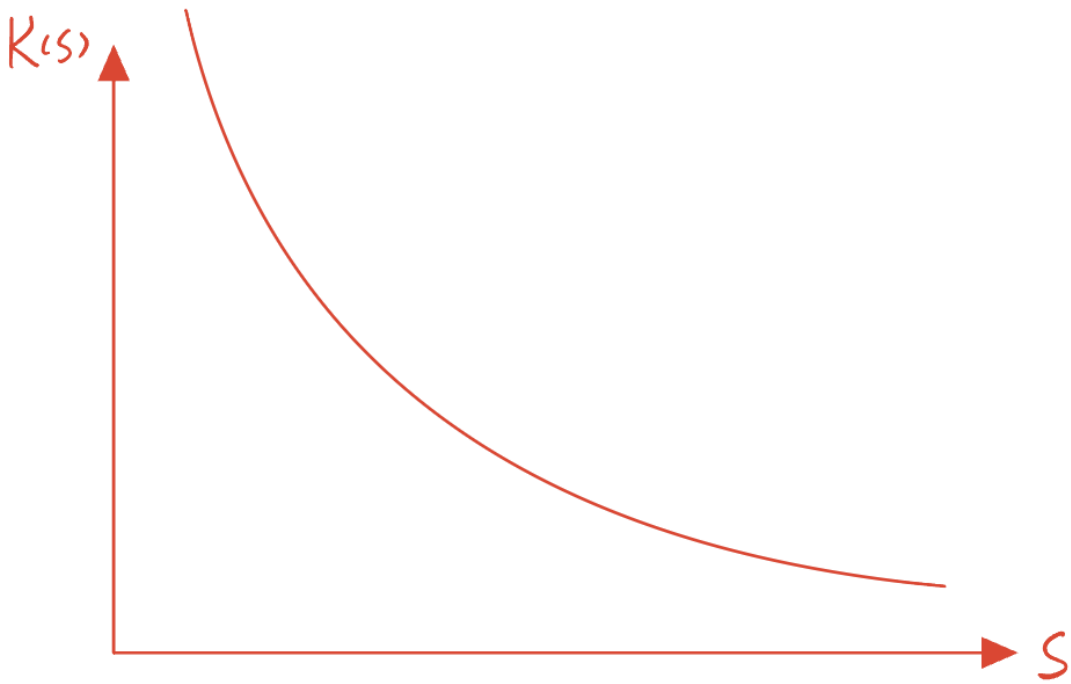


Non-Linear Migration Costs

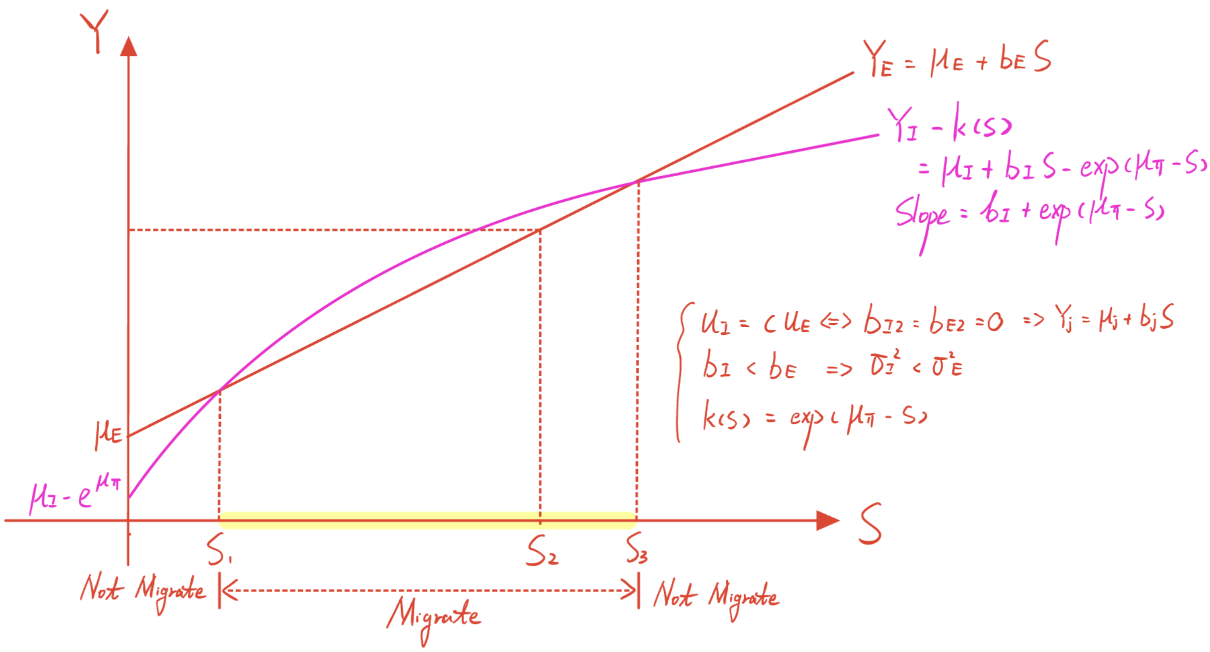
- To reconcile with findings in the previous section, the authors introduced non-linear migration costs:

$$k(S) = \exp(\mu_\pi - S)$$

  where μ_π is a constant
  - The migration costs look like



- As a result, only individual whose S satisfies $S_1 < S < S_3$ migrate



Week 3: Wage Impact of Immigration Using IV

- 3 Wage Impact Using IV Completed - A

Assessing the Effect of Immigration on Wages

2 Approach with IV Strategies: Overview

- We can slice labour market into sub-markets that *differ in the intensity of immigration* experienced (e.g. skill groups, regions, etc.)
- Then, we can exploit variation of immigrant shares across submarkets and regions
- Requires pre-allocation of immigrants to sub-markets, e.g. skill groups.
- Problems:
  - Immigrants self-select into sub-markets and regions
  - Natives leave sub-markets when immigrants enter
  - Pre-allocation may be problematic

Estimation Using Skill-Cell Approach and IV Strategies

$$\boxed{\text{For Each Group: First Difference} \rightarrow \text{IV}}$$

Setup

- Using panel data at group level:

$$\ln w_{irt} = a + a_r + a_i + a_t + a_r a_t + a_r a_i + a_i a_t + \gamma \ln X_{irt} + \epsilon_{irt}$$

- where

- a_i is a constant
- a_r, a_i, a_t are the region, group, and time fixed effects
- $a_r a_t + a_r a_i + a_i a_t$ are interaction terms
- $\ln X_{irt}$ is is the log of immigrant-to-natives ratio in that group, area, and year

First Difference Transformation

- First Difference the previous equation, anything without t subscript (time-invariant) will drop out:

$$\Delta \ln w_{irt} = A_t + A_r + A_i + \gamma \Delta \ln X_{irt} + \xi_{irt}$$

- where
  - $\Delta \ln w_{irt}$ is the change in the average wage in a particular group, area, and year
  - $\Delta \ln X_{irt}$ is the change in immigrant labour supply (immigrant-native ratio) in a particular group, area, and year
- Endogeneity Problem: immigrants self-select into sub-markets and regions with high wage growth ($Cov(\xi_{irt}, \Delta \ln X_{irt}) > 0$)

IV Strategy

- We need to find an IV Z_{irt} for $\Delta \ln X_{irt}$
- 2 *Requirements*:
  - Relevance: Z_{irt} can predict ΔX_{irt} (i.e. $Cov(Z_{irt}, \Delta X_{irt}) \neq 0$)
  - Exogeneity: Z_{irt} is uncorrelated with the error term ξ_{irt} (i.e. $Cov(Z_{irt}, \xi_{irt}) = 0$)
- Most commonly used instrument relies on the enclave pattern of immigrants (particular routes of immigration) (Card, 2009):
  - $Z_{irt} = \frac{\sum_c \lambda_{irc,\tau} \Delta I_{ict}}{X_{ir\tau}}$: *predicted growth rate of immigrants* into group i, region r based on historical patterns in the base period τ (past)
  - where:
    - $\lambda_{irc,\tau}$ is the share of immigrants from country c that fall into group i, region r, in period τ
    - ΔI_{ict} is the national-level change in number of immigrants from country c and group i between two points of time t and τ
    - $X_{ir\tau}$ is the number of immigrants in group i, region r in the base period

IV Estimation Steps

- 2SLS steps
  - Regress $\Delta \ln X_{irt}$ on Z_{irt} (first-stage):

$$\Delta \ln X_{irt} = \delta_t + \delta_r + \delta_i + \delta_1 Z_{irt} + \eta_{irt}$$

        - After estimation, we can compute the predicted value of $\widehat{\Delta \ln X_{irt}} = \hat{\delta}_t + \hat{\delta}_r + \hat{\delta}_i + \hat{\delta}_1 Z_{irt}$ and it is uncorrelated with ξ_{irt} because $Cov(Z_{irt}, \xi_{irt}) = 0$
    - Replace $\Delta \ln X_{irt}$ by $\Delta \widehat{\ln X_{irt}}$ in the main FD equation and estimate γ using OLS:

$$\Delta \ln w_{irt} = A_t + A_r + A_i + \gamma \Delta \widehat{\ln X_{irt}} + \xi_{irt}$$

Example

- Many papers have estimated wage impacts of immigration using this method, e.g. Card and Altonji (1991) for the US, Dustmann and Glitz (2003) for Germany.
- Dustmann and Glitz (2003) estimate *γ to be -0.41 for Germany*.
    - Interpretation: 1 percent increase in the labor supply of a particular skill group due to immigration leads to a decrease in the average wage for workers in that group by about 0.41 percent.
    - They estimate γ separately for non-tradable and tradable sectors. Their findings suggest that immigration *affected wages in the non-tradable sector, but not in the tradable sector*
        - Adjustments in the tradable sector may have taken place through other mechanisms than wages, e.g. changes in output mix, or adoption of new technology.

Problems

- *IV identifies a LATE*, which has limited interpretation
    - it is unlikely that a large increase, say 20%, in immigration in a particular region will result in $\gamma \times 20\%$ decrease in wages because other things may change too
        - General equilibrium effects
        - What if incumbent workers can respond by moving somewhere else?
        - How do we know which region r they will choose to go to?
            - Solution: We will model location choice together with price adjustment later (spatial equilibrium model)
- *Estimates are based on grouping of characteristics* (division of group matters)
    - What if high, medium and low skill workers differ in both efficiency units of labor and elasticities of substitution
    - What if males and females are not perfect substitutes?
    - How do we estimate the elasticities of substitution of different worker types?
    - Solution: we need to impose more structure on labour types (next section)

Estimation Using CES Production Functions (Region + Skill Cells)

Introduction

- A number of papers have used the nested constant elasticity of substitution (CES) production function to estimate elasticities of substitution between different labor types
  - They are easy to estimate
- The order of nesting can be subjective, though not necessarily sensitive.
- Examples of these papers include Ottaviano and Peri (2013) for US study, and Manacorda, Manning and Wadsworth (2013) for the UK.

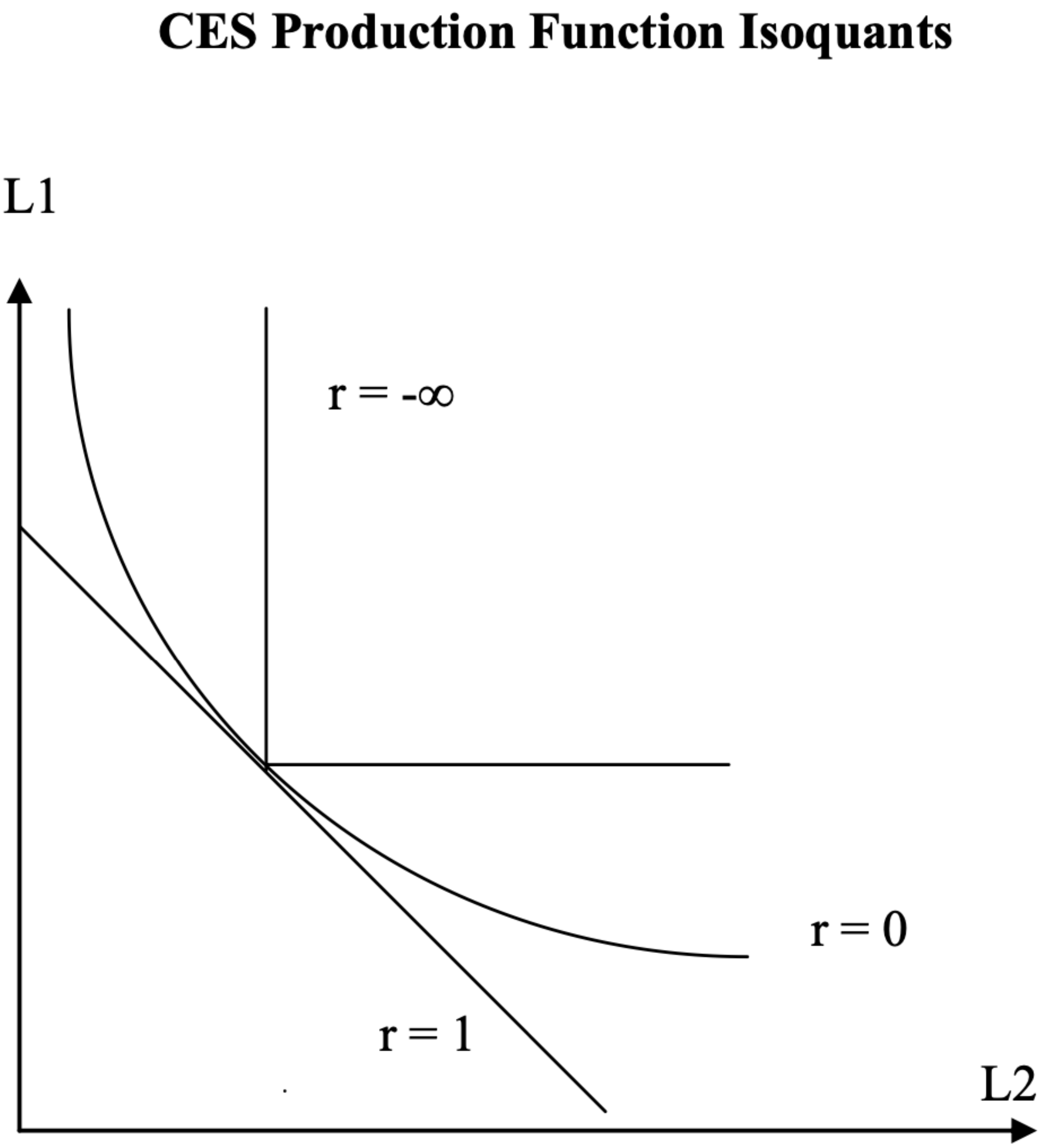
Constant Elasticity of Substitution (CES) Production Function

- CES is a flexible production function that displays *constant elasticity of substitution* between factors:

$$Q = a[s_1 L_1^r + s_2 L_2^r]^{\frac{1}{r}}$$

  - where
    - Q is output
    - L_1, L_2 are inputs
    - a is TFP (total factor productivity)
    - s_1, s_2 measure the relative importance of factors
- Properties
  - Elasticity of substitution $\rho = \frac{1}{1-r}$
  - If $r = 1$, then we have a linear / perfect substitute production function
  - If $r > 0 \iff \rho < 1$, then they are substitutes
  - If $r \to 0 \iff \rho \to 1$, then we have the Cobb-Douglas production function
  - If $r < 0 \iff \rho > 1$, then they are complements
  - If $r \to -\infty$, then we have the Leontief / perfect complements production function

- Isoquants:



CES Normalisation

- We can normalise one of the s_1, s_2 to be 1, and multiply L_1, L_2 by $1 = \frac{(s_2)^{\frac{1}{r}}}{(s_2)^{\frac{1}{r}}}$ so the quality still holds:

$$
\begin{aligned}
Q &= a[s_1 L_1^r + s_2 L_2^r]^{\frac{1}{r}} \\
&= \underbrace{a(s_2)^{1/r}}_{A} \left[\underbrace{\frac{s_1}{s_2}}_{\theta_1} L_1^r + L_2^r \right] \\
&= A[\theta_1 L_1 + L_2^r]^{1/r}
\end{aligned}
$$

- where

- $A = as_2^{1/r}$
- $\theta_1 = \frac{s_1}{s_2}$

2 Layers

- Manacorda, Manning and Wadsworth (2013):
  - Aggregate labour consists of high and low-skill workers who may be imperfect substitutes.
  - Within each skill group, we have labour of different ages who may be imperfect substitutes.
  - Within each age group, we have immigrants and natives who may also be imperfect substitutes.
  - Here one can add genders as one of the nesting too
  - The idea is that *workers become more substitutable as we move down the nest*.
- We'll focus on two characteristics: skill level and immigration status
  - Top layer: skill level
  - Bottom layer: immigration status

Firm's CES Production Function

Top Layer

- Assume that the firm produces output using *only labour* (but different kinds of labour):

$$Y_t = A_t L_t = A_t \underbrace{[\theta_{ht} L_{ht}^{\rho} + L_{lt}^{\rho}]^{1/\rho}}_{L_t}$$

  - where
    - L_{ht}, L_{lt} are labour inputs with high/low skill levels at time t
    - θ_{ht} is the relative productivity level of skilled to unskilled labour
  - $\sigma_E = \frac{1}{1-\rho}$ is the elasticity of substitution between high/low skill groups

Bottom/2nd Layer

- Within each skill group, workers differ by immigration status:

$$e \in \{h, l\} : L_{et} = \left(N_{et}^{\delta} + \beta_{et}^{M} M_{et}^{\delta}\right)^{1/\delta}$$

  - where
    - N are native inputs with skill level e at time t
    - M are immigrant inputs with skill level e at time t
    - β_{et}^{N} is the relative productivity of the immigrants relative to natives
  - $\sigma_I = \frac{1}{1-\delta}$ is the elasticity of substitution between immigrant/natives

- here, we assume it to be the same in the high-skill and low-skill group, but in reality we should make them different: Piyapromdee (2020) and Card (2009) find σ_I to be quite different for low-skill group from that of high-skill group

Labour Demand

- Assume *perfect competition*, wages are set at the marginal product of labour.
- Taking FOC, the wage of natives and immigrants in each skill cell is given by:

$$S \in \{N, M\}, e \in \{h, l\}:$$
$$\ln W_{et}^S = \ln A_t + \frac{1}{\sigma_E} \ln L_t + \ln \theta_{et} + \ln \beta_{et}^S + \left(\frac{1}{\sigma_I} - \frac{1}{\sigma_E}\right) \ln L_{et} - \frac{1}{\sigma_I} \ln S_{et}$$

Estimation: Overview

- The wage equation above is non-linear and may suffer from endogeneity, so we will do some transformations
- Step 1: Start from the bottom nest, estimate σ_I, β_{et}^S. Compute L_{et}
- Step 2: Proceed to the top level to estimate σ_E, θ_{et}

Estimation Step 1: Bottom Layer (estimate σ_I, β_{et}^S)

- *Subtract native's log wage ($\ln w_{et}^N$) from immigrant's log wage ($\ln w_{et}^M$)*:

$$\ln\left(\frac{w_{et}^M}{w_{et}^N}\right) = \ln \beta_{et}^M - \frac{1}{\sigma_I} \ln\left(\frac{M_{et}}{N_{et}}\right)$$

- Further *assume that $\ln \beta_{et}^M$ takes an additive form*: $\ln \beta_{et}^M = d_e + d_t$ where $d_e = \mathbb{1}[e = h], d_t = \mathbb{1}[t = ..]$ are skill and time dummies
  - This is because, within each skill-time cell, β_{eat}^M will be constant, but if we include all cell dummies, then they will be unidentified, so we simplify them into 2 dummies without interactions. *ALWAYS DECOMPOSE USING DUMMIES*! 有几个可以变的subscript/superscript就加几个dummy，可以把interaction terms也加上
  - we can do robustness check to see whether our estimates vary by our choice and specification of $\ln \beta_{et}^M$. A good estimate should be robust to different methods of proxy
- Then, we get the final estimation equation for the bottom level:

$$\ln\left(\frac{w_{et}^M}{w_{et}^N}\right) = \underbrace{d_e + d_t}_{\ln \beta_{et}^M} - \frac{1}{\sigma_I} \ln\left(\frac{M_{et}}{N_{et}}\right)$$

  - where
    - w_{et}^S can be computed as the average wage of workers with immigration status $S \in \{N, M\}$, skill level $e \in \{h, l\}$, and in time t

        - N_{et}, M_{et} are the labour inputs, which can be calculated as headcount or hours of work
        - d_e, d_t are skill and time dummies, can be easily compute
- Since all variables needed are observed, we can estimate: d_e, d_t, σ_I, and compute $\beta_{et}^M = \exp(d_e + d_t)$
- Given the estimates β_{et}^M, σ_I above, we can compute δ since $\sigma_I = \frac{1}{1-\delta}$
- But note that the validity of our estimates depend on the exogeneity of immigrant labour supply!
    - If immigrants or natives self-select into certain skill-age groups that have high wages, we need to find IVs for them
    - Cannot use the network IV mentioned in the Skill-Cell Approach, because we are estimating the national level impact rather than regional impact
    - Potential IV: distance from origin countries to destination interacted with push factors such as the number of wars in the source countries or GDP per capital
        - Relevance: predicts the inflow of immigrants in each cell.
        - Exogeneity: this is likely to be uncorrelated with the error term in the wage equation of receiving country
            - This might fail if the push factors also affect the host country's wages

Estimation Step 2: Top Layer

- Then, we can compute the effective labour supply of high/low skill groups:

$$L_{et} = \left(N_{et}^{\delta} + \hat{\beta}_{et}^{M} M^{\delta}\right)^{1/\delta}$$

- Then, *subtract low-skill worker's log wage ($\ln_{lt}^S$) from high-skill worker's log wage ($\ln_{ht}^S$) for group S at time t*:

$$S \in \{N, M\}:$$
$$\ln\left(\frac{w_{ht}^S}{w_{lt}^S}\right) = \ln \theta_{ht} + \ln\left(\frac{\beta_{ht}^S}{\beta_{lt}^S}\right) + \left(\frac{1}{\sigma_I} - \frac{1}{\sigma_E}\right)\ln\left(\frac{L_{ht}}{L_{lt}}\right) - \frac{1}{\sigma_I}\ln\left(\frac{S_{ht}}{S_{lt}}\right)$$

\- *red* represents unknowns here

- Rearrange:

$$S \in \{N, M\} : \ln\left(\frac{\hat{w}_{ht}^S}{\hat{w}_{lt}^S}\right) = \ln \theta_{ht} - \frac{1}{\sigma_E}\ln\left(\frac{L_{ht}}{L_{lt}}\right)$$

    - where
        - $\ln\left(\frac{\hat{w}_{ht}^S}{\hat{w}_{lt}^S}\right) = \ln\left(\frac{w_{ht}^S}{w_{lt}^S}\right) + \frac{1}{\sigma_I}\left(\ln\frac{S_{ht}}{S_{lt}} - \ln\frac{L_{ht}}{L_{lt}}\right) - \ln\left(\frac{\beta_{ht}^S}{\beta_{lt}^S}\right)$ can be computed from data and estimates in the bottom level
- With a further assumption: $\ln \theta_{ht} = \kappa_0 + \kappa_1 t$ (approximate the relative productivity by a linear time trend) (Card and Lemieux, 2001). We get the final estimation equation for

the top level:

$$\ln\left(\frac{\hat{w}^S_{ht}}{\hat{w}^S_{lt}}\right) = \kappa_0 + \kappa_1 t - \frac{1}{\sigma_E}\ln\left(\frac{L_{ht}}{L_{lt}}\right)$$

- where
  - $\ln\left(\frac{\hat{w}^S_{ht}}{\hat{w}^S_{lt}}\right) = \ln\left(\frac{w^S_{ht}}{w^S_{lt}}\right) + \frac{1}{\sigma_I}\left(\ln\frac{S_{ht}}{S_{lt}} - \ln\frac{L_{ht}}{L_{lt}}\right) - \ln\left(\frac{\beta^S_{ht}}{\beta^S_{lt}}\right)$ can be computed from data and estimates in the bottom level
  - t, L_{ht}, L_{lt} are observed
- We can estimate all unknown parameters in *red*, and compute $L_t = [\theta_{ht}L^\rho_{ht} + L^\rho_{lt}]^{1/\rho}$ where ρ is obtained from $\sigma_E = \frac{1}{1-\rho}$

Wage Impact Calculation

- Since now we have all parameters estimated, we can compute counterfactual wages using the formula in Labour Demand
  - Caveat: *TFP A_t assumed to be exogenous*. However, if we modify the model to have spillovers, e.g. having more high skill workers leads to higher TFP then A_t would depend on the composition of workers, and we need to estimate it.
- Empirical findings
  - Both Ottaviano and Peri (2013), and Manacorda, Manning and Wadsworth (2013) find that *immigrants and natives are imperfect substitutes within age/education cells*
  - This leads to the effect of immigration on natives to be mitigated, and leads to a larger effect of immigration on the wages of immigrants, i.e. *more competition among immigrants themselves*.

Problems

- Important assumption: immigrants compete with natives at exactly that part of the skill distribution to which they have been assigned, based on their observed age and education
  - What if we have downgrading in immigrants? E.g. a high skill immigrant might work in a low skill occupation when first arrived. Solution: one could define worker types based on where they are on the wage distribution
    - But notice that, in this model, we still allow natives with low skills to compete with immigrants with high skills

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Week 4: Wage Impact of Immigration Using Natural Experiments

- 4 Wage Impact Part2 Completed - A

Difference-in-Difference Method

- See 0 0 ECON0021 Microeconometrics Index > Week 4: Repeated Cross-Sections and Panel Data for detailed discussions

Setup

- We have 2 repeated cross-sectional datasets (before and after treatment)
- 2 Groups: treatment group (τ) and control group (c)
- Outcome: $i \in \{\tau, c\}, t \in \{0, 1\} : Y_{it}$

Estimation

- Important Assumption: Common Time Trend: without the treatment, the outcomes of the treatment and control group will evolve in the same way:

$$[Y_{c1} - Y_{c0}|c] = [Y_{c1} - Y_{c0}|\tau]$$

- Idea:
  - $Y_{c1} - Y_{c0}$: time effect / time trend
  - $Y_{\tau 1} - Y_{\tau 0}$: time effect + treatment effect
  - $(Y_{\tau 1} - Y_{\tau 0}) - (Y_{c1} - Y_{c0})$: treatment effect
- Regression Model:

$$Y_{it} = \beta_0 + \alpha T_{it} + \delta t + \gamma \mathbb{1}[i = \tau] + u_{it}$$

  - where:
    - α: DiD estimator
    - T_i: interaction term between the time dummy t and the treatment dummy $\mathbb{1}[i = \tau]$
    - δ: estimator for the time trend
    - t: time dummy
    - γ: estimator for pre-treatment difference in level
    - $\mathbb{1}[i = \tau]$: treatment dummy
  - We have:
    - $Y_{\tau 0} = \hat{\beta}_0 + \hat{\gamma}$
    - $Y_{\tau 1} = \hat{\beta}_0 + \hat{\alpha} + \hat{\delta} + \hat{\gamma}$
    - $Y_{c0} = \hat{\beta}_0$
    - $Y_{c1} = \hat{\beta}_0 + \delta$
    - Therefore: $\hat{\alpha} = (Y_{\tau 1} - Y_{\tau 0}) - (Y_{c1} - Y_{c0})$

Card (1990) - Regional Approach Using DiD

Setup: Mariel Boatlift

- Card 1990 uses a natural experiment setting: Mariel Boatlift

- Fidel Castro allowed all Cubans who wished to emigrate to U.S. to do so from the port of Mariel
- Around 125000 Cuban immigrants arrived in Miami in 1980, increasing Miami's LF by 7%, and even more so the percentage of low skilled workers
- Pre-treatment data: 1980 Census; Post-treatment data: 1979-1985 CPS data
- This circumvents the issue of selection, since the arrival city, Miami, was chosen politically, which is *exogenous* to its labour market conditions

Method

- Card uses a Diff-in-Diff estimator
- Outcome: wages, employment, and unemployment
- Treatment group: workers in Miami
- Control group: Atlanta, Houston, Los Angeles, and Tampa-St. Petersburg
    - and distinguish between effects on whites, blacks, Cubans, and Hispanics
- Assumption: Common Trend: in absence of Mariel Boatlift, Miami will experience the same change in labour market outcomes as Atlanta, Houston, Los Angeles, and Tampa-St. Petersburg

Findings

- The influx of immigrants had hardly any effect on the wage rates on neither the wages nor the unemployment rate of the low-skilled non-Cuban population in Miami
- This suggests a rapid absorption of immigrants into the labour market
- Influences:
    - It changed priors on the effects of immigration on labour markets
    - It made the use of natural experiments and DiD techniques popular

Discussions

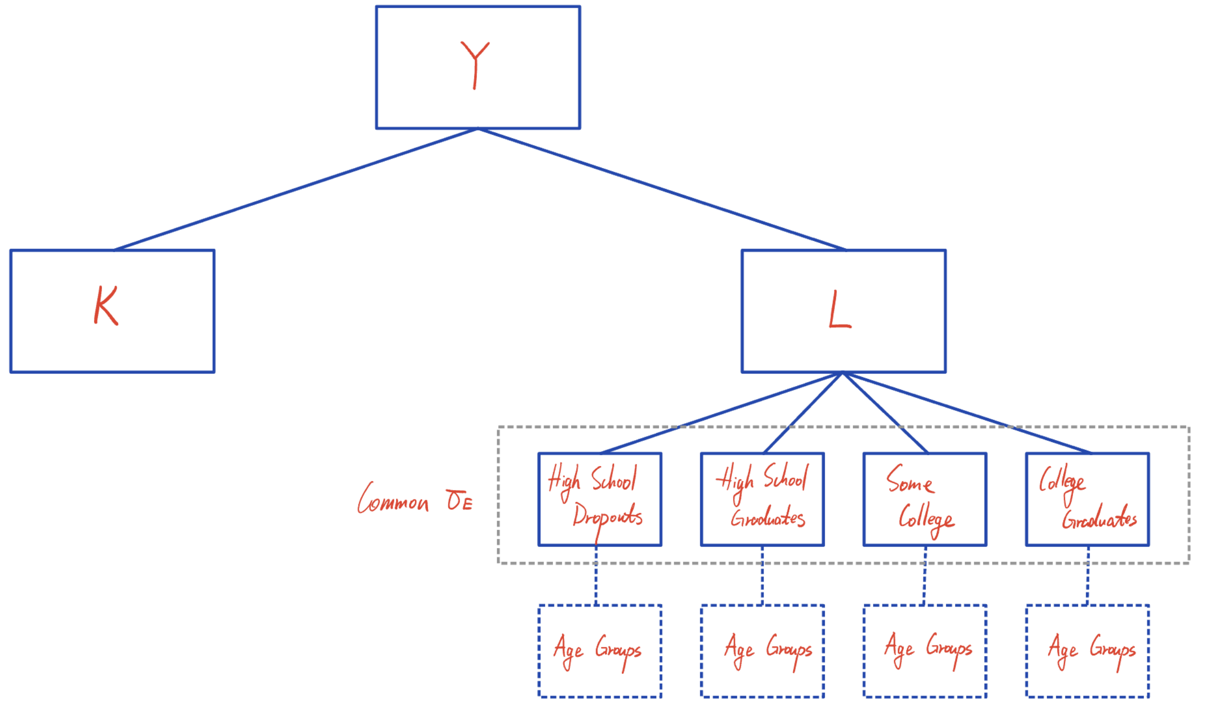
- Why 7% increase in LS in Miami led to little effects? (Note that a large number of other literature has the same conclusion).
- Here are 4 possible reasons:
- *The Miami labour market may be atypical of other local labour market in U.S.*
    - Miami's industry structure has a high concentration of apparel and textile industries, which is particularly well-suited to incorporate low-skill immigrants
    - High existing concentration of Hispanics could have facilitated integration
- *Internal Migration Response*: domestic native and earlier immigrant migration into Miami slowed down significantly after the Boatlift, hence the Mariel migrants may have partly displaced other potential migrants
- *Failure of Common Trend Assumption*: the treatment and control groups experienced different shocks

