Assignment#6

Sol^n. of Ordinary Differential Equations

Initial Value Problems

1. Using **Euler's method**, find y(0.2) from the equation: y' = x + y, y(0) = 0, take h = 0.1
2. **Derive Euler's formula** for solving initial value problem.
3. Solve y' = y/(x^2 + y^2), y(0) = 1 using **RK-2 method** in the range 0, 0.5, 1.
4. Solve y' = sinx + cosy subject to initial condition y(0) = 2 in the range 0(0.5)2 using the RK second order method.
5. Using the RK-2, obtain a solution of the equation y' = xy + y^2 with the initial condition y(0) = 1 for the range 0≤x≤0.6 with increments of 0.2.
6. Solve y' = 4$e^{0.8x}$ – 0.5y; subject to initial condition y(0) = 2, for y(0.5) and y(0.1) using Runge-Kutta 2nd order method.
7. Solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ using RK-4 method, for y(0.4). Given: y(0) = 1, h = 0.2
8. Write an **algorithm**, **pseudo-code** and a **program** in any high level language (C/C++/FORTRAN) to solve a first-order initial value problem using classical RK-4 method.
9. Solve the differential equation, dy/dx = (1 + x^2)y within x ≤ 0(0.2)0.4 and y(0) = 1 using RK 4th order method.
10. Solve y' = xy + y^2, y(0) = 1 for y(0.1) & y(0.2) using RK method of fourth order.
11. Solve the following simultaneous differential equations using RK 2nd order method at x = 0.1 & 0.2; $\frac{dy}{dx} = xz + 1$; $\frac{dz}{dx} = -xy$; with initial conditions y(0) = 0, z(0) = 1.
12. Solve by RK-2 method for x = 0(0.1)0.2

    $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$; y(0) = 1, y'(0) = 0
13. Using the RK-2 method, obtain a solution of the equation y" = y + xy' with the initial condition y(0) = 1, y'(0) = 0 to find y(0.2) and y'(0.2) [Take h = 0.1]
14. Solve the ordinary differential equation,y" = x$(y')^2$ – y^2 for x = 0.6 with the initial condition y(0) = y'(0) = 0 by using RK-2 method. [Take h = 0.3]
15. Solve the following differential equation within 0≤x≤0.4 using RK-2 method,

    $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 6x$; y(0) = y'(0) = 1, h = 0.2.
16. Solve:

    $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 4y = 3x$; y(0) = 0, y'(0) = 1, h = 0.5 within 0≤x≤1.0 using RK 4th order method.
17. Solve the following initial value problem for y(1.2) using the RK-4th order method:

    $y" - 3y' + y = sinx$; y(1) = 1.2, y'(1) = 0.5

Boundary Value Problems

1. Using the finite difference method, find y(0.25), y(0.5) and y(0.75) satisfying the differential equation xy" + y = 0, subject to the boundary conditions y(0) = 1, y(1) = 2.
2. Solve the following BVP using the finite difference method, by dividing the interval into four sub-intervals.

   $\frac{d^2y}{dx^2} = x + y$; y(0) = y(1) = 0

3. Solve the BVP: y" + 3y' = y' + x^2, y(0) = 2, y(2) = 5 at x = 0.5, 1, 1.5 using finite difference method.
4. Solve the following BVP using the finite difference method, by dividing the interval into four sub-intervals.

   y" = e^x + 2y' – y; y(0) = 1.5, y(2) = 2.5

5. Using Finite difference method to solve the BVP:

   y" = 4y' – 4y + e^{2x}; y(0) = 0, y(1) = 2 for three internal points in (0, 1)

6. Write an algorithm to solve two-point BVP using shooting method.
7. Solve the following BVP using shooting method:

   $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x$ with y(1) = 1 and y(2) = 5; Take h = 0.25

8. Using shooting method, solve the BVP:

   $\frac{d^2y}{dx^2} = 6y^2$ with y(0) = 1 and y(0.5) = 0.44;

   [Taking m0 = -1.8, m1 = -1.9, we get m2 = -2 & y(0.5) = 0.4441]