Production

Marginal Rate of Transformation (MRT) #flashcard

(from ECON0013, different from OX lecture)
MRT defines how much more of good j can be net supplied, if the firm decreases the net output of good i by one unit:

$$MRT_{i,j} = \frac{dy_j}{dy_i}_{F(y)=0} = -\frac{\frac{\partial F(y)}{\partial y_i}}{\frac{\partial F(y)}{\partial y_j}}$$

Marginal Rate of Technical Substitution (MRTS) #flashcard

(from ECON0013, different from OX lecture)
MRTS defines how much of input j will be needed, if the firm decreases in amount of input i by one unit, keeping the output and other inputs unchanged:

$$MRTS_{i,j} = -\frac{dz_j}{dz_i}_{q=f(z)} = \frac{\frac{\partial f}{\partial z_i}}{\frac{\partial f}{\partial z_j}}$$

Cost Minimisation #flashcard

$$c(w,q) = \min w \cdot z \; s.t. \; f(z) \geq q$$

This produces **conditional factor demand functions** $z(w,q)$

Shephard's Lemma #flashcard

$$\frac{\partial c(w,q)}{\partial w_l} = z_l(w,q)$$

Inferior Input #flashcard

An input l is inferior if $z_l(w,q)$ decreases with q:

$$\frac{dz_l}{dq} < 0$$

and the firm's marginal cost decreases with w_l if and only if l is inferior.
Also, if the technology exhibits CRS, then the inputs cannot be inferior.

Profit Maximisation #flashcard

$$\max p \cdot y \; s.t. \; y \in Y$$

this produces supply function/correspondence $y(p)$
In the **perfect competitive case**, the FOC is:

$$p = C'(q)$$

Symmetric Derivatives of Supply and Prices #flashcard

$$\frac{\partial y_l(p)}{\partial p_k} = \frac{\partial y_k(p)}{\partial p_l}$$

Weak Axiom of Profit Maximisation and Law of Supply #flashcard

WAPM:

$$p \cdot y(p) \geq p \cdot y' \; \forall \; y, y' \in Y$$

This implies:

$$\begin{cases} p \cdot y(p) & \geq p \cdot y(p') \\ p' \cdot y(p') & \geq p' \cdot y(p) \end{cases}$$

(i.e. mismatched price-supply combinations always yield less profit)
This also implies the **Law of Supply**:

$$(p' - p) \cdot (y(p') - y(p)) \geq 0$$

Hotelling's Lemma #flashcard

$$\frac{\partial \pi(p)}{\partial p_l} = y_l(p)$$

SR and LR Supply Functions of A Competitive Firm with Fixed Costs #flashcard

In the SR (fixed cost must be paid):

$$y(p) = \begin{cases} q \; s.t. \; p = C'(q) \text{ and } C''(q) > 0 & \text{if } p \geq \frac{C_{variable}(q)}{q} \\ 0 & \text{otherwise} \end{cases}$$