        - Angrist and Krueger (1999), Handbook of Labour Economics: they consider the counterfactual if the Mariel Boatlift did not happen
            - In 1994, the 2nd wave of boatlift was diverted to Guantanamo Bay instead of their planned destination Miami
            - Angrist and Krueger use this as a placebo test, and concluded that, the diverted Boatlift increased the black unemployment rate in Miami by 6.3%. Thus, at least in 1994, the common trend fails
- *Miami cannot be treated as an autarkic(isolated) labour market*: there are many ways through which the economy can adjust, such as changing output mix and technology
    - If factor price equalisation (a trade theory: free transportation $\rightarrow$ same wage across regions) holds, then the Mariel Boatlift would not affect the Miami labour market more than it affects the 4 cities in the control group
        - $\implies$ no cities can be used as the control group in DiD

Borjas (2003) - Skill-cell Approach

Method

- Assumptions:
    - Perfect substitutability between immigrants and native
    - Perfect substitutability within education-experience cells
    - No downgrading
    - Four skill groups are *equally imperfectly substitutable*
        - The substitutability between high school dropouts and college graduates is the same as that between high school dropouts and high school graduates
- 3-level CES Production Function:
    - Level 1: capital and labour
    - Level 2: 4 skill groups (high school dropouts, high school graduates, some college, college graduates)
    - Level 3: multiple age groups based on potential experience



Results

- Borjas estimates the elasticities, and then uses them to calculate the wage impact of immigrant inflow into U.S. between 1980 and 2000
- Results: a 10% increase in the immigrant share reduces the wages of competing workers by 3-4%
    - The actual immigration inflow between 1980 and 2000 increased the labour supply of working men by 11%, reducing the wages of:
        - the average native by 3.2%
        - the high school dropouts by 8.9%
        - the college graduates by 4.9%
        - the high school graduates by 2.6%
        - and barely changed the wages for workers with some college educations
    - These estimates imply that the immigration of the 1980s and 1990s has substantially worsened the labour market opportunities for most groups of natives, especially high school dropouts

Why Results Are Different from Card (1990)?

Borjas (2006)

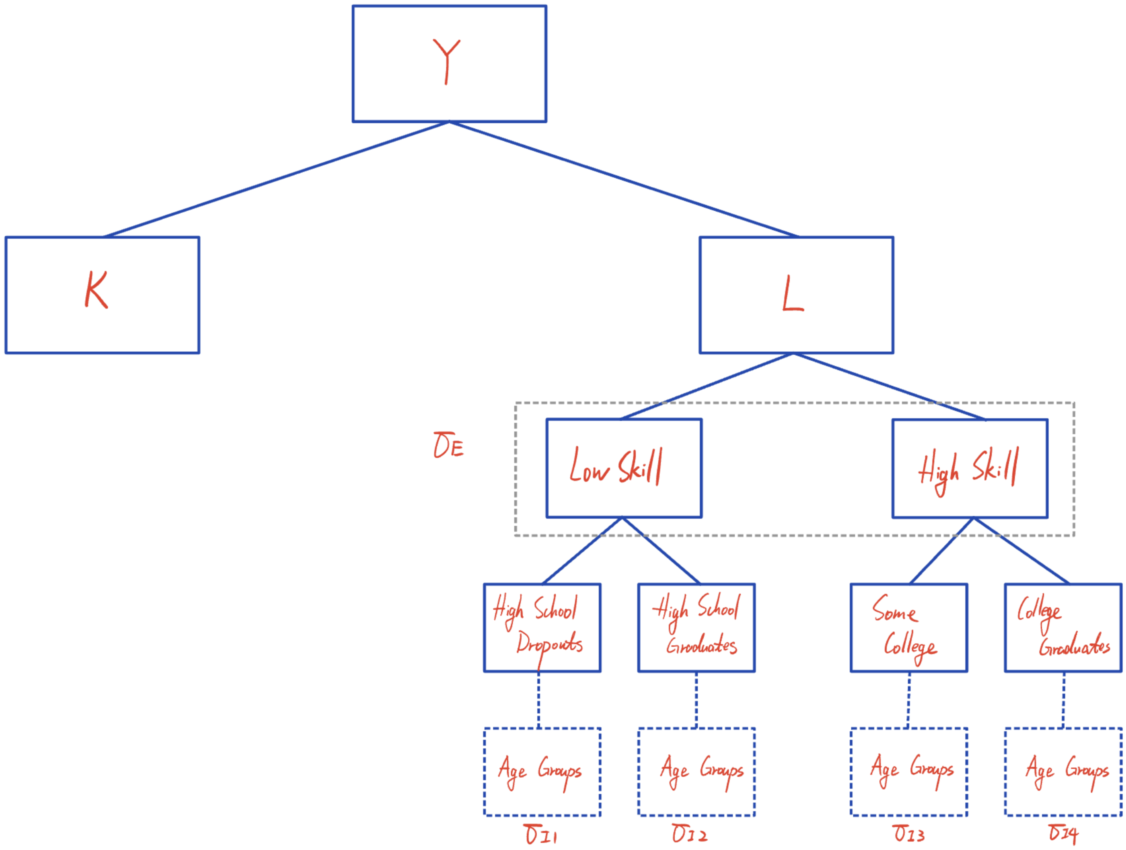
- Arguments:
    - Immigrants may *not be randomly distributed* across labour markets.
        - If immigrants tend to cluster in areas with thriving economies, this could attenuate the measurable negative effects immigrants on native workers

- Internal moving responses: Natives may respond to the entry of immigrants into a local labour market by *moving* to other localities until the native wages and returns to capital are again equalised across areas
    - Composition change → Common trend fails
    - Evidences:
        - Borjas 2006 performed a similar estimation as in Borjas 2003 but at different geographic levels: MSAs, state, census division, and nation.
            - He shows that the wage effects become more negative as the geographic level becomes larger
            - This suggests that the wage effects of immigration on local markets are attenuated if the *natives can easily move around*
            - However, studies on native migration responses provide mixed conclusions
- Another possibility is that data at a smaller geographic level is more subject to measurement errors, leading to attenuation bias
- *Both cause attenuation in effects (smaller absolute value) when looking at smaller regions*:
    - ME implies: zoom in → higher ME → attenuated effect
    - Mobility implies zoom in → higher mobility → attenuated effect

- Problems:
    - Internal migration flows are determined exogenously
    - In his framework, previous immigrants do not move internally when new immigrants arrive. Only natives move. However, literatures have shown that immigrants are more mobile
    - Not take into account the heterogeneity of labour. If natives and migrants are substitutes, then outflow of natives increase local wages; if they are complements, then outflow of natives depress local wages

Card (2009)

- Argument: Borjas' assumption on substitutability is not very reasonable
- Card estimates the elasticities using regional approach (at MSA level) and the network IV



-
- Findings:
  - High school dropouts are perfect substitutes for high school graduates
    - so the impact of low-skilled immigration is diffused across a wide segment of the labour market
  - Within broad education classes, immigrants and natives are imperfect substitutes
    - so, mostly, *new immigrants compete with existing immigrants, but less so with natives*

Week 5: Local Labour Markets I

- 5 Local Labour Markets I Completed - A

Motivation

- Analysis using the skill-cell regional approach typically involves changing the share of immigrants in a specific cell:

$$\Delta \ln w_{irt} = A_t + A_r + A_i + \gamma \Delta \ln \hat{X}_{irt} + \xi_{irt}$$

and use the estimate of γ to predict the change in wage corresponding to a change in LS of a particular skill group, but this requires us to know which region r the hypothetical immigrants will go to. Thus, we have to model internal location choices of immigrants

The Roback Model

- Related PS3 Question 1

Setup

General

- *Workers are perfectly mobile across areas*
  - Intercity commuting costs are prohibitive; this rules out the possibility of a person living in one city and working in another
  - Intracity commuting costs are ignored (to focus on the across-city allocation of workers and firms)
- *Workers are perfectly mobile across sectors*, so there is only 1 wage in each location (no arbitrage + perfect substitution -> same wage)
- *Homogenous workers*
- *Homogenous firms*
- *Only 1 good x* that is produced by firms and consumed by workers
  - and it is sold at a constant price normalised to $P = 1$

City

- Each city has an *amenity vector* $\boldsymbol{s} = (S_1, S_2, \ldots, S_n)$ and assumed to be *exogenous*
  - This amenity vector affects both workers' utility and firms' costs

The Worker's Problem

Setup

- *Each person supplies a single unit of labour* (no part-time job or unemployment)
- Taking the amenity vectors $\boldsymbol{s}$ as given, the representative worker solves the following *optimisation*:

$$\max_{x, l^c} U(x, l^c, s)$$

  subject to the *budget constraint*:

$$1 \times w = x \times 1 + l^c \times r \implies w = x + l^c r$$

  - where
    - w is the wage
    - x is the only national good
    - r is the rant of land
    - l^c is the residential land consumed

Equilibrium

- In equilibrium, the *indirect utility are equalised* across cities:

$$V(w, r, s) = V_0 \quad \forall \quad city$$

  - where $V(w, r, s)$ is the *indirect utility function* of workers given the wage, rent, and amenity of a city (the maximum utility workers can get given w, r, s)
    - $\frac{\partial V(w,r,s)}{\partial s} > 0$ if s is an amenity
    - $\frac{\partial V(w,r,s)}{\partial s} < 0$ if s is a dis-amenity

The Firm's Problem

Setup

- Assume the firm has *constant returns to scale (CRS) production function* (increase the inputs proportionally, output will increase for the same proportion):

$$X = f(l^p, N, s)$$

  - where
    - l^p is the land used in production
    - N is the total number of workers in the city
    - s is the amenity vector of the city

Equilibrium

- Since the production function has constant returns to scale, firms make *zero profit in equilibrium*.
  - $\implies$ *unit cost must equal product price* in all locations:

$$C(w, r, s) = P = 1 \quad \forall \quad \text{city}$$

  - Otherwise, firms move to more profitable areas
- Costs and amenities
  - $C_w = \frac{\partial C(w,r,s)}{\partial w} > 0$: cost increases as wage rises
  - $C_r = \frac{\partial C(w,r,s)}{\partial r} > 0$: cost increases as rent rises
  - $C_s = \frac{\partial C(w,r,s)}{\partial s} > 0$ if s is unproductive
  - $C_s = \frac{\partial C(w,r,s)}{\partial s} < 0$ if s is productive

General Equilibrium

Conditions

- Workers:

$$V(w, r, s) = V_0$$

- Firms (zero-profit condition):

$$C(w, r, s) = 1$$

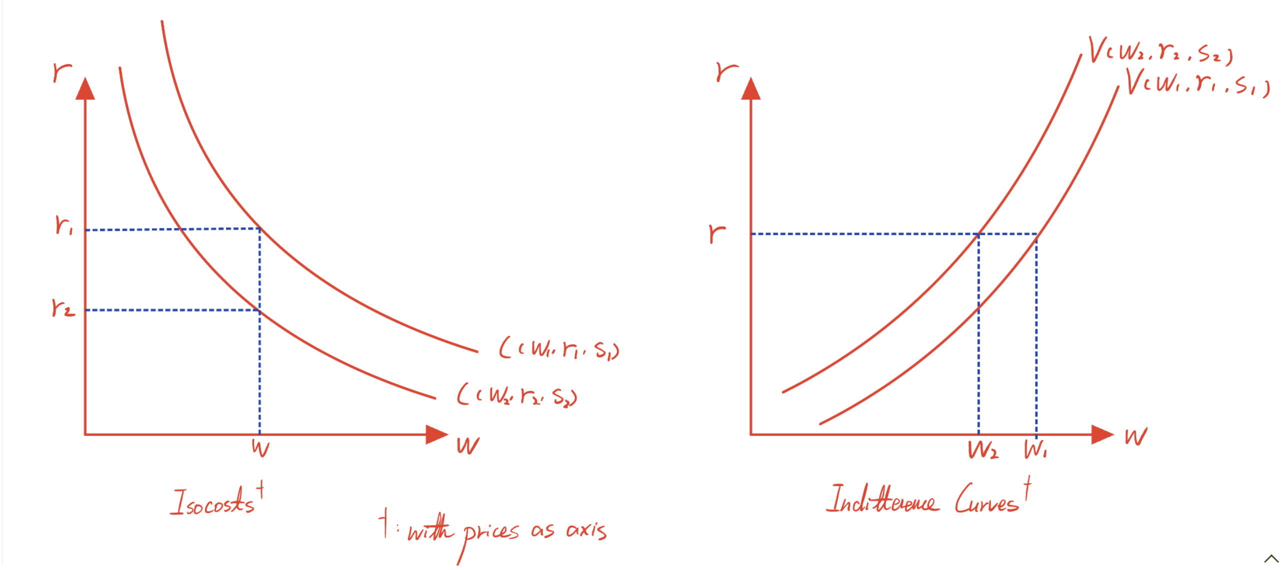
- Land adding-up constraint:

$$Ml^p + Nl^c = L$$

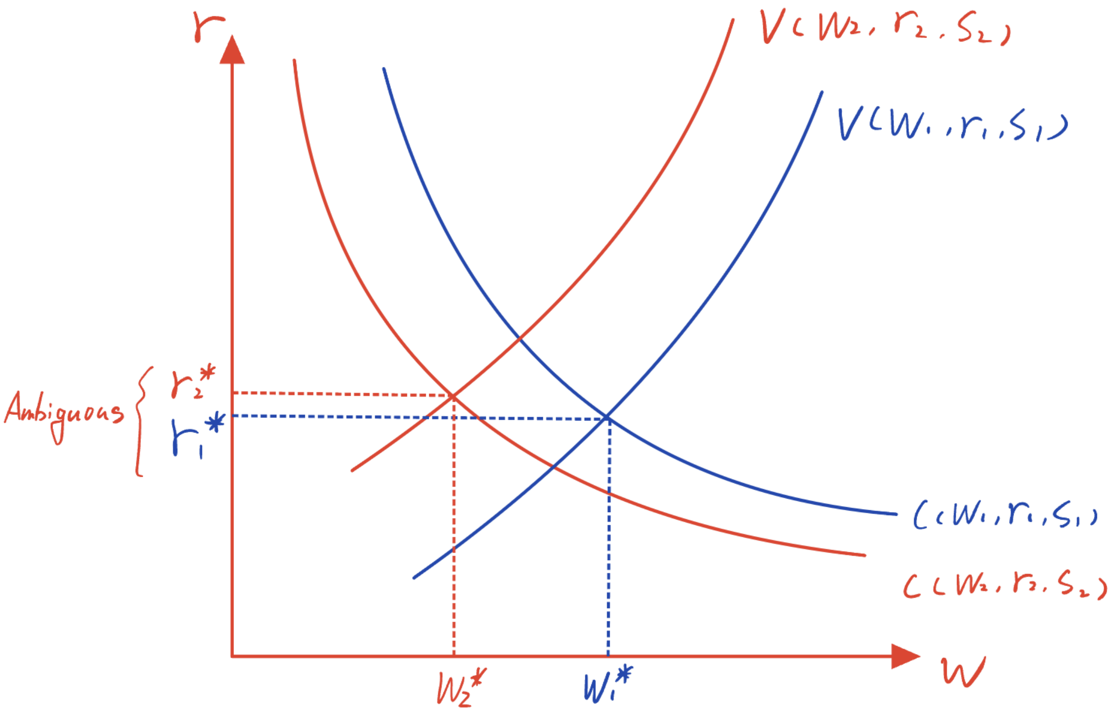
  - where:
    - L is the total amount of land
    - M is the number of firms
    - N is the number of workers
- Given V_0, w, r can be calculated as a function of s through the firms' and workers' equality constraints

