Assignment#7

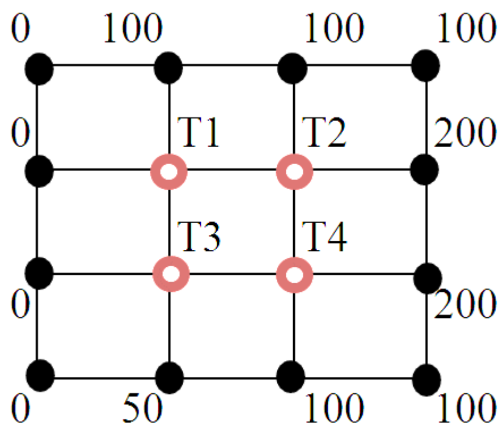
Numerical Sol^n. of Partial Differential Equations

1. The steady state two dimensions heat-flow in a metallic plate is given by

$$\frac{\partial^2 T}{\partial^2 x} + \frac{\partial^2 T}{\partial^2 y} = 0$$

Given the boundary conditions as shown in the figure below, find the temperatures T1, T2, T3 & T4. Solve the equations using Gauss-Seidel method.

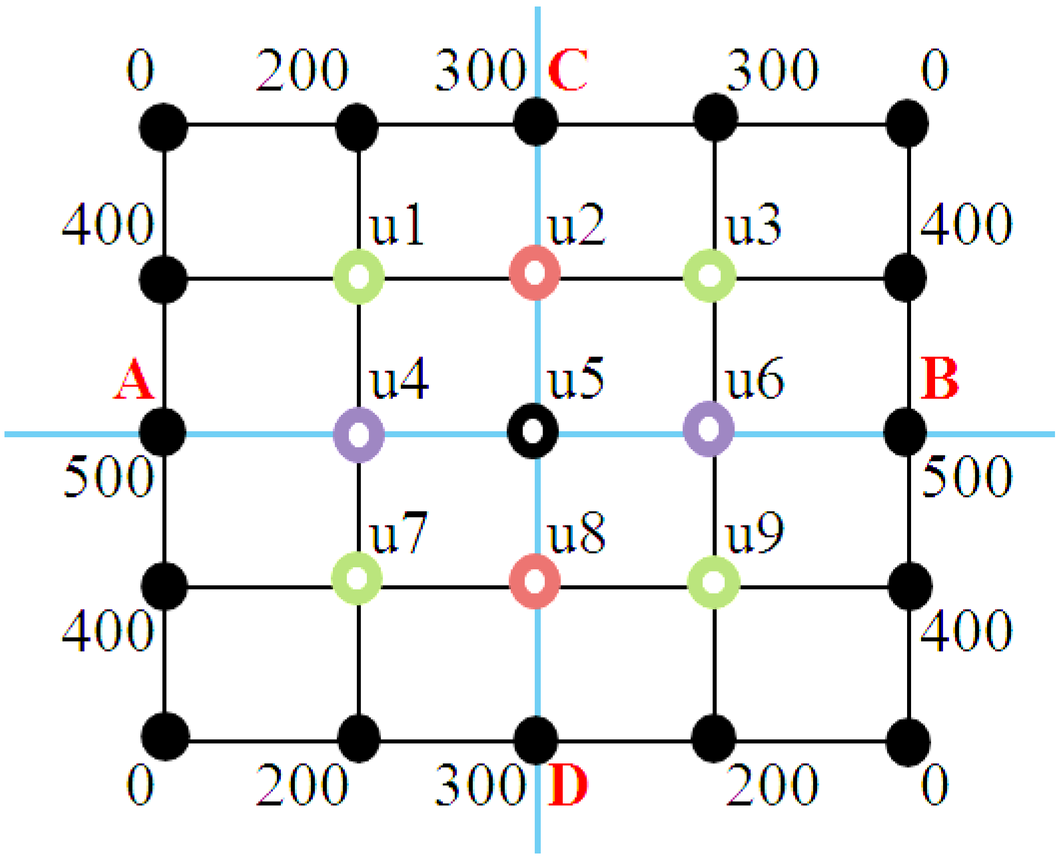
[Ans. $T_1 = 70.83, T_2 = 122.91, T_3 = 60.41, T_4 = 120.83$]



Torsion on a rectangular bar subject to twisting in governed by $\nabla^2 T = -4$. Given conditions: T = 0 on boundary, find T over a cross section of a bar of size **9cm x 9cm,** use the small grid size of **3cm x 3cm**. **[Ans. $T_1 = T_2 = T_3 = T_4 = 18$]**

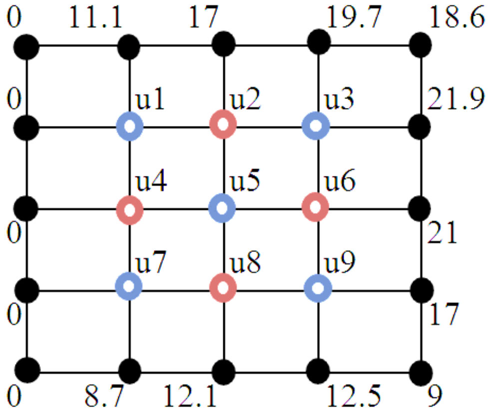
2. Solve the Laplace equation Uxx + Uyy = 0 given that

[Ans. $u_1 = u_3 = u_7 = u_9 = 325, u_2 = u_8 = 325, u_4 = u_6 = 375, u_5 = 350$]



3. Solve for the steady-state temperature in rectangular plate **8cm x 10cm**, if one 10cm side is held at 50°C, and the other 8cm side held at 30°C. Assume square grids of **2cm x 2cm**. [Hint: Create 4x5 data grid]
4. Solve the Laplace equation Uxx + Uyy = 0 given that

[Ans. $u_1 = 7.83, u_2 = 13.66, u_3 = 17.89, u_4 = 6.68, u_5 = 11.96, u_6 = 16.28, u_7 = 6.64, u_8 = 11.25, u_9 = 14.39$**]**



5. Solve the equation $\nabla^2 f = F(x, y)$ with F(x, y) = xy and f = 0 on boundary. The domain is a square with corners at (0, 0) & (3, 3). Use h=1.
6. Given $\frac{\partial^2 f}{\partial x^2} - \frac{\partial f}{\partial t} = 0$; f(0, t) = f(5, t) = 0, f(x, 0) = $x^2(25-x^2)$; find the values of f for x = ih (i = 0, 1, …, 5) and t = jk (j = 0, 1, …, 6) with h = 1 and k = ½, using the **Explicit method**.
7. Estimate the values at grid points of the following equations using recurrence formula [h = 1]:
   a. $f_{xx} - 0.5 f_t = 0$

      Given: f(0, t) = 0; f(5, t) = 0; f(x, 0) = x(5-x);

   b. $9 f_{xx} = f_t$

      Given: f(0, t) = -5; f(5, t) = 5;

      $$f(x, 0) = \begin{cases} -5 \; for \; 0 \le x \le 2.5 \\ 5 \; for \; 2.5 \le x \le 5 \end{cases}$$

8. Solve by relaxation method, the equation $\nabla^2 u = 0$ in the square region with square meshes starting with the initial values u1 = u2 = u3 = u4 = 1. **[Ans:** $\boldsymbol{u_1 = u_4 = 1, u_2 = 1.3, u_3 = 0.7}$**]**