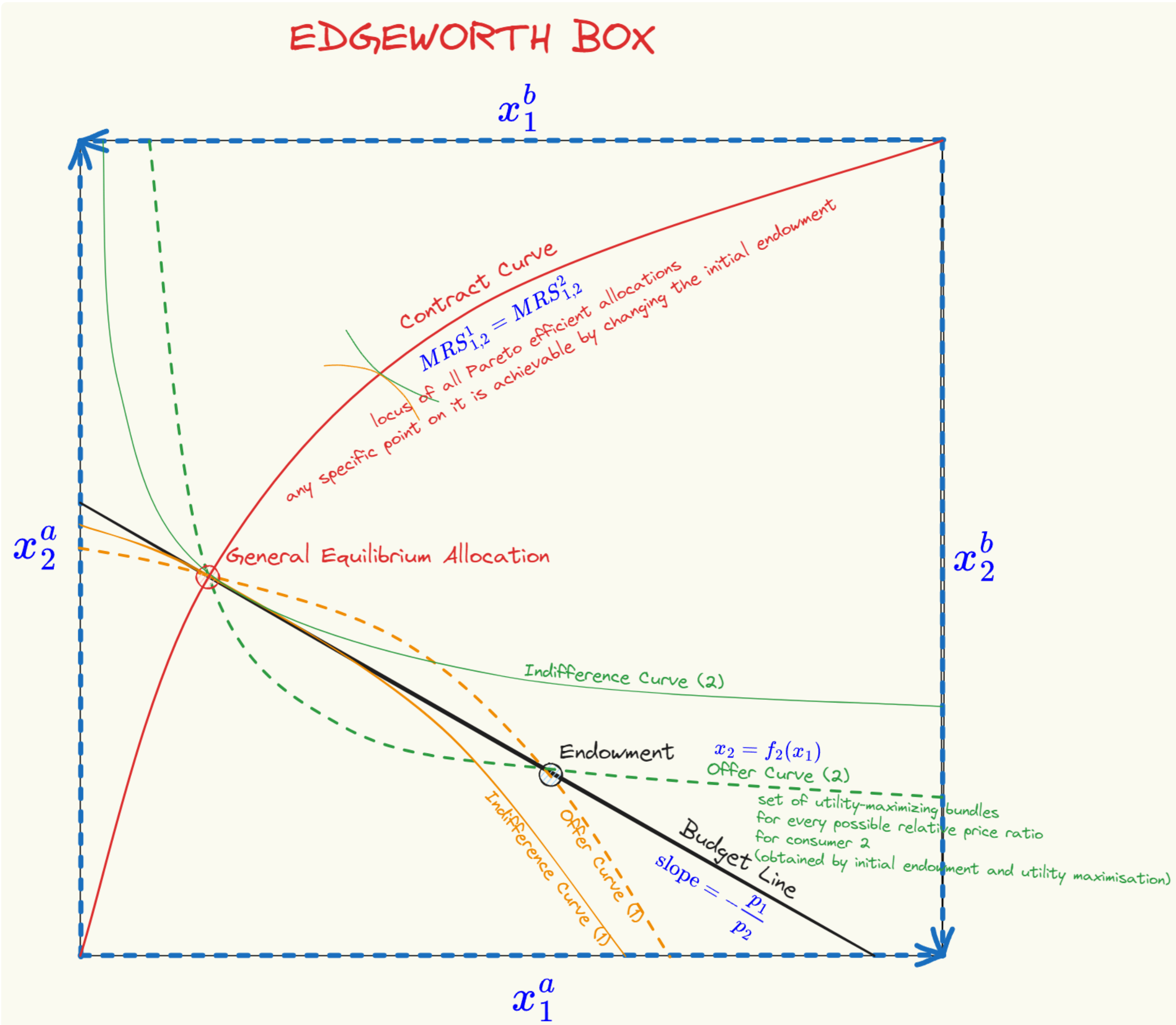
In the LR (fixed cost is zero if shutdown):

$$\text{ree Entry/Exit} \implies \pi = pq - C(q) = 0 \implies p = \frac{C(q)}{q}$$

General Equilibrium

2 Households, 2 Goods Exchange Economy

GE in an Edgeworth Box #flashcard



Excess Demand Functions in GE #flashcard

- The excess demand function for agent a is:

$$z^a(p) = x^a(p, {}^a) - {}^a$$

  where $x^a(.)$ is the demand function and ^{a} is its initial endowment
- Warlas Law implies that the *value of* aggregate excess demand function is zero:

$$p \cdot z(p) = 0$$

  - This also implies, if $L - 1$ markets clear, and the price in the remaining market is positive, then the remaining market clears.

Feasibility and Walrasian Equilibrium

- An allocation is **feasible** if it satisfies adding up:

$$\sum_{a \in A} x_i^a = \sum {}_i^a \; \forall \; i$$

- A price-allocation pair (p^*, x^*) is a **Walrasian Equilibrium** if:
  - the allocation is feasible

- each consumer is optimising given their budget set

The First Fundamental Theorem of Welfare Economics #flashcard

If (p^*, x^*) is a Walrasian Equilibrium, then it is Pareto Efficient.

- The only assumption on utility function is local non-satiation.
- In practice, there may be uncorrected market failures that lead to inefficiency: externalities, public goods, missing markets, market power..
- Pareto efficiency seems to be a minimal requirement.

The Second Theorem of Welfare Economics #flashcard

If utility functions are continuous, strictly increasing and strictly quasi-concave, then *every Pareto efficient allocation* can be a Walrasian equilibrium given appropriate redistribution.
i.e. *Any point on the contract curve* is achievable by selecting the initial endowment point.