Results (Unproductive but Desirable for Worker)

- 2 Cities $\{1, 2\}$: *Suppose $s_1 < s_2$ and s is unproductive, but desirable for workers*



- *Firm Side*:
  - Using prices w, r as axises, we can draw the *isocosts* of firms -- downward-sloping lines representing combinations of w, r that *equalise unit costs* at a given level of s
  - Since $s_1 < s_2$ and s is unproductive, the isocost in city 2 must be lower than the isocost in city 1 to equalise the costs -- with lower productivity, firms need to have lower rent at every wage level to have the same cost
- *Worker Side*:
  - Using prices w, r as axises, we can draw the *indifference curves* of workers -- upward-sloping lines representing combinations of w, r that *equalise utility* V_0 at a given level of s
  - Since $s_1 < s_2$ and s is desirable for workers, indifference curves in city 2 must be higher than in city 1 -- with higher amenity, people must pay higher rents at every wage level to be indifferent
- Equilibrium



-
- Combine the isocost and indifference curve, we can get the equilibrium point
- *(Assume amenities are unproductive but desirable for workers) The city with higher amenities must have lower wages, but the relative level of rent is uncertain*

Intuitions (Unproductive but Desirable for Worker)

- When s is unproductive, e.g. clean air, the firms prefer locations with lower s while workers prefer locations with higher s
- Meanwhile, w, r have to be such that both workers and firms are indifferent between different locations
- Since *high rents discourage both firms and workers*, there are *2 opposing forces on rent*
  - *Worker equilibrium requires high rents in locations with high amenity s to chock off immigration*
  - *Firm equilibrium requires low rents in locations with high amenity s to maintain firms*
- On the other hand, *since low wages discourage workers but attract business, wages must be lower in cities with higher amenities s*

Brief Summary

- Productive but Undesirable for Workers:

$$s \uparrow \rightsquigarrow \begin{cases} w & \uparrow \\ r & \text{Ambiguous} \end{cases}$$

- Unproductive but Desirable for Workers:

$$s \uparrow \rightsquigarrow \begin{cases} w & \downarrow \\ r & \text{Ambiguous} \end{cases}$$

- Productive and Desirable for Workers:

$$s \uparrow \rightsquigarrow \begin{cases} w & \text{Ambiguous} \\ r & \uparrow \end{cases}$$

- Unproductive and undesirable for Workers:

$$s \uparrow \rightsquigarrow \begin{cases} w & \text{Ambiguous} \\ r & \downarrow \end{cases}$$

Empirical Applications

- The Roback model implies a negative relationship between wages and amenities, and a positive relationship between wages and disamenities
- Use Census Bureau's Current Population Survey from May 1973, Roback estimates the effects of amenities and disamenities Z_c on wages:

$$\ln w_{ic} = x_i \beta + \gamma_w Z_c + \epsilon_{ic}$$

  - where x_i are individual characteristics
- Results: these are just correlations

  -
    Total crime rate (TCRIME 73) positive sign

    Local unemployment rate (UR 73) insignificant

    Particulate level (PART 73) positive sign

    Population density (DENSSMSA) insignificant

    Population growth (GROW GO70) strong positive coefficient

    Population size (POP73) strong positive coefficient

    Heating degree days (HDD) strong positive coefficient

    Total snowfall (TOTSNOW) strong positive coefficient

    Number of cloudy days (CLOUDY) strong positive coefficient

    Number of clear days (CLEAR) strong negative coefficient

Extensions

- Roback also extended the model by introducing a single non-traded local good
- A major limitation of Roback model is that *all workers are homogenous and indifferent between living in different cities*

- In reality, people may have different preferences over cities

---

Week 6: Local Labour Markets II

- 6 Local Labor Market Part2 Completed - A

Modelling Preferences for Cities (Discrete Choice Model)

- Similar to 0 0 ECON0021 Microeconometrics Index > Estimation with Aggregate Data
- Relevant question PS3 Q2

Setup

- Kline and Moretti (2013) extend the Roback Model by relaxing idiosyncratic preferences for cities
- *Worker's Utility*: for a given type of worker (where type could be defined by skill), assume:

$$u_{ic} = \underbrace{\beta_w(w_c - t) - \beta_r r_c + \beta_A A_c}_{\delta_c} + e_{ic}$$

  - where
    - w_c: wage in city c
    - r_c: rent in city c
    - A_c: amenities in city c (Unobserved)
    - t: common lump sum tax rate
    - β: preference parameters
    - e_{ic}: taste shock (random)
- Assume e_{ic} are *iid draws from Type-1 Extreme Value Distribution*
  - Generalised Extreme Value Type-1 Distribution (or the Gumbel Distribution) is used to model the distribution of maximum of a number of samples
  - Application: Suppose the there are J choices: $j = 1, \ldots, J$ and each of them has utility $v_j + b_j$ where v_j is deterministic and $b_j \sim$ Type I Extreme Value Distribution. Then, the choice probability is:

$$P_j = \frac{\exp(v_i)}{\sum_{j=1}^{J} \exp(v_j)}$$

- Applying this to our model, the *probability that person i chooses to live in city c* among all cities $k = 1, \ldots, J$ is:

$$P_c = \frac{\exp(\delta_c)}{\sum_{k=1}^{J} \exp(\delta_k)}$$

- The *expected population in city c* is the sum of the choice probability of all workers:

$$\sum_i \frac{\exp(\delta_{ic})}{\sum_{k=1}^J \exp(\delta_{ik})}$$

- Note that the indifference condition in original Roback model no long holds because workers now have heterogenous preferences

Mixed Logit

- Denote the city choice to be Y_i, which takes integer values $\{1, \dots, J\}$
- The mixed logit model specifies:

$$Pr(Y_i = j | X_{i1}, \dots, X_{iJ}) = \frac{\exp\left(X'_{ij}\beta\right)}{1 + \sum_{k=2}^J \exp\left(X'_{ik}\beta\right)}$$

  for choice $j = 2, \dots, J$
- For β to be identified, the base case $j = 1$ is *normalised to have zero utility*:

$$Pr(Y_i = 1 | X_{i1}, \dots, X_{iJ}) = \frac{\exp(0)}{1 + \sum_{k=2}^J \exp\left(X'_{ik}\beta\right)} = \frac{1}{1 + \sum_{k=2}^J \exp\left(X'_{ik}\beta\right)}$$

- The parameter vector β is common to all choices, and the value of $X'_{ik}\beta$ is interpreted as the *utility from choice k relative to the base case*

Estimation from Aggregate Level Data

- *Data available*: observed population shares in city city c (s_c)
- The model yields predicted population shares $\hat{s}_c$ as a function of $\beta_w, \beta_r, \beta_A A_c$
- We choose the values of $\beta_w, \beta_r, \beta_A A_c$ so that s_c and $\hat{s}_c$ are as close as possible (*MLE*)
- Given $e_{ic} \sim^{iid}$ Type-1 Extreme Value Distribution, the predicted population of a particular type of workers (e.g. college students) in city c is:

$$\begin{cases} \hat{s}_c & = \sum_{i=1}^N \left(\frac{\exp(\delta_c)}{1+\sum_{k=2}^J \exp(\delta_k)} \right) \forall\, c \neq 1 \\ \hat{s}_1 & = \sum_{i=1}^N \left(\frac{1}{1+\sum_{k=2}^J \exp(\delta_k)} \right) \end{cases}$$

- Berry (1994) suggests an *IV-based estimation approach* with 2 steps

Step 1: Back Out the Mean Utility (Inversion)

- Set $c = 1$ to be the base case
- Equate observed share to predicted share ($s = \hat{s}$) and take logs:

$$\begin{cases} \ln s_c & = \delta_c - \ln\left(1 + \sum_{k=2}^J \exp(\delta_k)\right) \forall\, c \neq 1 \\ \ln s_1 & = 0 - \ln\left(1 + \sum_{k=2}^J \exp(\delta_k)\right) \end{cases}$$

- Thus, we can back out δ_c from observed shares $s_1, \ldots, s_J$:

$$\delta_c = \ln s_c - \ln s_1$$

Step 2: Decompose the Mean Utility (Regression)

- Denote $\xi_c = \beta_A A_c$ since they are jointly identified as an unobserved component of the equation:

$$\delta_c = \beta_w(w_c - t) - \beta_r r_c + \xi_c$$

- The parameters of interest β_w, β_r represent the workers' demand elasticities for cities with respect to wages and rents.
  - They are *labour supply parameters*
- If ξ_c is uncorrelated with wages w_c and rent r_c, then we can just run *OLS*.
- However, amenities are likely to be correlated with local prices (recall Roback model), so we ned to use *IVs*
  - One common IV is a measure of firm's *labour demand shocks* at the national level interacted with cities' industrial composition in some base year
- Once we estimate β_w, β_r, we can back out $\xi_c = \beta_A A_c$ as residual $\xi_c = \delta_c - [\beta_w(w_c - t) - \beta_r r_c]$

Equilibrium Model

- We can extend the model to include multiple groups of labour and build in local labour demand using a nested CEF production function as in Week 5
  - Labour demand (from the production side):

$$w_c = f(\theta_c, N_c^d)$$

  - Labour supply (suppose each worker supplies one unit of labour):

$$N_c^s = \sum_i \left(\frac{\exp(\delta_c)}{\sum_{k=1}^J \exp(\delta_k)} \right)$$

    - where $\delta_c = \beta_w(w_c - t) - \beta_r r_c + \beta_A A_c$
- In equilibrium, wages adjust so that labour demand equals labour supply:

$$N_c^S = N_c^D$$

- Additionally, one can add housing demand and supply into the model.
  - Typically, it is assumed that housing rent is increasing in local population.
  - Why do we need housing rents? This is because we need a congestion force to choke o migration in this kind of model.
  - Suppose city 1 has very high local productivity $\theta \rightsquigarrow$ high wage. Then this will attract many people to move there but since housing rent is increasing in

population, the city becomes more expensive and hence less attractive.

- We can then use the model to simulate what would happen to wages and rents across cities if there is an influx of immigrants. The internal location choices of the new immigrants and price responses will be determined endogenously by the model.
- T*he full model is discussed in the next week*.

Week 7: Local Labour Markets III

- 7 Local Labour Market Part III - A

(Full) Static Spatial Equilibrium Model

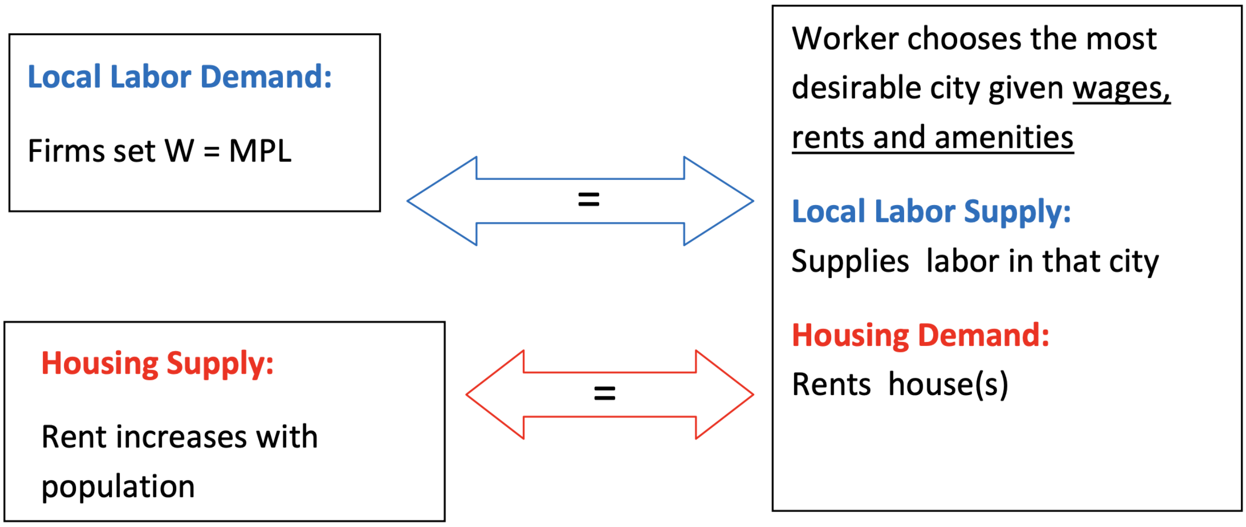
Introduction

- An inflow of immigrants may change more than wages. e.g.:
  - Housing rents: Saiz (2007)
  - Prices of immigrant-intensive services such as gardening: Cortes (2008)
  - Fiscal impacts: Storesletten (1999)
- To keep the model tractable, and because housing rent is the largest expenditure item of household's income, we focus on housing rents and wages here
- There will be equilibrium effects:
  - Changes in prices may induce workers to move
  - Overall impact also depends on migration responses:
    - Borjas, Freeman, and Katz (1997): the share of natives living in California stopped growing
    - Cadena and Kovak (2016): immigrants equilibrate local labour markets

Overview

- A static spatial equilibrium model (decision is made only once -- long-run choice)
- Workers differ in skill level, gender, and birth places
  - Heterogeneity in preferences for cities and place attachments
- Cities differ in productivity levels, housing prices, and amenities
- Only 1 production sector and 1 national good

- Structure:



I. Production Side: Local Labour Demand

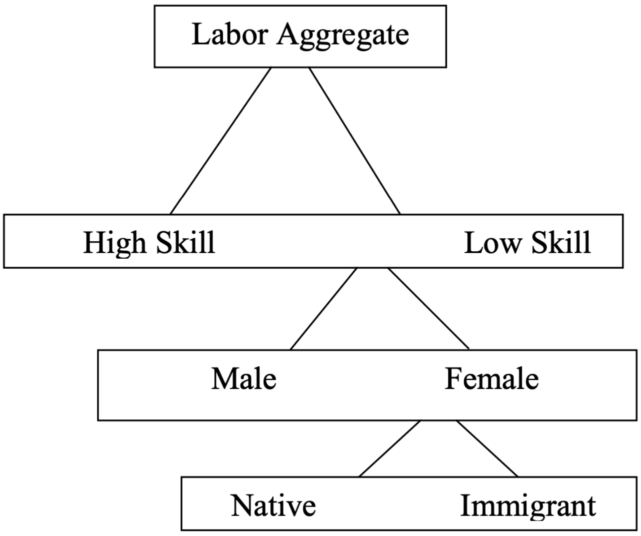
- Similar to Week 3 - Estimation Using Regional Approach and IV Strategies

