TUTORIAL SHEET → 1

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Ans → 1 → (3) $O(N+N)$ time
$O(1)$ space.

Ans → 2 → $T(n) = O(n)$, space $O(1)$.

Ans → 3 → $T(n) = O(\log_2 n)$, space $O(1)$.

Ans → 4 →
```
int sum = 0, i;
for(i=0; i*i <n; i++)
  {
    sum += i;
  }
```

$= n + (n-1) + (n-4) + (n-9) + \dots (n-k)$

$= n + (n*k) - (1^2 + 2^2 + 3^2 + \dots k^2)$

$= \sqrt{n}$

$i^2 < n$

$i < \sqrt{n}$

$T(n) = O(\sqrt{n})$, space $O(1)$

Ans → 5 →
```
int j=1 , i=0
while(i<=n)
  {
   i= i+j;
   j++;
  }
```

$0 <= n$. 1

$1 <= n$ 1

$3 <= n$

$(\underbrace{0, 1, 3, 6, 10, 15, 21, \dots n}_{k \text{ terms.}})$

K^{th} term $= \dfrac{(K * (K+1))}{2}$

$$n = \frac{K^2+K}{2}$$

$$K^2 + K = 2n$$

$$K^2 + K - 2n = 0$$

$$K = \frac{-1 \pm \sqrt{1+8n}}{2}$$

$$K = \frac{\sqrt{8n+1}}{2}$$

$$K = \frac{\sqrt{8n}}{2} = \sqrt{n}$$

$T(n) = \sqrt{n}$ space - $O(1)$

Ans-) 6=> void Recursion (int n) — $T(n)$

```
{
  if(n==1) return;
  recursion (n-1)      -> T(n-1)
  print (n);
  recursion (n-1) ;    -> T(n-1)
}
```

$$T(n) = \begin{cases} 1 & n=1 \\ 2T(n-1)+1 & n>1. \end{cases}$$

$$T(n) = 2T(n-1) + 1 \quad -① $$

$$T(n-1) = 2T(n-2) + 1.$$

$$T(n) = 2(2T(n-2)+1) + 1.$$

$$T(n) = 4T(n-2) + (1+2) \quad -② $$

$$T(n-2) = 2(T(n-3)) + 1.$$

$$T(n) = 4(2T(n-3)+1) + (1+2)$$

$$T(n) = 8T(n-3) + (1+2+4) \quad -③ $$

$$T(n) = 8[2(T(n-4)+1)] + (1+2+4)$$

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$T(n) = 16T(n-4) + (1+2+4+8)$ — (4)

$T(n) = 2^k T(n-k) + (1+2+4+8+ \cdots)$
k times.

$T(n-k) = T(1)$

$k = n-1$

$T(n) = 2^{n-1} T(1) + (1+2+4+8+ \cdots)$
(n-1) times.

$T(n) = \frac{2^n}{2} + (1+2+4+8 \cdots)$
(n-1) times.

$S_n = \frac{a(r^n-1)}{r-1}$ $a=1, r=2, n=n-1$

$T(n) = \frac{2^n}{2} + \left(\frac{2^{n-1} -1}{1}\right)$

$T(n) = \frac{2^n}{2} + \frac{2^n}{2} - 1$

$T(n) = \cancel{2}\left(\frac{2^n}{\cancel{2}}\right) - 1$

$T(n) = 2^n - 1$

$T(n) = O(2^n)$.

Ans.

Ans→7→ It is a binary search algorithm.

$T(n) = \log_2(n)$

$T(n) = T(\frac{n}{2}) + 1$

by using masters method (can't be solved)

$T(n) = aT(\frac{n}{b}) + f(n)$

so $a = 1$

$b = 2$

$f(n) = 1$

$c = \log_b a = \log_2(1) = 0$.

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$\simeq 1$

$n = f(n) = 1.$

$\therefore \; n^c = f(n)$

$T(n) := O(\log_2 n)$ Ans.

Ans→ ⑧ → ① $T(1) = 1.$

$T(n) = T(n-1) + 1 \quad -①$

$T(n) = T(n-2) + 2 \quad -②$

$T(n) = T(n-3) + 3 \quad -③$

$T(n) = T(n-k) + k \quad -④$

$n-k = 1$

$k = n-1$

$T(n) = T(1) + n-1$

$T(n) = n$

$T(n) = O(n)$ Ans.

⑪ $T(n) = T(n-1) + n \quad -①$

$T(n-1) = T(n-2) + (n-1)$

$T(n) = T(n-2) + (n + (n-1)) \quad -②$

$T(n) = T(n-3) + (n + (n-1) + (n-2)) \quad -③$

$T(n) = T(n-k) + (n + (n-1) + (n-2) + \cdots (n-k))$

$T(n-k) = T(1)$

$n = k+1$

$k = n-1$

$T(n) = T(1) + (n + (n-1) + (n-2) \cdots\cdots 0)$

$T(n) = 1 + (n + (n-1) + (n-2) + \cdots 1)$

$T(n) = 1 + \frac{n(n+1)}{2} \quad = \quad \frac{n^2+1}{2} + 1$

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$$T(n) = \frac{n^2+2}{2}$$

$$T(n) = O(n^2)$$

Ans.