Solving 2 Households, 2 Goods Exchange Economy: Reallocate Endowment to Get A Given Desired Distribution #flashcard

- Target: the exchange economy ends up at a specific point *on the contract curve*

1. Find the equation for the contract curve by equalising MRS and using the adding up constraint:

$$MRS^a_{1,2} = MRS^b_{1,2} \implies x_2 = g(x_1)$$

   - If the target allocation is on this contract curve, then it should be achievable.
2. Calculate the MRS at the target equilibrium
3. Calculate the relative price using the optimal condition (we can normalise the price of good 2 to be 1 $p_2 = 1$):

$$-\frac{p_1}{p_2} = MRS_{1,2}$$

4. Calculate the budget needed to purchase the target equilibrium, and use the relative prices to work out a reallocating plan (if both can be reallocated, just fix one and move another)

Existence of Walrasian Equilibrium in the Classical GE Theory #flashcard

- Key requirement:
  - the aggregate excess demand function is continuous
  - Walras' Law holds
- Then, we can use fixed-point theorems to prove the existence of a Walras' equilibrium.
- Further, if all goods are gross substitutes at all prices, then the Walras' equilibrium will be unique.

Robinson Crusoe Economy #flashcard

- #notes/tbd
- One person supplying labour to a firm, enjoying the rest of leisure time, and own all profits from the firm
- Solve the individual's utility maximisation e.g.

$$\max_{x,H} a \ln x + (1-a) \ln H$$

- Solve the firm's optimisation:

$$\max_{L} x = F(L)$$

note that if the production function is CRS, there will be no profit

- Solve the equilibrium

Small Open Economy: Stolper-Samuelson with Fixed Coefficients #flashcard

- Main idea: equal unit costs
- Assumptions:
    1. **Two-Good, Two-Factor Model**: There are 2 goods (e.g., cloth and food) and 2 factors of production (e.g., labor and capital).
    2. **Fixed Coefficients Production**: Each good is produced using fixed proportions of labor and capital (Leontief production), meaning no substitutability between factors.
    3. **Perfect Competition**: Goods and factor markets are perfectly competitive.
    4. **Full Employment**: All resources (labor and capital) are fully employed.
    5. **Closed Economy or Small Open Economy**: Often applied to both, but especially relevant for trade models.
    6. **Factor Intensity**: One good is labor-intensive, the other is capital-intensive.
    7. **Price Change**: Exogenous change in the price of one of the goods (e.g., due to trade).
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- Write the unit cost equations:

$$\begin{cases} q_1^w w + q_1^r r & = p_1 \\ q_2^w w + q_2^r r & = p_2 \end{cases}$$

- We can normalise the price of one good to be 1.

- Solve those equations to get the equilibrium $r^\star, w^\star$:

$$\boxed{w^\star \;=\; \frac{p_1\,q_2^r \;-\; p_2\,q_1^r}{q_1^w\,q_2^r - q_1^r\,q_2^w}, \qquad r^\star \;=\; \frac{q_1^w\,p_2 \;-\; q_2^w\,p_1}{q_1^w\,q_2^r - q_1^r\,q_2^w}.}$$

- Analyse the dynamics (we can also draw a graph of w, r)
- Key finding: *a rise in the relative price of a good will raise the real return to the factor used intensively in that good and reduce the real return to the other factor, even with fixed production coefficients.*

Factor Endowment: Rybczynski #flashcard

- Assumptions: 2 goods, 2 factors, good price fixed (small open economy, goods priced by the international market)
- Start from the factor use equation:

$$\begin{cases} a_{L1}q_1 + a_{L2}q_2 & = L \\ a_{K1}q_1 + a_{K2}q_2 & = K \end{cases}$$

- Solve those equations to get the equilibrium $q_1^\star, q_2^\star$ (best to solve using matrix inversion):

$$\boxed{q_1^\star = \frac{La_{K2} - Ka_{L2}}{a_{L1}a_{K2} - a_{L2}a_{K1}}, \qquad q_2^\star = \frac{Ka_{L1} - La_{K1}}{a_{L1}a_{K2} - a_{L2}a_{K1}}.}$$

- We can see that: *when the endowment of labour increases, the output of labour-intensive good will rise (with elasticity > 1), but the output of capital-intensive good will fall. v.v.*

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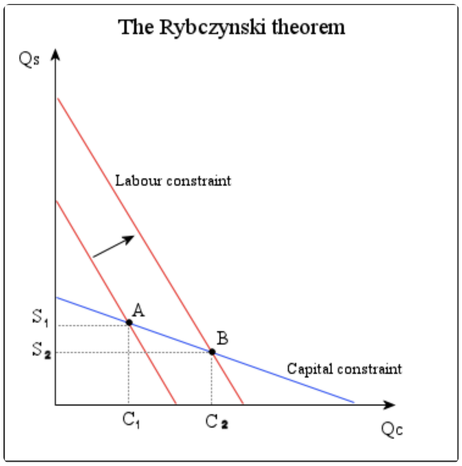

- *In general, an increase in a country's endowment of a factor will cause an increase in output of the good which uses that factor intensively, and a decrease in the output of the other good.*