Setup

- Each city c has many homogeneous and competitive firms in year t
- Firms produce identical nation goods with CES (indeed Cobb-Douglas) *Production Function*:

$$Y_{ct} = A_{ct} L_{ct}^{\alpha} K_{ct}^{1-\alpha}$$

  - where
    - A_{ct} is city-specific productivity (local TFP)
    - K_{ct} is capital
    - L_{ct} is a CES aggregate of different types of labour (with a nested structure)
    - $\alpha \in (0, 1)$ is the income share of labour
- *Nest Structure*: 3 layers, 8 cells



-
- *Level 1: High and Low Skills* $e \in \{H, L\}$:

$$L_{ct} = \left(\sum_e \theta_{ect} L_{ect}^{\rho_E} \right)^{\frac{1}{\rho_E}}$$

  - where $\theta_{Hct} + \theta_{Lct} = 1, \sigma_E = \frac{1}{1-\rho_E}$
- *Level 2: Male and Female* $g \in \{F, M\}$:

$$L_{ect} = \left(\sum_g \phi_{egct} L_{egct}^{\rho_G} \right)^{\frac{1}{\rho_G}}$$

  - where $\phi_{eFct} + \phi_{eMct} = 1, \sigma_G = \frac{1}{1-\rho_G}$
  - An alternative to this gender layer is to have a sector layer which models the full sector divisions, but that is too complicated, so we choose to model gender (female/male workers typically have different occupational choices)
- *Layer 3: Immigrants and Native* $s \in \{M, N\}$:

$$L_{egct} = \left(\sum_s \beta_{egct}^s S_{egct}^{\rho_{M,E}} \right)^{\frac{1}{\rho_{M,E}}}$$

  - where $\beta_{egct}^N + \beta_{egct}^M = 1, \sigma_{M,E} = \frac{1}{1-\rho_{M,E}}$
  - $\sigma_{M,E}$ varies by education since factors such as English skills may be less important for low-skill workers
- L_{ct} consists of all 3 levels of labour types where elasticity of substitution between immigrants and natives can vary by skill groups
- Assume further that
  - *Capital is perfectly elastically supplied at a common price* κ_t

- Firms operate in a *perfectly competitive output market*, so:

$$MPL = \frac{W}{P}$$

City Labour Demand Functions

- *City Labour Demand Functions*:

$$\begin{aligned}\ln \frac{W_{ct}^{egct}}{P_t} =& \frac{1}{\alpha}\ln A_{ct} + \eta_t + \ln \theta_{ect} + \frac{1}{\sigma_E}(\ln L_{ct} - \ln L_{ect}) \\ &+ \frac{1}{\sigma_G}(\ln L_{ect} - \ln L_{egct}) + \ln \phi_{egct} \\ &+ \frac{1}{\sigma_{M,E}}(\ln L_{egct} - \ln S_{egct}) + \ln \beta_{egct}^s\end{aligned}$$

- where $\eta_t = \ln\left(\alpha\left(\frac{1-\alpha}{\frac{\kappa}{P_t}}\right)^{\frac{1-\alpha}{\alpha}}\right)$

II. Worker Side: Location Choice

- Assume *immigration decision to be exogenous*
- z denotes worker characteristics $z = (e, g, s)$
  - where
    - e is skill level $e \in \{H, L\}$
    - g is gender $g \in \{F, M\}$
    - s is immigration status $s \in \{M, N\}$
- *A type z worker i inelastically supplies 1 unit of labour* and *maximises utility*:

$$U_{ict} = \max_{Q,G} \ln\left(Q_t^{\beta_z^r}\right) + \ln\left(G_t^{1-\beta_z^r}\right) + u_i(N_{ct})\ s.t.\ \underbrace{R_{ct}Q_t + P_tG_t \leq W_{ct}^z}_{\text{Budget Constraint}}$$

  - where
    - Q is quantity of housing
    - R is rent
    - G is quantity of national good consumed
    - P is price of the national good
    - N is amenities and networks (explained later)
- From the utility function, we can *derive the indirect utility function*:

$$V_{ict} = w_{ct}^z + u_i(N_{ct})$$

  - where
    - $w_{ct}^z = \ln\left(\frac{W_{ct}^z}{P_t}\right) - \beta_z^r \ln\left(\frac{R_{ct}}{P_t}\right)$ is *local real wage*, and β_z^r is the *income share of housing expenditure*, which is observed in data
- *Decompose $u_i(N_{ct})$*:

- $u_i(N_{ct})$ captures the value of amenities and networks:
  - *Amenities*:
    - Climate, quality of services, etc
    - We assume all residents have access to them, but they may value those amenities differently
  - *Networks*:
    - Immigrants value network size (a city's past number of immigrants from the same country)
    - Natives gain utility from living in or near their birth places
  - *Specifically*:

$$u_i(N_{ct}) = \underbrace{\beta_z^A x_{ct}^A}_{\text{Amenities}} + \underbrace{\underbrace{\beta_z^{st} x_{ic}^{st}}_{\text{Birth State}} + \underbrace{\beta_z^d x_{ic}^d}_{\text{Distance}}}_{\text{Only for Natives}} + \underbrace{\beta_z^{rb} x_{ic,t-\tau}^{rb}}_{\text{Network}} + \underbrace{\sigma^z \epsilon_{ict}}_{\text{Shock}}$$

  - *Assume that $\epsilon_{ict} \sim$ Type-I Extreme Value Distribution with variance σ^z*
  - Impose *Scale Normalisation*: we can normalise $\sigma^z = 1$ by dividing V_{ict} by σ^z because the absolute differences between choices are not identified $\implies$ $\epsilon_{ict} \sim$ Type-I Extreme Value Distribution with variance 1
- After the scale normalisation, *let Γ_{ct}^z denote the "mean utility" that is common across all type z workers in city c*:

$$V_{ict} = \underbrace{\lambda_z^w w_{ct}^z + \lambda_z^A x_{ct}^A}_{\Gamma_{ct}^z} + \lambda_z^{st} x_{ic}^{st} + \lambda_z^d x_{ic}^d + \lambda_z^{rb} x_{ic,t-\tau}^{rb} + \epsilon_{ict}$$

- The *probability of a person choosing to live in city c is*:

$$Pr_{ict} = \frac{\exp\left(\Gamma_{ct}^z + \left(\lambda_z^{st} x_{ic}^{st} + \lambda_z^d x_{ic}^d + \lambda_z^{rb} x_{ic,t-\tau}^{rb}\right)\right)}{1 + \sum_{k \in C}^{J-1} \exp\left(\Gamma_{kt}^z + \left(\lambda_z^{st} x_{ik}^{st} + \lambda_z^d x_{ik}^d + \lambda_z^{rb} x_{ik,t-\tau}^{rb}\right)\right)}$$

- The *labour supply of type z worker in city c in year t is*:

$$S_{zct} = \sum_{i \in z_t} Pr_{ict}$$

III. Land Side: Housing Market

- *Housing Production Function*:

$$Q_{ct} = a_{ct} l_{ct}^{\psi} m_{ct}^{1-\psi}$$

  - where
    - a measures productivity
    - l is land

        - m is material
- *Local Rents* are:

$$R_{ct} = i^t \times P_{h,ct}$$

    - where
        - i is interest rate
        - P is housing price
- *Housing Rent (Supply) Equation*:

$$\ln(R_{ct}) = \ln(CC_{ct}) + \gamma_c \ln Q_{ct}$$

    - where
        - CC is cost of construction
        - Q is *demand for housing* (from Cobb-Douglas utility function):

$$Q_{ct} = \frac{\sum_z \beta_z^r W_{ct}^z}{R_{ct}}$$

        - γ_c is *housing supply elasticity* which depends on geographic constraints x_c^{geo} and land use regulations x_c^{regu}:

$$\gamma_c = \gamma^{geo} x_c^{geo} + \gamma^{regu} \ln(x_c^{regu})$$

IV. Equilibrium

- *Equilibrium* is defined by a set of prices (W_{ct}^z, R_{ct}) ad populations (S_{zct}) such that:
    - Workers maximise utilities
    - Firms maximise profits
    - Labour demand equals labour supply
    - Housing demand equals housing supply

Data

- Main data: 5% samples of the 1980, 1990, 2000 U.S. Census, and 2005-7 ACS
- Locations: 114 MSA's which have all types of immigrants and the remaining MSA's are grouped as 1 outside option
- High skill is defined as some college or more
- Low skill is defined as high school graduates and dropouts
- Wages (by type) are the average hourly wage in a given MSA
- Rents are the average household monthly gross rents per member
- Price of national goods is the CPI in 2011 dollars
- also used shares of unavailable land and regulation index from Saiz (2010)

Estimation I: Production Side: Local Labour Demand

- This section introduces the estimation of Production Side: Local Labour Demand
- Similar to Week 3 - Estimation Using Regional Approach and IV Strategies
- Estimate from the bottom $\rightarrow$ top
    - *Step 1*:
        - *Estimate* $\beta^s_{egct}, \sigma_{M,E}$ in:

$$\ln\left(\frac{W^N_{egct}}{W^M_{egct}}\right) = \ln\left(\frac{\beta^N_{egct}}{1-\beta^N_{egct}}\right) - \frac{1}{\sigma_{M,E}}\ln\left(\frac{N_{egct}}{M_{egct}}\right) + \xi_{egct}$$

        - where
            - we use some *approximation by dummies*:

$$\ln\left(\frac{\beta^N_{egct}}{1-\beta^N_{egct}}\right) = d_g + d_e + d_t + d_c$$

            - and use the *IV*: predicted inflow of immigrants based on the 1980 enclave
        - Calculate $\rho_{M,E}$ from $\sigma_{M,E} = \frac{1}{1-\rho_{M,E}}$
    - *Step 2*:
        - With estimates of $\beta^s_{egct}, \rho_{M,E}$, *calculate* gender-skill specific labour supply:

$$L_{egct} = \left(\sum_s \beta^s_{egct} S^{\rho_{M,E}}_{egct}\right)^{\frac{1}{\rho_{M,E}}}$$

    - Then, estimate ϕ_{egct}, σ_G using similar methods above $\rightsquigarrow$ *proceed iteratively to the top level*

Estimation II: Worker Preferences

- This section introduces the estimation of Worker Preferences
- Use the *two-step method* from Berry, Levinsohn, and Pakes (1995, 2004)
- A more complicated version of Week 6 - Estimation location choice with aggregate data
- *Step 1*:
    - Use MLE to estimate Γ^z_{ct} and network parameters $\lambda^{st}_{zt}, \lambda^d_{zt}, \lambda^{rb}_{zt}$
        - The log-likelihood function is:

$$\mathcal{L}(\Gamma^z_{ct}, \lambda^{st}_{zt}, \lambda^d_{zt}, \lambda^{rb}_{zt}) = \sum_{i=1}^n \log\left(\frac{\exp\left(\Gamma^z_{ct} + \left(\lambda^{st}_z x^{st}_{ic} + \lambda^d_z x^d_{ic} + \lambda^{rb}_z x^{rb}_{ic,t-\tau}\right)\right)\mathbb{1}[c_i = c]}{1 + \sum_{k\in C}^{J-1} \exp\left(\Gamma^z_{kt} + \left(\lambda^{st}_z x^{st}_{ik} + \lambda^d_z x^d_{ik} + \lambda^{rb}_z x^{rb}_{ik,t-\tau}\right)\right)}\right)$$

- *Step 2*:
    - Decompose Γ^z_{ct} into values for w^z_{ct} and x^A_{ct} (treat $\lambda^A_z x^A_{ct}$ as a residual ξ^z_{ct}):

$$\Gamma^z_{ct} = \lambda^w_z w^z_{ct} + \xi^z_{ct}$$

- Use an *IV* to identify λ_z^w, Instrument: *Katz-Murphy (KM) index* which measures shifts in labour demand predicted by the city's industry structure:

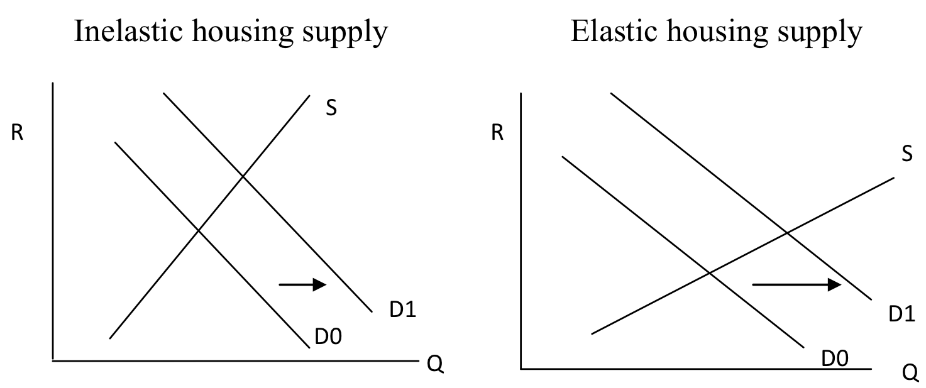
$$KM_{egct} = \sum_{i=1}^{ind} \omega_{i,c} \Delta L_{i,egct}$$

  - where
    - $\omega_{i,c}$ is the share of total hours worked in industry i in city c in year $t - \tau$
    - $\Delta L_{i,egct}$ is the change in log national hours worked in the same industry (excluding city c) between $t - \tau$ and t

Estimation III: Housing Supply

- This section introduces the estimation of Housing Market
- Recall the *Housing Supply Equation* derived earlier, and treat construction cost as residual $CC_{ct} = \epsilon_{ct}^{CC}$:

$$\ln(R_{ct}) = (\gamma^{geo} x_c^{geo} + \gamma^{regu} \ln(x_c^{regu})) \ln\left(\sum_z Z_{ct} \gamma_z^r W_{ct}^z\right) + \epsilon_{ct}^{CC}$$

  - KM Indices and their interaction with x_c^{regu} provide *exogenous variations* in housing demand, allowing us to identify housing supply (IV for demand $\rightsquigarrow$ identify supply)
  - 

Results

Baseline Estimates

Elasticity of Substitution			Housing Supply		
σ_E: skill	2.19	(0.11)	Geo γ^{geo}	0.91	(0.12)
σ_G: gender	1.97	(0.17)	Regulation γ^{reg}	0.53	(0.04)
$\sigma_{M,H}$: nativity	6.93	(0.15)			
$\sigma_{M,L}$: nativity	17.87	(0.82)			
Elasticity of Migration λ^w					
High skill male native	2.09	(0.25)	High skill male immi	3.84	(0.41)
High skill female native	1.02	(0.80)	High skill female immi	3.83	(0.22)
Low skill male native	1.32	(0.07)	Low skill male immi	1.23	(0.13)
Low skill female native	1.73	(0.06)	Low skill female immi	2.96	(0.19)

- Standard errors in parentheses
  - Higher elasticity of substitution $\Longleftrightarrow$ more substitutable
  - Higher elasticity of migration λ^w $\Longleftrightarrow$ more mobile

	High-skill male native	Low-skill male native
Birth state	2.78	3.63
Distance (1000 miles)	-0.63	-0.58
	High-skill male immi	Low-skill young immi
Previous immig. (million)	1.03	1.29

  - Low skill natives have stronger preferences to live in birth states than high skill natives
  - Low skill immigrants value city networks more than high skill immigrants