(iii) $T(n) = T(n/2) + 1$ — ①

$T(n/2) = T(n/4) + 1$

$T(n) = T(n/4) + 2$ — ②

$T(n/4) = T(n/8) + 1$

$T(n) = T(n/8) + 3$ — ③

$T(n) = T\left(\frac{n}{2^k}\right) + k$ — ④.

$\frac{n}{2^k} = 1$

$2^k = n$

$k = \log_2(n)$

$T(n) = T(1) + \log_2 n$

$T(n) = O(\log_2 n)$

Ans.

(iv) $T(n) = 2T\left(\frac{n}{2}\right) + 1$

$c = 1$

$n^c = n$

$f(n) = 1$

$n^c > f(n)$

$T(n) = O(n)$

(v) $T(n) = 2T(n-1) + 1$

$T(n) = O(2^n)$

Ans.

(vi) $T(n) = 3T(n-1)$, $T(0) = 1$.

$T(n) = 3(T(n-1))$ — (1)

$T(n-1) = 3T(n-2)$

$T(n) = 9T(n-2)$

$T(n) = 3^3 T(n-3)$

$T(n) = 3^k T(n-k)$

For $n-k = 0$

$n = k$

$T(n) = 3^n T(0)$

$T(n) = 3^n$

$T(n) = O(3^n)$ Ans.

(vii) $T(n) = T(\sqrt{n}) + 1$ — (1)

$T(\sqrt{n}) = T(n^{\frac{1}{4}}) + 1$

$T(n) = T(n^{\frac{1}{4}}) + 2$ — (2)

$T(n) = T(n^{1/8}) + 3$ — (3)

$T(n) = T(n^{(\frac{1}{2^k})}) + k$ }.

For $T((\sqrt{n})^{\frac{1}{k}}) = T(2)$

$n^{\frac{1}{2^k}} = 2$

$n^{\frac{1}{2^k}} = 2$.

$\frac{1}{2^k} \log n = 1$.

$2^k = \log n$

$k = \log_2(\log(n))$

$T(n) = O(\log(\log(n)))$ Ans.

(viii) $T(n) = T(\sqrt{n}) + n$

$T(\sqrt{n}) = T(n^{\frac{1}{4}}) + \sqrt{n}$

$T(n) = T(n^{\frac{1}{4}}) + (n + \sqrt{n})$

$T(n) = T(n^{\frac{1}{8}}) + (n + \sqrt{n} + n^{\frac{1}{4}})$

$T(n) = T(n^{\frac{1}{2^k}}) + (n + n^{\frac{1}{2}} + n^{\frac{1}{4}} + \text{----}$ k terms$)$

for $n^{\frac{1}{2^k}} = 2$

$\frac{1}{2^k} = \frac{1}{\log(n)}$

$2^k = \log(n)$

$k = \log(\log(n))$

$T(n) = 1 + (n + \sqrt{n} + \sqrt{n}\sqrt{n} + \text{----})$

$T(n) = 1 + \left(\begin{array}{l} \text{G.P } a = n \\ r = \sqrt{n} \\ \text{no of terms} = k \end{array} \right)$

$T(n) = 1 + \left(n \frac{(\sqrt{n})^k - 1}{k - 1} \right)$

$T(n) = 1 + n \left[\left(\frac{(\sqrt{n})^{\log(\log n)} - 1}{\log(\log n) - 1} \right) \right]$

$T(n) = n . \log \log(n)$ {by neglecting other values}.

$T(n) = O(n . \log(\log(n)))$

Ans→ 9→ int sum=0, i ;

for (i=0 ; i<n ; i++)

{

sum += i ;

}

0, 1, 2, ---- n

so T(n) = O(n) ,, , space O(1)

Ans→10→ $O(N*(N, N-1, ----1))$

$O(N* \frac{(N+1)}{2})$

(4) $O(N*N)$

Ans→11→ $O(\frac{n}{2} * (\log_2 N))$

$= O(n \log n)$

Ans.

Ans→12→ x will always be a better choice for large input.

Ans→13→ $O(\log N)$

Ans→14→ $T(n) = 7(T(\frac{n}{2})) + (3n^2+2)$

$f(n) = 3n^2+2$

$a = 7$

$b = 2$

$c = \log_b a = \log_2(7) = 2.807$

$n^c = n^{2.8} \approx n^{2.8}$

$f(n) = 3n^2+2$

so $n^c > f(n)$

so $T(n) = O(n^{2.8})$ or (c) $\Theta(n^{2.8})$

(a) $O\, n^{(2.8)}$

(a) $\Theta(n^3)$

Ans→15→ $f_1(n) = n^{\sqrt{n}}$

$f_2(n) = 2^n$

$f_3(n) = (1.000001)^n$

$f_4(n) = n^{(10*2^{\frac{n}{2}})}$

a) $f_2(n) > f_4(n) > f_3(n) > f_1(n)$

Ans→16→ $f(n) = 2^{2n}$

$\log f(n) = 2n \log_2 2$

$\log f(n) = 2n$ or

$f(n) = 2^n . 2^n$

$\Omega(2^n)$ Ans.

Ans→17→ $T(n) = 2T(\frac{n}{2}) + n^2$

$c = 1$

$n^c = n$

$n^2 > n$

$f(n) > n^c$

$T(n) = \Theta(n^2)$

Ans→18→ $O(\log N)$ [Its a G.C.D function when n keeps on decreasing by n/2]

Ans→19→ $T(n) = O[N^2 + N]$

$T(n) = O(N^2)$ Ans.

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