Progetto MPC monociclo

1 Identificazione modello

$$\dot{x} = f(x,u) \quad , \quad \dot{x} = \begin{bmatrix} v\cos\vartheta \\ v\sin\vartheta \\ \omega \end{bmatrix} \quad , \quad \dot{x}_c = \begin{bmatrix} V_d\cos\vartheta_d \\ V_d\sin\vartheta_d \\ \omega_d \end{bmatrix} = \begin{bmatrix} V_c\cos\vartheta_c \\ V_c\sin\vartheta_c \\ \omega_c \end{bmatrix}$$

$$\delta x = x - x_c \; , \quad \delta u = u - u_c \qquad \omega_c = \frac{V_c}{R} \leftarrow$$

$$\delta\dot{x} = f(x_c,u_c) + \underbrace{\frac{\partial f}{\partial x}\bigg|_{x_c,u_c}}_{A_c} \cdot \delta x + \underbrace{\frac{\partial f}{\partial u}\bigg|_{x_c,u_c}}_{B_c} \delta u$$

$\rightarrow$ Controllo $u = [V_x, \omega] = [v\cos\vartheta, \omega]$

$$A = \begin{bmatrix} 0 & 0 & -v\sin\vartheta \\ 0 & 0 & v\cos\vartheta \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad P_c = f(x_c,u_c) = \begin{bmatrix} V_c\cos\vartheta_c \\ V_c\sin\vartheta_c \\ \omega_c \end{bmatrix}$$

$$\delta\dot{x} = \frac{\delta x_{k+1} - \delta x_k}{\tau} \Rightarrow \delta x_{k+1} = \delta x_k + \delta\dot{x}\,\tau = \delta x_k + (f(x_c,u_c) + A_c\delta x_k + B_c\delta u_k)\tau =$$
$$= \delta x_k + P_c\tau + A_c\delta x_k\tau + B_c\delta u_k\tau$$

$$\delta x_{k+1} = (\underbrace{A_c T_s + I}_{A_f})\,\delta x_k + \underbrace{B_c T_s}_{B_f}\,\delta u_k + P_c\tau$$

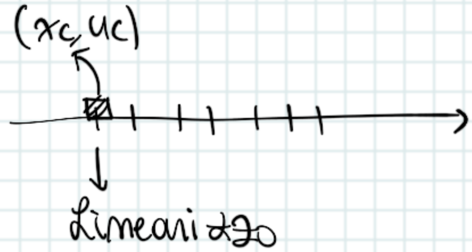
$$J = \int_0^{+\infty} (x(t) - x_d(t))^T Q\,(x(t) - x_d(t)) + u(t)^T R u(t)\, dt$$

$$J = \sum_{i=1}^{N_u} (\delta x_i - \underset{\substack{\| \\ x_d(t_i)}}{x_{d,i}})^T Q (\delta x_i - x_{d,i}) + \delta u_i^T R \delta u_i$$

$$\dot{x} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \end{bmatrix} \leftarrow \dot{x} = f(x,u)$$

s.t. $x(t_c) = x_c$

u_c ad mentulam canis



- Parto da un punto (x_c, u_c)
- Linearizzo intorno a (x_c, u_c)

↳ $\delta x = x - x_c ,\ \delta u = u - u_c$

↳ $\delta \dot{x} = f(x_c, u_c) + A_c \delta x + B_c \delta u$

↳ $\underbrace{\delta x_{k+1}} = (\underbrace{I + \tau A_c}_{A})\delta x_k + \underbrace{\tau B_c}_{B} \delta u_k + \tau P_c$

↳ $x_{k+1} = A x_k + B u_k$

$$U = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N_u) \end{bmatrix} = \begin{bmatrix} \delta u(0) \\ \vdots \\ \delta u(N_u) \end{bmatrix}$$

$t_c + \tau$

$u_c = u_c + \delta u(0)$

MPC

- $t_c = t_c + \tau$

  $u_c = u_c + \delta u(0)$

  $x_c \to (x_c, u_c)$