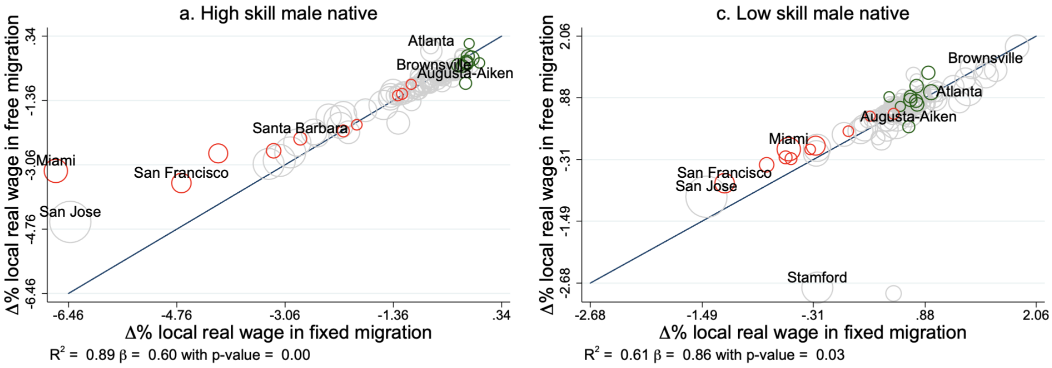
Model Fit

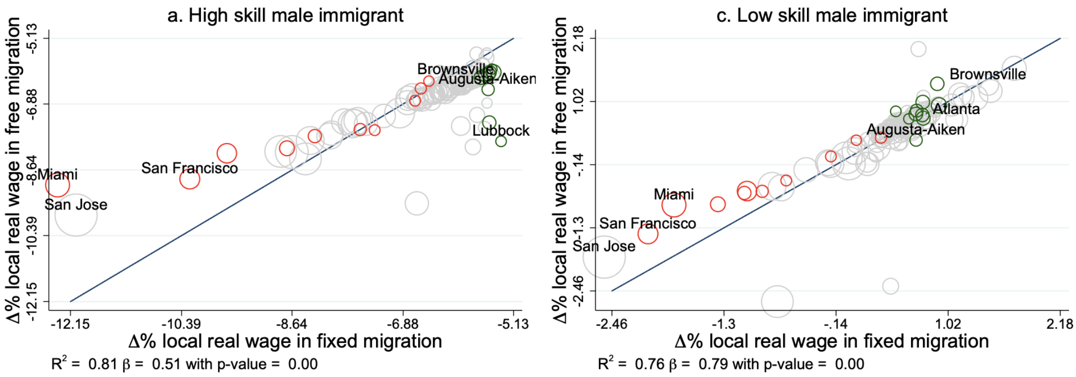
- The model could replicate the effects of Mariel boatlift on wages and rents well.
- The model could match the fraction of natives living outside their birth states well.
- The model could match the fraction of immigrants from major sending countries across cities well.

Predictions and Counterfactuals

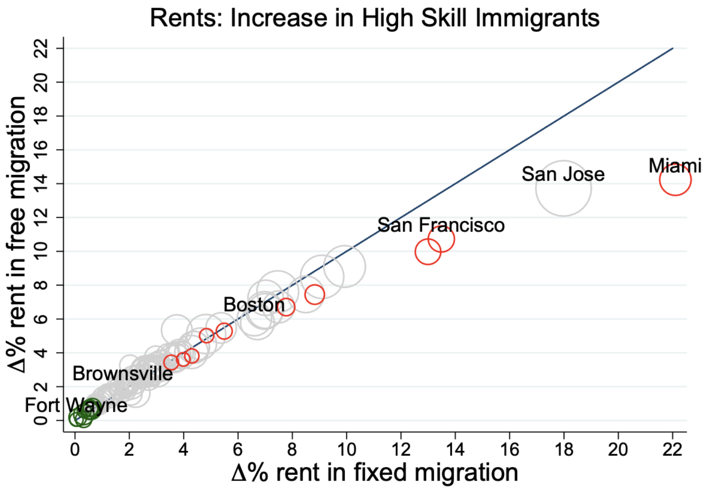
- *Predictions*
  - A city with the following characteristics would have a *larger outow of incumbent workers* in response to immigration
    - higher share of natives who have already left their birthplaces and immigrants with dispersion of large networks
    - more inelastic housing supply

- lower productivity
- lower amenities

- *Counterfactual 1: Change in the Skill Mix of Immigrants*
  - Consider 3.6 million new high skill immigrants
  - This raises the ratio immigrants to natives among high skill workers from 0.17 to 2.5 (roughly the UK ratio in 2003-2005)
  - Effect on native real wages
    - 

    - Red (green) bubbles are the ten cities with most inelastic (elastic) housing supply. Size of a bubbles reflects the number of new immigrants as a proportion of local population.
    - Fixed migration: low-skill native local real wages ↓ in some locations and high skill native local real wages ↓ in most locations
    - Free migration: Mean reversion of impacts (i.e. impacts are more equalised across locations)
  - Effect on immigrant real wages
    - 

    - Fixed migration: low skill immigrant local real wages ↓ in some locations and high skill immigrant real wages substantially ↓
    - Free migration: negative impacts are attenuated in some locations
  - Effect on rents



-
- Fixed migration: rent ↑ \$1308 in gateway cities and ↑ \$224 in other cities
- Free migration: rent ↑ \$1095 in gateway cities and ↑ \$226 in other cities

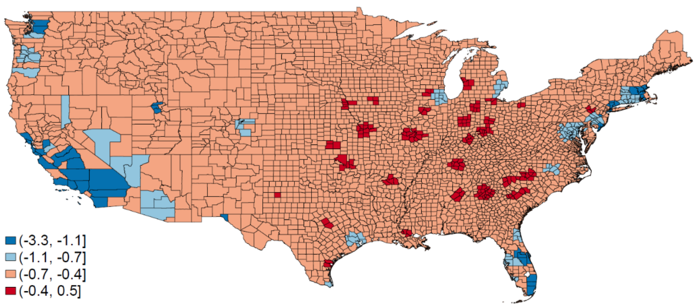
- Welfare
  - Welfare is measured by utility expressed in annual wage units (*Compensated Variation*)
  - Utility depends on wage, rent, amenities and home/networks
  - Gateway cities

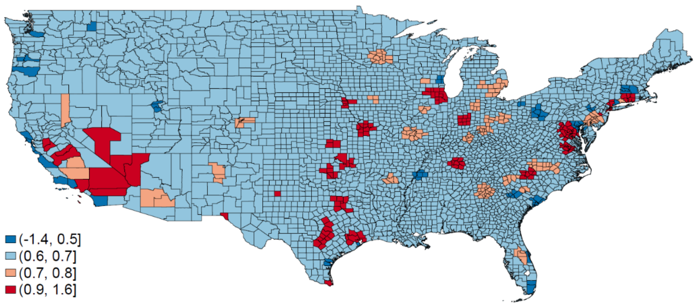
Gateway cities	Fixed migration	Free migration
	△ welfare	△ welfare
No rent redistribution		
High-skill male native	-3,304	-2,696
High-skill female native	-2,114	-1,691
Low-skill male native	569	493
Low-skill female native	463	395
High-skill male immigrant	-7,497	-6,711
High-skill female immigrant	-5,344	-4,817
Low-skill male immigrant	146	139
Low-skill female immigrant	116	118

- Other cities

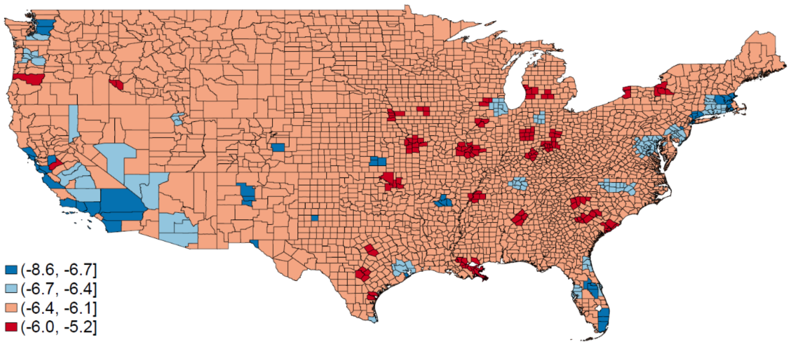
Other cities	Fixed migration	Free migration
	△ welfare	△ welfare
No rent redistribution		
High-skill male native	-831	-787
High-skill female native	-435	-451
Low-skill male native	347	351
Low-skill female native	288	289
High-skill male immigrant	-4,855	-4,964
High-skill female immigrant	-3,520	-3,622
Low-skill male immigrant	176	188
Low-skill female immigrant	144	152
Ave. net gain/loss without rental income	-723	-686
Ave. net gain/loss with rental income	57	94

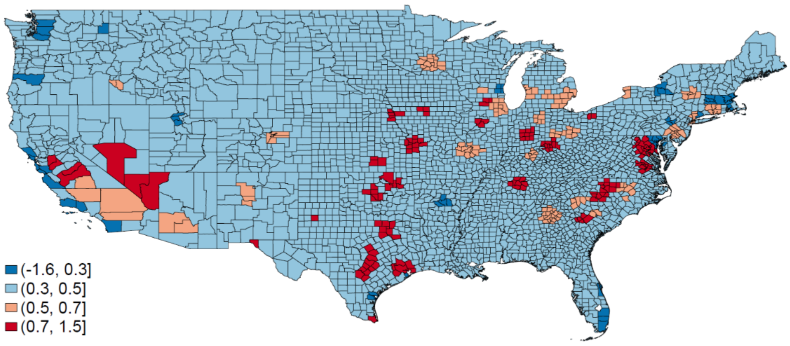
- Welfare effect on high-skill native (top) and low skill native (bottom)





- Welfare effect on high-skill immigrants (top) and low skill immigrants (bottom)





- *Counterfactual 2: 25% Increase in Immigrant Stock (skill mix unchanged)*:
  - Average rent increases by \$779 in gateway and \$156 in other cities (0.83-0.81 % for a 1% increase in population)
  - Positive wage effects on high-skill natives and less negative wage effects on high-skill immigrants relative to the first counterfactual
  - The average change in welfare is
    - no rent redistribution: −\$443
    - with rent redistribution: \$98
- *Counterfactual 3: Mexico-US Border Walls*:
  - We approximate the effects by removing 20, 59, 80 % of potential illegal immigrants in the states adjacent to Mexico

MSA	Percent of removal		
	20%	50%	80%
Phoenix, AZ	5,200	13,001	20,801
Fresno, CA	1,833	4,583	7,331
Los Angeles-Long Beach, CA	21,770	54,422	87,075
	:		
	:		
Kileen-Temple, TX	102	258	413
Lubbock, TX	48	122	195
San Antonio, TX	1,440	3,603	5,764
Total	73,045	182,621	292,192

  -

- Fixed Migration:
    - Rents fall by $< 1\%$.
    - Wages of low-skill natives in those states $\uparrow$
    - For low-skill immigrants in those states, wages $\uparrow$
    - Annual wages of all high skill natives and immigrants $\downarrow$
    - But all of these wage effects are very small
  - Free Migration:
    - As workers reallocate, the impacts become even more negligible
  - In all cases, the potential benefits of the border wall are considerably lower than the estimated construction cost.
- *Conclusions*:
  - The arrival of new immigrants has larger impacts on incumbent immigrants
  - Migration responses may reduce the negative impacts of immigration Significant gains for landlords
  - Skill-selective immigration policy makes low skill workers better off, but high skill workers worse off
    - Overall welfare gains
    - But larger net gains under non-skill selective policy
  - The potential benefits of border walls are substantially smaller than the proposed construction cost.

Week 8: Wage Assimilation

- 8 Wage Assimilation completed - A

Motivations

- Why should we care about how fast wages of immigrants grow in the host country?
  - Tax contributions depend on earnings
  - Education of immigrants' children may depend on their parent' income

Empirical Literature on Immigrant Assimilations

Early Literature (Chiswick 1978)

Introduction

- One of the first papers on immigrants' wage assimilation is Chiswick 1978
- Those papers assume migrations to be permanent, and use earnings regressions to measure the progress immigrants made after migrated to the host country

Empirical Implementation

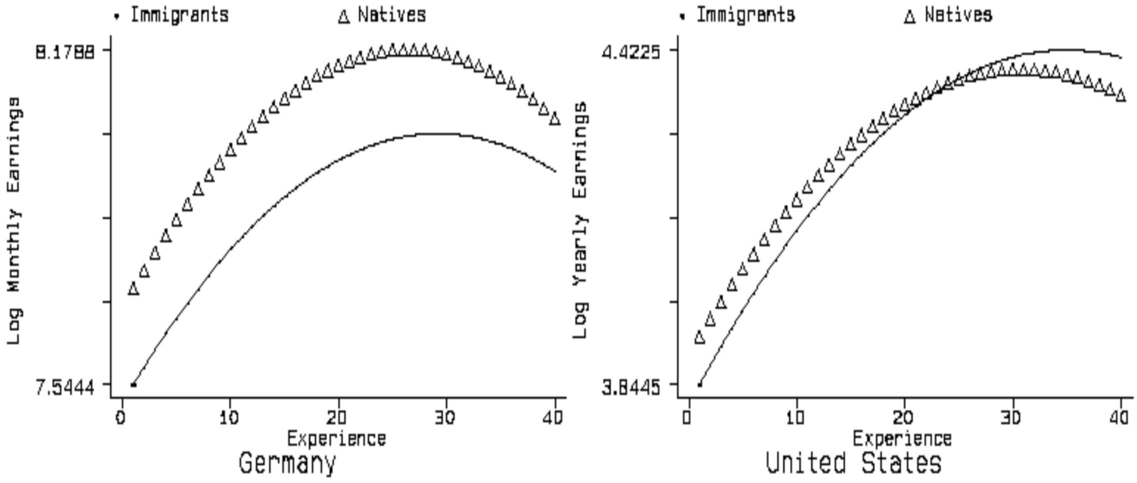
- Two log-wage equations for immigrants (I) and natives (N):

$$\begin{cases} \ln w_i^I & = b_1^I X_i + b_2^I EX_i + b_3^I YSM_i + e_i^I \\ \ln w_i^N & = b_1^N X_i + b_2^N EX_i + e_i^N \end{cases}$$

- where
  - X_i is a vector of individual characteristics (e.g. gender or occupations)
    - b_i measures the returns to those characteristics
  - $EX_i = \textbf{Age} - \textbf{Years of schooling} - 6$ is potential experience, which is extrapolated because it's not available in the dataset
    - b_2^I measures the return to one year of total working experience (experience in host & home country)
  - YSM_i is the number of years since migration / years spent in the host country
    - b_3^I measures the return to each year spent in the host country
  - $b_2^I + b_3^I$ measures the overall returns to labour market experience for immigrants
  - b_2^N measures the return to each year of labour market experience for natives
- *If $b_2^I + b_3^I > b_2^N$, then earnings of immigrants grow faster in the host country than earnings of natives.*

Results

- Chiswick (1978) uses 1970 US census data and estimates a regression similar to the one above
  - His main findings are:
    - Immigrants have an earning disadvantage of about 17% upon arrival
    - After about 10-15 years in the US labour market, earnings of immigrants overtake those of native workers
    - Chiswick explains this finding with the reason that "immigrants have more innate ability, and are more motivated towards labour market success, or self-finance larger investments in post-school training"
- Dustmann (1993) performs the same analysis for Germany
  - He does not find any evidence for earnings assimilation
  - Explanations:
    - The temporary nature of migrations in Germany leads to lower investments in host-country specific human capital
    - Migration to Germany may be "negatively selected"
    - The labour market in Germany night be less favourable for immigrants

- 

Cohort Effect (from Borjas 1985)

Why Biased?

