Demand System

Utility Maximization → (Lagrangian / Tangency Condition) → Uncompensated / Marshallian Demand $d_i(y,p)$

$$MRS_{ij} = -\frac{\partial u/\partial q_i}{\partial u/\partial q_j} = -\frac{p_i}{p_j}$$

$$w_i = \frac{p_i d_i(y,p)}{y}$$ → Budget Share (in terms of y)

Uncompensated / Marshallian Demand $d_i(y,p)$

- Substitute into Utility Function: $u(d(y,p))$
- Roy's Identity: $d_i(y,p) = -\frac{\partial v(y,p)/\partial p_i}{\partial v(y,p)/\partial y}$
- Substitute into Hicksian Demand: $d_i(y,p) = g_i(v(y,p), p)$

Indirect Utility Function $v(y,p)$

- Inverse
- Inverse

Expenditure / Cost Function $c(v,p)$

- Substitute into Marshallian Demand: $g_i(v,p) = d_i(c(v,p), p)$
- Shephard's Lemma: $g_i(v,p) = \frac{\partial c(v,p)}{\partial p_i}$
- Multiple by Prices: $c(v,p) = p'g(v,p)$

$$w_i = \frac{p_i g_i(v,p)}{c(v,p)}$$ → Budget Share (in terms of v)

Cost Minimization → (Lagrangian / Tangency Condition) → Compensated / Hicksian Demand $g_i(v,p)$

$$MRS_{ij} = -\frac{\partial u/\partial q_i}{\partial u/\partial q_j} = -\frac{p_i}{p_j}$$