- Borjas (1985) argues that *estimations based on a cross-section of data (such as the ones above) may lead to misleading conclusions*

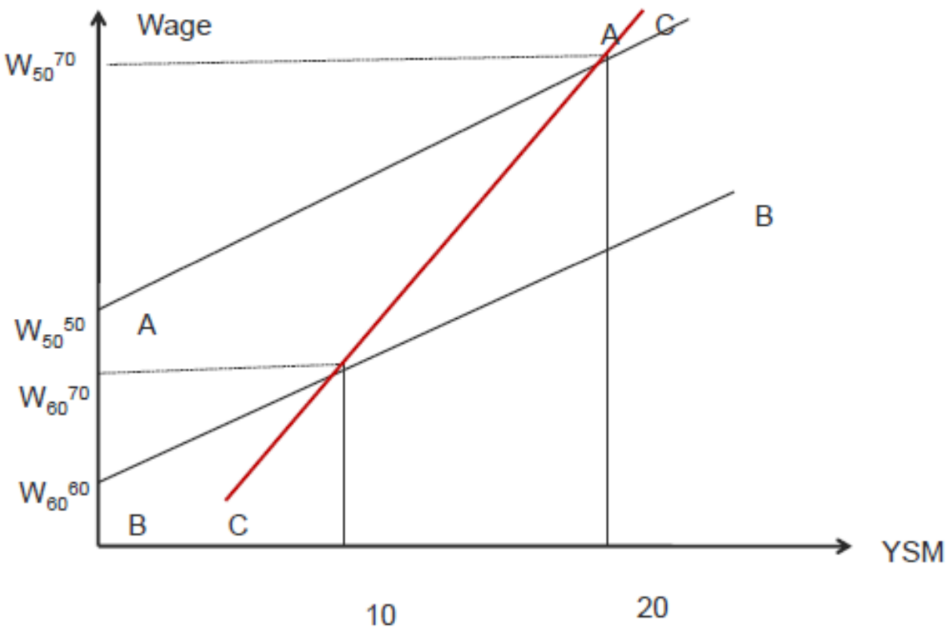
  - This is because immigrants with different years of residence in the host country also arrived in different years

  - If the entry wages of immigrants change over time, then the "YSM" coefficient will pick up this confounding effect

  - Specifically, the coefficient on "YSM" will consist of both returns to years spent in host labour market and differences in migrant cohort quality

  - *Graphically and algebraically*:

  -



- Let w_t^T denote the average log-wage of immigrants in year T who entered the host country in year t

- Assume $w_{50}^{50} > w_{60}^{60}$ (*cohort effect*) and both cohorts have the same speed of assimilation
- If we only have access to 1970 census data, then we can only observe w_{60}^{70}, w_{50}^{70}
- Base on this, one would estimate the wage growth per year in the host country as:

$$\Delta w = \frac{w_{50}^{70} - w_{60}^{70}}{20 - 10}$$

- This is the slope of the red line, which *overestimates the speed of assimilation due to confounding cohort effect*
- *Lower initial wages of subsequent cohorts (quality↓) could lead to an overestimate of immigrant assimilation, while higher wages of subsequent cohorts (quality↑) could lead to an underestimate of immigrant assimilation*
- Algebraically:

$$\frac{w_{50}^{70} - w_{60}^{70}}{10} = \underbrace{\frac{w_{60}^{80} - w_{60}^{70}}{10}}_{\text{Wage Growth for the 1960 Cohort}} + \underbrace{\frac{w_{50}^{70} - w_{60}^{80}}{10}}_{\text{Cohort Effect}}$$

- We can easily see that

$$\begin{cases} \text{Quality improves over time} & \Longrightarrow \text{Cohor Effect} < 0 \Longrightarrow \text{Underestimatin of convergence} \\ \text{Quality worsens over time} & \Longrightarrow \text{Cohor Effect} > 0 \Longrightarrow \text{Overestimation of convergence} \end{cases}$$

Idea 1: Add Another Cross-Section (Cohort Effects)

- One way to address the problem is to *include further census year*, so that the same cohort of immigrants can be observed at 2 different time points
- For example, if we also have 1980 census date, then we can estimate:

$$\Delta w = \frac{w_{50}^{70} - w_{50}^{80}}{30 - 20}$$

- However, this FD estimator can only eliminate time-unvarying fixed effect (cohort effect), but *this cannot control for differences in macroeconomic conditions (macro effects)* in 1970 and 1980

Idea 2: Estimate Cohort Effect under Assumptions (Cohort + Time + Macro Effects)

Why It's Hard to Estimate Cohort Effect?

- Consider log-wage regressions where we control for both cohort and year effects:

$$\begin{cases} \ln w_{it}^I & = b_1^I X_{it} + b_2^I EX_{it} + b_3^I YSM_{it} + b_m^I C_{im} + \gamma_t^I T_{it} + e_{it}^I \\ \ln w_{it}^N & = b_1^N X_{it} + b_2^N EX_{it} + \gamma_t^N T_{it} + e_{it}^N \end{cases}$$

- where
  - t is an index indicating the year in which individual i is observed

- X_{it} is a vector of individual characteristics, e.g. gender, marital status and educational level
- EX_{it} is potential work experience extrapolated from data
- YSM_{it} is the number of years since migration / years spent in the host country
  - b_3^I measures the time effect
- $T_{it} = \mathbb{1}[\text{Year of observation} = t]$ is a cross-section time indicating dummy which equals to 1 if individual i is drawn from the cross-section in year t
  - γ_t^I, γ_t^N measure the macro effects on log wages of immigrants and natives
- C_{im} is an indicator function, which equals to the calendar year m in which immigrant i arrived
  - b_m^I captures the cohort effects

- *Problem: macro effect and time effect cannot be separately identified with repeated cross-sections* $\implies$ *cannot estimate return to YSM*

  - We have $YSM_{it} = T_{it}(t - C_{im})$
  - Substitute this equality into the log-wage for immigrants:

$$\begin{aligned}\ln w_{it}^I &= b_1^I X_{it} + b_2^I EX_{it} + b_3^I YSM_{it} + b_m^I C_{im} + \gamma_{it}^I T_{it} + e_{it}^I \\ &= b_1^I X_{it} + b_2^I EX_{it} + b_3^I T_{it}(t - C_{im}) + b_m^I C_{im} + \gamma_{it}^I T_{it} + e_{it}^I\end{aligned}$$

  - In the data from a given year t, $T_{it} = 1$:

$$\ln w_{it}^I = b_1^I X_{it} + b_2^I EX_{it} + (b_m^I - b_3^I) C_{im} + \underbrace{b_e^I t + \gamma_t^I}_{\text{both vary by time}} + e_{it}^I$$

  - We cannot separately identify b_3^I (time/YSM effect) and γ_t^I (macro effect), hence we cannot separate b_m^I from b_3^I
  - $\rightsquigarrow$ b_3^I (time/YSM effect), b_m^I (cohort effect), γ_t^I (macro effect) cannot be separately identified simultaneously
  - However, if we can somehow back out $\{\gamma_t^I\}_{t=1}^T$, then we can identify b_3^I, b_m^I

Solution 1: Same Cohort Effect within Group

- *Fix γ_t^I to be the same for immigrants who arrived over a number of years* (e.g. a decade)
- In the extreme case where there is no time nor cohort effect, this brings us back to Chiswick 1978
- Assumptions like this need to be carefully justified by data: if one has strong reason to believe that the inflow of immigrants over a particular period is of roughly the same quality (for instance because immigrants all arrived from one particular source country) then this may be a plausible assumption

Solution 2: Same Macro Effect for Immigrants and Natives

- Borjas (1985) assumes that *the macro effect is the same for immigrants and natives*: $\gamma_t^I = \gamma_t^N$

- Then, we can first estimate the macro effect γ_t^N using data of natives, and use this (assume $\gamma_t^I = \gamma_t^N$) to identify cohort effects b_m^I in the immigrant equation
- Nevertheless, there are evidences suggesting that change in macroeconomic conditions is likely to have different effects on wages of natives and immigrants (Dustmann, Glitz and Vogel, EER 2010)

Solution 2+: Parameterising Macro Effects

- Bratsberg et al. (2005) show that the common macro effect assumption leads to serious bias in assimilation profiles for the US
- The provide an alternative method of estimating γ by *parameterising the macro effect at regional level*, allowing for different variations for immigrants and natives depending on local unemployment rates:

$$\begin{cases} \gamma_{rt}^I & = \gamma_t^0 + \eta^I \ln u_{rt} \\ \gamma_{rt}^N & = \gamma_t^0 + \eta^N \ln u_{rt} \end{cases}$$

- where
  - u_{rt} is the unemployment rate in year t and region r in which individual i lives in
- Indeed, according to their estimation, macro effects are different for natives and migrants -- wages of immigrants are more responsive to changes in local unemployment than wages of natives:

$$\begin{cases} \eta^I & = -0.14 \\ \eta^N & = -0.02 \end{cases}$$

Sources of Wage Assimilation

Importance of Understanding the Mechanism

- We know that the wage gap between immigrants and natives fall with labour market experience in the US, but what are the sources of wage assimilation and why it's important to understand the mechanism?
  - In the short run, income gap could lead to high costs of welfare programmes used by immigrants
  - In the long run, there could be inter-generational effects if migrated parents are unable to afford their children's education
  - Some policies may be used to speed up this convergence or eliminate the initial gap

Lessem and Sanders (2014)

Possible Channels

- Lessem and Sanders (2014) focus on two channels:

- *Returns to experience*: work experience in the host country may be more valuable than work experience in the home country because immigrants learn skills specific to the host country (e.g. languages and institutional knowledge)
- *Job search*: new immigrants may not be able to find their preferred / best-matched job instantly. As they spend more time in the host country, job matching improves and they move up the ladder.

Evidences

- Lessem and Sanders (2014) simulate the model to find each person's long-run (10-years) occupation in the US, and *calculate a counterfactual by assigning this best-matched job to each immigrant right in the first year of entry* and holding it fixed.
- Therefore, all wage increases in this counterfactual can be attributed to returns to experience. Comparing such increase with the reality where wage increases are caused by both experience and job search, they find that:
    - The slopes are similar, but the initial immigration-native wage gap reduces by 7%
- They conclude that: *in the early years after entry, job search frictions play an important role in the native-immigrant wage gap, but most of the assimilations later are attributed to returns to experience*

Other Channel: Spatial Assimilation

- Another channel through which the wage gap narrows is *spatial assimilation*:
- When migrating, immigrants tend to locate initially in the same city as their previous immigrants.
- However, over time, immigrants move to other cities and catch up on earnings without changing occupations.
- It could be that they move to cities where skill compositions are more favourable (i.e. cities where there are more workers with complementary skills) and hence their wages rise

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Week 9/10: Job Search Models

- 9 Unemployment, Search and Matching Completed - A

Introduction

Search and Matching Frictions

- Many markets are characterised by search and matching frictions. It takes time and effort and may cost money for buyers and sellers to find one another (*search costs*), and sometimes when a buyer and seller meet, they aren't a good match (*matching frictions*).

- In the labor market, job seekers (immigrants) may have trouble getting employers to consider their applications, and firms with vacant jobs may have trouble attracting applicants. And, once a job seeker and a prospective employer meet, one or both may decide that it isn't worth going forward with the match.
- The housing market is another significant market in which search and matching frictions are important, as is the marriage "market."

Why the Walrasian (Competitive) Model Doesn't Work in Markets with Search Frictions

- In markets with search frictions, a supply/demand (Walrasian) perspective is inconsistent with the *coexistence of excess supply and excess demand*.
- The supply/demand model would explain unemployment by saying that wages are too high to clear the market, but it would also say that employers can't fill all of their vacancies because wages are too low. So, the Walrasian perspective leads to a contradiction.

2 Key Questions

- In the labor market, taking the environment as given, how do workers search, how do they decide which job offer to accept, etc? Similarly, how do firms search for workers?
- What is the appropriate concept of equilibrium, i.e., how do we endogenize the market constraints that individual decision makers face?

A Chronological Outline of Job Search Literature

- *One-sided search models* (individual decision making) – optimal stopping problems.
- *1st-generation equilibrium models* – random search models with wage (or price) posting
  - In the labor market, these models were designed to explain (i) unemployment and (ii) wage dispersion for observationally similar workers. In these models, unemployment is explained as a result of workers rationally rejecting unattractive job offers in favor of waiting for something better. The equilibrium question is where the distribution of wage offers comes from.
- *Macro labor models* (especially the Diamond, Mortensen, Pissarides model)
  - The DMP model is essentially a labor demand-side explanation of unemployment. The key decision in the model is employers' choice of how many vacancies to post, and this is determined in equilibrium by a simple free-entry condition.
  - the more vacancies there are, the longer it takes for firms to fill their vacancies, so the expected value of vacancy becomes smaller. Firms keep entering until the expected value of vacancy is zero.
- *Directed (competitive) search models*
  - In random search models, job seekers "bump into" firms seeking workers at some probability. In directed search models, job seekers see posted wage offers and decide

where to apply (and firms choose their wage offers anticipating the offers that other firms will make and how workers will react to these offers).

Using Job Search Models

- *Applied Macro questions*:
  - How to understand differences across countries in the steady-state rate of unemployment?
  - unemployment insurance – level, duration
  - firing costs – advance notification, severance pay,
  - “temporary” versus “permanent” jobs minimum wage
  - payroll taxation
  - job training and search subsidies subsidies for job creation
  - How to understand wages, employment and unemployment over the business cycle?
- *Applied Micro questions*:
  - How to understand wage inequality across “observationally equivalent” workers?
  - search frictions as a source of wage dispersion
  - on-the-job search and wages over the life cycle competition between current and future employers wage-tenure contracts
  - How does the housing market work? what is the role of the asking price?
  - Who marries whom – assortative matching in the marriage market

McCall Model of Job Search (Optimal Sequential Search)

Objective

- Write down an equilibrium model for markets with search and matching frictions
- Basic framework: unemployed workers receive job offers continuously, and decide whether to accept
- Key: optimal stopping rule in a sequential search environment

Setup

Time

- Our simple model characterises sequential job searching in a stationary environment: *assume continuous time with discount rate r and infinite horizon*.
  - Reasons/meanings for these assumptions:
    - We choose continuous time instead of discrete time for tractability
    - Discounting reflects the idea that cost of job search is a time cost -- keeping other things equal, a worker would rather have a job now than later
    - Finally, we assume infinite horizon to make the problem stationary -- the worker's accept/decline decision is time-irrespective (i.e. irrespective of age,

length of unemployment, etc.). In a finite-horizon problem, the worker's decision decision rule will change as he/she approaches the end of life horizon

- *Remember to multiply dt whenever calculating probability or interest returns!!!*

Distribution of Wage Offers

- We assume *the distribution of wage offers is known, stationary, and exogenous with: wage range $w \in [\underline{w}, \overline{w}]$, pdf/pmf $f(w) = Pr(w = W)$, and cdf $F(w) = Pr(w < W)$*
    - Reasons/meanings for these assumptions:
        - "Known" means that workers have good information about the distribution of offers available in the market, but specific wage offers are stochastic.
        - Stationarity means this wage offer distribution does not change over time
        - The support of $F(w)$ being bounded $w \in [\underline{w}, \overline{w}]$ indicates that wage offers cannot be arbitrarily large or small, which is realistic and makes the model tractable
        - Also, the assumption that the distribution is exogenous simplifies this model to a single-agent decision problem
- Specific distribution (functional form) assumption: *wage offers arrive at at Poisson rate α, which is also exogenous*
    - Reasons/meanings for these assumptions:
        - Poisson process is the simplest stochastic process: let X_t be the number of offers received in an time interval t
        - The *probability of getting 1 offer in an time interval dt is approximately αdt, and this approximation gets more accurate as $dt \to 0$*
        - The assumption that α is exogenous implies that workers cannot affect α by changing searching efforts

Worker's Income and Utility

- *Worker's flow utility:*

$$U = \text{income} = \begin{cases} b & \text{when unemployed} \\ w & \text{when employed at wage } w \end{cases}$$

- *The worker's income in a short time interval dt is bdt when unemployed and wdt when employed at wage w*
    - We can interpret b as the income equivalence of home production, leisure, and/or unemployment benefits, etc.

Accept/Decline Offer

- *A job offer lasts forever once accepted, and there is not on-the-job search*
- We want to derive the optimal stopping rule for this

Derivation Workflow

$$\boxed{\left.\begin{array}{l}\text{Employed} : N(w) \\ \text{Unemployed} : U\end{array}\right\} \implies \text{Reservation Wage} : R \; s.t. \; N(R) = U}$$

Derivation 1: Continuous-Time Dynamic Programming

- The key decision problem is whether to accept an offer
- Here, we apply dynamic programming to evaluate the *expected discounted lifetime utility* of employment at wage w (denoted as $N(w)$) and of unemployment (denoted as U)

Expected Discounted Lifetime Utility of Employment ($N(w)$)

- Move from discrete time interval dt to continuous time:

$$\begin{aligned} N(w) &= \frac{1}{1+rdt}[wdt + N(w)] \qquad \text{(Stationary } N(w)\text{)} \\ (1+rdt)N(w) &= wdt + N(w) \\ rN(w) &= w \\ N(w) &= \frac{w}{r} \end{aligned}$$

- Interpretation: this is similar to the idea of pricing the PV of infinite-horizon cash flow with per-period payment c and discount rate r. This is also known as the Bellman Equation for Employment:

$$N(w) = \frac{w}{r}$$

Expected Discount Lifetime Utility of Unemployment (U)

- Derive the expected discount lifetime utility of unemployment using the same method (α is the Poisson rate):

$$\begin{aligned} U &= \frac{bdt + \alpha dt \cdot (\text{Utility}|\text{Receive Offer}) + (1-\alpha dt) \cdot (\text{Utility}|\text{Not R}}{1+rdt} \\ U &= \frac{bdt + \alpha dt E[\max\{N(w), U\}] + (1-\alpha dt)U}{1+rdt} \\ U + Urdt &= bdt + \alpha dt E\left[\max\left\{\frac{w}{r}, U\right\}\right] + U - \alpha dt U \\ Urdt &= bdt + \alpha dt E\left[\max\left\{\frac{w}{r}, U\right\}\right] - \alpha dt U \\ rU &= b + \alpha E\left[\max\left\{\frac{w}{r}, U\right\}\right] - \alpha U \\ rU &= b + \alpha\left\{E\left[\max\left\{\frac{w}{r}, U\right\}\right] - U\right\} \end{aligned}$$

- Caveat: $N(w)$ enters this equation, so if its different, U will also be different
- Interpretation: Again, this can be intuitively understood as a flow asset pricing exercise: the instantaneous/flow value associated with an asset has 2 components: the dividend

(payment generated) and capital gain/loss in the future. This is known as the Bellman Equation for Unemployment:

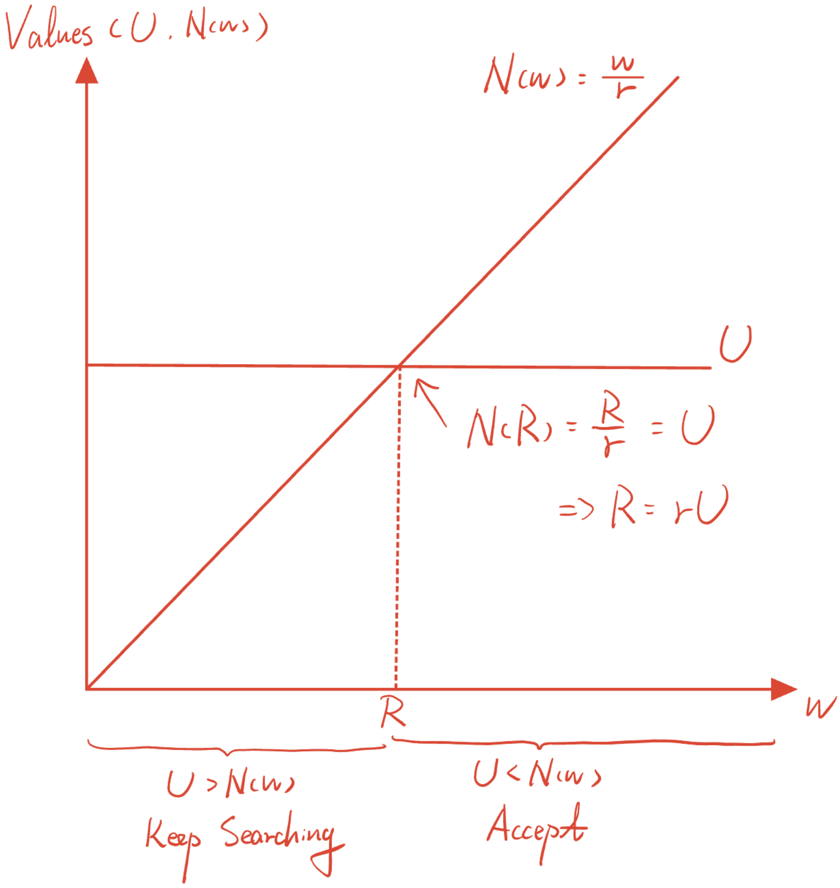
$$rU = \underbrace{b}_{\text{"Dividend"}} + \underbrace{\alpha \left\{ E\left[\max\left\{ \frac{w}{r}, U \right\} \right] - U \right\}}_{\text{"Expected Capital Gain"}}$$

Derivation 2: Reservation Wage Recursion

- *The worker accept an offer with wage w if and only if $N(w) \geq U$. Put differently, accept w if and only if $w \geq R$ where R denotes the reservation wage at which* $N(w) = N(R) = U$

$$\implies N(R) = \frac{R}{r} = U \implies R = rU$$

- Graphically:



- We can then solve the reservation wage recursively, starting from the Bellman equation for unemployment:

$$
\begin{aligned}
rU &= b + \alpha \left\{ E\left[\max\left\{ \frac{w}{r}, U \right\} \right] - U \right\} \\
rU + \alpha U &= b + \alpha E\left[\max\left\{ \frac{w}{r}, U \right\} \right] \\
U &= \frac{b}{r+\alpha} + \frac{\alpha}{r+\alpha} E\left[\max\left\{ \frac{w}{r}, U \right\} \right] \\
rU &= \frac{br}{r+\alpha} + \frac{\alpha}{r+\alpha} E\left[\max\left\{ w, rU \right\} \right] \\
R &= \frac{br}{r+\alpha} + \frac{\alpha}{r+\alpha} E\left[\max\left\{ w, R \right\} \right] && \text{(Beca} \\
\frac{r+\alpha}{r} \cdot R &= b + \frac{\alpha}{r} E\left[\max\left\{ w, R \right\} \right] && \text{(Multiply } \frac{r\cdot}{} \\
R &= b + \frac{\alpha}{r} E[\max\left\{ w, R \right\}] - \frac{\alpha}{r} R \\
R &= b + \frac{\alpha}{r} E[\max\left\{ w - R, 0 \right\}] && \text{(Put in}
\end{aligned}
$$

- Expand the expectation and use integration by parts:

$$
E[\max\{w - R, 0\}] = \int_R^{\overline{w}} (w - R) f(w)\, dw = \int_R^{\overline{w}} (1 - F(w))\, dw
$$

- Therefore, we have the *expression for reservation wage*:

$$
R(\alpha, b, r) = b + \frac{\alpha}{r} \int_{R(\alpha,b,r)}^{\overline{w}} (1 - F(w))\, dw
$$

Comparative Statics

- We want to study how R and expected unemployment duration vary with b, α.

Unemployment Duration, Reservation Wage, Hazard Rate

- Recall that $F(R) = Pr(w < R) \implies 1 - F(R) = Pr(w \geq R)$. Firstly, we defined the *hazard rate out of unemployment*

$$
h(z) = \alpha(1 - F(R))
$$

  as the probability of exiting unemployment at any instant conditioning on being unemployment up to that point of time. This is *decreasing in reservation wage R*:

$$
\frac{\partial h(z)}{\partial R} < 0
$$

- Expected unemployment duration, denoted as $E[T]$, is *decreasing in the hazard rate $h(z)$ and increasing in reservation wage R*:

$$
\frac{\partial E[T]}{\partial h(z)} < 0, \frac{\partial h(z)}{\partial R} < 0 \implies \frac{\partial E[T]}{\partial R} > 0
$$

- Intuitively, the lower the reservation wage is or the faster a worker can leave the unemployment pool, the shorter his/her duration of unemployment will be. (

$R \downarrow, h(z) \uparrow \implies E[T] \downarrow$)

How Reservation Wage / Unemployment Duration Varies with Unemployment Benefit

- Recall that reservation wage is:

$$R(\alpha, b, r) = b + \frac{\alpha}{r}\int_R^{\overline{w}} (1 - F(w))\, dw$$

- To calculate $\frac{\partial R}{\partial b}$, we need to manipulate our expression for R using the Leibniz Integral Rule, which shows us how to compute integrations when the domains of integration can also vary:

$$\frac{d}{dz}\int_{a(z)}^{b(z)} f(x, z)\, dz = \int_{a(z)}^{b(z)} \frac{df}{dz}\, dx + f(b(z), z)\frac{db}{dz} - f(a(z), z)\frac{da}{dz}$$

- Applying the Leibniz rule (we only need the 3rd term because only the lower bound is a function of w) where $x = w, z = b, f(w, b) = (1 - F(w)), a(z) = R(b)$:

$$R'(b) = \frac{\partial R(\alpha, b, r)}{\partial b} = 1 - \frac{\alpha}{r}(1 - F(R))R'(b)$$

- This implies *the reservation wage R is increasing in b*:

$$R'(b) = \frac{\partial R(\alpha, b, r)}{\partial b} = \frac{r}{r + \alpha(1 - F(R))} > 0$$

- Since the expected unemployment duration $E[T]$ is increasing in R, we know $E[T]$ is increasing in b:

$$\frac{\partial E[T]}{\partial R} > 0, \frac{\partial R}{\partial b} > 0 \implies \frac{\partial E[T]}{\partial b} > 0$$

- In other words:

$$b \uparrow \implies R \uparrow \implies E[T] \uparrow$$

How Reservation Wage / Unemployment Duration Vary with Probability of Getting Job Offer

- Recall that reservation wage is:

$$R(\alpha, b, r) = b + \frac{\alpha}{r}\int_R^{\overline{w}} (1 - F(w))\, dw$$

- Again, we can calculate $\frac{\partial R}{\partial \alpha}$ using Product Rule and Leibniz Rule:

$$
\begin{aligned}
R'(\alpha) &= \frac{\partial R(\alpha, b, r)}{\partial \alpha} \\
&= \frac{1}{r}\int_R^{\overline{w}} (1 - F(w))\, dw + \frac{\alpha}{r} \frac{\partial \int_R^{\overline{w}} (1 - F(w))\, dw}{\partial \alpha} && \text{(Product Rule)} \\
&= \frac{1}{r}\int_R^{\overline{w}} (1 - F(w))\, dw - \frac{\alpha}{r}(1 - F(R))R'(\alpha) && \text{(Lebniz Rule)}
\end{aligned}
$$

- This implies *reservation wage R is increasing α:

$$
R'(\alpha) = \frac{\partial R(\alpha, b, r)}{\partial \alpha} = \frac{\int_R^{\overline{w}} (1 - F(w))\, dw}{r + \alpha(1 - F(R))} > 0
$$

- Intuitively, as job offers arrive more frequently, workers become more picky.
- However, the *effect on expected unemployment duration $E[T]$ is ambiguous*: $E[T]$ is inversely related to the hazard rate $h(z) = \alpha(1 - F(R))$, which means α also has a direct effect on expected duration:
  - *Direct effect*: $\alpha \uparrow$ means offers arrive more quickly, so $E[T]$ decreases
  - *Indirect effect*: $\alpha \uparrow$ means job seekers adjust reservation wage R upwards, so $E[T]$ increases
- In summary:

$$
\alpha \uparrow \implies \begin{cases} R \uparrow \\ E[T] \text{ ambigous: } \begin{cases} \text{Direct Effect:} & h(z) \downarrow \implies E[T] \downarrow \\ \text{Indirect Effect:} & R \uparrow \implies E[T] \uparrow \end{cases} \end{cases}
$$

Extension: Exogenous Job Destruction

Setup

- Suppose now there is *exogenous job destruction* with a Poisson rate λ
- According to asset value formulation:

$$
\begin{cases} rN(w) & = w + \lambda(U - N(w)) \\ rU & = b + \alpha E\left[\max\left\{N(w) - U, 0\right\}\right] \end{cases}
$$

Reservation Wage

- Define the *reservation wage* as $N(R) = U \iff R = rU$, then we manipulate the first equation:

$$
\begin{aligned}
rN(w) - rU &= w + \lambda(U - N(w)) - rU \\
r(N(w) - U) + \lambda(N(w) - U) &= w - rU \\
N(w) - U &= \frac{w - R}{r + \lambda}
\end{aligned}
$$

- Substitute into the second equation:

$$
\begin{aligned}
R &= b + \frac{\alpha}{r+\lambda} E[\max\{w - R, 0\}] \\
&= b + \frac{\alpha}{r+\lambda} \int_R^{\overline{w}} (1 - F(w))\, dw
\end{aligned}
$$

- Implication: $\frac{\partial R}{\partial \lambda} < 0$: if jobs does not last long, then it is not worthy to search for a long time for a job with high wage

Steady State Unemployment

- Let u denote the *steady state unemployment rate* where outflow is equal to inflow:

$$
\underbrace{\overbrace{\alpha}^{\text{Pr of offer}} \times \overbrace{(1 - F(R))}^{\text{Pr of accept}} \times \overbrace{u}^{\text{\% of unemployed}}}_{\text{inflow}} = \underbrace{\lambda(1-u)}_{\text{outflow}}
$$

- This implies:

$$
u = \frac{\lambda}{\alpha(1 - F(R)) + \lambda}
$$

Comparative Statics

- How does the steady state unemployment rate changes with λ?

$$
\frac{\partial u}{\partial \lambda} = \frac{\alpha(1 - F(R)) + \lambda + \lambda\alpha f(R)\frac{\partial R}{\partial \lambda}}{[\alpha(1 - F(R)) + \lambda]^2}
$$

- The sign is ambiguous because $\frac{\partial R}{\partial \lambda} < 0$
- Intuitively, there are two channels:
  - Direct effect: higher λ indicates bigger flow from employed $\rightarrow$ unemployed
  - Indirect effect: higher λ indicates lower reservation wage, so there will be also a bigger flow from unemployed $\rightarrow$ employed

Other Possible Extensions

- Endogenous search intensity (α depends on worker's effort)
- Search with a finite horizon (special case of non-stationary search)
- On-the-job search (Burdett and Mortensen, IER 1998)

Summary

- W1: Migration decision in a simple model
- W2: Roy's model:
  - Positive selection: upper distribution $\rightarrow$ upper distribution
  - Negative selection: lower dist $\rightarrow$ lower dist

    - Neither
    - Extensions
- W3/4: Wage impacts of immigration
    - 1. Diff-in-diff / Natural experiment (Marial boatlift)
    - 2. Reduced-form IV approach using skill/area/combined cells
    - 3. Simulation approach using an equilibrium model
- 4. W5/6/7: Spatial equilibrium model
    - Estimate parameters using Berry 1994
- W8: Wage assimilations
- W9: Job search model